2. $p \wedge \{\neg(q \wedge r) \vee (p \rightarrow q)$.

Soln:

$$p \wedge \neg(q \wedge r) \vee (p \rightarrow q)$$

$$\Leftrightarrow p \wedge \neg(q \wedge r) \vee (\neg p \vee q) \quad , p \rightarrow q \Leftrightarrow \neg p \vee q.$$

$$\Leftrightarrow p \wedge (\neg q \vee \neg r) \vee (\neg p \vee q) \quad \text{demorgan's}$$

$$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r) \vee \neg p \vee q \quad \text{distribut.}$$

which is the sum of product.

and is the required DNF.

*) Find the conjunctive normal forms for the following:

(i) $(p \wedge \neg(q \wedge r)) \vee (p \rightarrow q)$.

Soln:

$$(p \wedge \neg(q \wedge r)) \vee (p \rightarrow q)$$

$$\Leftrightarrow (p \wedge \neg(q \wedge r)) \vee (\neg p \vee q) \quad , p \rightarrow q \Leftrightarrow \neg p \vee q$$

$$\Leftrightarrow (p \wedge (\neg q \vee \neg r)) \vee (\neg p \vee q) \quad \text{demorgan's}$$

$$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q) \quad \text{distribut.}$$

$$\Leftrightarrow (p \vee (p \wedge \neg R)) \wedge ((\neg q \vee (p \wedge \neg R)) \vee (\neg p \vee q) \quad \text{distrib.}$$

$$\Leftrightarrow (p \vee p) \wedge (p \vee \neg R) \wedge (\neg q \vee p) \wedge (\neg q \vee \neg R) \vee (\neg p \vee q) \quad \text{distrib.}$$

$$\Leftrightarrow (p \vee p) \wedge (p \vee \neg R) \wedge (\neg q \vee p) \wedge (\neg q \vee \neg R \vee \neg p \vee q) \quad \text{Associativ.}$$

$\Leftrightarrow p \wedge (p \vee \neg R) \wedge (\neg q \vee p) \wedge (\neg q \vee q \vee \neg R \vee \neg p)$

$\Leftrightarrow p \wedge (p \vee \neg R) \wedge (\neg q \vee p) \wedge (T \vee (\neg R \vee \neg p))$

$\Leftrightarrow p \wedge (p \vee \neg R) \wedge (\neg q \vee p) \wedge T$

$\Leftrightarrow p \wedge (p \vee \neg R) \wedge (\neg q \vee p)$.

which is the required CNF.

2. $(q \vee (p \wedge q)) \wedge \neg ((p \vee r) \wedge q)$

Soln:

$(q \vee (p \wedge q)) \wedge \neg((p \vee r) \wedge q)$

$\Leftrightarrow q \wedge \neg ((p \vee r) \wedge q)$ Absorption law

$\Leftrightarrow q \wedge \neg(p \vee r) \vee \neg q$ Demorgan's

$\Leftrightarrow q \wedge (\neg p \wedge \neg r) \vee \neg q$ demorgan's

$\Leftrightarrow q \wedge (\neg p \vee \neg q) \wedge (\neg r \vee \neg q)$ distributive

is the required CNF.

Principal Disjunctive and Principal Conjunctive Normal Forms (PDNF) & (PCNF)

Definitions:

Given a number of variables, the products in which each of its variable or its negation, but not both, occurs only once are called the minterms.

$\Leftrightarrow p \wedge (p \vee \neg R) \wedge (\neg q \vee p) \wedge (\neg q \vee q \vee \neg R \vee \neg p)$ Absorption Law

$\Leftrightarrow p \wedge (p \vee \neg R) \wedge (\neg q \vee p) \wedge (T \vee (\neg R \vee p))$ $p \vee \neg p = T$

$\Leftrightarrow p \wedge (p \vee \neg R) \wedge (\neg q \vee p) \wedge T$ $p \wedge T = p$

$\Leftrightarrow p \wedge (p \vee \neg R) \wedge (\neg q \vee p)$.

which is the required CNF.

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2. $(q \vee (p \wedge q)) \wedge \neg ((p \vee r) \wedge q)$

Soln:

$(q \vee (p \wedge q)) \wedge \neg ((p \vee r) \wedge q)$

$\Leftrightarrow q \wedge \neg ((p \vee r) \wedge q)$ Absorption law

$\Leftrightarrow q \wedge \neg (p \vee r) \vee \neg q$ Demorgan's

$\Leftrightarrow q \wedge (\neg p \wedge \neg r) \vee \neg q$ demorgan's

$\Leftrightarrow q \wedge (\neg p \vee \neg q) \wedge (\neg r \vee \neg q)$ distributive

is the required CNF.

Principal Disjunctive and Principal Conjunctive Normal Forms (PDNF) & (PCNF)

Definitions:

Given a number of variables, the products in which each of its variable or its negation, but not both, occurs only once are called the minterms.

For 2 variables p and q, the possible minterms are $p \wedge q$, $\neg p \wedge q$, $p \wedge \neg q$, $\neg p \wedge \neg q$.

For 3 variables p, q and r, the possible minterms are.

$p \wedge q \wedge r$, $\neg p \wedge q \wedge r$, $p \wedge \neg q \wedge r$, $p \wedge q \wedge \neg r$, $\neg p \wedge \neg q \wedge r$, $\neg p \wedge q \wedge \neg r$, $p \wedge \neg q \wedge \neg r$, $\neg p \wedge \neg q \wedge \neg r$.

Defn:

Given a number of variables, the sums in which each of its variable or its negation, but not both occurs only once are called the maxterms.

For 2 variables p and q, the possible max terms are $p \vee q$, $\neg p \vee q$, $p \vee \neg q$, $\neg p \vee \neg q$.

For 3 variables p, q and r, the possible max terms are

$p \vee q \vee r$, $\neg p \vee q \vee r$, $p \vee \neg q \vee r$, $p \vee q \vee \neg r$, $\neg p \vee \neg q \vee r$, $\neg p \vee q \vee \neg r$, $p \vee \neg q \vee \neg r$, $\neg p \vee \neg q \vee \neg r$.

Defns:

A formula consisting of sum of minterms in the variables only is known as PDNF. (Sum of Products canonical form)

A formula consisting of product of maxterms in the variables only is known as PCNF. (Product of Sums canonical form)

Working Procedure:

PCNF

① To find the PDNF, first find the DNF.

CNF

VF

② Introduce $\wedge T$ in missing terms.

$F \equiv P \wedge \neg P$

③ Apply $T \equiv P \vee \neg P$.

④ Apply Distributive law.

⑤ Identical terms are deleted (eg: $P \vee P \Leftrightarrow P$)

Problems:

Obtain the PDNF and PCNF of the following statements using truth tables.

1. $(\neg p \vee \neg q) \rightarrow (p \leftrightarrow \neg q)$.
2. $p \vee (\neg p \rightarrow (q \vee (\neg q \rightarrow r)))$.

1.

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$ (a)	$p \leftrightarrow \neg q$ (b)	$a \to b$
T	T	F	F	F	F	T ✓
T	F	F	T	T	T	T ✓
F	T	T	F	T	T	T ✓
F	F	T	T	T	F	F

PDNF: $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge q)$

Now PDNF of $\neg(a \to b)$ is $\neg p \wedge \neg q$.

$\therefore$ PCNF is $\neg(\neg p \wedge \neg q) = p \vee q$.

Let s : gn. formula.

2.

p	q	r	$\neg p$	$\neg q$	$\neg q \to r$	$q \vee (\neg q \to r)$	$\neg p \to (q \vee (\neg q \to r))$	s
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	F	T	T	T	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	F	F

PDNF $(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r)$
$\vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r)$
$\vee (\neg p \wedge \neg q \wedge r)$.

PDNF of $\neg(S) \equiv \neg(\neg p \wedge \neg q \wedge \neg r)$.

$\therefore$ PCNF $= p \vee q \vee r$.

*) Without constructing the truth tables, find the PDNF of the following statements.

1. $(p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$.

Soln :

Gn. statement is the DNF, but not PDNF.

To find PDNF :

$$(p \wedge q) \vee (\neg p \wedge q) \vee (q \wedge r)$$

$$\Leftrightarrow (p \wedge q \wedge T) \vee (\neg p \wedge q \wedge T) \vee (q \wedge r \wedge T)$$

$$\Leftrightarrow [p \wedge q \wedge (r \vee \neg r)] \vee [\neg p \wedge q \wedge (r \vee \neg r)] \vee [q \wedge r \wedge (p \vee \neg p)]$$

$$\Leftrightarrow (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (q \wedge r \wedge p) \vee (q \wedge r \wedge \neg p)$$

$$\Leftrightarrow (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r)$$

which is the required PDNF.

2. $p \wedge \neg(q \wedge r) \vee (p \rightarrow q)$

Soln:

$p \wedge \neg(q \wedge r) \vee (p \rightarrow q)$

$\Leftrightarrow p \wedge (\neg q \vee \neg r) \vee (\neg p \vee q)$

$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \vee q)$.

$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r) \vee [(\neg p \wedge T) \vee (q \wedge T)]$

$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \wedge (q \vee \neg q) \vee$
$q \wedge (p \vee \neg p)]$

$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \wedge \neg q) \vee$
$(\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg p)$

$\Leftrightarrow (p \wedge \neg q) \vee (p \wedge \neg r) \vee (\neg p \wedge \neg q)$
$\vee (p \wedge q) \vee (q \wedge \neg p)$

$\Leftrightarrow (p \wedge \neg q \wedge T) \vee (p \wedge \neg r \wedge T) \vee$
$(\neg p \wedge \neg q \wedge T) \vee (p \wedge q \wedge T) \vee (q \wedge \neg p \wedge T)$

$\Leftrightarrow [(p \wedge \neg q) \wedge (r \vee \neg r)] \vee [(p \wedge \neg r) \wedge (q \vee \neg q)$
$\vee (\neg p \wedge \neg q \wedge (r \vee \neg r)] \vee (p \wedge q \wedge (r \vee \neg r)$
$\vee ((q \wedge \neg p) \wedge (r \vee \neg r)]$

$\Leftrightarrow (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg r \wedge q)$
$\vee (p \wedge \neg r \wedge \neg q) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$
$\vee (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (q \wedge \neg p \wedge r) \vee (q \wedge \neg p \wedge \neg r)$

$\Leftrightarrow (p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$
$\vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$
$\vee (\neg p \wedge \neg q \wedge \neg r) \vee (p \wedge q \wedge r)$.

Note:

Since all the minterms are present in the above problem, it is a tautology.

*) Without constructing the truth tables, find the PCNF of the following statements.

1. $(p \vee q) \wedge (r \vee \neg p) \wedge (q \vee \neg r)$

Soln:

Let $S \Leftrightarrow (p \vee q) \wedge (r \vee \neg p) \wedge (q \vee \neg r)$

Since it is a CNF, we directly find PCNF.

$S \Leftrightarrow [(p \vee q) \vee F] \wedge [(r \vee \neg p) \vee F] \wedge [(q \vee \neg r) \vee F]$

$\Leftrightarrow [(p \vee q) \vee (r \wedge \neg r)] \wedge [(r \vee \neg p) \vee (q \wedge \neg q)]$

$\wedge [(q \vee \neg r) \vee (p \wedge \neg p)]$

$\Leftrightarrow (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (r \vee \neg p \vee q)$
$\wedge (r \vee \neg p \vee \neg q) \wedge (q \vee \neg r \vee p) \wedge$
$(q \vee \neg r \vee \neg p)$

$\Leftrightarrow (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge$
$(\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r)$ is the required PCNF.

2. $\cancel{p\vee}\ (p\wedge q)\vee(\neg p\wedge q\wedge r)$

Soln:

Let $S \Leftrightarrow (p\wedge q)\vee(\neg p\wedge q\wedge r)$

$\Leftrightarrow (p\wedge q\wedge T)\vee(\neg p\wedge q\wedge r)$

$\Leftrightarrow [(p\wedge q)\wedge(r\vee\neg r)]\vee(\neg p\wedge q\wedge r)$

$\Leftrightarrow (p\wedge q\wedge r)\vee(p\wedge q\wedge\neg r)\vee(\neg p\wedge q\wedge r)$

is the ~~required~~ PDNF.

To find PCNF:

$\neg S \Leftrightarrow \cancel{\neg q}\ (p\wedge\neg q\wedge r)\vee(\neg p\wedge\neg q\wedge r)$
$\vee(\neg p\wedge q\wedge\neg r)\vee(p\wedge\neg q\wedge\neg r)\vee$
$(\neg p\wedge\neg q\wedge\neg r)$

$\neg\neg S \Leftrightarrow (\neg p\vee q\vee\neg r)\wedge(p\vee q\vee\neg r)$
$\vee(p\vee\neg q\vee r)\wedge(\neg p\vee q\vee r)\wedge$
$(p\vee q\vee r)$

which is the required PCNF.