

# Number System - MPES

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# Number Systems

Decimal

Binary →

Octal

Hexadecimal →

Octal (8)

Hexadecimal (16)

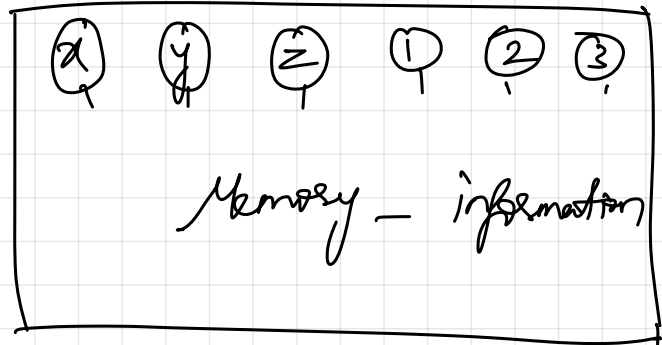
0 to 9

A to F

A  
B  
C

D  
E  
F

$$\underline{2 + 3 = 5}$$



Decimal (10)

0 to 9

Binary (2)

0 1

## Decimal Number System

0 to 9

$$5984 = 5000 + 900 + 80 + 4$$

$$= 5 \times 10^3 + 9 \times 10^2 + 8 \times 10^1 + 4 \times 10^0$$

$$\text{nth digit} \rightarrow 10^{n-1}$$

$$\text{nth digit} \times 10^{n-1} \Rightarrow \text{nth digit} \times (\text{base})^n$$

## Binary number System

0 & 1

base - 2

bit  $\rightarrow$  these binary digits

11011

$$\begin{array}{r} 2 \overline{) 8} \\ 2 \overline{) 4} - 0 \\ 2 \overline{) 2} - 0 \\ 1 - 0 \end{array}$$

$$\begin{array}{r} 10 \\ \hline 10 \end{array}$$

$\rightarrow 1000 \rightarrow$  binary no for 8 (dec)

$$= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$\rightarrow$  decimal number.

Decimal No

Binary No

0

0

1

1

2

10

3

11

4

100

5

101

↙

1111

16

1 0000

## Exercise:

What is the binary equivalent for the following

(i)  $39 \rightarrow \underline{100111}$

(ii)  $25 \rightarrow 11001$

$$\begin{array}{r} 2 \overline{) 39} \\ \underline{19} - 1 \\ 2 \overline{) 19} \\ \underline{9} - 1 \\ 2 \overline{) 9} \\ \underline{4} - 1 \\ 2 \overline{) 4} \\ \underline{2} - 0 \\ 1 - 0 \end{array}$$

## Binary addition

Ex - 1

$$\begin{array}{r} \text{Carry} \\ \textcircled{1} \\ 1 \quad 1 \quad 0 \quad 1 \\ + 0 \quad 0 \quad 0 \quad 1 \\ \hline 1 \quad 1 \quad 1 \quad 0 \end{array}$$

Rule

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	10

1+0+0  $\rightarrow 1$

$1+1 \rightarrow 10+1$

(11)

## Exercise

$$\begin{array}{r} 1 \quad 1 \quad 1 \quad 1 \rightarrow 15 \\ + 1 \quad 0 \quad 0 \quad 1 \rightarrow 9 \\ \hline 11 \quad 000 \quad 24 \end{array}$$

$$\begin{array}{r} 2 \overline{) 24} \\ \underline{12} - 0 \\ 2 \overline{) 12} \\ \underline{6} - 0 \\ 2 \overline{) 6} \\ \underline{3} - 0 \\ 1 - 1 \end{array}$$

## Binary Subtraction

$$\begin{array}{cccc}
 & 0 & & \\
 & \textcircled{1} & \rightarrow & 0 \\
 1 & & & 1 \\
 \hline
 (-) & 0 & 0 & 1 & 1 \\
 \hline
 & 1 & 0 & 1 & 0 \\
 \hline
 \end{array}$$

Example

$$\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 1 & \\ (-2) & 0 & 1 & 0 & 1 & 1 \\ \hline & 1 & 0 & 0 & 0 & 0 \end{array} \rightarrow \begin{array}{c} 7 \\ 8 \end{array}$$

15

## Negative number representation?

# Complement

A diagram illustrating the decomposition of the number 10. On the left, the number 10 is enclosed in a circle. An arrow points from this circle to two separate circles on the right. The top circle contains the number 2, and the bottom circle contains the number -2.

9's Complement  $\rightarrow$  decimal numbers

$$\begin{array}{r} 99 \\ - 45 \\ \hline 54 \\ + 1 \end{array} \rightarrow 55$$

9's Compl  
234

$$999 - 234 \rightarrow 765$$

10's complement  $\rightarrow$  9's complement + 1

$$\begin{array}{l} 10's \text{ comp of } 45 \rightarrow 9's \text{ of } 45 + 1 \\ 54 + 1 = 55 \end{array}$$

Exercise

What is the 10's complement of the following

i) 523 ? 477

$$\begin{array}{r} 0 \text{ to } 9 \\ \hline \end{array}$$

$$\begin{array}{r} 9's \\ - 52 \\ \hline 38 \\ + 1 \\ \hline 39 \end{array}$$

$$\begin{array}{r} 45 \\ - 52 \\ \hline 80 \end{array}$$

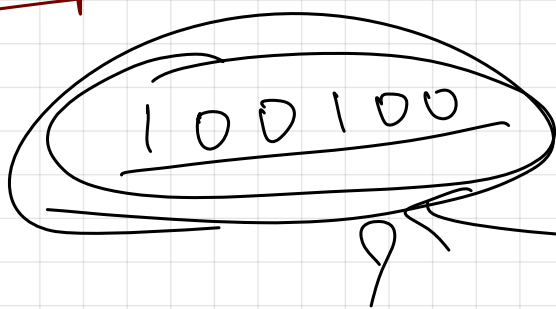
$$\begin{array}{r} 45 \\ + 55 \\ \hline 100 \\ \text{Carry} = 1 \end{array}$$

The 10's complement of a decimal number =  $-$  decimal number

The negative decimal no is represented by its 10's complement. (9's & +1)  $\rightarrow$  10's

The negative binary no is represented by its 2's complement.

1's complement



what is complement of

011011  $\rightarrow$  1's complement.

$$\begin{array}{r} 011011 \\ + 000001 \\ \hline 011100 \end{array}$$

2's complement  $\rightarrow$  1's complement + 1

$$011011 + 1$$

$$\Rightarrow \underline{011100}$$

Exercise

Find 2's complement of the following?

(i)  $101101 \rightarrow 010010 \text{ (1's)} \xrightarrow{+1} 010011 \text{ (2's)}$

(ii)  $111 \rightarrow 000 \text{ (1's)} \xrightarrow{+1} 001$

(iii)  $1001 \rightarrow 0110 \rightarrow 0111$



$$\text{div } \underline{00001001} \rightarrow \underline{1110111}$$

1001

4-bit

$$(+)\ 0111 \rightarrow 2's \text{ complement}$$

8-bit

16-bit

① 0000

8-bit processor 2's

Example

Add +5 & -7

$$+5 \rightarrow 00000101$$

-7

$$\begin{array}{r} 0 \\ 1 \\ 10 \\ 11 \\ 100 \\ \underline{0101} \\ 110 \\ 111 \end{array}$$

$$7 \rightarrow \begin{array}{r} 00000111 \\ 11111000 + \end{array}$$

$$\underline{11111001}$$

-2

$$+5 \rightarrow 00000101$$

$$-7 \rightarrow 11111001$$

$$\underline{11111110}$$

2

$$\begin{array}{r} 00000010 \\ 11111101 \\ \hline 11111110 \end{array}$$

# Binary Subtraction using 2's complement

The addition of 2's complement of a number is equivalent to the subtraction of the number.

(10)

1+1 → 10

$$\begin{array}{r} 0101 \quad (5) \\ - 0010 \quad (-2) \\ \hline \end{array}$$

$$\begin{array}{r} 0011 \rightarrow 3 \\ \hline \end{array}$$

5 →

0 1 0 1

-2 →

0 0 1 0

1 1 0 1 +1

1 1 1 0

Negative

0 1 0 1

1 1 1 0

Carry

(+)

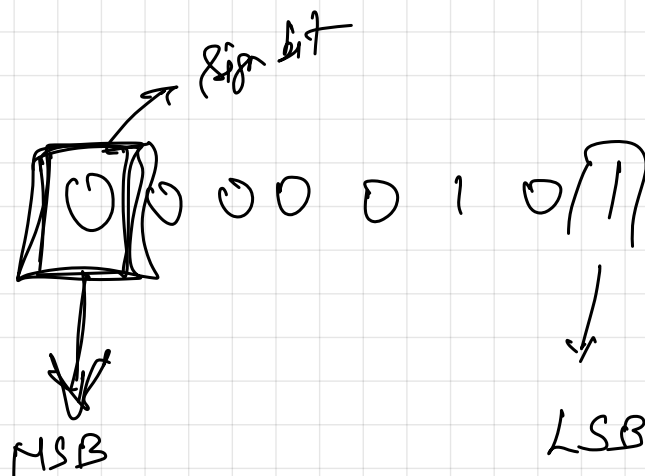
4

1

0 0 1 1

# Exercise

Add decimal numbers 69 & -18



(2)

16

(4)

(15)

8 bit

7 bit

69 →

-18 →

100110011

18

00010010

11101101 +

11101110

2 | 18  
2 | 9 - 0  
2 | 4 - 1  
2 | 2 - 0  
1 - 0

10010

## Conversion of Real number

$$\begin{aligned}\textcircled{0.565} &= 0.5 + 0.06 + 0.005 \\ &= \frac{5}{10} + \frac{6}{100} + \frac{5}{1000} \\ &= 5 \times 10^{-1} + 6 \times 10^{-2} + 5 \times 10^{-3}\end{aligned}$$

binary real no.

$$\begin{aligned}\textcircled{0.1011} &= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} \\ &\rightarrow \underline{0.6875 \text{ (decimal)}}$$

Exercise

Convert

1011.010 to decimal real number

$$\begin{array}{r} 1011 \\ \begin{array}{l} \rightarrow 1 \times 2^0 \\ \rightarrow 1 \times 2^1 \\ \rightarrow 0 \times 2^2 \\ \rightarrow 1 \times 2^3 \end{array} \\ \hline 11 \end{array}$$

.010

$$\begin{array}{r} 0 \times 2^{-1} \\ 1 \times 2^{-2} \\ \hline 0 \times 2^{-3} \end{array}$$

0.25

11.25 ✓

Conversion of decimal fraction to binary fraction

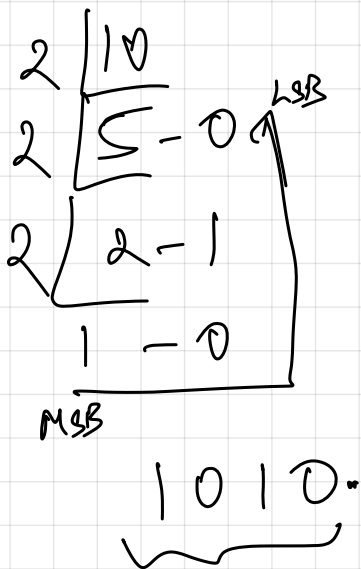
0.375  $\rightarrow$  0.011 (binary fraction)

Fraction	Fraction $\times 2$	Remainder fraction	Integ
0.375	$0.375 \times 2 = 0.75$	0.75 (MSB)	0
0.75	$0.75 \times 2 = 1.50$	0.50	1
0.50	$0.50 \times 2 = 1.00$	0.00 (LSB)	1

# Exercise

10.7 (decimal)

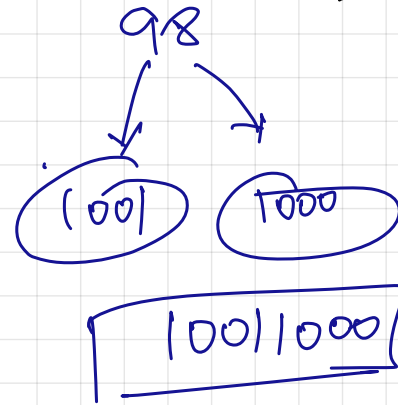
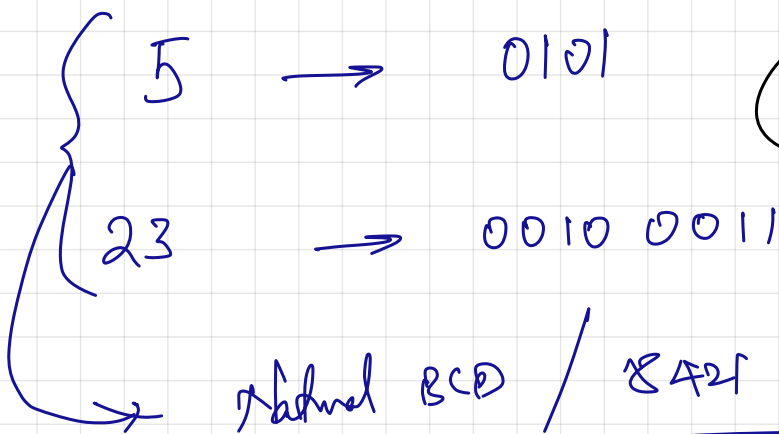
convert it into binary.



fraction	2x fraction	R.F	Integer
0.7	1.4	0.4	1
0.4	0.8	0.8	0
0.8	1.6	0.6	1
0.6	1.2	0.2	1
0.2	0.4	0.4	0
0.4	0.8	0.8	0
0.8	1.6	0.6	1

1010.1011001

## Binary Coded Decimal (BCD)



→ 2 4 21 BCD code  
→ Excess 3 BCD code

	Decimal	2	4	2	1	
9's comp	0	0	0	0	0	1's complement
	1	0	0	0	1	
	2	0	0	1	0	
	3	0	0	1	1	
	4	0	1	0	0	
		1	0	1	1	
	5	1	1	0	0	
	6	1	1	0	1	
	7	1	1	1	0	
	8	1	1	1	1	
	9					

Excess - 3 BCD

Decimal

Excess - 3 BCD

0

0011

1

0100

2

0101

3

4

12

0000

-3

-3

0000

-3 to 12

Gray code → reflected binary

A	B	o/p
0	0	0
0	1	1
1	0	1
1	1	0

Ex-02

<u>Binary</u>	<u>Gray code</u>
0 0 0 0	0 0 0 0
0 0 0 1	0 0 0 1
0 0 1 0	0 0 1 1
0 0 1 1	0 0 1 0
0 1 0 0	0 1 1 0
0 1 0 1	0 1 1 1
1 1 1 1	1 0 0 0

Hexadecimal Number System

0 to 9

0 to 9

16 → 10  
17 → 11

10 → A  
11 → B  
12 → C  
13 → D  
14 → E  
15 → F



00	0C	19
01	0D	1A
02	0E	1B
03	0F	1C
04	10	1D
05	11	1E
06	12	1F
07	13	20
08	14	
09	15	
- 0A	16	
- 0B	17	
	18	

Hexa decimal  
numbers

(F) → 1111

(B) → 1011

$$\begin{array}{r}
 2 \overline{) 12} \\
 2 \overline{) 6} = 0 \\
 2 \overline{) 3} = 0 \\
 1 - 1
 \end{array}$$

BSP

01011100

BSP?

0011 1010 1111 1011

Hex  
5C →

Shenik

3AFB

# Conversion of Hexadecimal to Decimal

$$\begin{array}{l} \text{5C} \\ \swarrow \quad \searrow \\ C \times 16^0 = 12 \times 16^0 = 12 \\ 5 \times 16^1 = 5 \times 16^1 = 80 \\ \hline 92 \end{array}$$

Decimal		Hex		Binary representation of Hexadecimal	
0	8	0	8	0000	1000
1	9	1	9	0001	1001
<del>2</del>	10	2	A	0010	1010
3	11	3	B	0011	1011
4	12	4	C	0100	1100
5	13	5	D	0101	1101
6	14	6	E	0110	1110
7	15	7	F	0111	1111

Exercises

convert.

i) 5A9 to decimal  $\rightarrow$  ?

ii) 1A5D to decimal  $\rightarrow$  ? 6749 ✓

$$\begin{array}{l} 5A9 \\ \left. \begin{array}{l} \xrightarrow{16 \times 9} \\ \xrightarrow{A \times 16} \\ \xrightarrow{5 \times 16^2} \end{array} \right\} 1449 \checkmark \end{array}$$

Decimal to Hexadecimal

①

16	35
16	2 (MS) - 3 (LSB)

23  $\rightarrow$  Hex

②

16	10767
16	672 $\rightarrow$ F (LSB)
16	42 $\rightarrow$ 0
2	A

Hex  
2A0F

## Conversion of Binary to Hexadecimal

$$\begin{array}{ccccccc} 0001 & 0111 & 1011 & 0101 & & & 0101 \\ \hline \downarrow & \downarrow & \downarrow & \downarrow & & & \\ 1 & 7 & B & 5 & & & \end{array}$$

17B5

Exercise

$$(1101011010)_2 \rightarrow (35A)_{16}$$

?

## Conversion of Hexadecimal to Binary

$$(\underline{5})CB8_{16} \rightarrow (0101) (1100) (1011)$$

(1000)

$$(0101110010111000)_2$$

Exercise

$$(4F2D)_{16} = (0100111100101101)_2$$

?

# Octal Number System

<u>Decimal</u>	<u>Octal</u>
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	10
9	11

## Conversion of Octal to Decimal No:

$$(75)_8$$

$$\begin{array}{l} 75 \\ \left. \begin{array}{l} \rightarrow 5 \times 8^0 = 5 \\ \rightarrow 7 \times 8^1 = 56 \end{array} \right\} \end{array}$$

$$\underline{(61)} \rightarrow \text{decimal}$$

### Exercise

$$(645)_8 \rightarrow ? \quad 421$$

$$(1258)_8 \rightarrow ?$$

$$(5273)_8 \rightarrow ? \quad 2747 \quad (1258)_8 \text{ is not an octal no.}$$

## Conversion of Decimal to Octal

$$\begin{array}{r|l} 8 & 67 \\ \hline 8 & 8 - 3 \text{ (LSB)} \\ \hline & 1 - 0 \text{ (MSB)} \end{array}$$

103

Exercise

$$(461)_{10} \downarrow$$

$$(715)_{10} \downarrow$$

$$(78)_{10} \rightarrow (116)_8$$

$$(715)_8$$

$$(1313)_8$$

$$(2747)_{10} \rightarrow (5273)_8$$

## Binary representation of Octal numbers

$$05 \rightarrow 101$$

$$(245)_8 \rightarrow (100\ 101)_2$$

Exercise

Convert octal to binary

$$(715)_8 \rightarrow (111001101)_2$$

$$(1313)_8 \rightarrow (011001011)_2$$

Conversion of binary to octal

$$(001101110)_2 \rightarrow$$

$$\begin{array}{ccc} 001 & 101 & 110 \\ \downarrow & \downarrow & \downarrow \\ 1 & 5 & 6 \end{array}$$

$$(156)_8$$

Example

$$(1011111001)_2 \rightarrow (1371)_8$$

## Conversion of Octal to Hexadecimal

$$(456)_8 \rightarrow (10010110)_2$$

$000 \mid 00 \mid 0 \mid 110$   
 $\downarrow \quad \downarrow \quad \downarrow$   
 $1 \quad 2 \quad 13$

## Conversion of Hexadecimal to Octal

$$(41DF)_{16} = (0100\ 1101\ 1111)_2$$

$\begin{array}{ccccccc} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} & \underbrace{\hspace{1cm}} \\ 2 & 3 & 3 & 7 \end{array}$

$(2337)_8$



## Exercises

(i)  $(572)_8$  to  $(17A)_{16}$

(ii)  $(166)_8$  to  $(7b)_b$

(iii)  $(5CE)$  to  $(271b)_8$

(iv)  $12ED$  to  $(11355)_8$