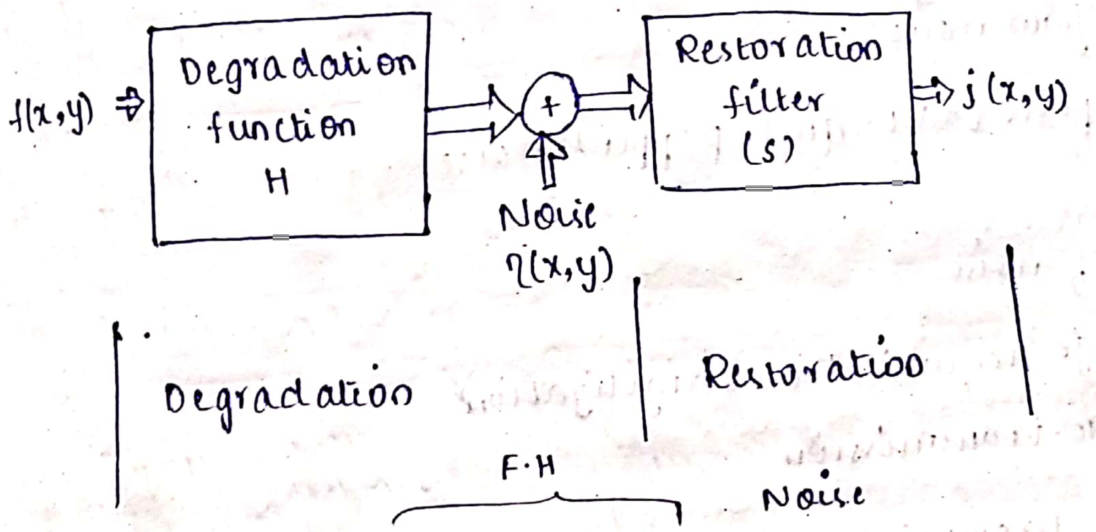
Image Restoration

* A model of the image degradation | restoration Process



degraded o/p : $g(x,y) = \underbrace{f(x,y) * h(x,y)}_{F \cdot H} + \underbrace{\eta(x,y)}_{\text{Noise}}$

$$G(u,v) = F(u,v)H(u,v) + N(u,v)$$

$f(x,y)$ = Input Image

$f(x,y) \xleftrightarrow{F.T} F(u,v)$

$g(x,y)$ = Degraded o/p

$g(x,y) \longleftrightarrow G(u,v)$

$h(x,y)$ = Impulse response of degradation filter / Network

$H(u,v)$ = Transfer function of degradation network

$h(x,y) \xleftrightarrow{F.T} H(u,v)$

$\eta(x,y)$ = Noise signal added in channel

$\eta(x,y) \xleftrightarrow{F.T} N(u,v)$

* Noise models:

→ Gaussian noise
→ Rayleigh noise
→ Gamma noise
→ Exponential noise
→ uniform noise
→ Impulse (salt-and-pepper) noise

* Source of noise

→ Image acquisition (digitization)
→ Image transmission

* Spatial properties of noise

→ Statistical behaviour of the gray-level values of pixels

→ Noise parameters, correlation with the image

* Frequency parameters

* Noise propability density functions:

→ Noises are taken as random variables.

→ Random Variables
- propability density function (PDF)

1.) Gaussian noise

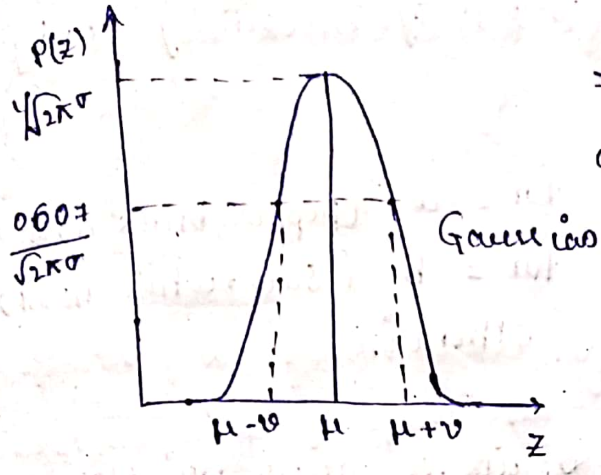
→ Electronic ckt noise & sensor noise. due to poor illumination/ high temp

→ Math. tractability in spatial & freq. domain

$$P(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

μ → mean, σ^2 → variance

Note: $\int_{-\infty}^{\infty} P(z)\,dz = 1$



70% in $[(\mu-\sigma), (\mu+\sigma)]$

95% in $[(\mu-2\sigma), (\mu+2\sigma)]$

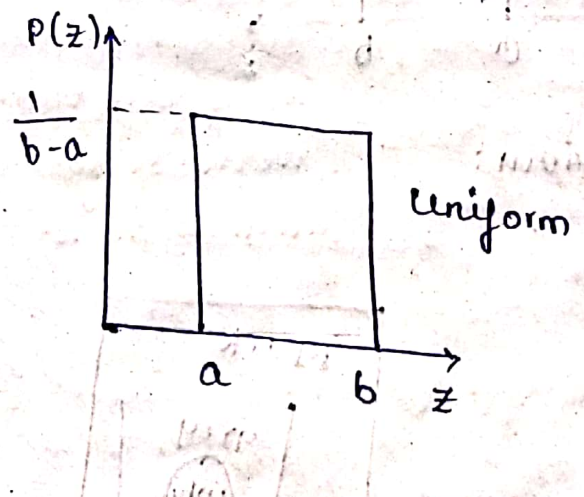
2) uniform Noise

→ in practical, used for random number generator that are used in simulation.

$$P(z) = \begin{cases} 1/b-a, & \text{if } a \le z \le b \\ 0 & \text{otherwise} \end{cases}$$

Mean: $\mu = \frac{a+b}{2}$

Variance: $\sigma^2 = \frac{(b-a)^2}{12}$



3) Impulse (salt & pepper) noise:

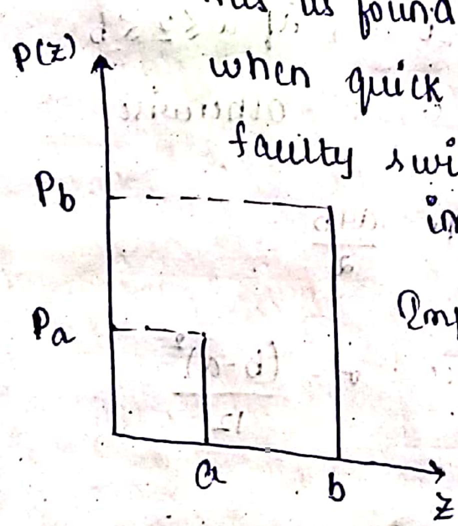
→ Quick transients, such as faulty switching during imaging

$$P(z) = \begin{cases} P_a & \text{for } z = a \quad \text{(pepper, black noise)} \\ P_b & \text{for } z = b \quad \text{(salt, white noise)} \\ 0 & \text{otherwise} \end{cases}$$

If either P_a or P_b is 0, it is called unipolar. Otherwise, it is called bipolar.

→ In practical, impulses are usually stronger than image signals. Ex.: $a = 0$ (black) and $b = 255$ (white) in 8-bit image

This is found ~~when~~ in quick situations when quick transients such as faulty switching takes place during imaging.

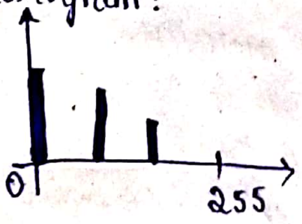


Impulse

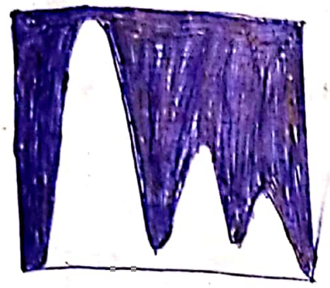
* Test for noise behaviour:

Test pattern

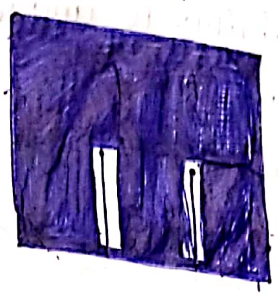
Its histogram:



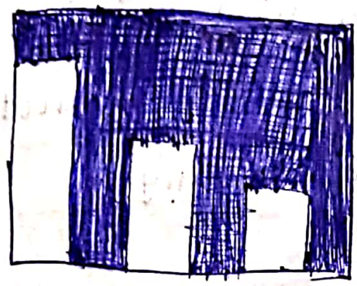
If Gaussian noise is added then:



Salt & Pepper



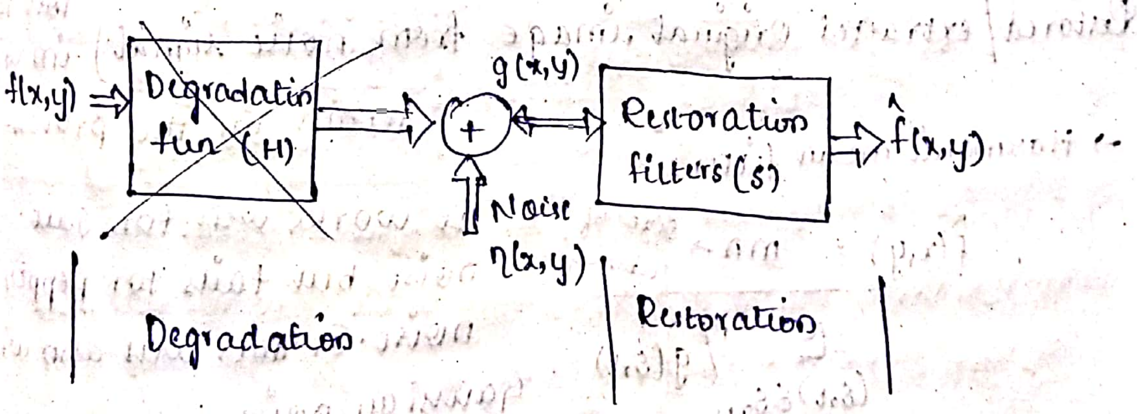
uniform



* Periodic noise

→ Arise from electrical or electromechanical interference during image acuquitisation.

* Additive noise only:



$$\begin{cases} g(x,y) = f(x,y) + \eta(x,y) \\ G(u,v) = F(u,v) + N(u,v) \end{cases}$$

* Spatial filters for de-noising additive noise:

→ Skills similar to image enhancement

→ Mean filters

→ Order-Statistics filters

→ Adaptive filters

1) Mean filters: Mean filters smoothes local variations in an image & noise is reduced as a result of blurring.

→ Arthemetic mean

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

window centered at (x,y)

→ Geometric mean:

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{1/mn}$$

This achieves smoothing comparable to the AM mean filter but tends to lose less image details in the process.

Restored/extracted original image from noise signal

→ Harmonic mean filter:

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} 1/g(s,t)}$$

mn → size of the subimage

It works well for salt noise but fails for pepper noise. It does well also with Gaussian noise.

→ Contra-harmonic mean filter:

$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

eliminates the salt noise

Q = -1 harmonic

Q = 0, arthemetic mean

Q = +, ? it elimates the pepp noise

order of the filter

It can't be done simultaneously.

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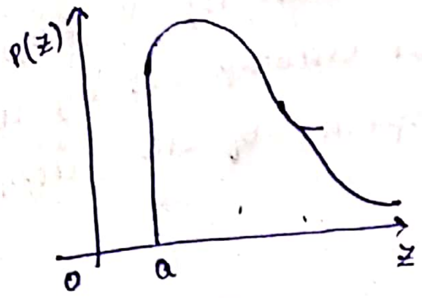
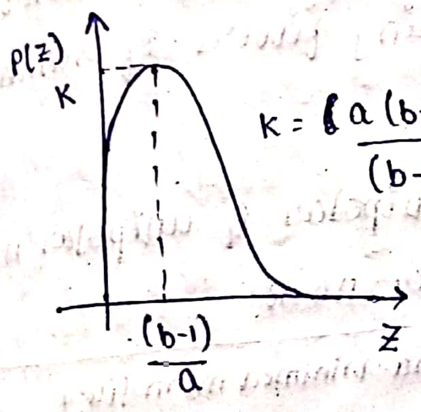
4.) Rayleigh Noise :



$$P(z) = \begin{cases} \frac{2}{b}(z-a)\, e^{-\frac{(z-a)^2}{b}} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

Rayleigh density is useful for approximation skewed histogram. This is helpful in charaterising noise phenomenon in range imagining.

5.) Erlang (gamma) Noise :



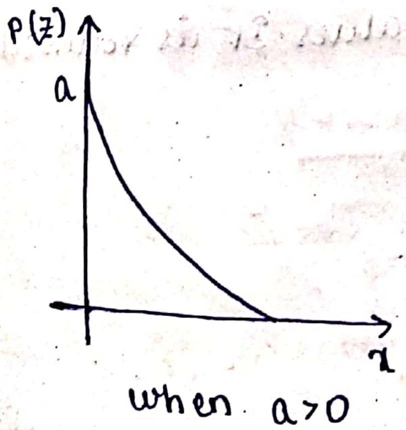
$$K = \frac{a(b-1)^{b-1}}{(b-1)!} e^{-(b-1)}$$

$$P(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$\bar{z}$ = mean = $\frac{b}{a}$

$\sigma^2 = \frac{b}{a^2}$

6.) Exponential Noise : find application in lazer imaging



$$P(z) = \begin{cases} a e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

when $a > 0$

mean = $\bar{z} = \frac{1}{a}$

Variance = $\sigma_z^2 = \frac{1}{a^2}$

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2) Order Static filters:

whose response is based on ordering (ranking) the pixels contained in the image or encompassed by the filter. Ranking determines the response of the filter.

→ median filter:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$$

For certain types of random noises they provides excellent noise reduction propabilities with considerably less bluring than linear smoothing filters of similar size.

It is effective in presence of both bipolar & unipolar noise. It is well suited for salt-&-pepper noise.

Alpha-trimmed mean filter

- Delete the d/2 lowest & d/2 highest gray level pixels. middle (mn-d) pixels

$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

→ Max filter:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\max}\{g(s,t)\}$$

It is useful for finding the brightest point in an image. Pepper noise has very low values. It is reduced by the max filter.

→ Min filter:

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\}$$

It is useful for finding the darkest point in an image. It reduces the salt noise.

→ Mid point filter:

$$\hat{f}(x,y) = \frac{1}{2}\left\{ \max_{(s,t)\in S_{xy}} \{g(s,t)\} + \min \{g(s,t)\} \right\}$$

It works wells for randomly differientied noise, guasian and uniform noise.

* Periodic noise reduction by frequency Domain Filter:

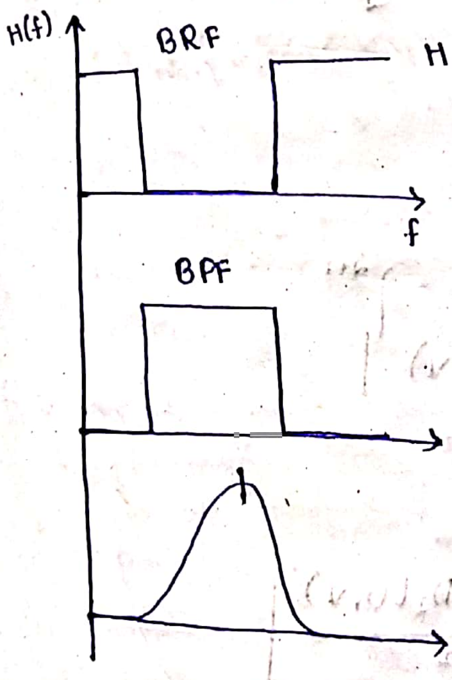
There are 3 types of Selective filters:

1.) Band Reject filter - reject an istropic freq.

2.) Band Pass filter

3.) Notch filter. - reject (or pass) frequencies in predefined neighborhood

1.) Band reject filter:



1.) Ideal Band Reject Filter:

$$H(u,v) = \begin{cases} 1 & D(u,v) < D_0 - W/2 \\ 0 & D_0 - \frac{W}{2} \leq D(u,v) \leq D_0 + W/2 \\ 1 & D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

2.) Gaussian BRF:

$$H(u,v) = 1 - e^{-\left\{\frac{D^2(u,v) - D_0^2}{D(u,v)W}\right\}}$$

3.) Butterworth BRF:

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)W}{D^2(u,v) - D_0^2}\right]^{2n}}$$

W = width of Band

D_0 = cut off freq.

$D(u,v)$ = Distance from the center of freq.

2.) Band Pass Filter:

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

3.) Notch filter:

1.) Ideal notch Reject filter:

$$H(u,v) = \begin{cases} 0 & D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1 & \text{Otherwise} \end{cases}$$

$$D_1(u,v) = \left[\left(u - \frac{M}{2} - u_0\right)^2 + \left(v - \frac{N}{2} - v_0\right)^2\right]^{1/2}$$

$$D_2(u,v) = \left[\left(u - \frac{M}{2} + u_0\right)^2 + \left(v - \frac{N}{2} + v_0\right)^2\right]^{1/2}$$

2.) Butterworth Notch reject filter:

$$H(u,v) = \frac{1}{1 + \left\{\frac{D_0^2}{D_1(u,v)\, D_2(u,v)}\right\}^{2n}}$$

3.) Gaussian Notch reject filter:

$$H(u,v) = 1 - e^{-\left[\frac{D_1(u,v)\, D_2(u,v)}{D_0^2}\right]}$$

4.) Notch pass filter:

$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

* Linear - Position Invariant System:

$h(x,y) \rightarrow$ Impulse func.

$$f(x,y),\ F(u,v) \longrightarrow \boxed{\begin{matrix} \text{LTI} \\ H(u,v) \end{matrix}} \longrightarrow \begin{matrix} g(x,y) = f(x,y) * h(x,y) \\ G(u,v) = F(u,v)\,H(u,v) \end{matrix}$$

$H(u,v) \rightarrow$ Transfer func.

$$f(x,y) \Longleftrightarrow F(u,v)$$
$$h(x,y) \Longleftrightarrow H(u,v)$$
$$g(x,y) \Longleftrightarrow G(u,v)$$

If H is linear,

$$f(x,y) \longrightarrow \boxed{H} \longrightarrow H[f(x,y)]$$

$$H[a f_1(x,y) + b f_2(x,y)] = a[H(f_1(x,y))] + bH[f_2(x,y)]$$

$a = b = 1$

$$H[f_1(x,y) + f_2(x,y)] = H[f_1(x,y)] + H[f_2(x,y)]$$

Additive property | Superposition

$f_2(x,y) = 0$, $a \neq 1$

$$H[a f_1(x,y)] = aH[f_1(x,y)] \rightarrow \text{Homogenity property}$$

linear $\rightarrow$ Superposition + Homogenity

Response of sum of different signals is equal to sum of their individual respone is linearity.

Position Invarient Property:

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

Impulse:

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\,\delta(x-\alpha, y-\beta)\,d\alpha\,d\beta.$$

Spark light

$$g(x,y) = H[f(x,y)]$$

$$g(x,y) = H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\,\delta(x-\alpha, y-\beta)\,d\alpha\,d\beta\right]$$

$$g(x,y) = a f_1 + f_2 \longrightarrow \boxed{H} \longrightarrow aH[f_1] + H[f_2]$$

linear property

$$g(x,y) = \iint f(\alpha,\beta)\,H\left[\delta(x-\alpha, y-\beta)\,d\alpha\,d\beta\right]$$

Sum of individual responses

$$H[\delta(x-\alpha, y-\beta)] = h(x-\alpha, y-\beta) = h(x,y,\alpha,\beta)$$

↓ pre-defined

$$g(x,y) = \iint f(\alpha,\beta)\,h(x-\alpha, y-\beta)\,d\alpha\,d\beta$$

= convolution of $f(x,y)$ & $h(x,y)$

$$g(x,y) = f(x,y) * h(x,y)$$

* Estimating the degradation func.
  - → Estimation by Image observation
  - → Estimation by experimentation
  - → Estimation by modeling

1.) Estimation by Image Observation:

Take a window in the image

- Simple structure

Strong signal content

Estimate the original image in the window

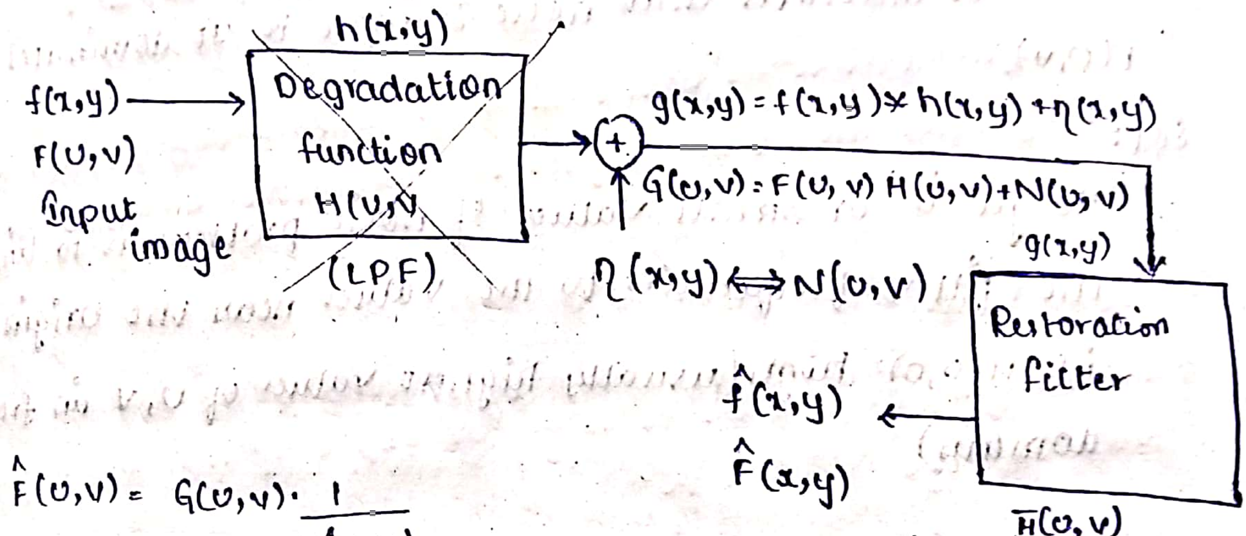
$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)} \quad \begin{array}{l} \leftarrow \text{Known} \\ \leftarrow \text{estimate} \end{array}$$

2.) Estimation by experimentation:

→ If the image accquitisation system is ready.

→ Obtain the impulse response

* Inverse Filtering:



$$\hat{F}(U,V) = G(U,V) \cdot \frac{1}{H(U,V)}$$

$$= \frac{F(U,V)H(U,V) + N(U,V)}{H(U,V)}$$

$f(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$

$= f(t) * h(t)$

$$\boxed{\hat{F}(U,V) = F(U,V) + \frac{N(U,V)}{H(U,V)}}$$

At Origin LPF will have higher Values

limitations:

1.) $H(U,V) \rightarrow$ unknown

$N(U,V) \rightarrow$ unknown

$H(U,V) = \infty$

$$\frac{N(U,V)}{H(U,V)} = 0 \Rightarrow \hat{F} = F = \hat{F}(U,V) = F(U,V)$$

→ Even if we know the degradation func we cannot recover the undergraded image [inverse F.T of F(u,v)]

→ If the degradation func has 0 or very small values i.e $H(u,v)=0$

$$\Rightarrow \frac{N(u,v)}{H(u,v)} = \infty$$

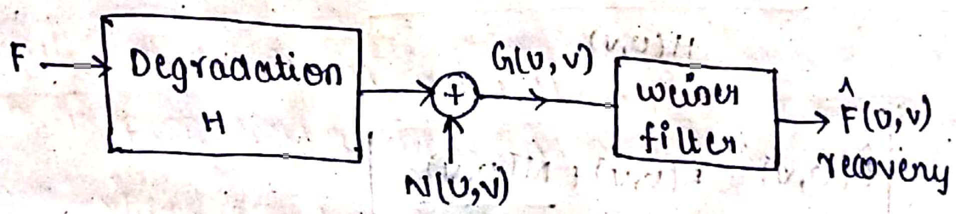
$$\hat{F}(u,v) = F(u,v) + \infty$$

It indicates that noise is more i.e It dominates F(u,v)

Sol:

To get '0' or small values of noise problem is to limit the filter frequencies to the values near the origin i.e $H(0,0)$ = high (usually higher values of u, v in freq. domain)

* Weiner Filter (or) min mean square Filter (MMSF)
least " " " (LMSF)



$$Error = \hat{F}(u,v) - F(u,v)$$

error should be zero $(\hat{f} = f)$
↓
min

$e = \hat{f} - f$
$= 0.2$
$e^2 = 0.04$ → more

Statistical avg or mean value of error

$$\bar{e} = E[e] = E[f - \hat{f}]$$

$\bar{e}^2 = E[e^2] = E[(f-\hat{f})^2]$

$\downarrow$

statistical mean square error

$E(x) = \int x\, f(x)\, dx$

$\downarrow$

mean values

$E[x^2] = \int x^2 f(x)\, dx$

$\downarrow$

mean square

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)\, S_f(u,v)}{S_f(u,v)\,|H(u,v)|^2 + S_\eta(u,v)}\right] G(u,v)$$

$$= |H(u,v)|^2 = H^*(u,v)\, H(u,v)$$

$S_f(u,v)$ = power spectral density of $F(u,v)$

$S_\eta(u,v)$ = -u- - - - -u u u $N(u,v)$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right] G(u,v)$$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right] G(u,v)$$

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UNIT-IV

* Segmentation:

- Segmentation attempts to partition the pixels of an image into groups that strongly correlate with the objects in an image.

- The purpose of image segmentation is to partition an image into meaningful regions with respect to a particular application.

- The segmentation is based on measurements taken from the image and might be grey level, colour, texture, depth or motion.

* Applications

- Identifying objects in a scene for object-based measurements such as size and shape.

- Identifying objects in a moving scene for object-based viedo compression.

* Segmentation algorithms generally are based on one of two basic properties of intensity values. Discontinuity to partition an image

* Detection of Discontinous:

There are 3 basic types of grey level discontinuites that we tend to look for in digital images:

- points
- Lines
- Edges

we typically find discontinuites using masks and correlation.

1) Point detection:

- Note that the mask is the same as the mask of Laplacian operation.
- The only differences that are considered of interest are those large enough (as determined) to be considered isolated points

$$\boxed{|R| > T}$$

It can be achieved simply using the mask below:

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Points are detected at those pixels in the subsequent filtered image that are above a set threshold.

i/p

$$\begin{bmatrix} z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 \\ z_7 & z_8 & z_9 \end{bmatrix}$$

mask

$$\begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix}$$

$$R = w_1 z_1 + w_2 z_2 + \cdots + w_9 z_9$$

$$R = \sum_{i=1}^{N} w_i z_i$$

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2) Line detection:

- The nxt level of complexity is to try to detect lines.
- The masks below will extract lines that are one pixel thick and running in a particular direction.

$$\begin{bmatrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & -1 \end{bmatrix} \quad \begin{bmatrix} -1 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & 2 & -1 \end{bmatrix} \quad \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Horizontal R_1 | $+45°$ R_2 | vertical R_3 | $-45°$ R_4

- Apply every masks on the image.
- Let R_1, R_2, R_3, R_4 denotes the response of the horizontal.. respectively.
- If, at a certain point in the image

$|R_i| > |R_j|$, for all $j \neq i$, that point is said to be more likely associated with a line in the directi: of mask i.

3) Edge Detection:

- Segmentation by finding pixels on a region boundary.
- Derivatives are used to find discontinuities.

  1st derivative tells us where an edge is

  2nd derivative can be used to show edge direction.

Given a 3*3 region of an image the following edge detection filters can be used.

$$\begin{bmatrix} z_1 & z_2 & z_3 \\ z_4 & z_5 & z_6 \\ z_7 & z_8 & z_9 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Prewitt

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Roberts

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

Sobel

Often, problems arise in edge detection.

* Laplacian edge detection:

We encountered the 2nd order derivative based laplacian filter already.

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

- The laplacian is typically not used by itself as it is to sensitive to noise.

Arithmetic

Huffman Coding Steps:

	x_1	x_2	x_3	
x_i	2	6	7	
$P(x)$	$6/25 = 0.24$	$14/25 = 0.56$	$5/25 = 0.2$	

Steps:

Descending Order:

Message	Prob
$x_3 - 7$	0.56 0
$x_2 - 6$	0.24 10
$x_1 - 2$	0.2 11

$0.56 = 0 \quad P(b)$

$0.44 = 1 \quad P(a)$

$P(c)$

$L_{avrg} = (0.56 \times 1) + (0.24 \times 2) + (0.2 \times 2)$

$= 1.44$ bits $\qquad L = \Sigma P(r_k) l_k$

Compression Ratio $C = \frac{b}{b'} = 8$ no. of bits $= \frac{3}{1.4} = 2.14$

$L = 8^{1/3} = 2$

$R = 1 - \frac{1}{C} = 1 - \frac{1}{2.14}$

0.06×0.6

036
000x
0036