Constants:

Mass = 1 kg
Unstretched Length = 1 m
g = 9.81 m/s^2
k = 44 N/m

Time Domain

0 s <= t <= 10 s

Proof that RK4 and derivation work:

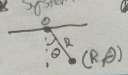
- Look at fixed points of system

Fixed points:

- When no initial motion is present and with initial angle at 0. The initial stretch of the spring must be equal to mg/k + L_i (-mg/k + L_i when initial angle is pi
- For this system if the length of the spring dosent change the angle also dosent change

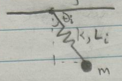
$EL: \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\rho}} = \frac{\partial \mathcal{L}}{\partial \rho}$

System



$-\pi \le \theta_i < \pi$

$U_g = 0$



Givens

k, L_i, m, θ_i

functions

$L(t)$: Length : L

$\theta(t)$: Angle : θ

Expansion (spring) - $L = L_i + \Delta h$

$\frac{dL}{dt} = V$

$U = \frac{1}{2}k(L - L_i)^2 - mgL\cos\theta$

$\frac{d\theta}{dt} = \omega$

$I = mL^2$

$K = \frac{1}{2}I(\omega)^2 + \frac{1}{2}m(v)^2 = \frac{1}{2}mL^2\omega^2 + \frac{1}{2}mv^2$

$\mathcal{L} = \frac{1}{2}mL^2\omega^2 + \frac{1}{2}mv^2 - \frac{1}{2}k(L - L_i)^2 + mgL\cos\theta$

$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial v}\right) = \frac{d}{dt}(mv) = ma = mL\omega^2 - k(L - L_i) + mg\cos\theta$

$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \omega} = \frac{d}{dt}(mL^2\omega) = 2mLv\omega + mL^2\alpha = -mgL\sin\theta$

$$\boxed{\begin{aligned} \frac{d^2\theta}{dt^2} &= -\frac{g}{L}\sin\theta - 2v\omega \\ \frac{d^2L}{dt^2} &= L\omega^2 - \frac{k}{m}(L - L_i) + g\cos\theta \end{aligned}}$$

System:

Fixed Point ↓

$$f = \left\{ \begin{aligned} \frac{dL}{dt} &= V \\ \frac{d\theta}{dt} &= \omega \\ \frac{dv}{dt} &= L\omega^2 - \frac{k}{m}(L - L_i) + g\cos\theta \\ \frac{d\omega}{dt} &= -\frac{g}{L}\sin\theta - 2v\omega \end{aligned} \right. \qquad \begin{pmatrix} V \\ \omega \\ L\omega^2 - \frac{k}{m}(L - L_i) + g\cos\theta \\ -\frac{g}{L}\sin\theta - 2v\omega \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Fixed point (x_0) at when

$$X_{01} = \begin{pmatrix} V \\ \omega \\ R \\ \theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{mg}{k} + L_i \\ 0 \end{pmatrix}, \quad X_{02,3} = \begin{pmatrix} V \\ \omega \\ R \\ \theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ L_i - \frac{mg}{k} \\ \pm\pi \end{pmatrix}$$

How to find Bifurcation values for the system by:

Constant: $\theta_i, L_i, V_i, \omega_i, g$

Variable: k, m

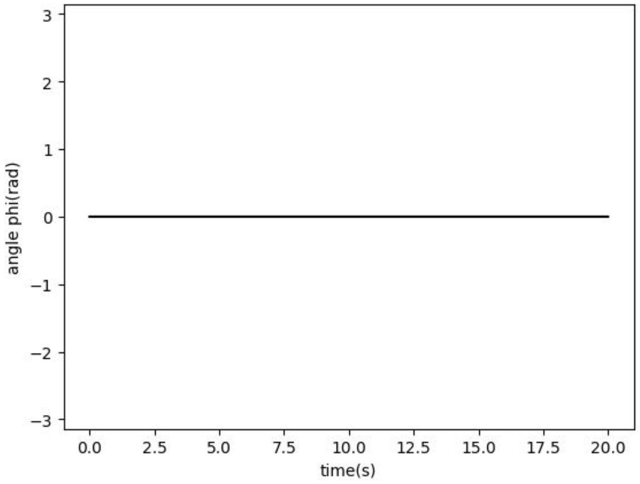
$\varepsilon = \frac{k}{m}$

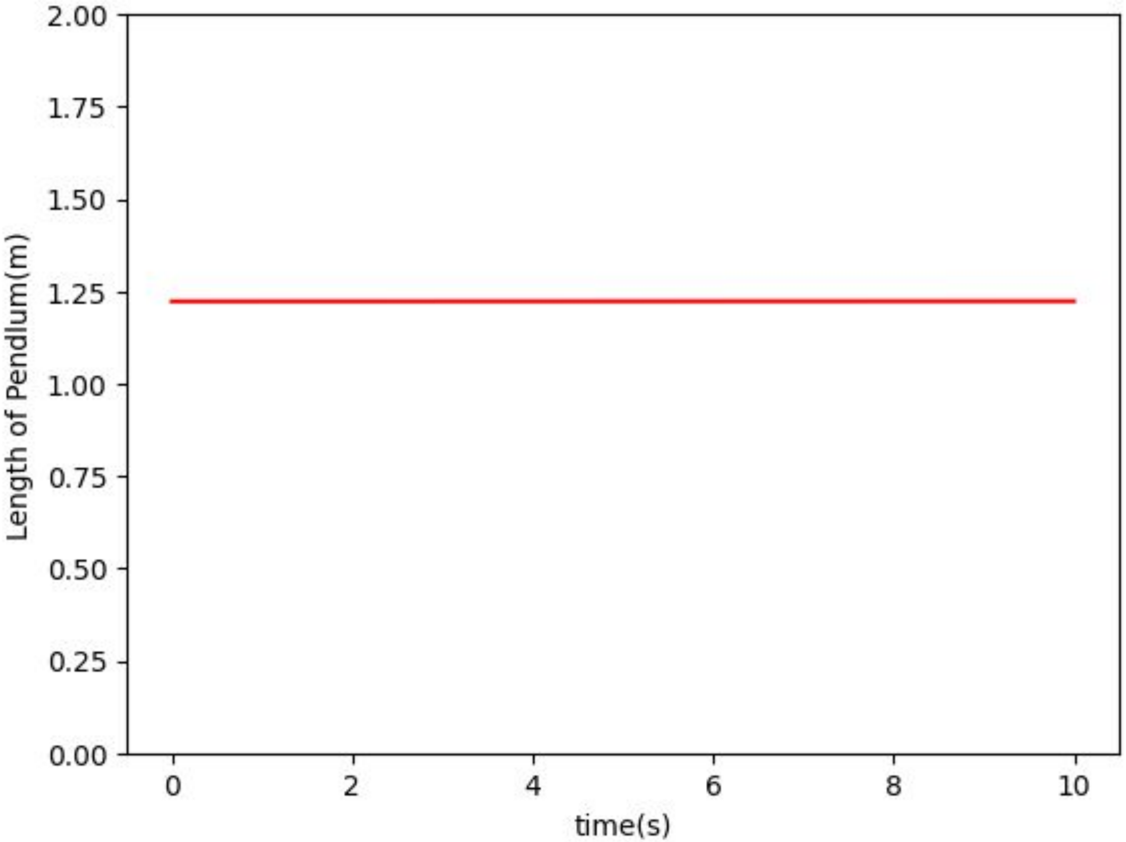
What does a Bifurcation look like?

$$f_\varepsilon = \left\{ \begin{aligned} \frac{d^2\theta}{dt^2} &= -\frac{g}{L}\sin\theta - 2v\omega \\ \frac{d^2L}{dt^2} &= L\omega^2 - \varepsilon(L - L_i) + g\cos\theta \end{aligned} \right.$$

Initial Conditions:

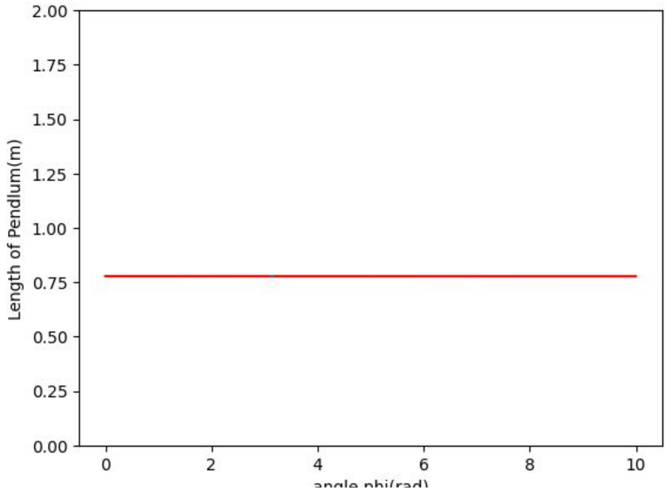
t_old = 0
R_old = (mg/k) + L_i
phi_old = 0
v_old = 0
w_old = 0



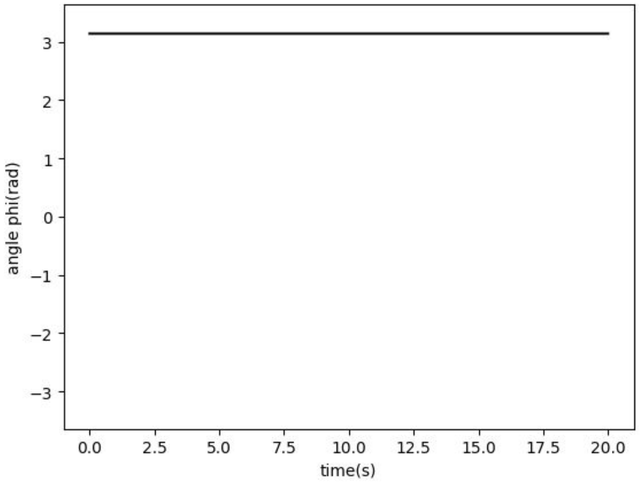


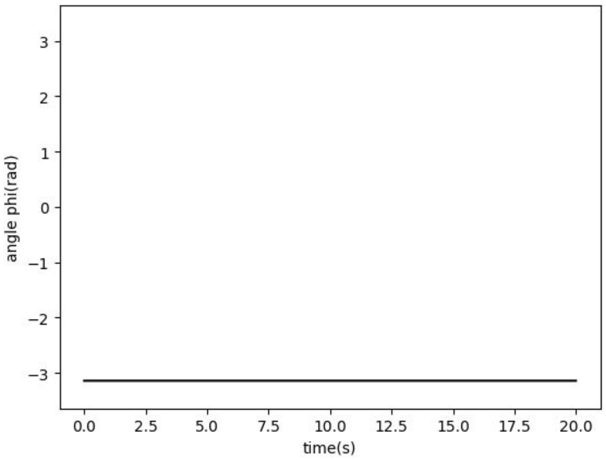
Initial Conditions:

t_old = 0
R_old = -(mg/k) + L_i
phi_old = pi, -pi
v_old = 0
w_old = 0



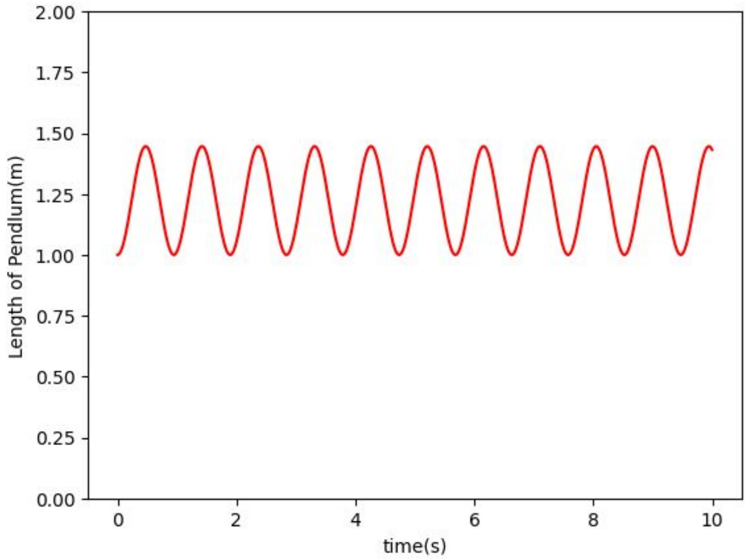
*only one length graph is used because both pi and -pi produce the same graph



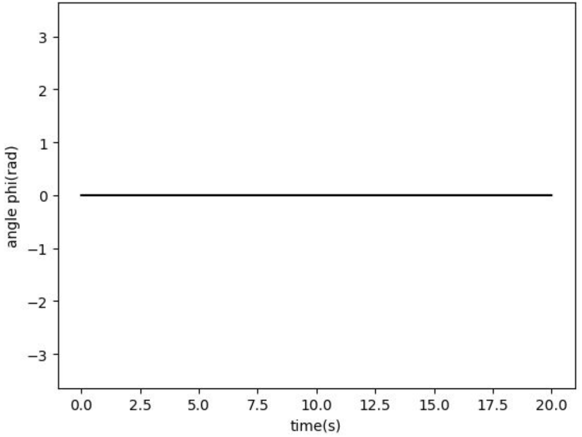


Initial Conditions:

t_old = 0
R_old = L_i
phi_old = 0
v_old = 0
w_old = 0



When all initial condition but the initial length is zero the system acts as a normal mass spring under gravity



Constants:

Mass = 1 kg
Unstretched Length = 1 m
G = 9.81 m/s^2

Initial Conditions:

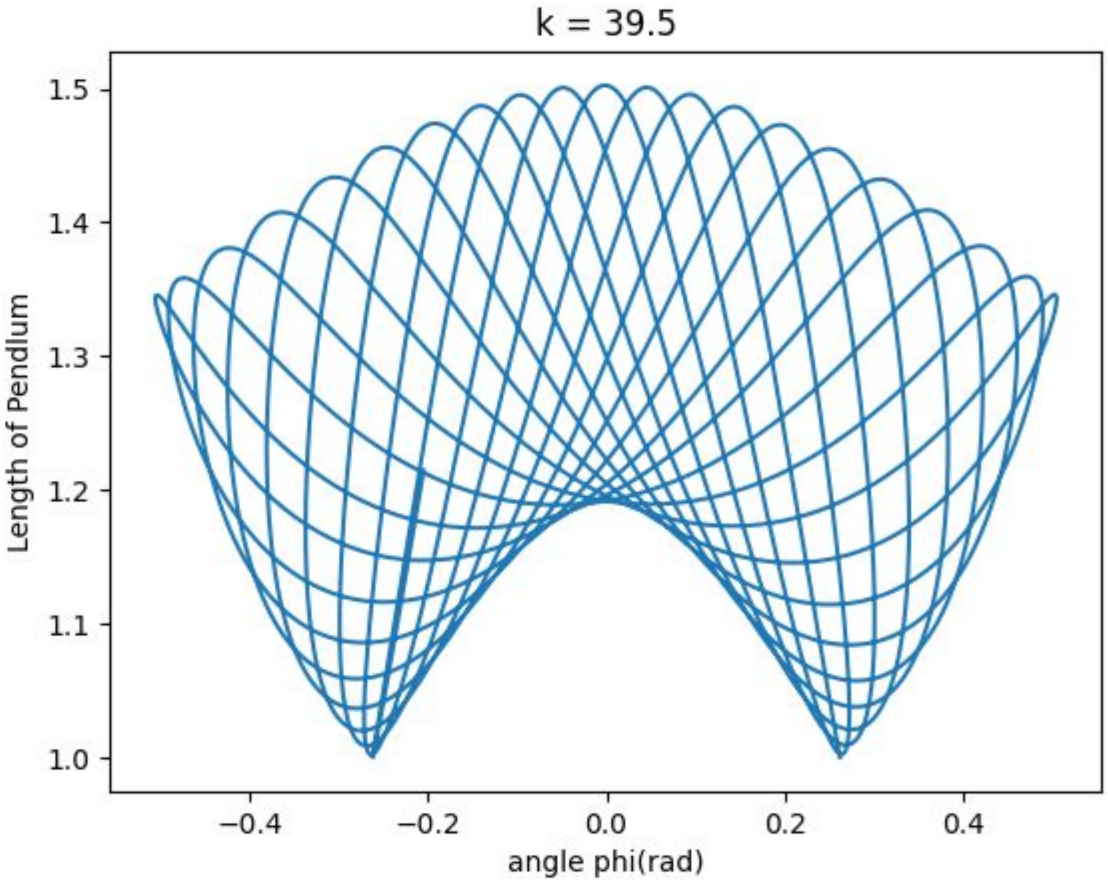
Initial Length = L_i = 1m
Initial Angle = pi/12 rad

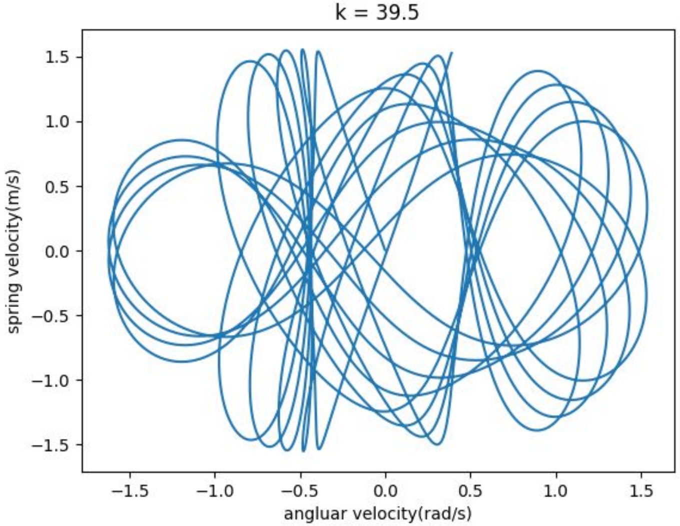
Time Domain:

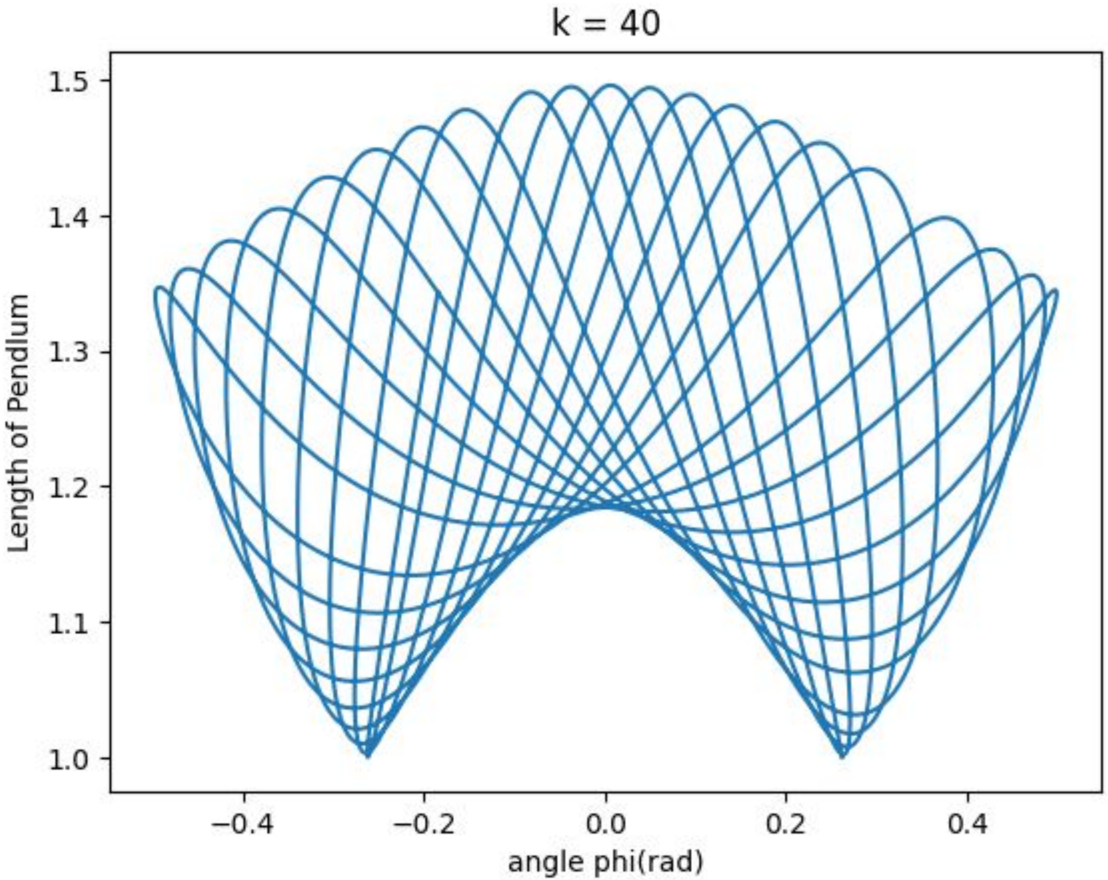
0 s <= t <= 20 s

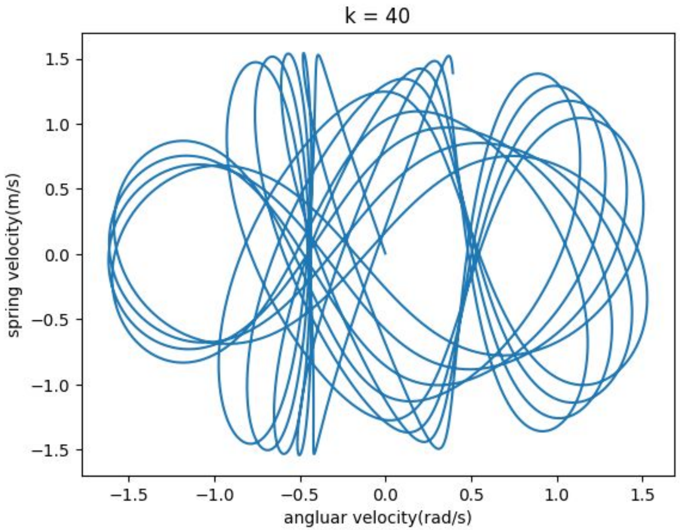
Q: For what k value does a bifurcation appear?(still working on it)

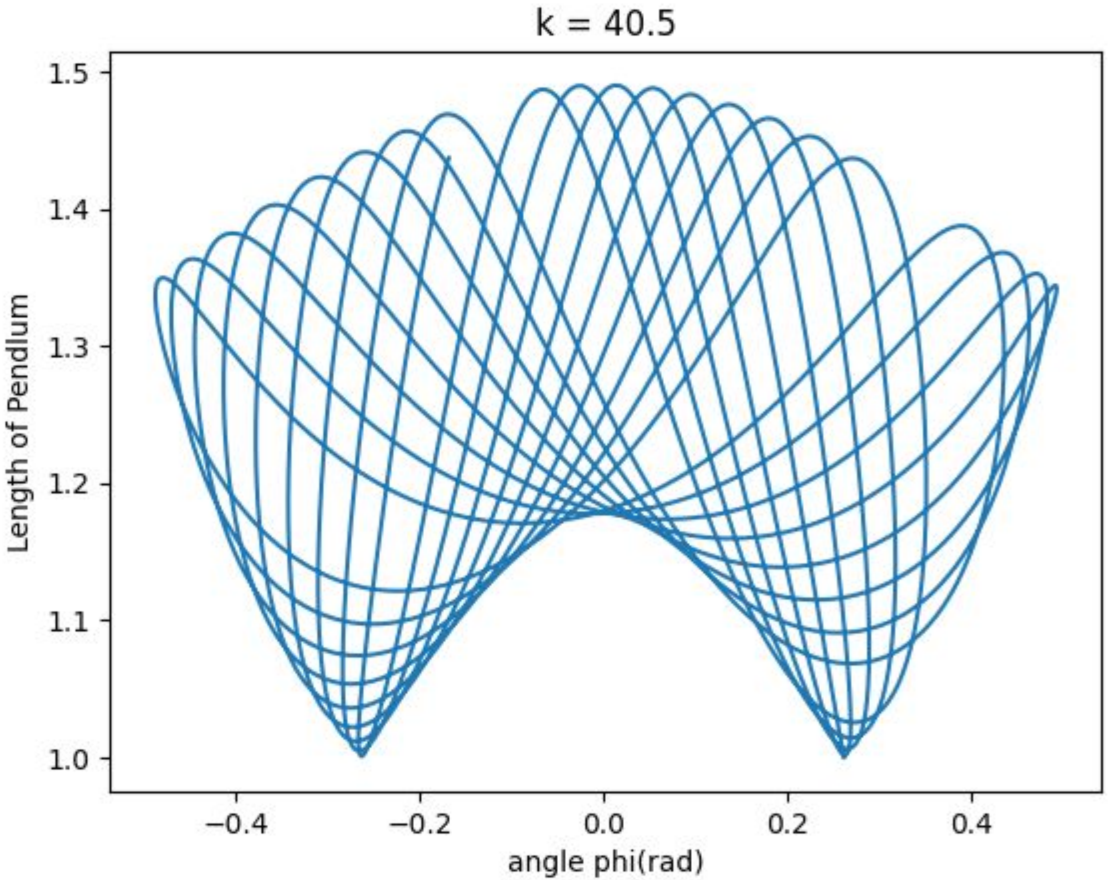
Something cool. Click through the next few slides to see how the system progresses while the spring constant increases

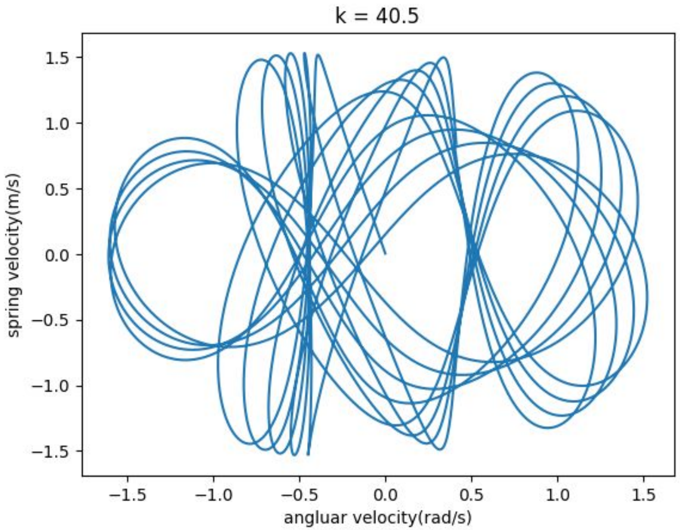

k = 39.5
Length of Pendlum
1.5
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4
angle phi(rad)

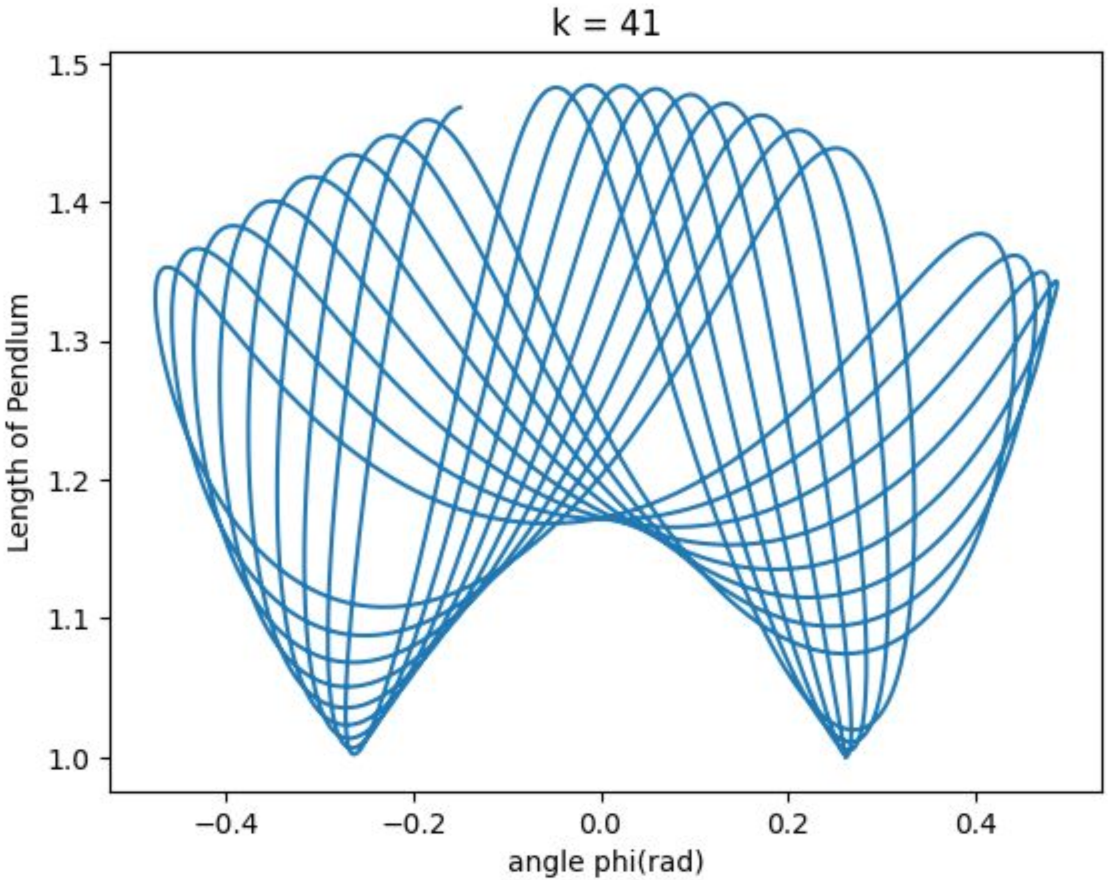

k = 39.5
spring velocity(m/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
angluar velocity(rad/s)

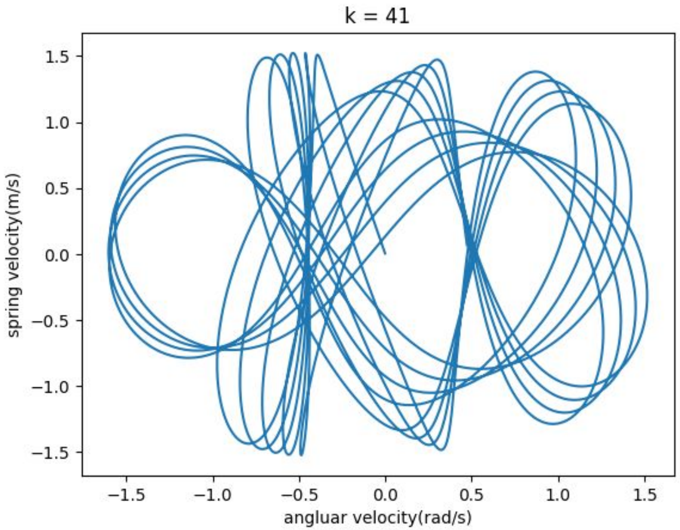

k = 40
Length of Pendulum
angle phi(rad)
1.0
1.1
1.2
1.3
1.4
1.5
−0.4
−0.2
0.0
0.2
0.4

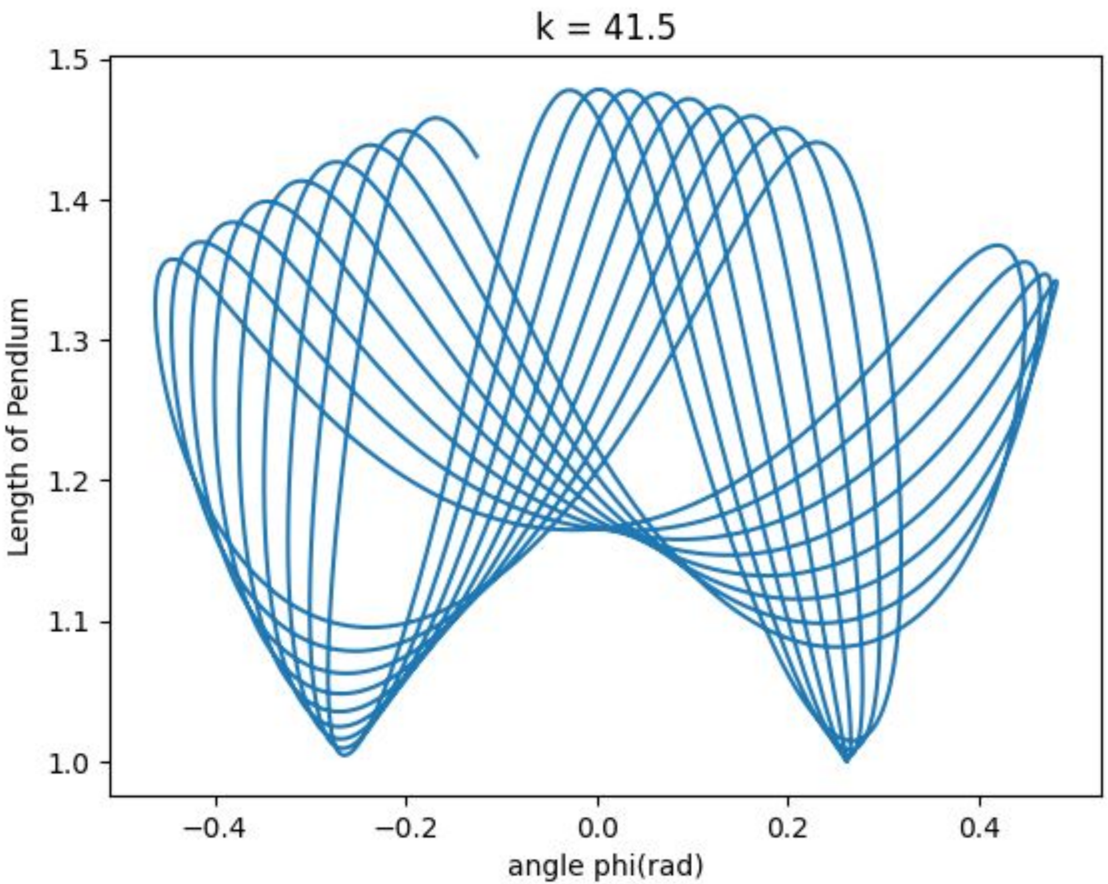

k = 40
spring velocity(m/s)
angluar velocity(rad/s)
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

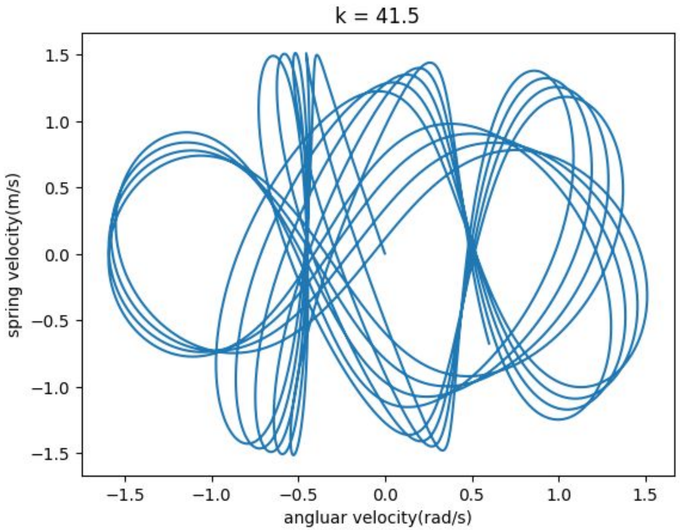

k = 40.5
Length of Pendlum
angle phi(rad)
1.5
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4

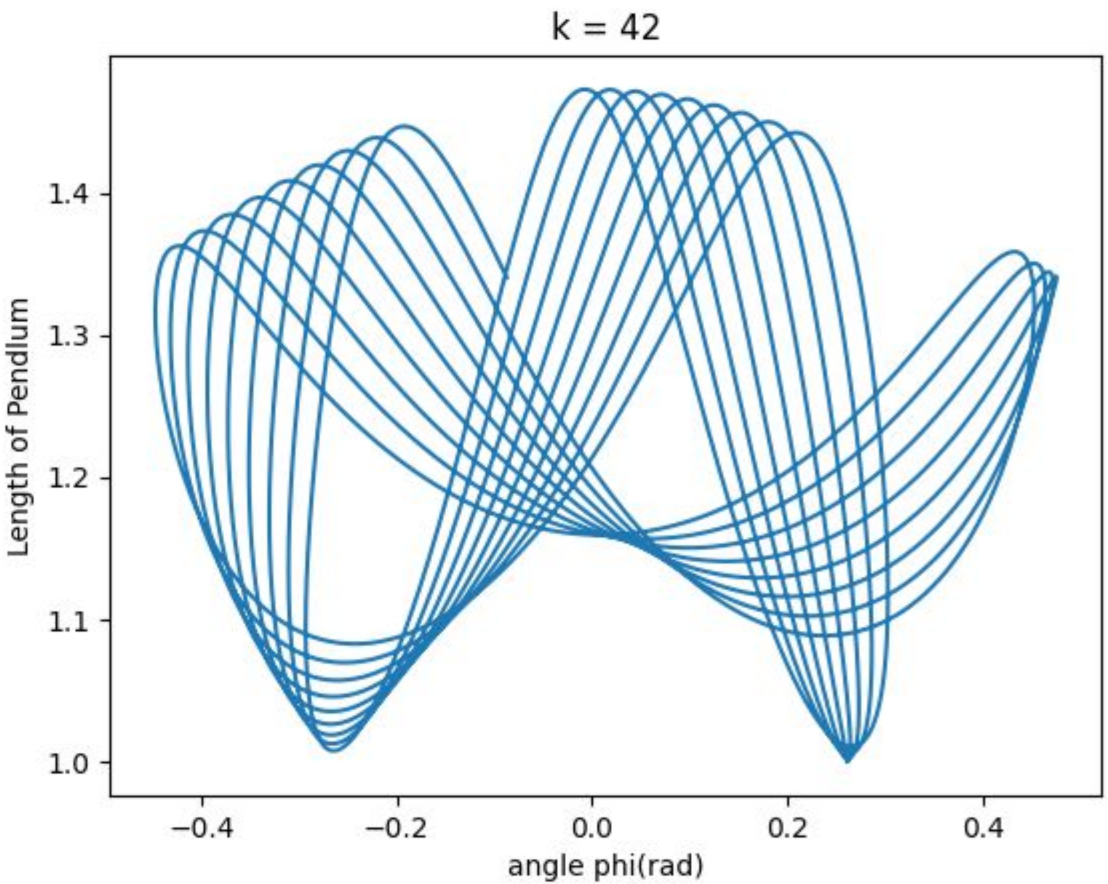

k = 40.5
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

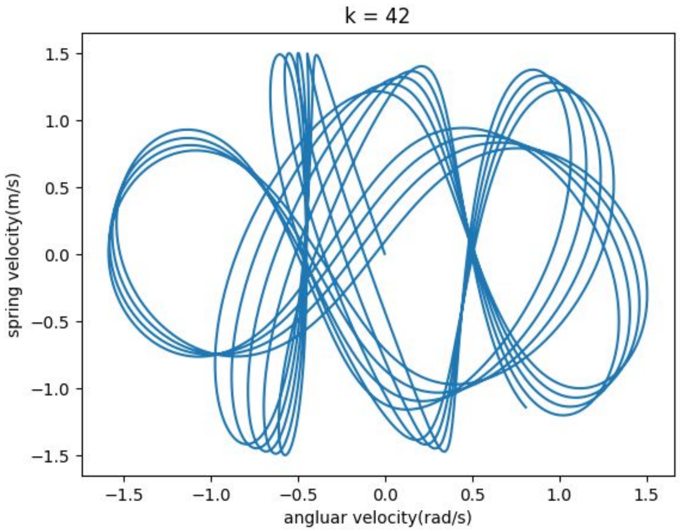

k = 41
Length of Pendulum
angle phi(rad)
1.0
1.1
1.2
1.3
1.4
1.5
−0.4
−0.2
0.0
0.2
0.4

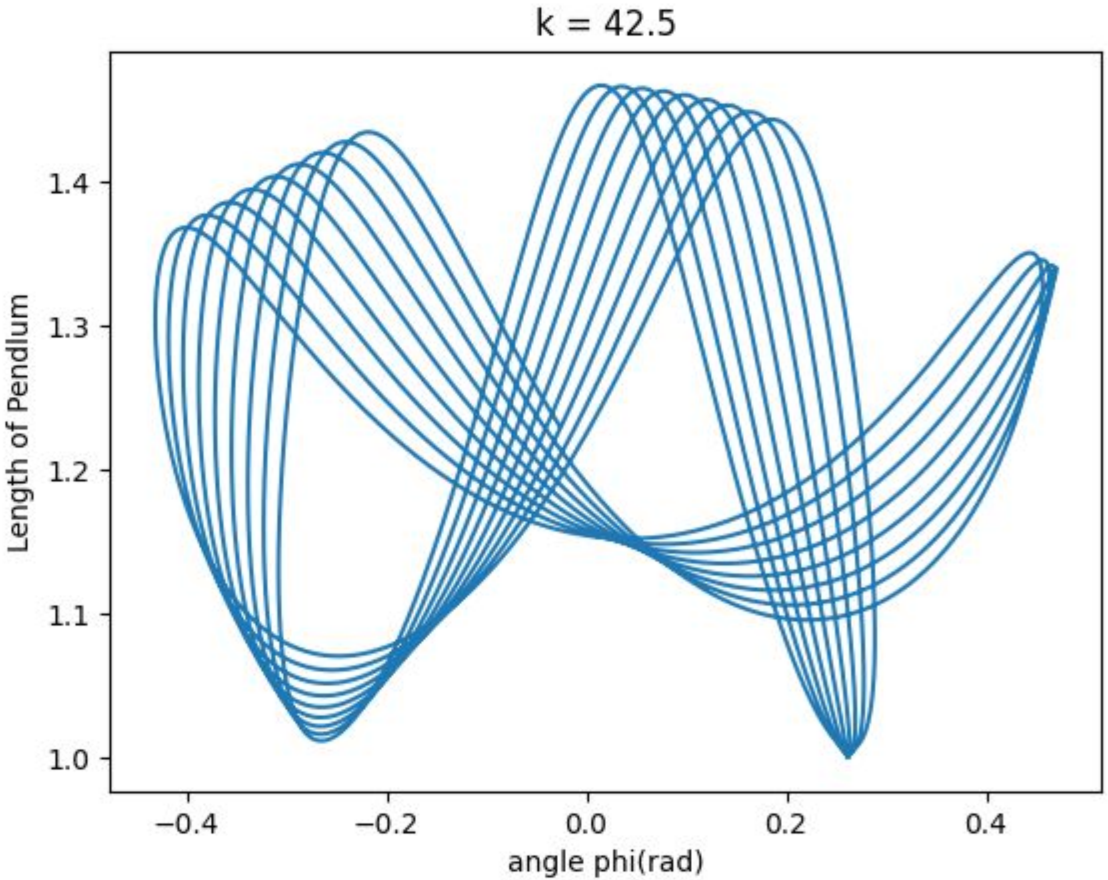

k = 41
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

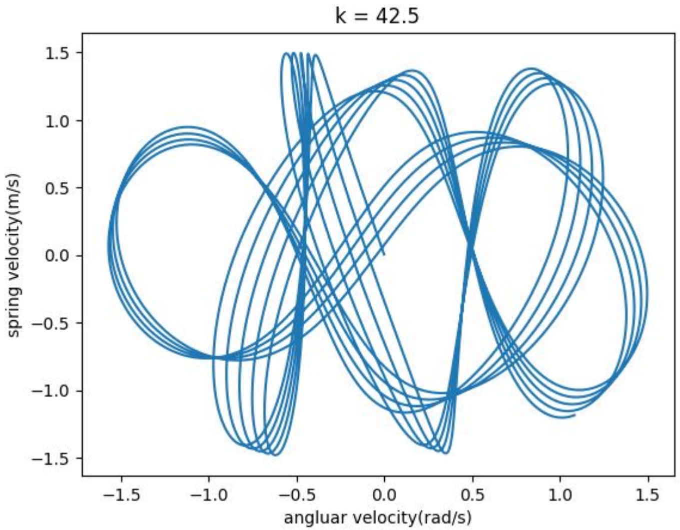

k = 41.5
Length of Pendlum
angle phi(rad)

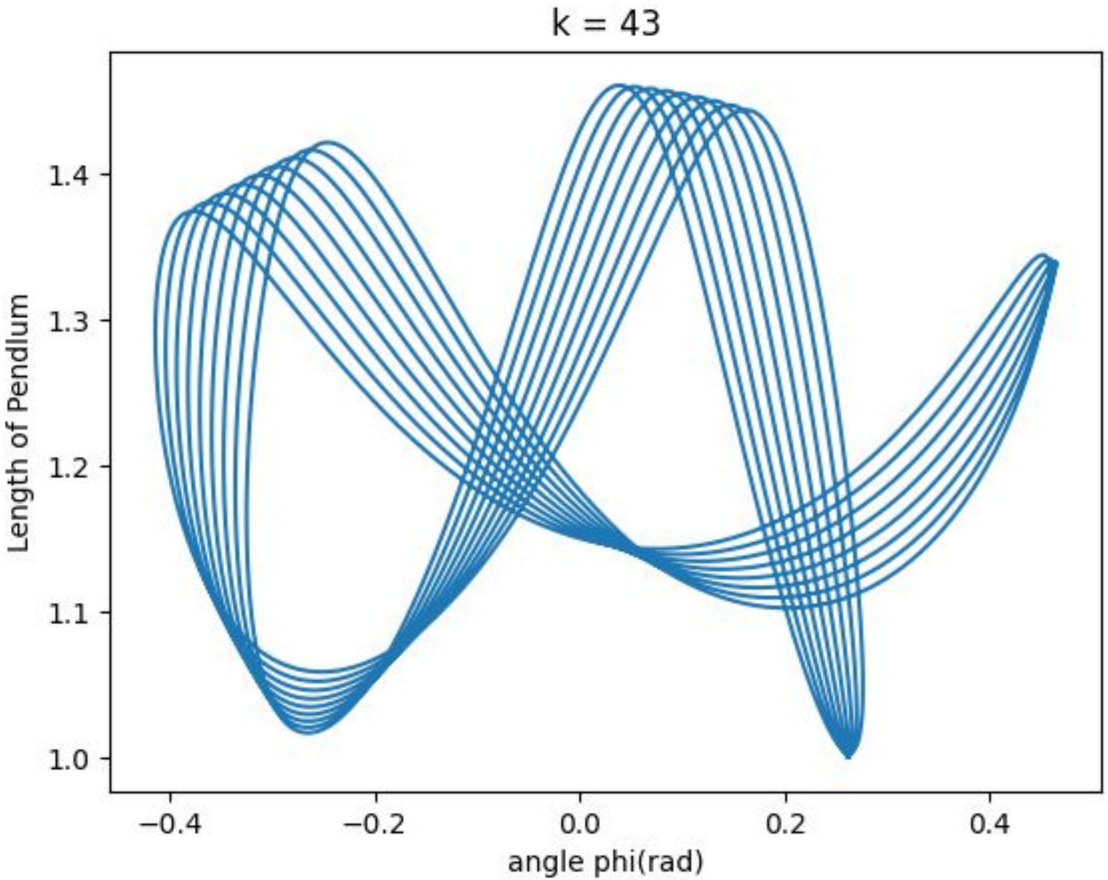

k = 41.5
spring velocity(m/s)
angluar velocity(rad/s)

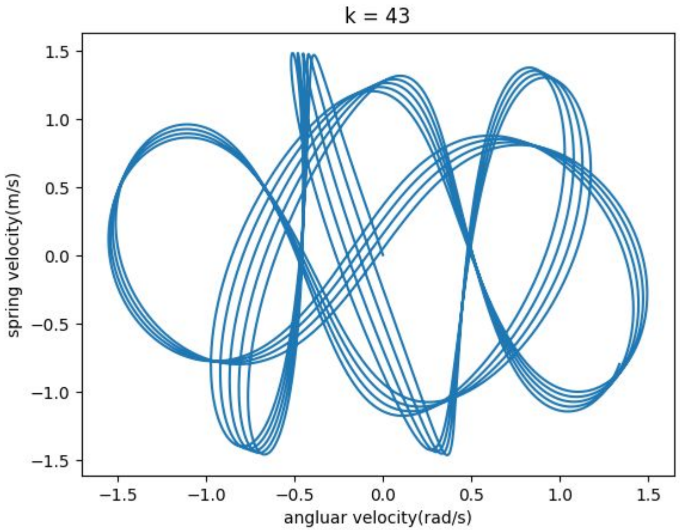

k = 42
Length of Pendlum
angle phi(rad)
1.0
1.1
1.2
1.3
1.4
−0.4
−0.2
0.0
0.2
0.4

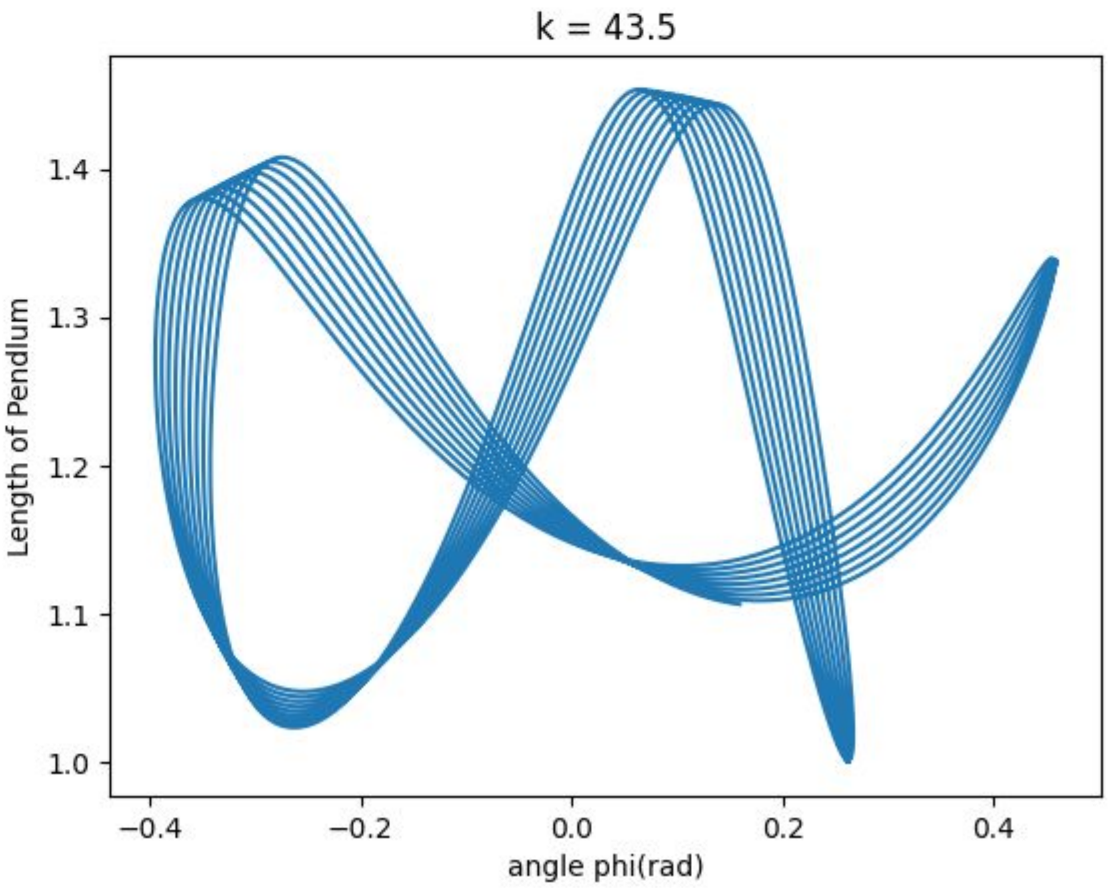

k = 42
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

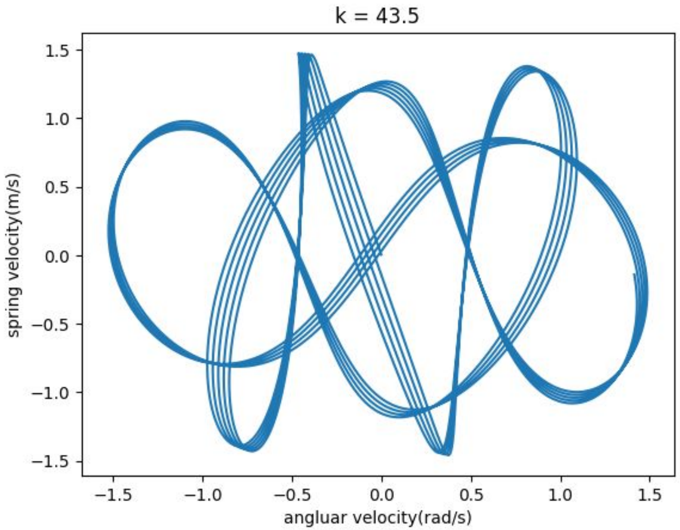

k = 42.5
Length of Pendlum
angle phi(rad)
1.0
1.1
1.2
1.3
1.4
−0.4
−0.2
0.0
0.2
0.4

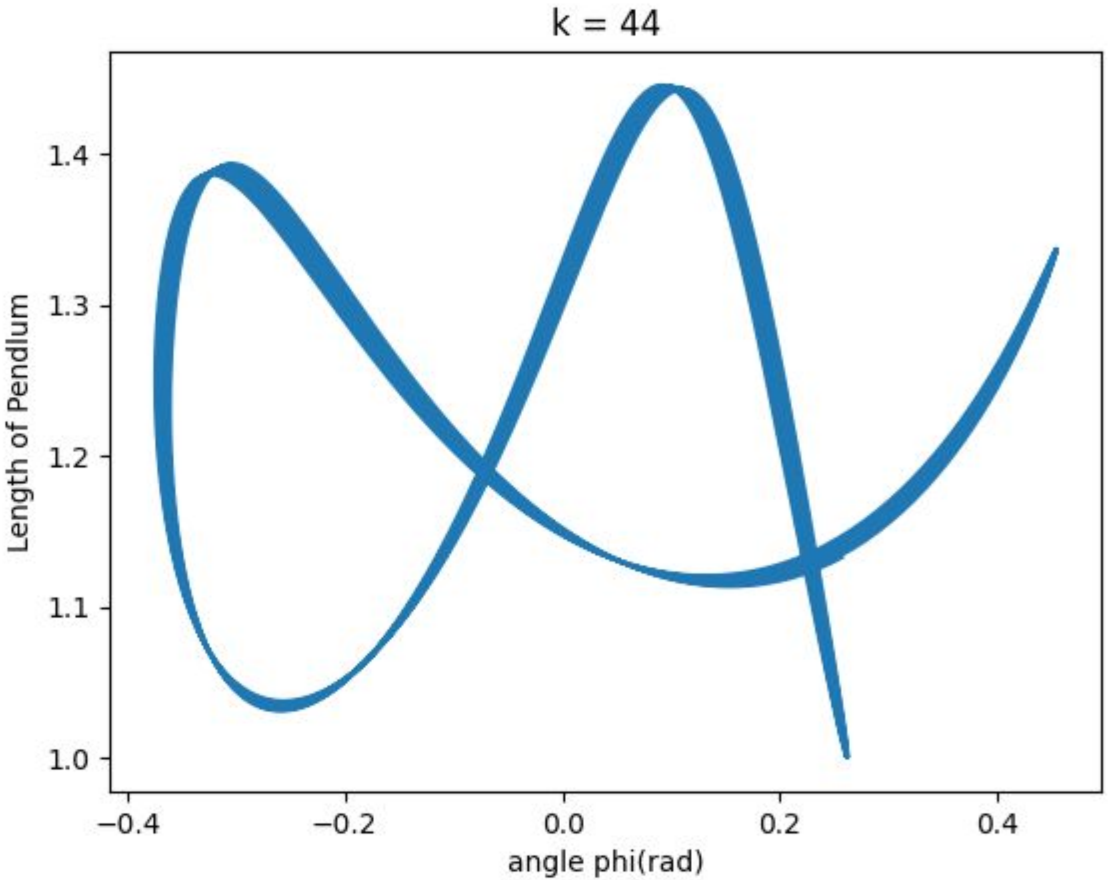

k = 42.5
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

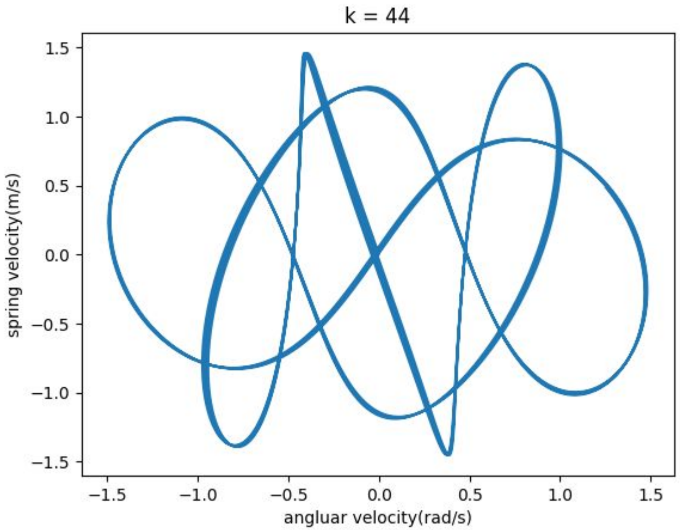

k = 43
Length of Pendlum
angle phi(rad)
1.0
1.1
1.2
1.3
1.4
−0.4
−0.2
0.0
0.2
0.4

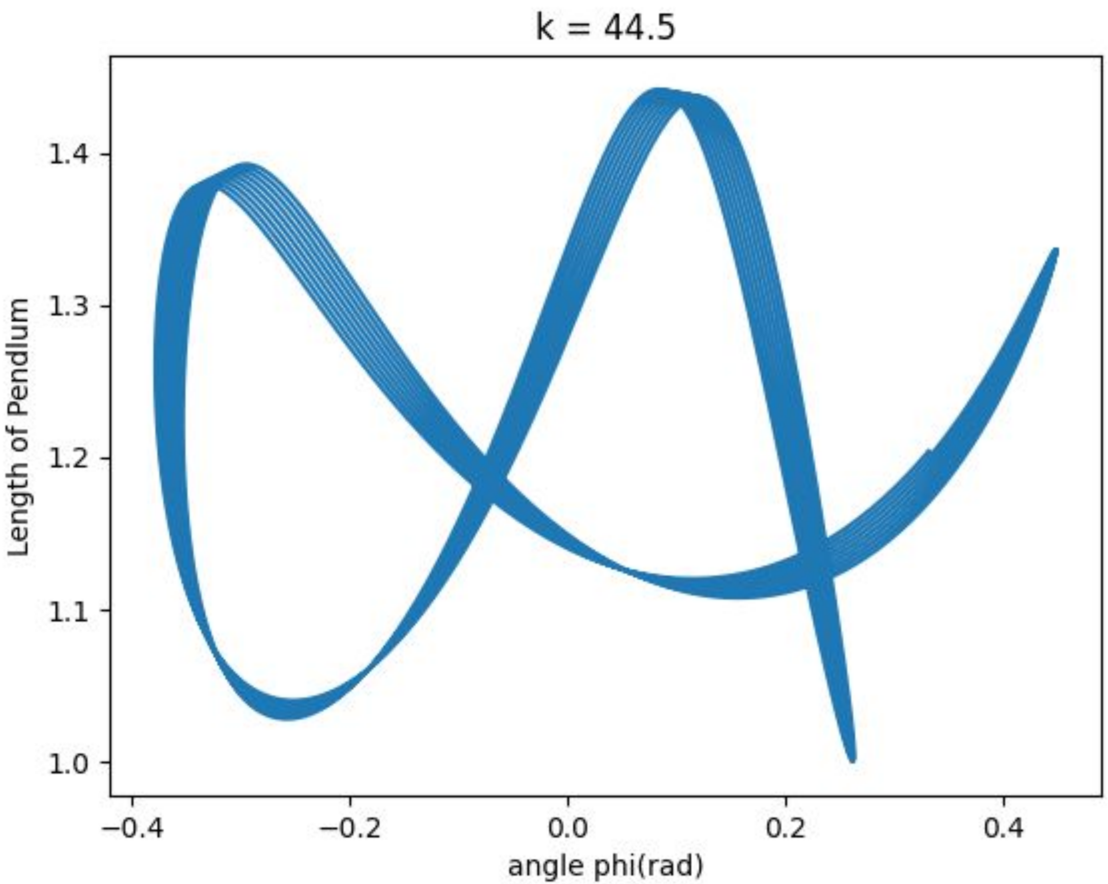

k = 43
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

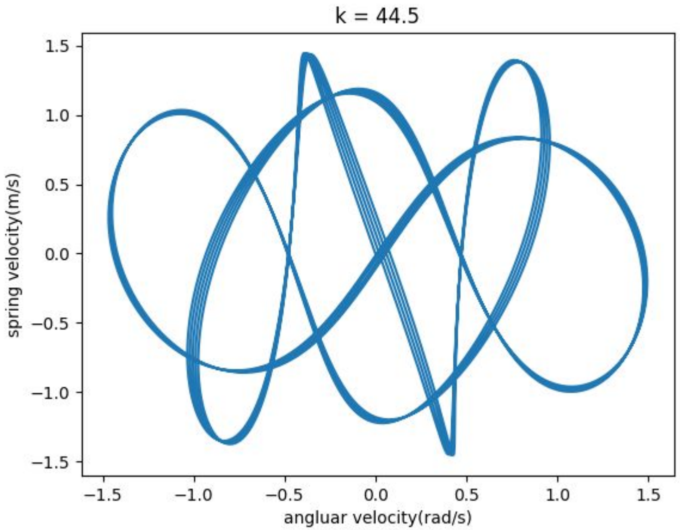

k = 43.5
Length of Pendlum
angle phi(rad)
1.0
1.1
1.2
1.3
1.4
−0.4
−0.2
0.0
0.2
0.4

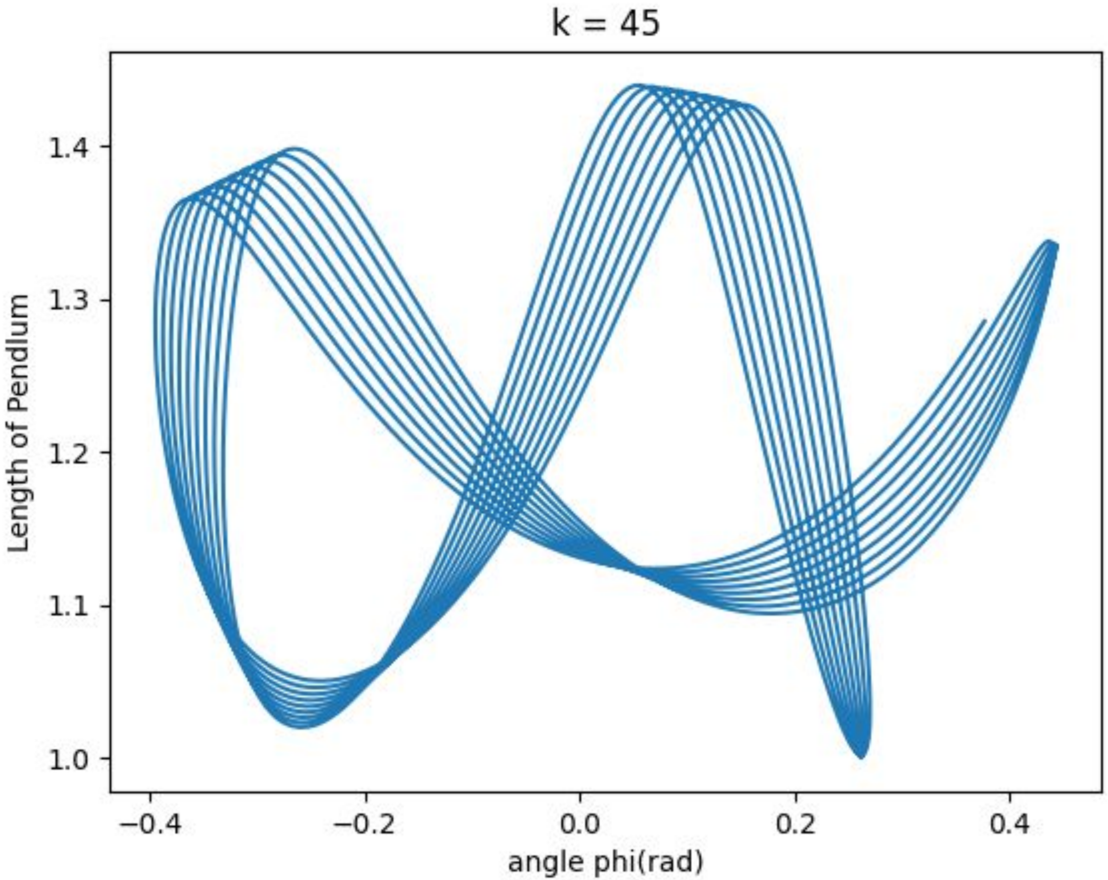

k = 43.5
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

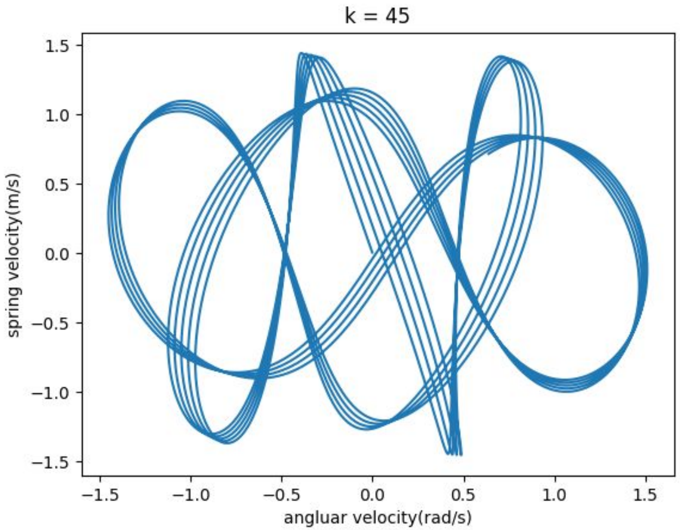

k = 44
Length of Pendlum
angle phi(rad)
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4

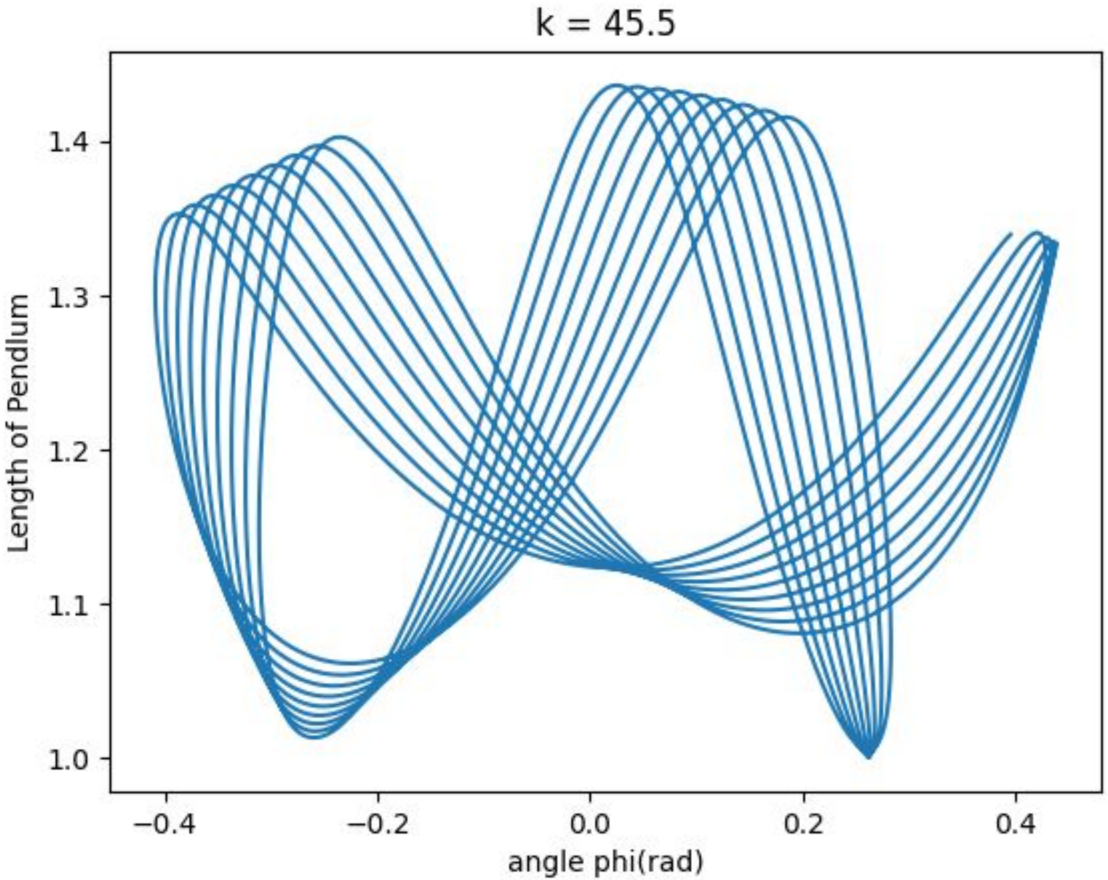

k = 44
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


k = 44.5
Length of Pendlum
angle phi(rad)
1.0
1.1
1.2
1.3
1.4
−0.4
−0.2
0.0
0.2
0.4

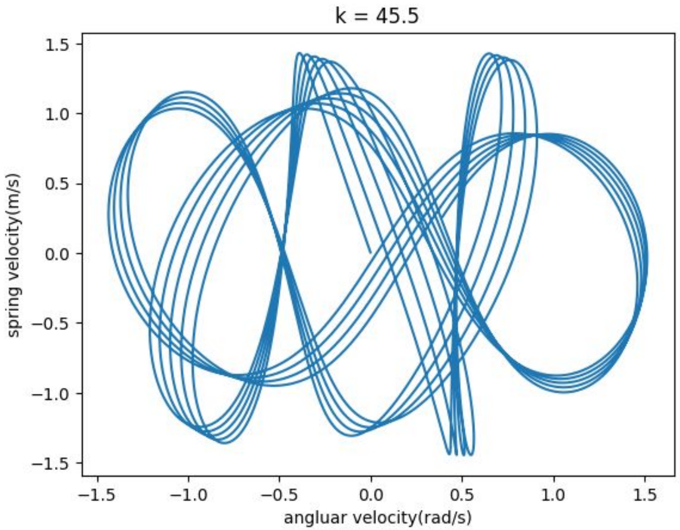

k = 44.5
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


k = 45
Length of Pendlum
angle phi(rad)
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4

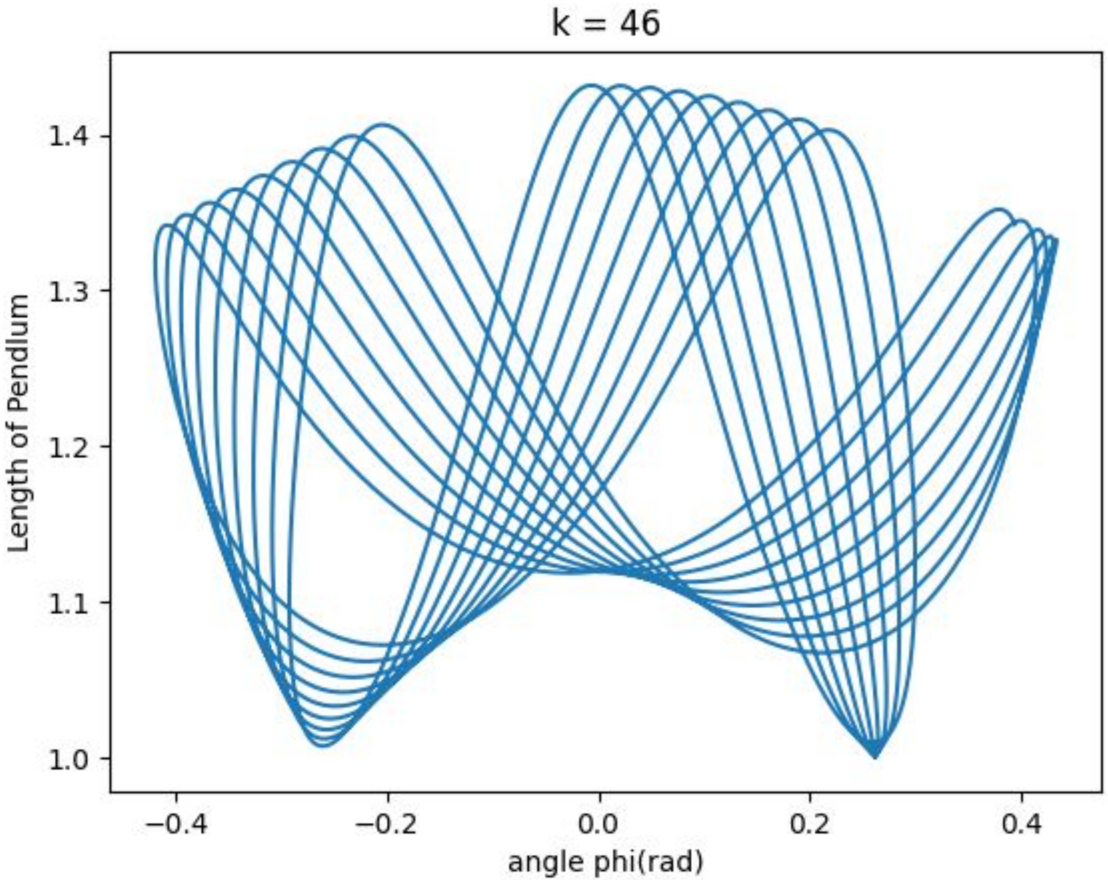

k = 45
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

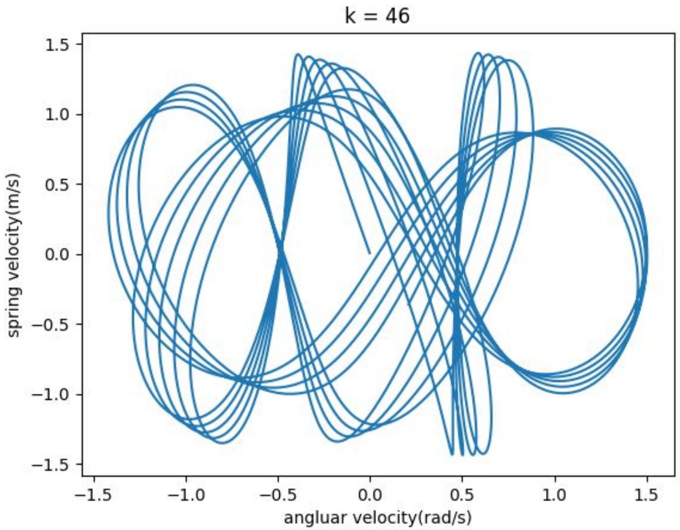


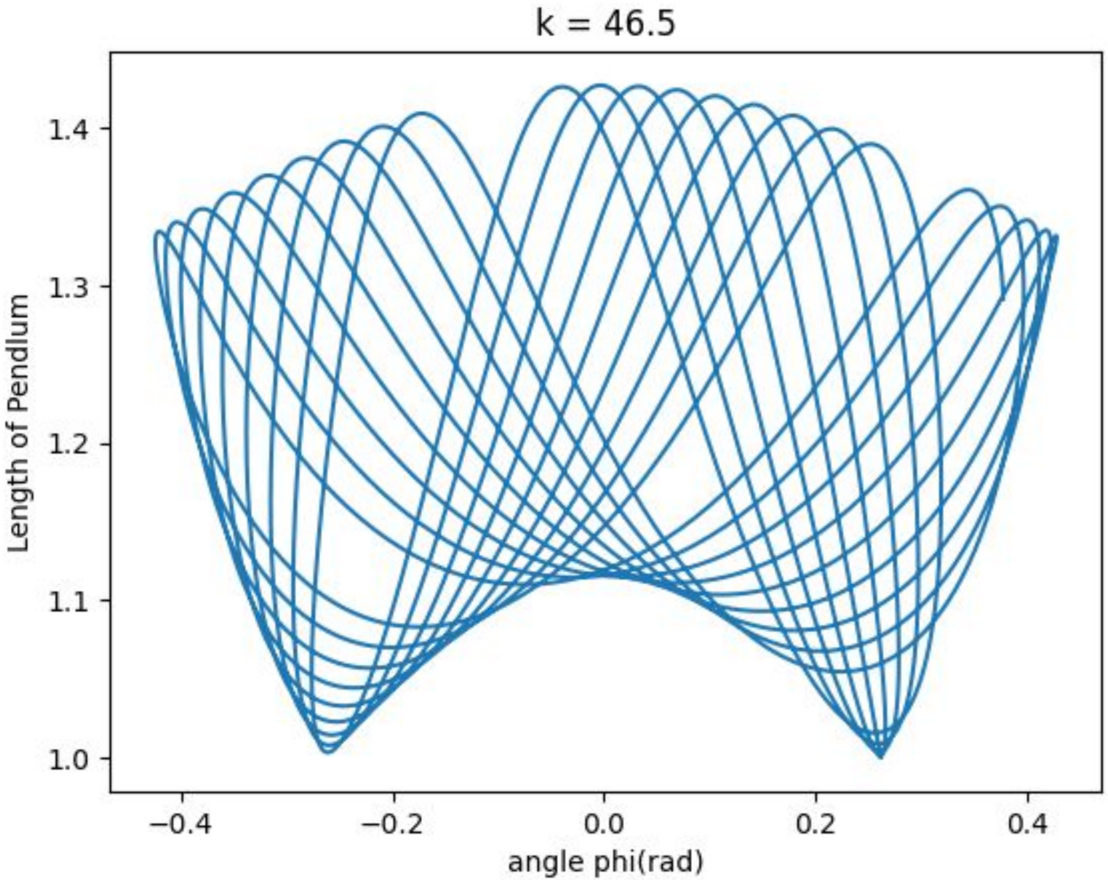
k = 45.5
Length of Pendlum
angle phi(rad)
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4

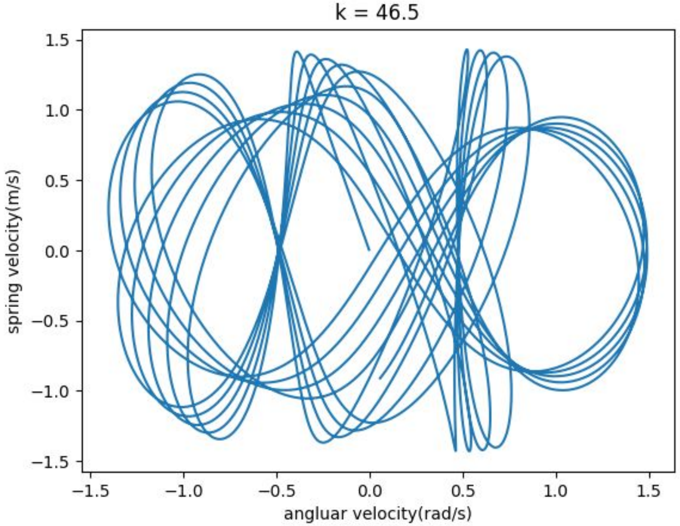


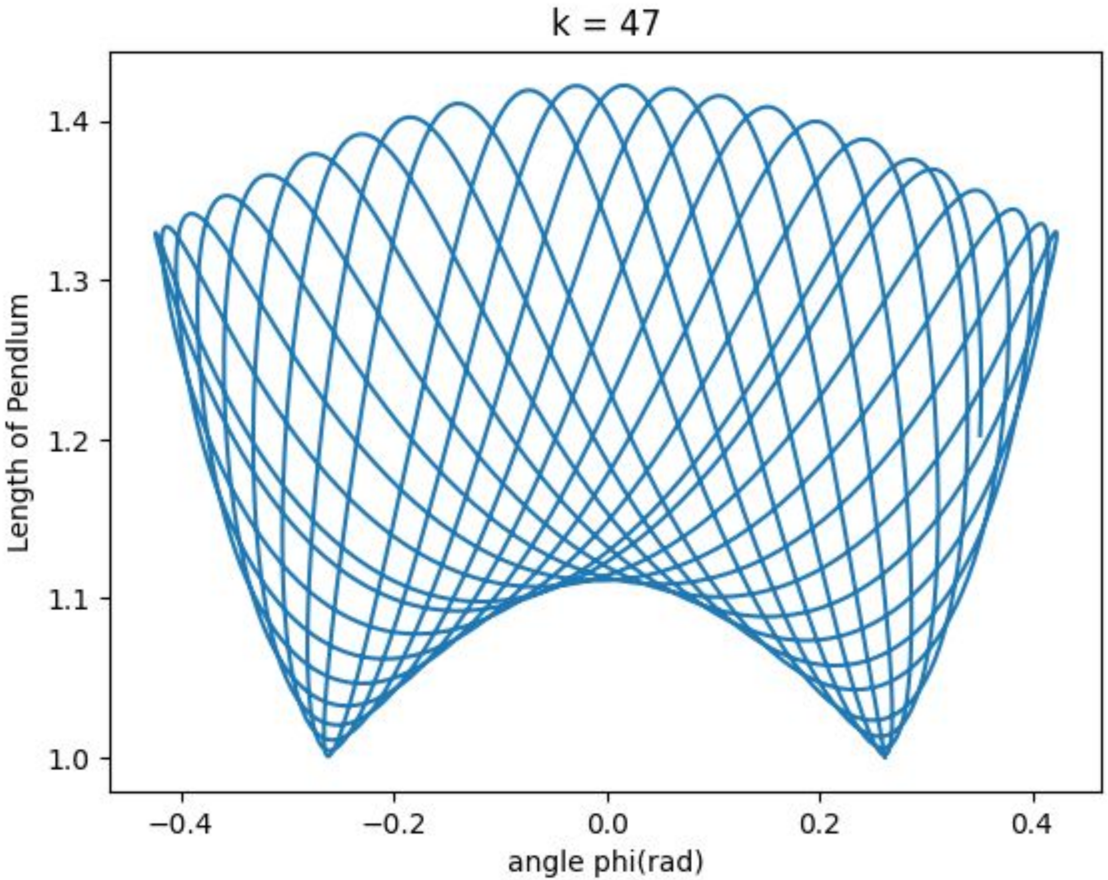
k = 45.5
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

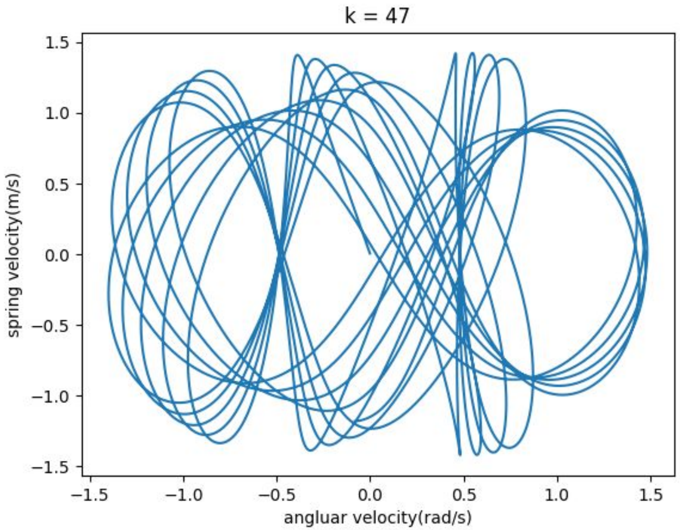

k = 46
Length of Pendlum
angle phi(rad)
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4

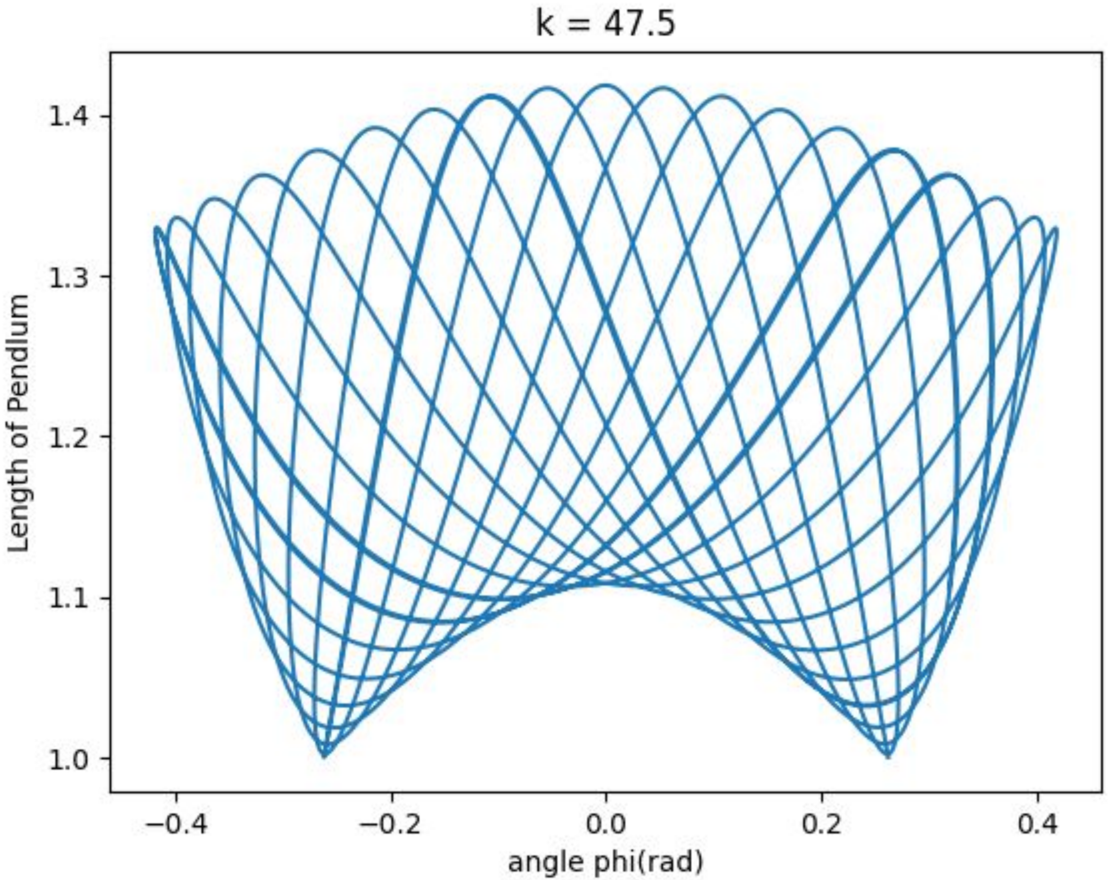

k = 46
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

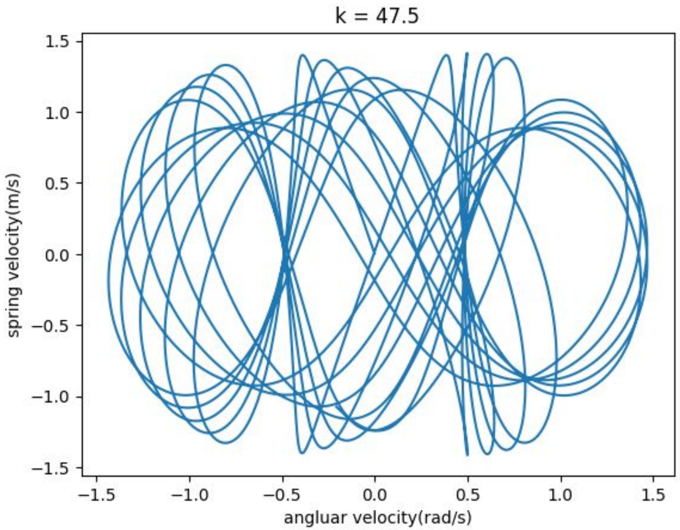

k = 46.5
Length of Pendlum
angle phi(rad)
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4


k = 46.5
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


k = 47
Length of Pendulum
angle phi(rad)
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4


k = 47
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


k = 47.5
Length of Pendlum
angle phi(rad)
1.4
1.3
1.2
1.1
1.0
−0.4
−0.2
0.0
0.2
0.4


k = 47.5
spring velocity(m/s)
angluar velocity(rad/s)
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5