Unit-II :- Logic

Logic : Logic is a discipline that deals with the methods of reasoning.

Propositions : A declarative Sentence which is True / False but not both. is Called Propositions (statements).

The statements p, q, r, s, - - - are used to denote the propositions.

1) NewDelhi is in London (F)
2) 2 + 2 = 4 (T)

Notation :-

→ If the proposition is true then its truth value is denoted by T (or) 1.
→ If the proposition is false, its truth Value is 0. (or) F.

→ **Connectives** :-

1) If p & q are two propositions then the proposition "p and q" is denoted by p∧q. is Called the Conjunction and is defined as, if it is true, when p & q are True otherwise False.

the truth table for p∧q as follows.

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

2) If p and q are two propositions, then the proposition p or q denoted by "p∨q" is called the disjunction and is defined as If it is false then both p & q are false otherwise True.

the truthtable for p∨q are as follows

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

③ Let p be any proposition then its negation is denoted by ¬p (or) ~p.

the truth table for negation p as follows

p	¬p
T	F
F	T

④ If p and q are two propositions then "if p then q" is denoted by p → q is called the Conditional proposition, which is false, when p is true and q is false otherwise True.

p

the truth table $p \rightarrow q$ as follows -

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(5) If p & q are two propositions, then "p if and only if q" is denoted by ~~$p \Rightarrow q$~~ $p \leftrightarrow q$ is called the Biconditional proposition. which is true, when both p & q have the same values. otherwise False

the truth table $p \leftrightarrow q$ as follows.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Tautology and Contradiction :-

A compound proposition which is always true for every truthvalues is called Tautology.

A Compound proposition which is always false for every truthvalues is called Contradiction.

Eg :-

Find i) $p \vee \neg p$ ii) $p \wedge \neg p$

sol:- i) $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	T	T	
T	F	T	
F	T	T	
F	F		

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

$p \vee \neg p$ is tautology & $p \wedge \neg p$ is a Contradiction.

<u>Contigency</u> :-

A proposition which is (Contigency) neither tautology nor a Contradiction is called Contigency.

① Construct the truth table for $(p \longrightarrow q) \longrightarrow (q \longrightarrow p)$.

sol:- Let $s = (p \rightarrow q) \rightarrow (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

1) Construct the truth table for

$(q \rightarrow \neg p) \longleftrightarrow (p \longleftrightarrow q)$

let $S = (q \rightarrow \neg p) \rightarrow (p \leftrightarrow q)$

sol:-

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	S
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

2) Prove that, $(\neg p \wedge p) \wedge q$ is Contradiction.

sol:-

p	q	$\neg p$	$\neg p \wedge p$	$(\neg p \wedge p) \wedge q$
T	T	F	F	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

$\therefore$ All the truth values for $(\neg p \wedge p) \wedge q$ are false.

$\therefore$ $(\neg p \wedge p)$ is a Contradiction.

3) Prove that ~~$(p \& q)$~~ $(p \wedge q) \longrightarrow (p \vee q)$ is a tautology

sol: Let $S = (p \wedge q) \longrightarrow (p \vee q)$

sol:-

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

$\therefore$ S is a Tautology

Equivalence of propositions :-

Two compound propositions $A(P_1, P_2, \ldots, P_n)$ and $B(P_1, P_2, \ldots, P_n)$ are said to be logically equivalent or simply equivalent. If they have identical truth table. It is denoted by $A \Longleftrightarrow B$ or $A \equiv B$.

① prove that, $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$.

sol:- let $S = \underbrace{\neg(p \vee q)}_{LHS} \Leftrightarrow \underbrace{\neg p \wedge \neg q}_{RHS}$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

$\therefore$ LHS = RHS

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\therefore$ They are logically (every) equivalent.

Duality Law :-

The dual of a compound proposition is obtained by replacing each $\wedge$ by $\vee$, each $\vee$ by $\wedge$. Each T by F and each F by T.

→ The dual of A is denoted by A^*.

Duality Theorem :-

If $A(p_1, p_2, \ldots p_n)$ ~~is~~ (or) $\equiv B(p_1, p_2 \ldots p_n)$ where A & B are Compound-propositions then $A^*(p_1, p_2 \ldots p_n) \equiv B^*(p_1, p_2 \ldots p_n)$.

① construct the truth table for $\overbrace{(p \to (q \to r))}^{A =} \to \overbrace{((p \to q) \to (p \to r))}^{B}$

sol:

p	q	r	q → r	p → q	p → r	A	B
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	T	F	T	T	T
T	F	F	T	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

S = A → B
T
T
T
T
T
T
T
T

∴ $S = A \to B$ is a tautology.

Tautological Implication

A compound proposition 'A' is said to tautologically imply to B. If $A \to B$ is tautology. It is denoted by $A \Rightarrow B$

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① prove that, $p \Rightarrow (p \vee q)$

sol:- To prove that $p \to (p \vee q)$ is a tautology

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$\therefore p \to (p \vee q)$ is a tautology.

$\therefore p \Rightarrow (p \vee q)$.

② Prove that $p \to (q \to r) \Rightarrow (p \to q) \to (p \to r)$

sol:- Let $A = p \to (q \to r)$;

$B = (p \to q) \to (p \to r)$

[p q r]

To prove that, $A \Rightarrow B$; $A \to B$ is tautology

$\therefore p \to (q \to r) \Rightarrow (p \to q) \to (p \to r)$

1st Sum in Duality theorem

↓

Turn Back.

(3) prove that $(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \Rightarrow r$

sol:- Let $A = (p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)$

$B = r$

we have to prove that, $A \rightarrow B$.

p	q	r (B)	$p \vee q$	$p \rightarrow r$	$(p \vee q) \wedge (p \rightarrow r)$	$q \rightarrow r$	A	$A \rightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F	T
F	F	T	F	T	F	T	F	T
F	F	F	F	T	F	T	F	T

$\therefore A \rightarrow B$ is a tautology

Hence $A \Rightarrow B$.

$\therefore (p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r) \Rightarrow r$

Algebra of propositions :-

S.No	Name of the Law	Primary Form	Dual Form
1)	Idempotent Law	$p \vee p \Leftrightarrow p$	$p \wedge p \Leftrightarrow p$
2)	Identity Law	$p \vee F \Leftrightarrow p$	$p \wedge T \Leftrightarrow p$
3)	Dominant law	$p \vee T \Leftrightarrow T$	$p \wedge F \Leftrightarrow F$
4)	Complement Law	$p \vee \neg p \Leftrightarrow T$	$p \wedge \neg p \Leftrightarrow F$
5)	Commutative law	$p \vee q \Leftrightarrow q \vee p$	$p \wedge q \Leftrightarrow q \wedge p$
6)	Associative law	$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$	$(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$
7)	Distributive law	$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
8)	Absorption law	$p \vee (p \wedge q) \Leftrightarrow p$	$p \wedge (p \vee q) \Leftrightarrow p$
9)	Demargon's law	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$	$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

Equivalences involving Conditionals :-

1) $p \longrightarrow q \Longleftrightarrow \neg p \vee q$

2) $p \longrightarrow q \Longleftrightarrow \neg q \rightarrow \neg p$

3) $(p \rightarrow q) \wedge (p \rightarrow r) \Leftrightarrow p \longrightarrow (q \wedge r)$

4) $(p \rightarrow q) \vee (p \rightarrow r) \Leftrightarrow p \longrightarrow (q \vee r)$

5) $(p \rightarrow r) \wedge (q \rightarrow r) \Leftrightarrow (p \vee q) \longrightarrow r$

6) $(p \rightarrow r) \vee (q \rightarrow r) \Leftrightarrow (p \wedge q) \longrightarrow r$

Equivalences involving BiConditionals :-

1) $p \longleftrightarrow q \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)$

2) $p \longleftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \longrightarrow p)$

3) $p \longleftrightarrow q \Longleftrightarrow \neg p \longrightarrow \neg q$

① Using truth table, prove that $(p \longrightarrow (q \longrightarrow s)) \wedge (\neg r \vee p) \wedge q \Longrightarrow r \longrightarrow s$

sol:- To prove that, $\{[p \longrightarrow (q \longrightarrow s)] \wedge [\neg r \vee p] \wedge q\} \longrightarrow \{r \longrightarrow s\}$ is a tautology.

Let : $a = [p \longrightarrow (q \longrightarrow s)]$;

$b = [(\neg r \vee p) \wedge q]$;

$A = a \wedge b$;

$B = r \longrightarrow s$.

p	q	s	r	$\neg r$	$q\to s$	$\neg pr \vee p$	a	b	A	B	$A\to B$
T	T	T	T	F	T	T	T	T	T	T	T
T	T	T	F	T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	F	T	F	F	T
T	T	F	F	T	F	T	F	T	F	T	T
T	F	T	T	F	T	T	T	F	F	F	T
T	F	T	F	T	T	T	T	F	F	F	T
T	F	F	T	F	T	T	T	F	F	F	T
T	F	F	F	T	T	T	T	F	F	T	T
F	T	T	T	F	T	F	T	T	T	T	T
F	T	T	F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T	T	T	F	T
F	T	F	F	T	F	T	T	T	T	T	T
F	F	T	T	F	T	F	T	F	F	T	T
F	F	T	F	T	T	T	T	F	F	T	T
F	F	F	T	F	T	F	T	F	F	F	T
F	F	F	F	T	T	T	T	F	F	T	T

$\therefore [p \to (q \to s)] \wedge [\neg p \vee p] \wedge q \Rightarrow r \to s$ is a tautology.

2) $\neg(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \to r))$

Sol:- Let $A = (p \vee (q \wedge r))$

$B = [(p \vee q) \wedge (p \to r)]$

$C = \neg[p \vee (q \wedge r) \leftrightarrow ((p \vee q) \wedge (p \to r))$

or

$A \leftrightarrow B$

p	q	r	$q \wedge r$	A	$\neg A$	$p \vee q$	$p \rightarrow r$	B	$A \leftrightarrow B$
T	T	T	T	T	F	T	T	T	F
T	T	F	F	T	F	T	F	F	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	F	T	F	F	T
F	T	T	T	T	F	T	T	T	F
F	T	F	F	F	T	T	T	T	T
F	F	T	F	F	T	F	T	T	F
F	F	F	F	F	T	F	T	T	F

$\therefore \neg(p \vee (q \wedge r)) \longleftrightarrow ((p \vee q) \wedge (p \rightarrow r))$

prove that

① $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \Leftrightarrow p \wedge q$. without using truth table.

sol:- Consider,

$(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \Leftrightarrow p \wedge q$.

$(\neg p \vee q) \wedge (p \wedge (p \wedge q))$

LHS $\Leftrightarrow (\neg p \vee q) \wedge (p \wedge (p \wedge q))$

$\Leftrightarrow (\neg p \vee q) \wedge ((p \wedge p) \wedge q)$ [$\because$ Associative]

$\Leftrightarrow [\neg p \vee q] \wedge [p \wedge q]$ [$\because p \wedge p = p$]

$\Leftrightarrow [\neg p \wedge (p \wedge q)] \vee [q \wedge (p \wedge q)]$

[$\because$ Distributive]

$\Leftrightarrow [(\neg p \wedge p) \wedge q)] \vee [(q \wedge q) \wedge p]$

($\because$ Associative, Commutative)

$\Leftrightarrow (F \wedge q) \vee (q \wedge p)$

$\Leftrightarrow F \vee (p \wedge q) \Leftrightarrow p \wedge q$.

= R.H.S //

$\therefore$ Hence proved.

② prove that $\neg(p \leftrightarrow q) \Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$ without using truth table.

sol:- $\neg(p \leftrightarrow q) \Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$.

$\neg(p \leftrightarrow q) \Leftrightarrow \neg[(p \rightarrow q) \wedge (q \rightarrow p)]$

$\Leftrightarrow \neg[(p \rightarrow q) \wedge (\neg q \vee p)]$

$\Leftrightarrow \neg((\neg p \vee q) \wedge (\neg q \vee p)]$

$\Leftrightarrow \neg(\neg p \vee q) \vee \neg(\neg q \vee p)$

$\Leftrightarrow (p \wedge \neg q) \vee (q \wedge \neg p)$

$\Leftrightarrow (p \wedge \neg q) \vee (\neg p \wedge q)$

$[\because p \rightarrow q \Leftrightarrow \neg p \vee q]$

$\because p \leftrightarrow q \Leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow p)]$

Normal Forms :-

when the number of propositions $P_1, P_2, \ldots$ P_n increases, better method be used to reduce into standard forms is Called Normal Forms.

there are 2 types of normal forms

1) Disjunctive normal Form (DNF)

2) Conjunctive normal Form (CNF)

Elementary product :-

A product of the Variables and their negations are Called Elementary product

eg:- $P, \neg P, P \wedge q, \neg p \wedge q \ldots$

Elementary Sum :-

A Sum of Variables and their negations are called Elementary Sum.

eg :- p, $\neg p$, $p \vee q$, $p \vee \neg q$ - - - -

DNF (Disjunctive Normal Form) :-

Sum of Elementary product.

CNF

Product of Elementary Sum.

① Find the DNF of $q \rightarrow (q \rightarrow p)$.

sol:- Given ; $q \rightarrow (q \rightarrow p) \Leftrightarrow q \rightarrow (\neg q \vee p)$

$\Leftrightarrow \neg q \vee (\neg q \vee p)$

$\Leftrightarrow (\neg q \vee \neg q) \vee p$ [∴ Commutative]

$\Leftrightarrow \neg q \vee p$

which is the required DNF

[∵ $p \rightarrow q \Leftrightarrow \neg p \vee q$]

② Find the CNF of $\neg(p \vee q) \leftrightarrow (p \wedge q)$

sol:- $\neg(p \vee q) \leftrightarrow (p \wedge q) \Leftrightarrow [\neg(p \vee q) \wedge (p \wedge q)]$

$\vee [\neg\neg(p \vee q) \wedge \neg(p \wedge q)]$

$\boxed{\therefore p \leftrightarrow q \Leftrightarrow (p \wedge q) \vee (\neg p \wedge \neg q)}$

$\Leftrightarrow [(\neg p \wedge \neg q) \wedge (p \wedge q)] \vee [(p \vee q) \wedge (\neg p \vee \neg q)]$

$\Leftrightarrow [(\neg p \wedge p) \wedge (\neg q \wedge q)] \vee [(p \vee q) \wedge (\neg p \vee \neg q)]$

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$\Leftrightarrow (F \wedge F) \vee [(p \vee q) \wedge (\neg p \vee \neg q)]$.

$\Leftrightarrow F \vee (p \vee q) \wedge (\neg p \vee \neg q)$

$\Leftrightarrow (p \vee q) \wedge (\neg p \vee \neg q)$

which is the required CNF.

PDNF and PCNF [Principle Conjunctive Normal Form & Principle Disjunctive Normal Form].

Find PDNF & PCNF of $(q \vee (p \wedge r)) \wedge \neg((p \vee r) \wedge q))$

:- Let $S = (q \vee (p \wedge r)) \wedge \neg((p \vee r) \wedge q)$

Let $A = q \vee (p \wedge r)$; $B = \neg((p \vee r) \wedge q)$

p	q	r	$p \wedge r$	A	$p \vee r$	$(p \vee r) \wedge q$	B	$A \wedge B$	Min Terms	Max Terms
T	T	T	T	T	T	T	F	F		$\neg p \vee \neg q \vee \neg r$
T	T	F	F	T	T	T	F	F		$\neg p \vee \neg q \vee r$
T	F	T	T	T	T	F	T	T	$p \wedge \neg q \wedge r$	
T	F	F	F	F	T	F	T	F		$\neg p \vee q \vee r$
F	T	T	F	T	T	T	F	F		$p \vee \neg q \vee \neg r$
F	T	F	F	T	F	F	T	T	$\neg p \wedge q \wedge \neg r$	
F	F	T	F	F	T	F	T	F		$p \vee q \vee \neg r$
F	F	F	F	F	F	F	T	F		$p \vee q \vee r$

PCNF :- $(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee q \vee \neg r) \wedge (p \vee q \vee r)$.

PDNF :- $(p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge \neg r)$

② Find the PDNF & CDNF of $(p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$

Sol:- Let $A = p \wedge q$; $B = \neg p \wedge q$; $C = q \wedge r$

we have to find $A \vee B \vee C$

p	q	r	$\neg p$	A	B	C	$A \vee B \vee C$	Min Terms	Max Terms
T	T	T	F	T	F	T	T	$p \wedge q \wedge r$	
T	T	F	F	T	F	F	T	$p \wedge q \wedge \neg r$	
T	F	T	F	F	F	F	F		$\neg p \vee q \vee \neg r$
T	F	F	F	F	F	F	F		$\neg p \vee q \vee r$
F	T	T	T	F	T	T	T	$\neg p \wedge q \wedge r$	
F	T	F	T	F	T	F	T	$\neg p \wedge q \wedge \neg r$	
F	F	T	T	F	F	F	F		$p \vee q \vee \neg r$
F	F	F	T	F	F	F	F		$p \vee q \vee r$

PDNF :- $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r)$

PCNF :- $(\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (p \vee q \vee r)$

③ Find the PDNF & PCNF of $p \vee (\neg p \longrightarrow (q \vee (\neg q \rightarrow r)))$.

sol:- Let $A = q \vee (\neg q \longrightarrow r)$; $B = \neg p \longrightarrow q \vee (\neg q \longrightarrow r)$

We have to find $p \vee B$

p	q	r	$\neg p$	$\neg q$	$\neg q \rightarrow r$	A	B	$p \vee B$	Min Terms	Max Terms
T	T	T	F	F	T	T	T	T	$p \wedge q \wedge r$	
T	T	F	F	F	T	T	T	T	$p \wedge q \wedge \neg r$	
T	F	T	F	T	T	T	T	T	$p \wedge \neg q \wedge r$	
T	F	F	F	T	F	F	T	T	$p \wedge \neg q \wedge \neg r$	
F	T	T	T	F	T	T	T	T	$\neg p \wedge q \wedge r$	
F	T	F	T	F	T	T	T	T	$\neg p \wedge q \wedge \neg r$	
F	F	T	T	T	T	T	T	T	$\neg p \wedge \neg q \wedge r$	
F	F	F	T	T	F	F	F	F		$p \vee q \vee r$

PDNF :- $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee$
$(\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

PCNF :- $p \vee q \vee r$