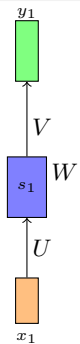# CS7015 (Deep Learning) : Lecture 14

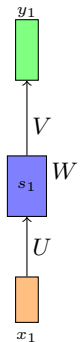## Long Short Term Memory Cells (LSTMs), Gated Recurrent Units (GRUs)

Mitesh M. Khapra

Department of Computer Science and Engineering
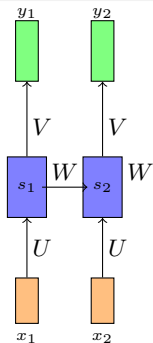Indian Institute of Technology Madras

Module : Selective Read, Selective Write, Selective Forget - The Whiteboard Analogy
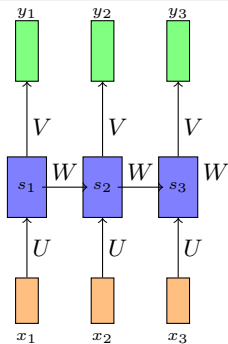
- The state (s_i) of an RNN records information from all previous time steps

- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input

- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input

- The state (s_i) of an RNN records information from all previous time steps

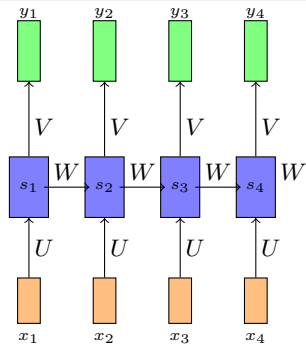- At each new timestep the old information gets morphed by the current input

- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input

- The state (s_i) of an RNN records information from all previous time steps
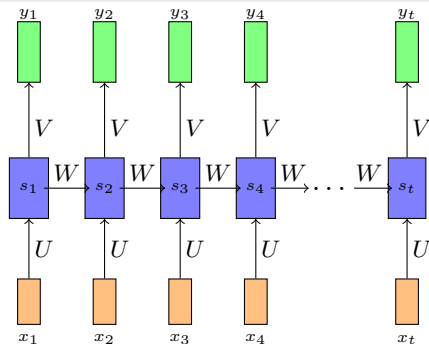- At each new timestep the old information gets morphed by the current input

- The state (s_i) of an RNN records information from all previous time steps
- At each new timestep the old information gets morphed by the current input
- One could imagine that after t steps the information stored at time step $t-k$ (for some $k < t$) gets completely morphed
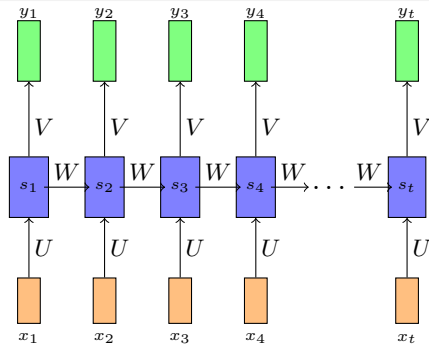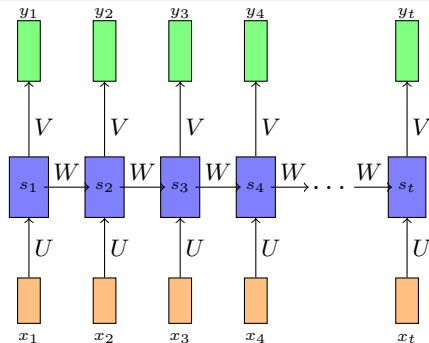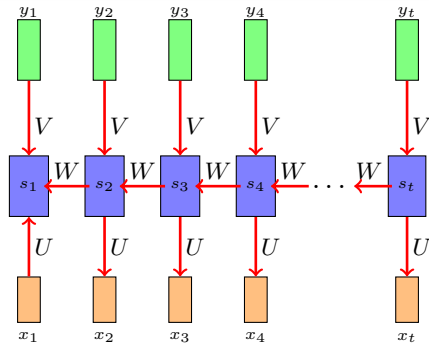
- The state (s_i) of an RNN records information from all previous time steps

- At each new timestep the old information gets morphed by the current input

- One could imagine that after t steps the information stored at time step $t - k$ (for some $k < t$) gets completely morphed

  so much that it would be impossible to extract the original information stored at time step $t - k$

- A similar problem occurs when the information flows backwards (backpropagation)

- A similar problem occurs when the information flows backwards (backpropagation)
- It is very hard to assign the responsibility of the error caused at time step t to the events that occurred at time step $t - k$
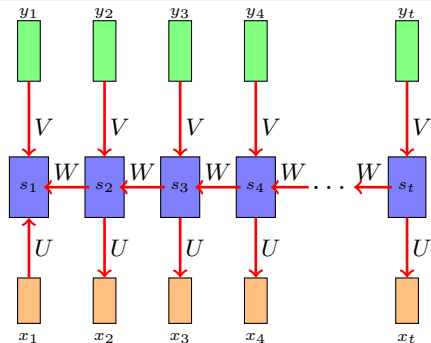
- A similar problem occurs when the information flows backwards (backpropagation)
- It is very hard to assign the responsibility of the error caused at time step t to the events that occurred at time step $t - k$
- This responsibility is of course in the form of gradients and we studied the problem in backward flow of gradients
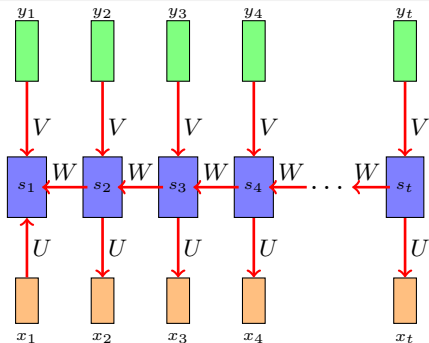
- A similar problem occurs when the information flows backwards (backpropagation)
- It is very hard to assign the responsibility of the error caused at time step t to the events that occurred at time step $t - k$
- This responsibility is of course in the form of gradients and we studied the problem in backward flow of gradients
- We saw a formal argument for this while discussing vanishing gradients

- Let us see an analogy for this

- Let us see an analogy for this
- We can think of the state as a fixed size memory

- Let us see an analogy for this
- We can think of the state as a fixed size memory
- Compare this to a fixed size white board that you use to record information

- Let us see an analogy for this
- We can think of the state as a fixed size memory
- Compare this to a fixed size white board that you use to record information
- At each time step (periodic intervals) we keep writing something to the board

- Let us see an analogy for this
- We can think of the state as a fixed size memory
- Compare this to a fixed size white board that you use to record information
- At each time step (periodic intervals) we keep writing something to the board
- Effectively at each time step we morph the information recorded till that time point

- Let us see an analogy for this
- We can think of the state as a fixed size memory
- Compare this to a fixed size white board that you use to record information
- At each time step (periodic intervals) we keep writing something to the board
- Effectively at each time step we morph the information recorded till that time point
- After many timesteps it would be impossible to see how the information at time step $t - k$ contributed to the state at timestep t

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step

Mitesh M. Khapra   CS7015 (Deep Learning) : Lecture 14

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step
- Selectively write on the board

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step
- Selectively write on the board
- Selectively read the already written content

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step
- Selectively write on the board
- Selectively read the already written content
- Selectively forget (erase) some content

- Continuing our whiteboard analogy, suppose we are interested in deriving an expression on the whiteboard
- We follow the following strategy at each time step
- Selectively write on the board
- Selectively read the already written content
- Selectively forget (erase) some content
- Let us look at each of these in detail

Mitesh M. Khapra   CS7015 (Deep Learning) : Lecture 14

Selective write

- There may be many steps in the derivation but we may just skip a few

Selective write

- There may be many steps in the derivation but we may just skip a few
- In other words we select what to **write**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

Selective write

- There may be many steps in the derivation but we may just skip a few

- In other words we select what to **write**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$

Selective write

- There may be many steps in the derivation but we may just skip a few
- In other words we select what to **write**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$

Selective write

- There may be many steps in the derivation but we may just skip a few

- In other words we select what to **write**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$

Selective read

- While writing one step we typically read some of the previous steps we have already written and then decide what to write next

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$

Selective read

- While writing one step we typically read some of the previous steps we have already written and then decide what to write next

- For example at Step 3, information from Step 2 is important

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$

Selective read

- While writing one step we typically read some of the previous steps we have already written and then decide what to write next
- For example at Step 3, information from Step 2 is important
- In other words we select what to **read**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$
$$bd + a = 34$$

Selective read

- While writing one step we typically read some of the previous steps we have already written and then decide what to write next
- For example at Step 3, information from Step 2 is important
- In other words we select what to **read**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$bd = 33$$
$$bd + a = 34$$

Selective forget

- Once the board is full, we need to delete some obsolete information

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information

$$ac = 5$$
$$bd = 33$$
$$bd + a = 34$$

Mitesh M. Khapra   CS7015 (Deep Learning) : Lecture 14

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

Selective forget

- Once the board is full, we need to delete some obsolete information

- But how do we decide what to delete? We will typically delete the least useful information

- In other words we select what to **forget**

$$ac = 5$$
$$bd = 33$$
$$bd + a = 34$$

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$

$$bd + a = 34$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$ac(bd + a) = 170$$
$$bd + a = 34$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ac = 5$$
$$ac(bd + a) = 170$$
$$ad = 11$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ad + ac(bd + a) = 181$$
$$ac(bd + a) = 170$$
$$ad = 11$$

Selective forget

- Once the board is full, we need to delete some obsolete information
- But how do we decide what to delete? We will typically delete the least useful information
- In other words we select what to **forget**

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ad + ac(bd + a) = 181$$
$$ac(bd + a) = 170$$
$$ad = 11$$

- There are various other scenarios where we can motivate the need for selective write, read and forget

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

① ac

② bd

③ $bd + a$

④ $ac(bd + a)$

⑤ ad

⑥ $ac(bd + a) + ad$

$$ad + ac(bd + a) = 181$$
$$ac(bd + a) = 170$$
$$ad = 11$$

- There are various other scenarios where we can motivate the need for selective write, read and forget

- For example, you could think of our brain as something which can store only a finite number of facts

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ad + ac(bd + a) = 181$$
$$ac(bd + a) = 170$$
$$ad = 11$$

- There are various other scenarios where we can motivate the need for selective write, read and forget
- For example, you could think of our brain as something which can store only a finite number of facts
- At different time steps we selectively read, write and forget some of these facts

$$a = 1 \quad b = 3 \quad c = 5 \quad d = 11$$

Compute $ac(bd + a) + ad$

Say "board" can have only 3 statements at a time.

1. ac
2. bd
3. $bd + a$
4. $ac(bd + a)$
5. ad
6. $ac(bd + a) + ad$

$$ad + ac(bd + a) = 181$$
$$ac(bd + a) = 170$$
$$ad = 11$$

- There are various other scenarios where we can motivate the need for selective write, read and forget

- For example, you could think of our brain as something which can store only a finite number of facts

- At different time steps we selectively read, write and forget some of these facts

- Since the RNN also has a finite state size, we need to figure out a way to allow it to selectively read, write and forget

Module : Long Short Term Memory(LSTM) and Gated Recurrent Units(GRUs)

Questions

- Can we give a concrete example where RNNs also need to selectively read, write and forget ?

Questions

- Can we give a concrete example where RNNs also need to selectively read, write and forget ?
- How do we convert this intuition into mathematical equations ?
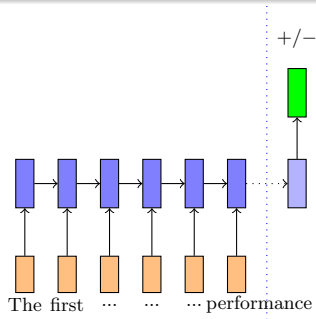
Questions

- Can we give a concrete example where RNNs also need to selectively read, write and forget ?

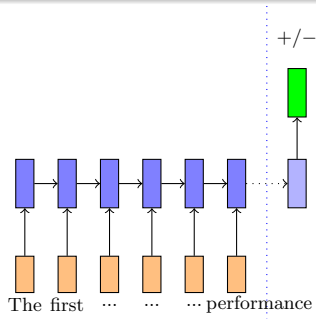- How do we convert this intuition into mathematical equations ? We will see this over the next few slides

+/−

The first ⋯ ⋯ ⋯ performance

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state

+/−

The first ··· ··· ···performance

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance
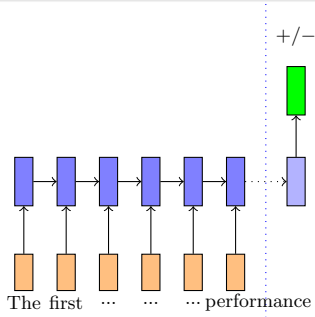
- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost

+/−

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost
- Ideally we want to

+/−

The first ⋯ ⋯ ⋯ performance

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost
- Ideally we want to
  - **forget** the information added by stop words (a, the, etc.)

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost
- Ideally we want to
  - **forget** the information added by stop words (a, the, etc.)
  - **selectively read** the information added by previous sentiment bearing words (awesome, amazing, etc.)

+/−

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Consider the task of predicting the sentiment (positive/negative) of a review
- RNN reads the document from left to right and after every word updates the state
- By the time we reach the end of the document the information obtained from the first few words is completely lost
- Ideally we want to
  - **forget** the information added by stop words (a, the, etc.)
  - **selectively read** the information added by previous sentiment bearing words (awesome, amazing, etc.)
  - **selectively write** new information from the current word to the state

Questions

- Can we give a concrete example where RNNs also need to selectively read, write and forget ?
- How do we convert this intuition into mathematical equations ?

+/−

The first $\cdots$ $\cdots$ $\cdots$ performance

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance
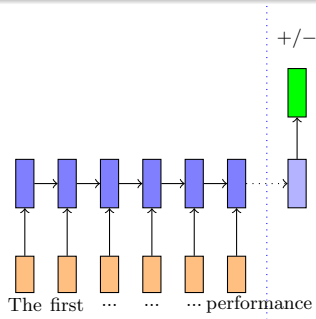
- Recall that the blue colored vector (s_t) is called the state of the RNN

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Recall that the blue colored vector (s_t) is called the state of the RNN
- It has a finite size $(s_t \in \mathbb{R}^n)$ and is used to store all the information upto timestep t

+/−

The first ··· ··· ···performance

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Recall that the blue colored vector (s_t) is called the state of the RNN
- It has a finite size $(s_t \in \mathbb{R}^n)$ and is used to store all the information upto timestep t
- This state is analogous to the whiteboard and sooner or later it will get overloaded and the information from the initial states will get morphed beyond recognition

Review: The first half of the movie was dry but the second half really picked up pace. The lead actor delivered an amazing performance

- Recall that the blue colored vector (s_t) is called the state of the RNN
- It has a finite size ($s_t \in \mathbb{R}^n$) and is used to store all the information upto timestep t
- This state is analogous to the whiteboard and sooner or later it will get overloaded and the information from the initial states will get morphed beyond recognition
- **Wishlist:** selective write, selective read and selective forget to ensure that this finite sized state vector is used effectively
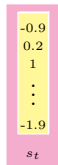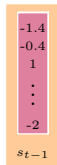
-1.4
-0.4
1
⋮
-2

s_{t-1}

0.7
-0.2
1.1
⋮
-0.3

x_t

-0.9
0.2
1
⋮
-1.9

s_t

- Just to be clear, we have computed a state s_{t-1} at timestep $t-1$ and now we want to overload it with new information (x_t) and compute a new state (s_t)
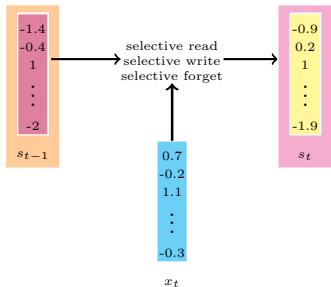
- Just to be clear, we have computed a state s_{t-1} at timestep $t-1$ and now we want to overload it with new information (x_t) and compute a new state (s_t)

- While doing so we want to make sure that we use selective write, selective read and selective forget so that only important information is retained in s_t
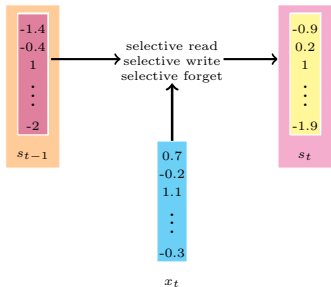
- Just to be clear, we have computed a state s_{t-1} at timestep $t-1$ and now we want to overload it with new information (x_t) and compute a new state (s_t)

- While doing so we want to make sure that we use selective write, selective read and selective forget so that only important information is retained in s_t

- We will now see how to implement these items from our wishlist
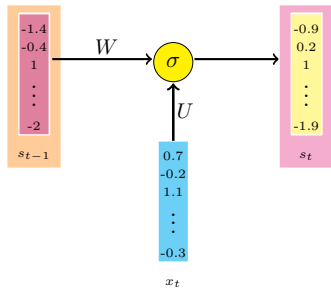
Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t
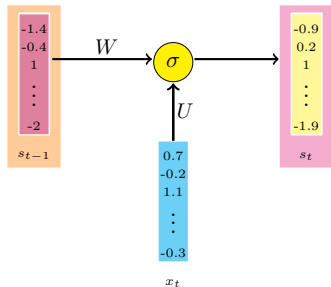
Selective Write
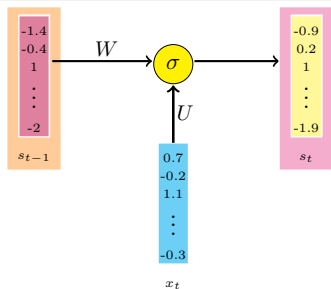
- Recall that in RNNs we use s_{t-1} to compute s_t

  $s_t = \sigma(Ws_{t-1} + Ux_t)$ *(ignoring bias)*

Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t
  $s_t = \sigma(Ws_{t-1} + Ux_t)$ *(ignoring bias)*
- But now instead of passing s_{t-1} as it is to s_t we want to pass (write) only some portions of it to the next state
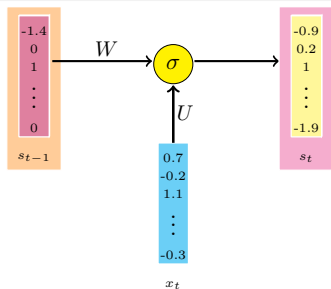
Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t
  $s_t = \sigma(W s_{t-1} + U x_t)$ *(ignoring bias)*
- But now instead of passing s_{t-1} as it is to s_t we want to pass (write) only some portions of it to the next state
- In the strictest case our decisions could be binary (for example, retain 1st and 3rd entries and delete the rest of the entries)
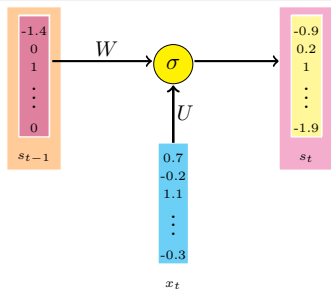
Selective Write

- Recall that in RNNs we use s_{t-1} to compute s_t
  $s_t = \sigma(W s_{t-1} + U x_t)$ *(ignoring bias)*
- But now instead of passing s_{t-1} as it is to s_t we want to pass (write) only some portions of it to the next state
- In the strictest case our decisions could be binary (for example, retain 1st and 3rd entries and delete the rest of the entries)
- But a more sensible way of doing this would be to assign a value between 0 and 1 which determines what fraction of the current state to pass on to the next state

Selective Write

- We introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state

selective write

x_t

Selective Write

- We introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state

- Each element of o_{t-1} gets multiplied with the corresponding element of s_{t-1}

Selective Write

- We introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state

- Each element of o_{t-1} gets multiplied with the corresponding element of s_{t-1}

- Each element of o_{t-1} is restricted to be between 0 and 1

Selective Write

- We introduce a vector o_{t-1} which decides what fraction of each element of s_{t-1} should be passed to the next state

- Each element of o_{t-1} gets multiplied with the corresponding element of s_{t-1}

- Each element of o_{t-1} is restricted to be between 0 and 1

- But how do we compute o_{t-1}? How does the RNN know what fraction of the state to pass on?

Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)

Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)
- We compute o_{t-1} and h_{t-1} as
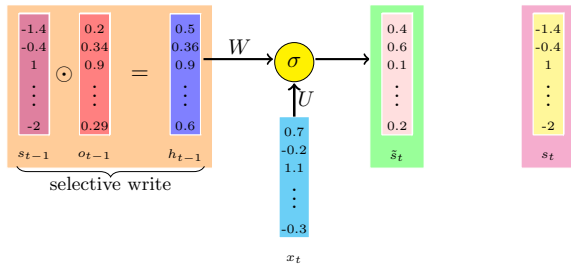
selective write

Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)

- We compute o_{t-1} and h_{t-1} as

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$

Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)

- We compute o_{t-1} and h_{t-1} as

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$
$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

selective write

Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)

- We compute o_{t-1} and h_{t-1} as

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$
$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

- The parameters W_o, U_o, b_o need to be learned along with the existing parameters W, U, V

Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)

- We compute o_{t-1} and h_{t-1} as

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$
$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

- The parameters W_o, U_o, b_o need to be learned along with the existing parameters W, U, V

- The sigmoid (logistic) function ensures that the values are between 0 and 1

selective write

x_t

Selective Write

- Well the RNN has to learn o_{t-1} along with the other parameters (W, U, V)

- We compute o_{t-1} and h_{t-1} as

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$
$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

- The parameters W_o, U_o, b_o need to be learned along with the existing parameters W, U, V

- The sigmoid (logistic) function ensures that the values are between 0 and 1

- o_t is called the output gate as it decides how much to pass (write) to the next time step

Selective Read

- We will now use h_{t-1} to compute the new state at the next time step
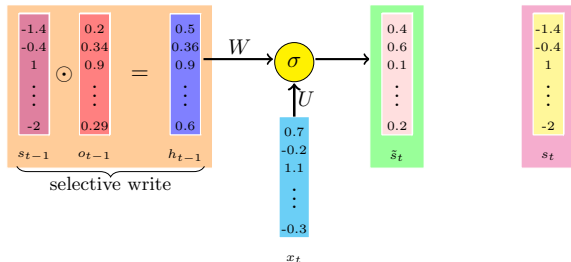
selective write

x_t

Selective Read

- We will now use h_{t-1} to compute the new state at the next time step
- We will also use x_t which is the new input at time step t
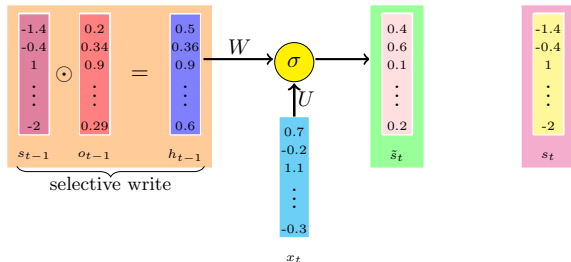
$$\tilde{s}_t = \sigma(Wh_{t-1} + Ux_t + b)$$

Selective Read

- We will now use h_{t-1} to compute the new state at the next time step

- We will also use x_t which is the new input at time step t

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$

- Note that W, U and b are similar to the parameters that we used in RNN (for simplicity we have not shown the bias b in the figure)
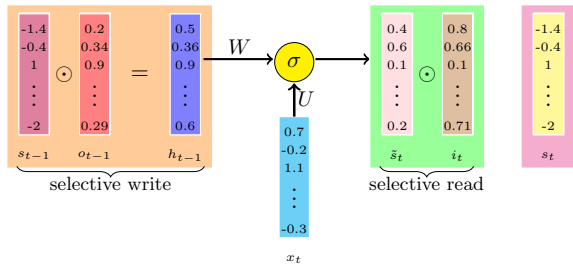
Selective Read

- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t

selective write

x_t
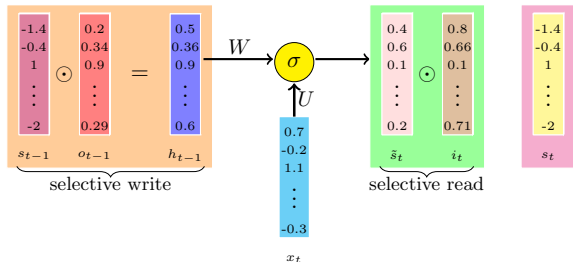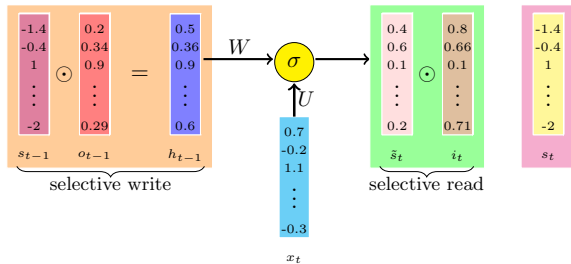
selective read

Selective Read

- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t

- However, we may not want to use all this new information and only selectively **read** from it before constructing the new cell state s_t

selective write

selective read

x_t

Selective Read

- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t

- However, we may not want to use all this new information and only selectively **read** from it before constructing the new cell state s_t

- To do this we introduce another gate called the input gate

selective write

selective read

x_t

Selective Read

- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t

- However, we may not want to use all this new information and only selectively **read** from it before constructing the new cell state s_t

- To do this we introduce another gate called the input gate

$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

selective write

selective read

x_t
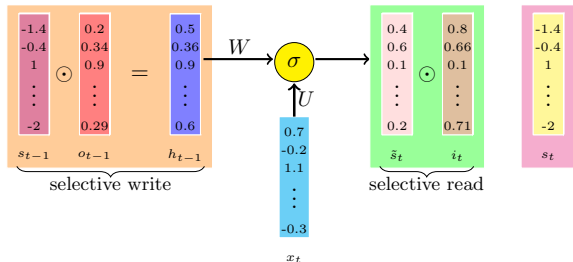
Selective Read

- $\tilde{s}_t$ thus captures all the information from the previous state (h_{t-1}) and the current input x_t

- However, we may not want to use all this new information and only selectively **read** from it before constructing the new cell state s_t

- To do this we introduce another gate called the input gate

$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

- and use $i_t \odot \tilde{s}_t$ as the selectively read state information
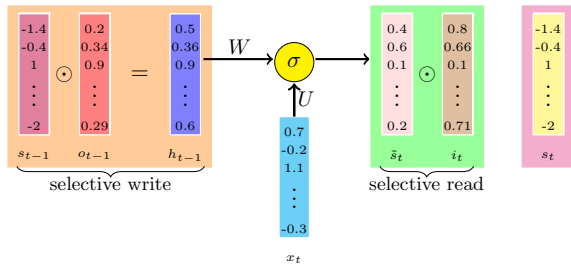
- So far we have the following

- So far we have the following

  Previous state:

  s_{t-1}

- So far we have the following
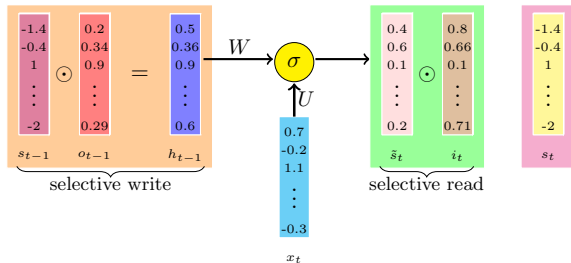
  Previous state:

  s_{t-1}

  Output gate:

  $o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$

- So far we have the following
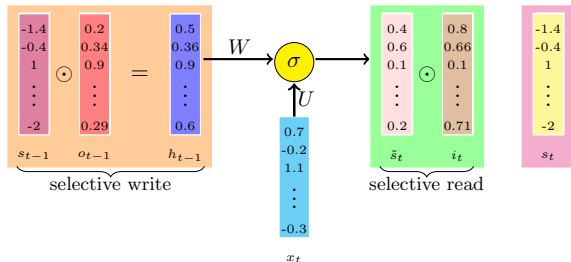
  Previous state:

  s_{t-1}

  Output gate:

  $o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$
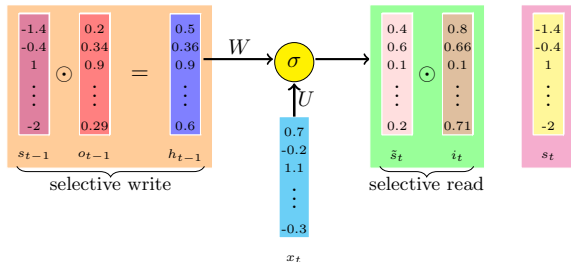
  Selectively Write:

  $h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$

- So far we have the following

  Previous state:

  s_{t-1}
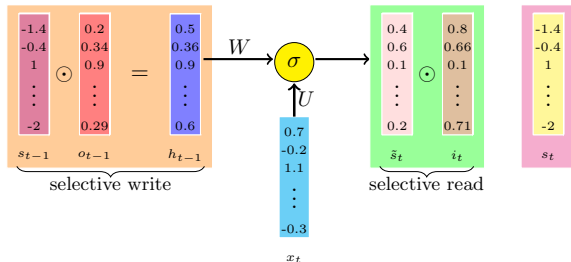
  Output gate:

  $o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$

  Selectively Write:

  $h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$

  Current (temporary) state:

  $\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$

- So far we have the following

  Previous state:

  s_{t-1}

  Output gate:

  $o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$

  Selectively Write:

  $h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$

  Current (temporary) state:

  $\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$

  Input gate:

  $i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$

- So far we have the following

  Previous state:

  s_{t-1}

  Output gate:

  $o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$

  Selectively Write:

  $h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$

  Current (temporary) state:

  $\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$

  Input gate:

  $i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$

  Selectively Read:

  $i_t \odot \tilde{s}_t$

Selective Forget
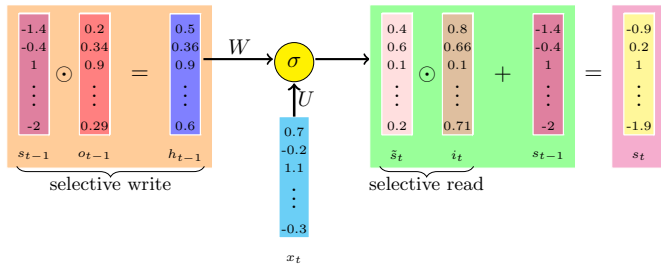
- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state

Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state
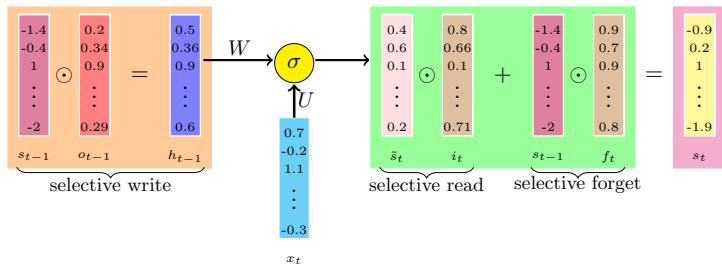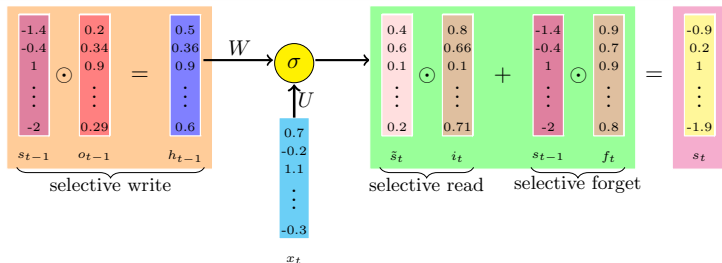- Here is one simple (but effective) way of doing this:
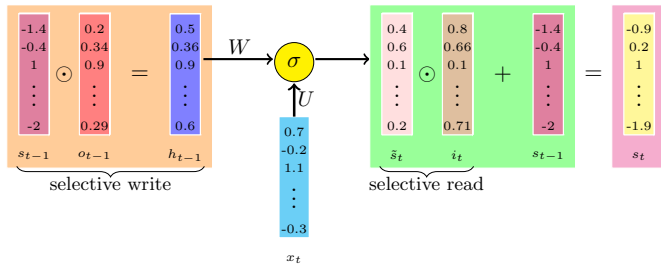
Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state

- Here is one simple (but effective) way of doing this:
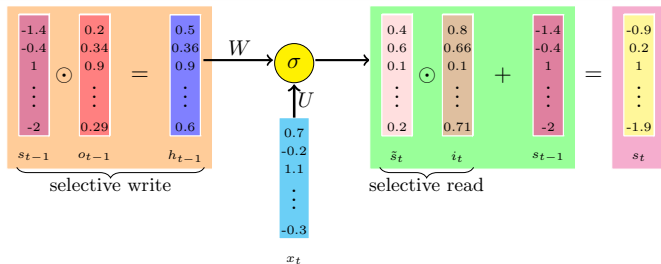
$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$
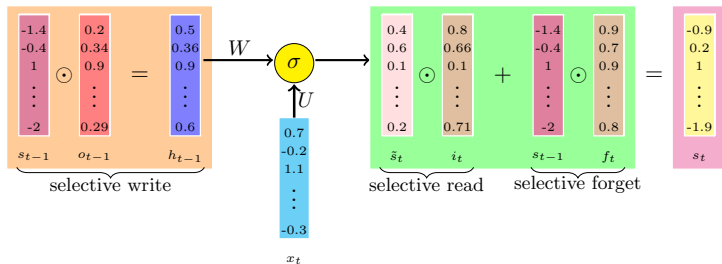
Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state

- Here is one simple (but effective) way of doing this:

$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it

Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state

- Here is one simple (but effective) way of doing this:

$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it

- To do this we introduce the forget gate

Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state

- Here is one simple (but effective) way of doing this:
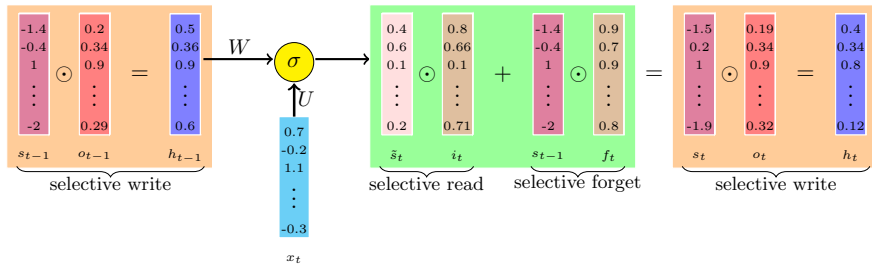
$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it

- To do this we introduce the forget gate

$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

Selective Forget

- How do we combine s_{t-1} and $\tilde{s}_t$ to get the new state
- Here is one simple (but effective) way of doing this:

$$s_t = s_{t-1} + i_t \odot \tilde{s}_t$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it
- To do this we introduce the forget gate

$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$
$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$
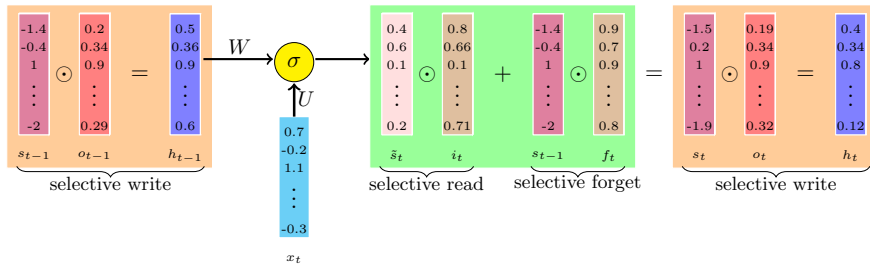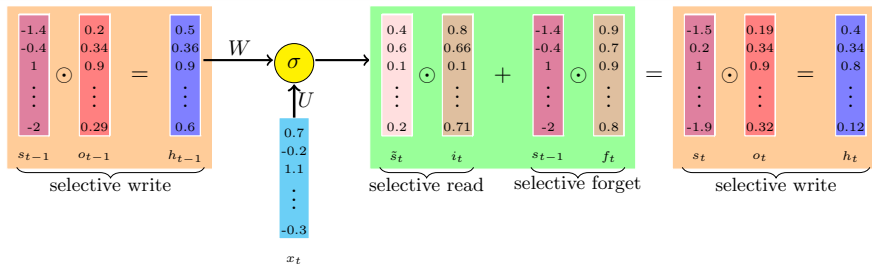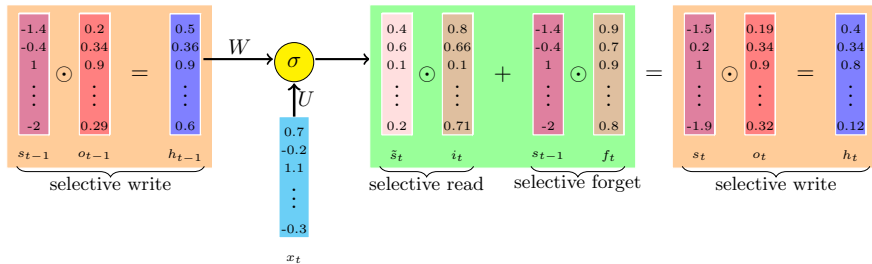
- We now have the full set of equations for LSTMs

- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

States:

- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t
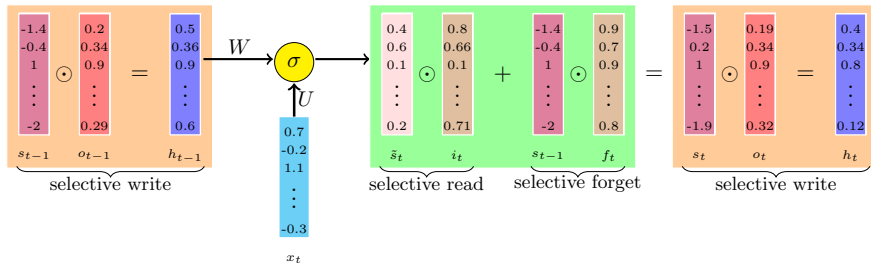
Gates: **States:**

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$

- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates: **States:**

$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$

$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$

- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t
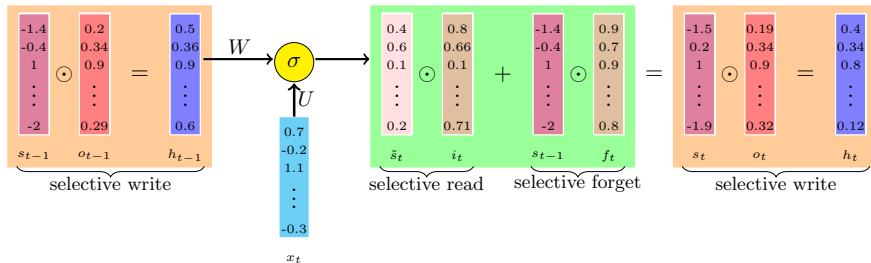
Gates: **States:**

$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$

$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$

$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$

- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

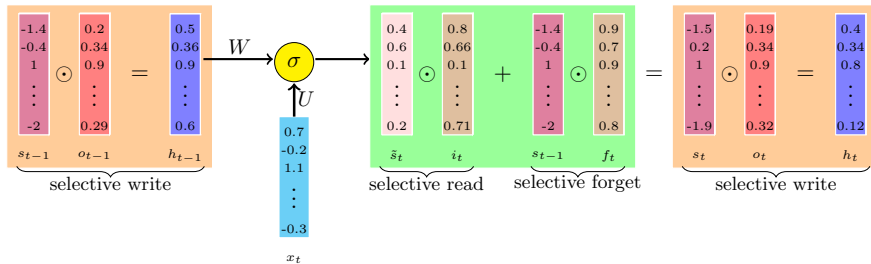**Gates:**

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$

- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
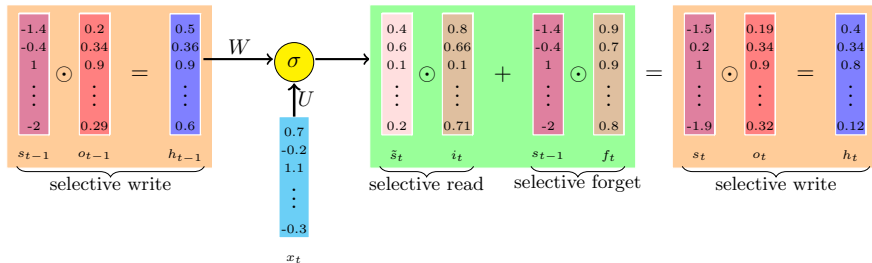$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$
$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$

- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$
$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$
$$h_t = o_t \odot \sigma(s_t)$$

- We now have the full set of equations for LSTMs
- The green box together with the selective write operations following it, show all the computations which happen at timestep t

Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$
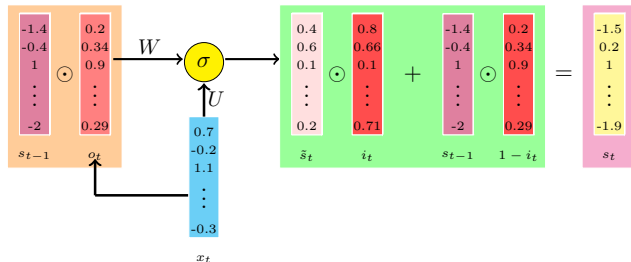$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$
$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$
$$h_t = o_t \odot \sigma(s_t) \text{ and } rnn_{out} = h_t$$

- LSTM has many variants which include different number of gates and also different arrangement of gates
- The one which we just saw is one of the most popular variants of LSTM
- Another equally popular variant of LSTM is Gated Recurrent Unit which we will see next

$$\begin{bmatrix} -1.4 \\ -0.4 \\ 1 \\ \vdots \\ -2 \end{bmatrix}$$

s_{t-1}

The full set of equations for GRUs

$$\begin{bmatrix} -1.4 \\ -0.4 \\ 1 \\ \vdots \\ -2 \end{bmatrix}$$

s_{t-1}

The full set of equations for GRUs
Gates:
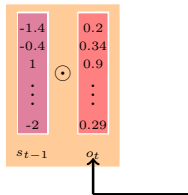
States:

The full set of equations for GRUs
Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

States:

The full set of equations for GRUs

Gates:
$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

States:
$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$
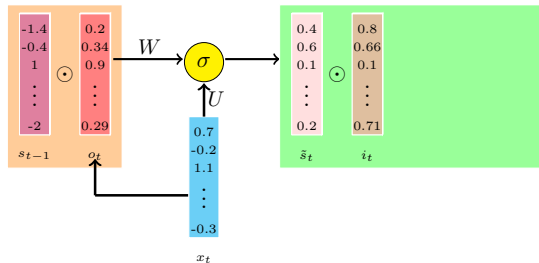
The full set of equations for GRUs

Gates:

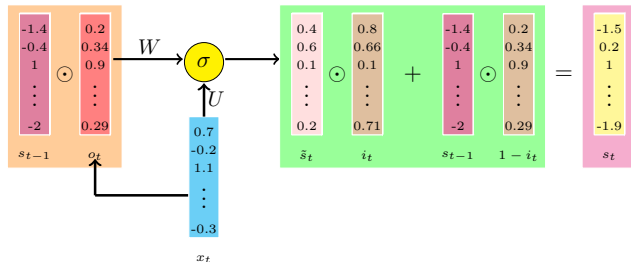$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$

The full set of equations for GRUs

Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$
$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s}_t$$

The full set of equations for GRUs

Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

- No explicit forget gate (the forget gate and input gates are tied)

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s}_t$$

The full set of equations for GRUs
Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$$
$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s}_t$$

- No explicit forget gate (the forget gate and input gates are tied)
- The gates depend directly on s_{t-1} and not the intermediate h_{t-1} as in the case of LSTMs
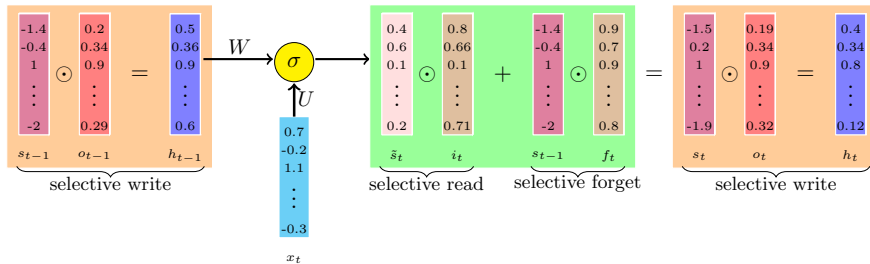
Module : How LSTMs avoid the problem of vanishing gradients
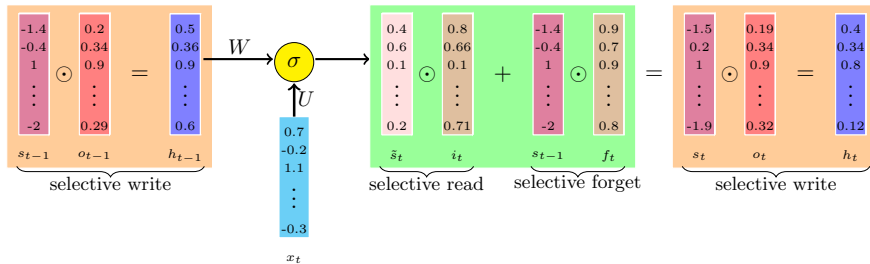
Intuition

- During forward propagation the gates control the flow of information
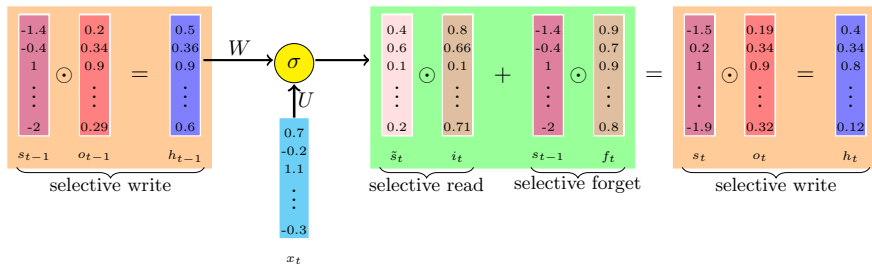
Intuition

- During forward propagation the gates control the flow of information

- They prevent any irrelevant information from being written to the state

Intuition

- During forward propagation the gates control the flow of information

- They prevent any irrelevant information from being written to the state

- Similarly during backward propagation they control the flow of gradients
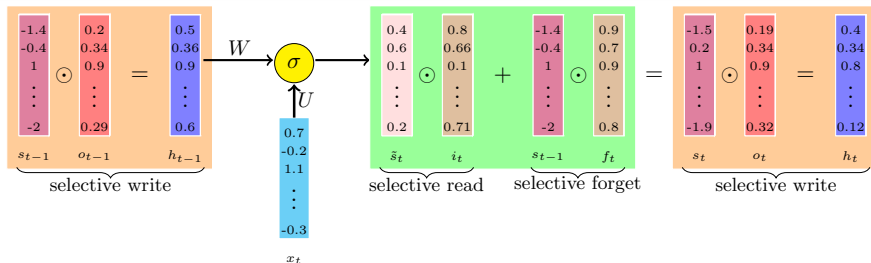
Intuition

- During forward propagation the gates control the flow of information

- They prevent any irrelevant information from being written to the state

- Similarly during backward propagation they control the flow of gradients

- It is easy to see that during backward pass the gradients will get multiplied by the gate
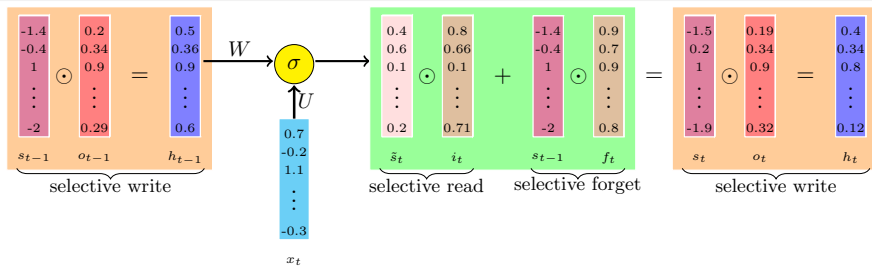
- If the state at time $t-1$ did not contribute much to the state at time t (i.e., if $\|f_t\| \to 0$ and $\|o_{t-1}\| \to 0$) then during backpropagation the gradients flowing into s_{t-1} will vanish
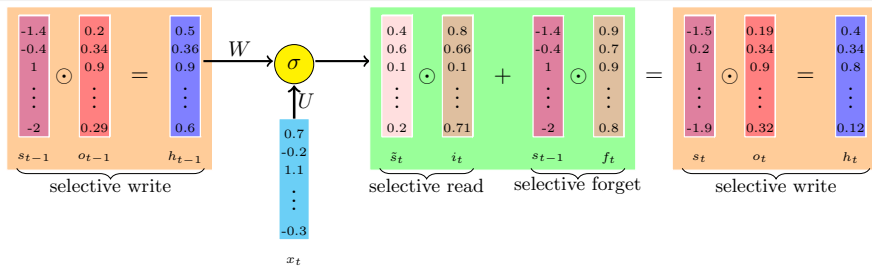
- If the state at time $t - 1$ did not contribute much to the state at time t (i.e., if $\|f_t\| \to 0$ and $\|o_{t-1}\| \to 0$) then during backpropagation the gradients flowing into s_{t-1} will vanish

- But this kind of a vanishing gradient is fine (since s_{t-1} did not contribute to s_t we don't want to hold it responsible for the crimes of s_t)
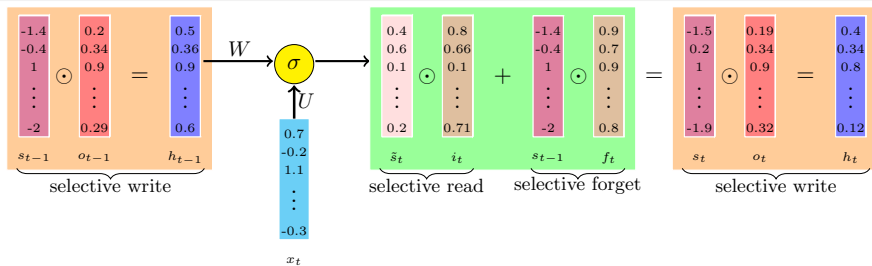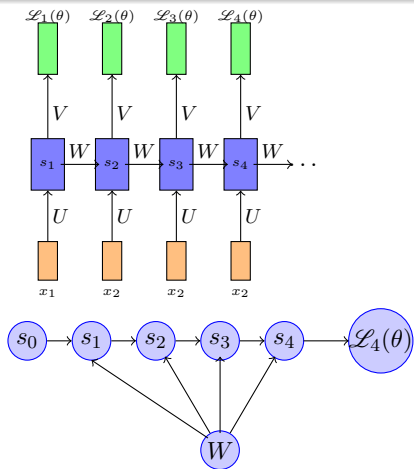
- If the state at time $t-1$ did not contribute much to the state at time t (i.e., if $\|f_t\| \to 0$ and $\|o_{t-1}\| \to 0$) then during backpropagation the gradients flowing into s_{t-1} will vanish

- But this kind of a vanishing gradient is fine (since s_{t-1} did not contribute to s_t we don't want to hold it responsible for the crimes of s_t)

- The key difference from vanilla RNNs is that the flow of information and gradients is controlled by the gates which ensure that the gradients vanish only when they should (i.e., when s_{t-1} didn't contribute much to s_t)

We will now see an illustrative proof of how the gates control the flow of gradients
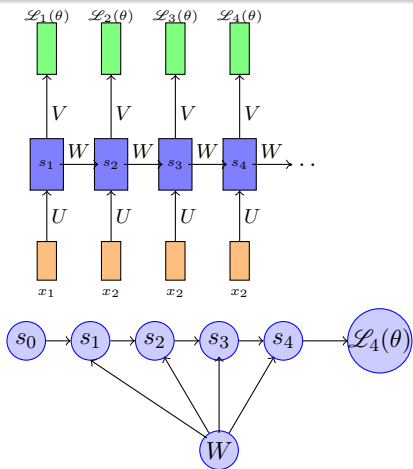
- Recall that RNNs had this multiplicative term which caused the gradients to vanish

$$\frac{\partial \mathscr{L}_t(\theta)}{\partial W} = \frac{\partial \mathscr{L}_t(\theta)}{\partial s_t} \sum_{k=1}^{t} \prod_{j=k}^{t-1} \frac{\partial s_{j+1}}{\partial s_j} \frac{\partial^+ s_k}{\partial W}$$

- Recall that RNNs had this multiplicative term which caused the gradients to vanish

$$\frac{\partial \mathscr{L}_t(\theta)}{\partial W} = \frac{\partial \mathscr{L}_t(\theta)}{\partial s_t} \sum_{k=1}^{t} \prod_{j=k}^{t-1} \frac{\partial s_{j+1}}{\partial s_j} \frac{\partial^+ s_k}{\partial W}$$

- In particular, if the loss at $\mathscr{L}_4(\theta)$ was high because W was not good enough to compute s_1 correctly then this information will not be propagated back to W as the gradient $\frac{\partial \mathscr{L}_t(\theta)}{\partial W}$ along this long path will vanish
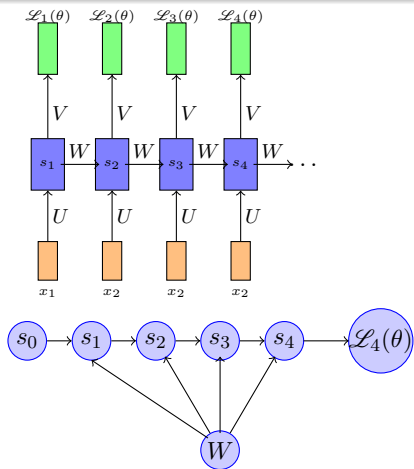
- In general, the gradient of $\mathscr{L}_t(\theta)$ w.r.t. θ_i vanishes when the gradients flowing through **each and every path** from $L_t(\theta)$ to θ_i vanish.
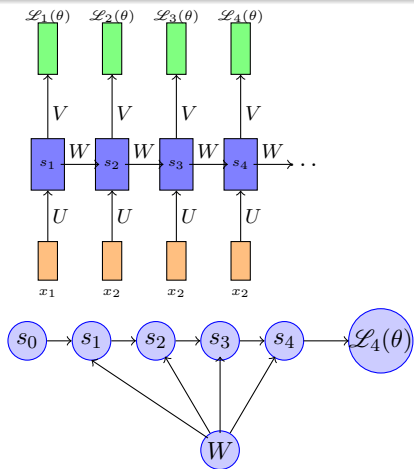
- In general, the gradient of $\mathscr{L}_t(\theta)$ w.r.t. θ_i vanishes when the gradients flowing through **each and every path** from $L_t(\theta)$ to θ_i vanish.

- On the other hand, the gradient of $\mathscr{L}_t(\theta)$ w.r.t. θ_i explodes when the gradient flowing through **at least one path** explodes.
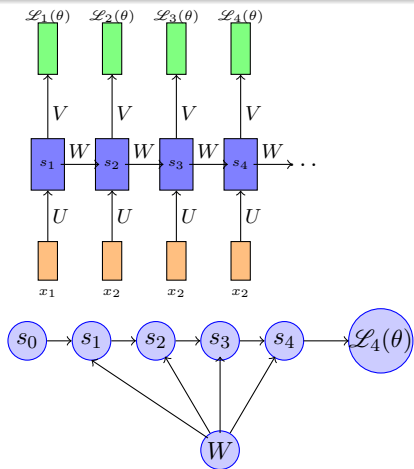
- In general, the gradient of $\mathscr{L}_t(\theta)$ w.r.t. θ_i vanishes when the gradients flowing through **each and every path** from $L_t(\theta)$ to θ_i vanish.

- On the other hand, the gradient of $\mathscr{L}_t(\theta)$ w.r.t. θ_i explodes when the gradient flowing through **at least one path** explodes.

- We will first argue that in the case of LSTMs there exists at least one path through which the gradients can flow effectively (and hence no vanishing gradients)

- We will start with the dependency graph involving different variables in LSTMs

- We will start with the dependency graph involving different variables in LSTMs
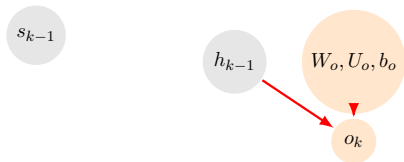- Starting with the states at timestep $k - 1$

- We will start with the dependency graph involving different variables in LSTMs
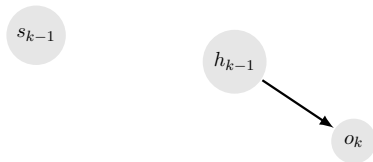- Starting with the states at timestep $k-1$

s_{k-1}

h_{k-1}

- We will start with the dependency graph involving different variables in LSTMs
- Starting with the states at timestep $k - 1$

$$o_k = \sigma(W_o h_{k-1} + U_o x_k + b_o)$$

- We will start with the dependency graph involving different variables in LSTMs
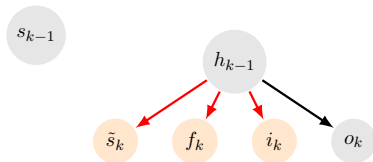- Starting with the states at timestep $k - 1$

$$o_k = \sigma(W_o h_{k-1} + U_o x_k + b_o)$$

- For simplicity we will omit the parameters for now and return back to them later

s_{k-1}

h_{k-1}

o_k

- We will start with the dependency graph involving different variables in LSTMs
- Starting with the states at timestep $k-1$

$$o_k = \sigma(W_o h_{k-1} + U_o x_k + b_o)$$

- For simplicity we will omit the parameters for now and return back to them later
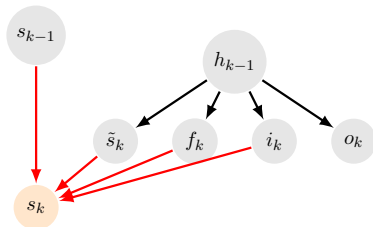
$$i_k = \sigma(W_i h_{k-1} + U_i x_k + b_i)$$
$$f_k = \sigma(W_f h_{k-1} + U_f x_k + b_f)$$
$$\tilde{s}_k = \sigma(W h_{k-1} + U x_k + b)$$

- We will start with the dependency graph involving different variables in LSTMs

- Starting with the states at timestep $k-1$

$$o_k = \sigma(W_o h_{k-1} + U_o x_k + b_o)$$

- For simplicity we will omit the parameters for now and return back to them later

$$i_k = \sigma(W_i h_{k-1} + U_i x_k + b_i)$$
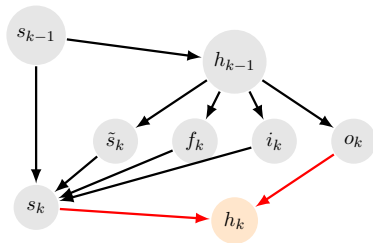$$f_k = \sigma(W_f h_{k-1} + U_f x_k + b_f)$$
$$\tilde{s}_k = \sigma(W h_{k-1} + U x_k + b)$$
$$s_k = f_k \odot s_{k-1} + i_k \odot \tilde{s}_k$$

- We will start with the dependency graph involving different variables in LSTMs
- Starting with the states at timestep $k - 1$

$$o_k = \sigma(W_o h_{k-1} + U_o x_k + b_o)$$

- For simplicity we will omit the parameters for now and return back to them later
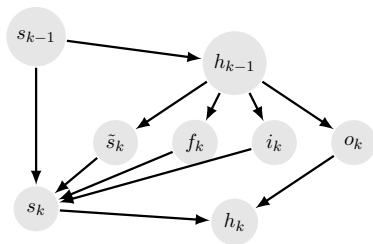
$$i_k = \sigma(W_i h_{k-1} + U_i x_k + b_i)$$
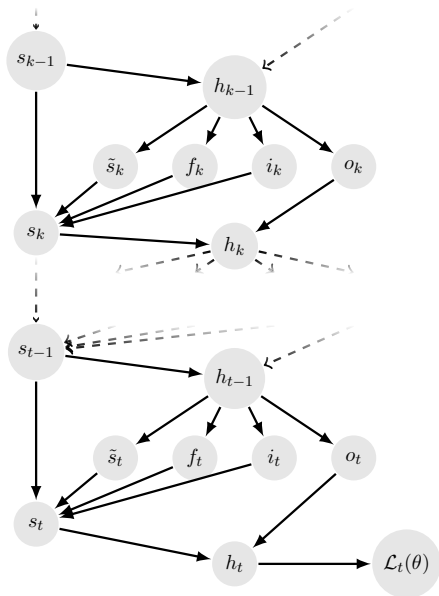$$f_k = \sigma(W_f h_{k-1} + U_f x_k + b_f)$$
$$\tilde{s}_k = \sigma(W h_{k-1} + U x_k + b)$$
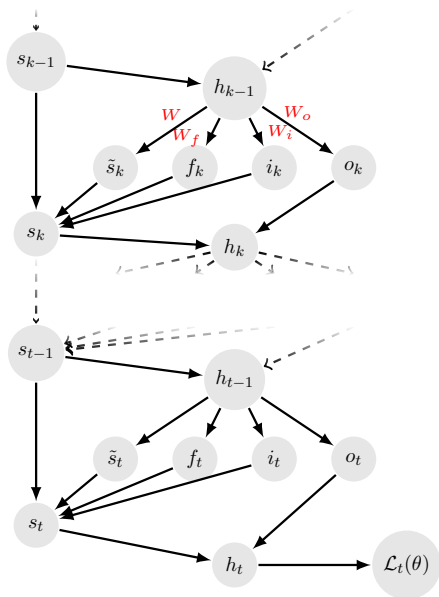$$s_k = f_k \odot s_{k-1} + i_k \odot \tilde{s}_k$$
$$h_k = o_k \odot \sigma(s_k)$$

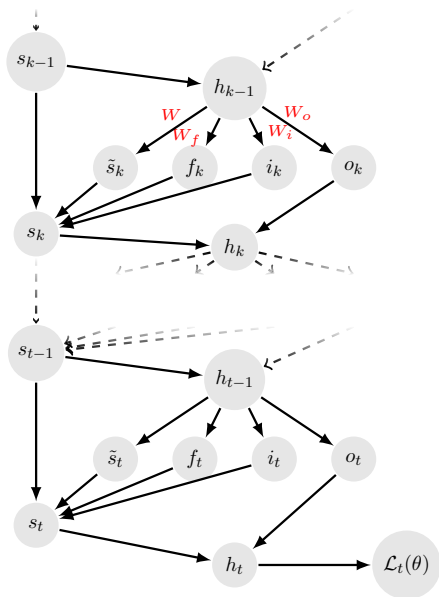- Starting from h_{k-1} and s_{k-1} we have reached h_k and s_k

- Starting from h_{k-1} and s_{k-1} we have reached h_k and s_k
- And the recursion will now continue till the last timestep

- Starting from h_{k-1} and s_{k-1} we have reached h_k and s_k
- And the recursion will now continue till the last timestep
- For simplicity and ease of illustration, instead of considering the parameters (W, W_o, W_i, W_f, U, U_o, U_i, U_f) as separate nodes in the graph we will just put them on the appropriate edges. (We show only a few parameters and not all)

- Starting from h_{k-1} and s_{k-1} we have reached h_k and s_k
- And the recursion will now continue till the last timestep
- For simplicity and ease of illustration, instead of considering the parameters (W, W_o, W_i, W_f, U, U_o, U_i, U_f) as separate nodes in the graph we will just put them on the appropriate edges. (We show only a few parameters and not all)
- We are now interested in knowing if the gradient from $\mathscr{L}_t(\theta)$ flows back to an arbitrary timestep k

- For example, we are interested in knowing if the gradient flows to W_f through s_k
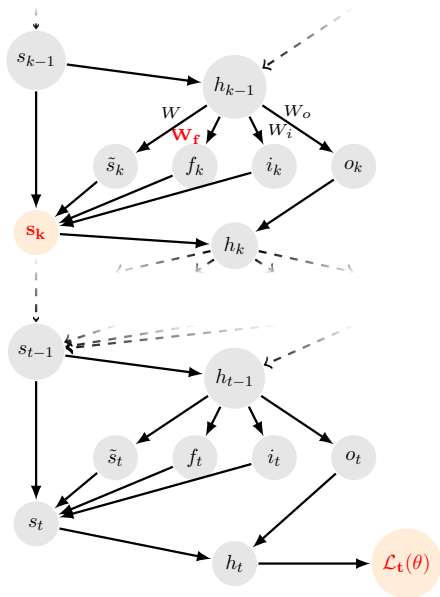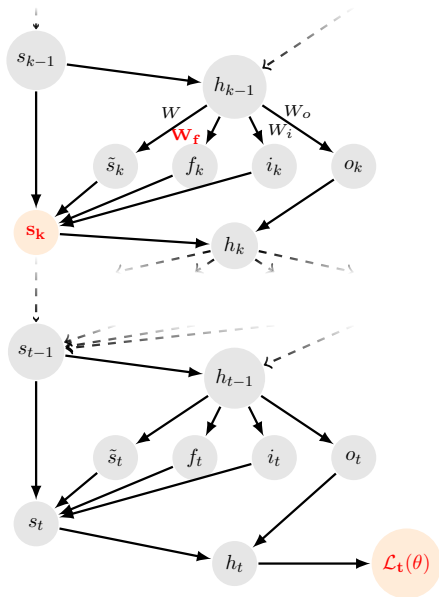
- For example, we are interested in knowing if the gradient flows to W_f through s_k
- In other words, if $\mathscr{L}_t(\theta)$ was high because W_f failed to compute an appropriate value for s_k then this information should flow back to W_f through the gradients
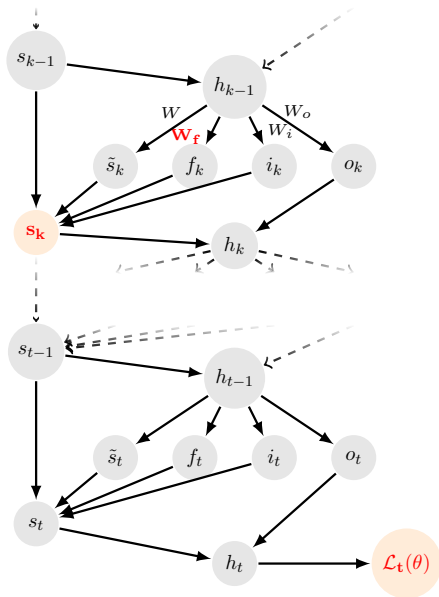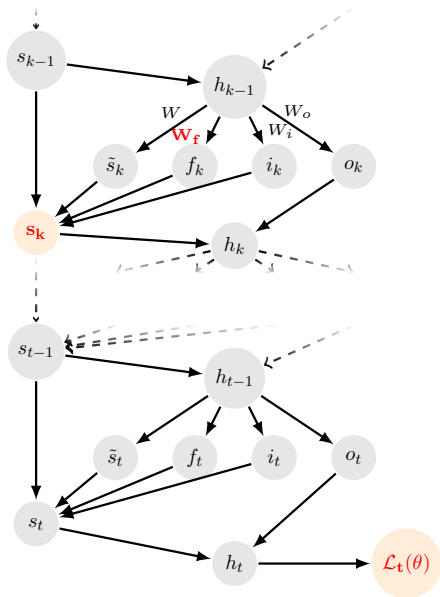
- For example, we are interested in knowing if the gradient flows to W_f through s_k
- In other words, if $\mathscr{L}_t(\theta)$ was high because W_f failed to compute an appropriate value for s_k then this information should flow back to W_f through the gradients
- We can ask a similar question about the other parameters (for example, W_i, W_o, W, etc.)

- For example, we are interested in knowing if the gradient flows to W_f through s_k
- In other words, if $\mathscr{L}_t(\theta)$ was high because W_f failed to compute an appropriate value for s_k then this information should flow back to W_f through the gradients
- We can ask a similar question about the other parameters (for example, W_i, W_o, W, etc.)
- How does LSTM ensure that this gradient does not vanish even at arbitrary time steps? Let us see

- It is sufficient to show that $\frac{\partial \mathcal{L}_t(\theta)}{\partial s_k}$ does not vanish (because if this does not vanish we can reach W_f through s_k)
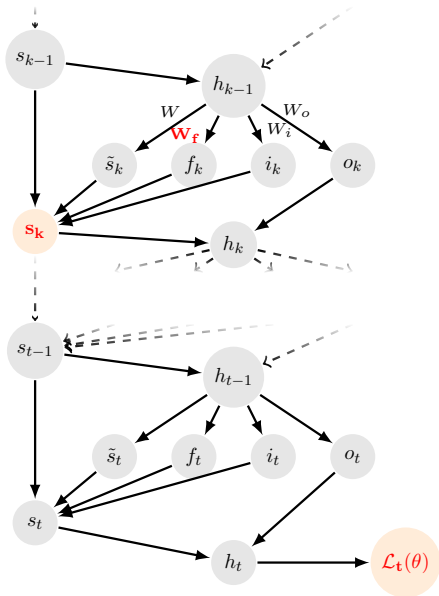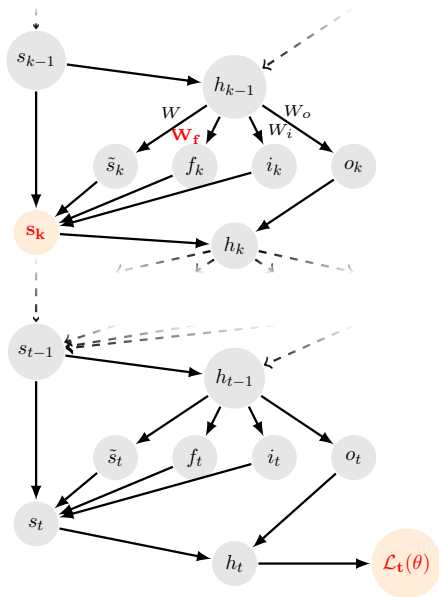
- It is sufficient to show that $\frac{\partial \mathcal{L}_t(\theta)}{\partial s_k}$ does not vanish (because if this does not vanish we can reach W_f through s_k)
- First, we observe that there are multiple paths from $\mathscr{L}_t(\theta)$ to s_k (you just need to reverse the direction of the arrows for backpropagation)
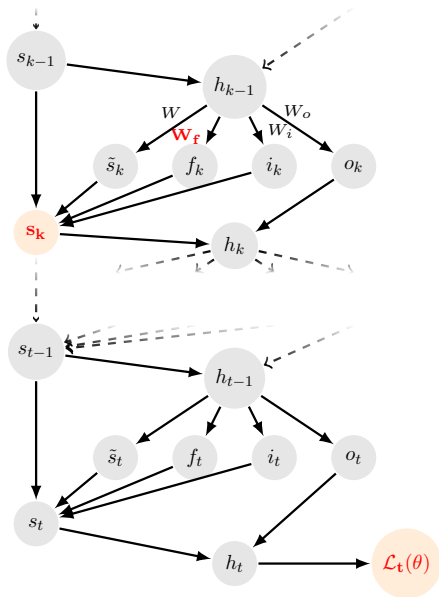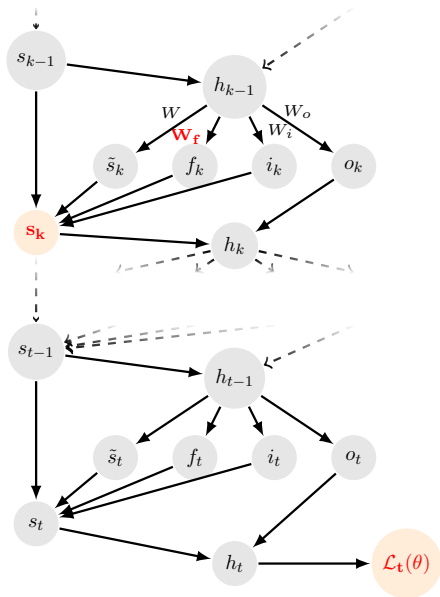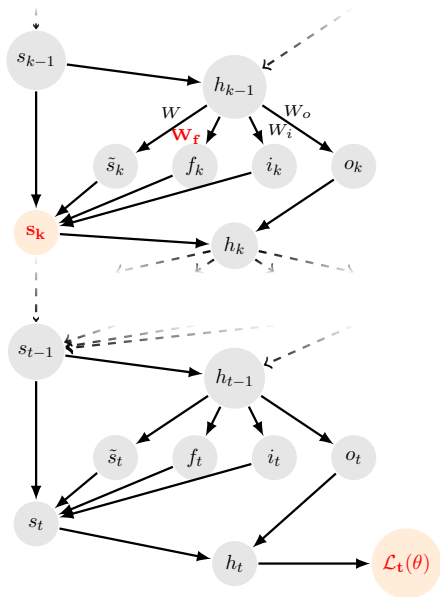
- It is sufficient to show that $\frac{\partial \mathcal{L}_t(\theta)}{\partial s_k}$ does not vanish (because if this does not vanish we can reach W_f through s_k)

- First, we observe that there are multiple paths from $\mathcal{L}_t(\theta)$ to s_k (you just need to reverse the direction of the arrows for backpropagation)

- For example, there is one path through s_{k+1}, another through h_k

- It is sufficient to show that $\frac{\partial \mathcal{L}_t(\theta)}{\partial s_k}$ does not vanish (because if this does not vanish we can reach W_f through s_k)
- First, we observe that there are multiple paths from $\mathscr{L}_t(\theta)$ to s_k (you just need to reverse the direction of the arrows for backpropagation)
- For example, there is one path through s_{k+1}, another through h_k
- Further, there are multiple paths to reach to h_k itself (as should be obvious from the number of outgoing arrows from h_k)
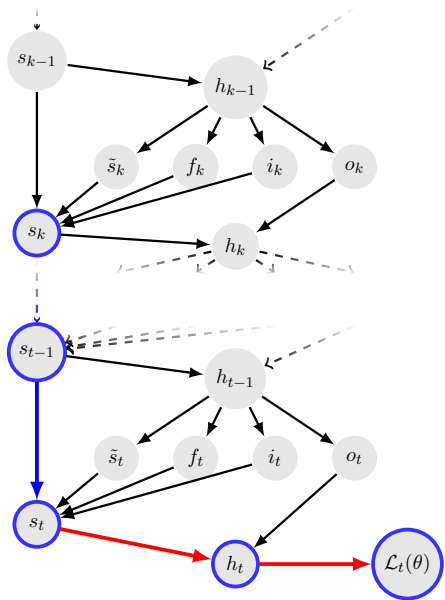
- It is sufficient to show that $\frac{\partial \mathcal{L}_t(\theta)}{\partial s_k}$ does not vanish (because if this does not vanish we can reach W_f through s_k)

- First, we observe that there are multiple paths from $\mathscr{L}_t(\theta)$ to s_k (you just need to reverse the direction of the arrows for backpropagation)

- For example, there is one path through s_{k+1}, another through h_k

- Further, there are multiple paths to reach to h_k itself (as should be obvious from the number of outgoing arrows from h_k)

- So at this point just convince yourself that there are many paths from $\mathscr{L}_t(\theta)$ to s_k

- Consider one such path (highlighted) which will contribute to the gradient
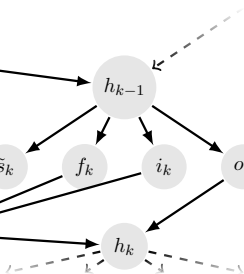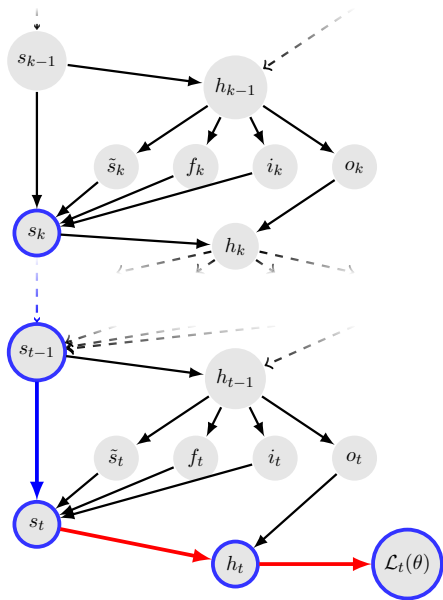
- Consider one such path (highlighted) which will contribute to the gradient
- Let us denote the gradient along this path as t_0

- Consider one such path (highlighted) which will contribute to the gradient
- Let us denote the gradient along this path as t_0

$$t_0 = \frac{\partial \mathscr{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k}$$
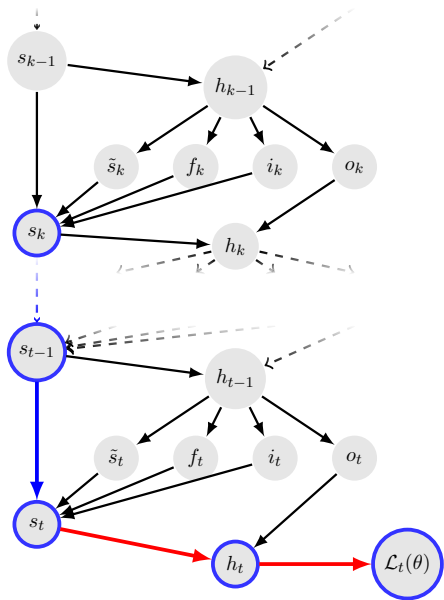
- Consider one such path (highlighted) which will contribute to the gradient
- Let us denote the gradient along this path as t_0

$$t_0 = \frac{\partial \mathscr{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k}$$

- The first term $\frac{\partial \mathscr{L}_t(\theta)}{\partial h_t}$ is fine and it doesn't vanish (h_t is directly connected to $\mathscr{L}_t(\theta)$ and there are no intermediate nodes which can cause the gradient to vanish)
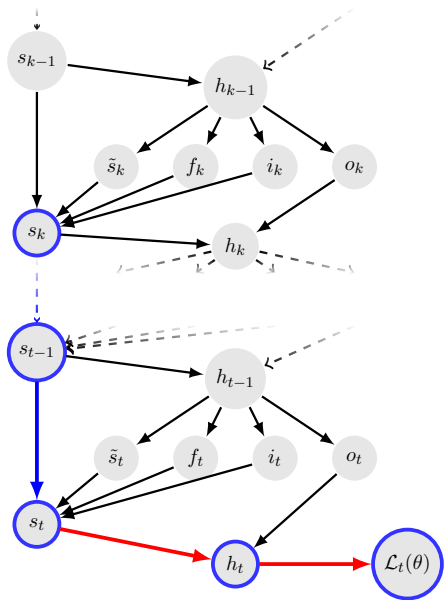
- Consider one such path (highlighted) which will contribute to the gradient
- Let us denote the gradient along this path as t_0

$$t_0 = \frac{\partial \mathscr{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k}$$

- The first term $\frac{\partial \mathscr{L}_t(\theta)}{\partial h_t}$ is fine and it doesn't vanish (h_t is directly connected to $\mathscr{L}_t(\theta)$ and there are no intermediate nodes which can cause the gradient to vanish)
- We will now look at the other terms $\frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}}$ $(\forall t)$
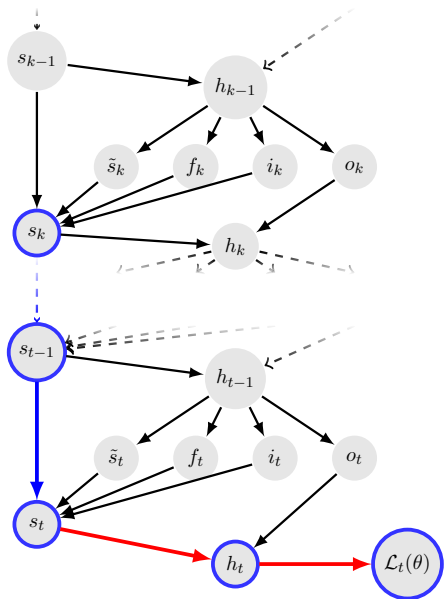
- Let us first look at $\frac{\partial h_t}{\partial s_t}$
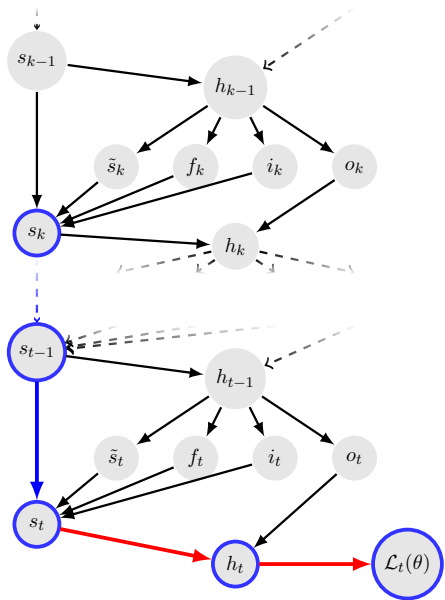
- Let us first look at $\frac{\partial h_t}{\partial s_t}$
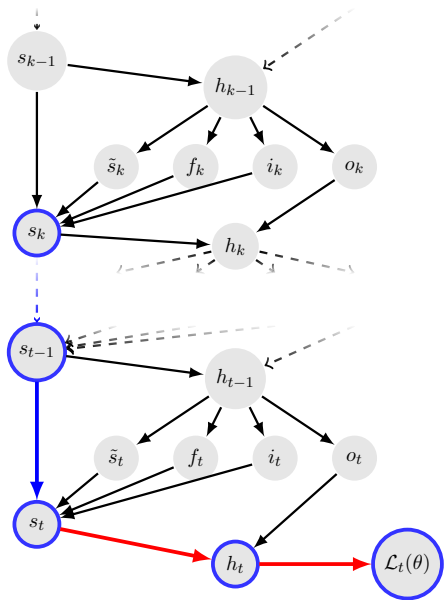- Recall that

$$h_t = o_t \odot \sigma(s_t)$$

- Let us first look at $\frac{\partial h_t}{\partial s_t}$
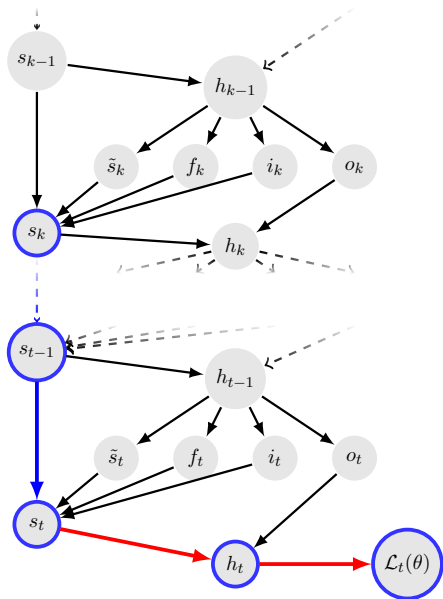- Recall that

$$h_t = o_t \odot \sigma(s_t)$$

- Note that h_{ti} only depends on o_{ti} and s_{ti} and not on any other elements of o_t and s_t

- Let us first look at $\frac{\partial h_t}{\partial s_t}$
- Recall that

$$h_t = o_t \odot \sigma(s_t)$$

- Note that h_{ti} only depends on o_{ti} and s_{ti} and not on any other elements of o_t and s_t
- $\frac{\partial h_t}{\partial s_t}$ will thus be a square diagonal matrix $\in \mathbb{R}^{d \times d}$ whose diagonal will be $o_t \odot \sigma'(s_t) \in \mathbb{R}^d$ (see slide 35 of Lecture 14)

- Let us first look at $\frac{\partial h_t}{\partial s_t}$
- Recall that

$$h_t = o_t \odot \sigma(s_t)$$

- Note that h_{ti} only depends on o_{ti} and s_{ti} and not on any other elements of o_t and s_t
- $\frac{\partial h_t}{\partial s_t}$ will thus be a square diagonal matrix $\in \mathbb{R}^{d \times d}$ whose diagonal will be $o_t \odot \sigma'(s_t) \in \mathbb{R}^d$ (see slide 35 of Lecture 14)
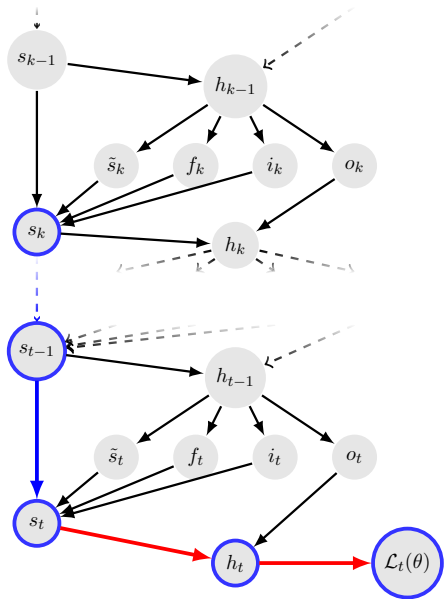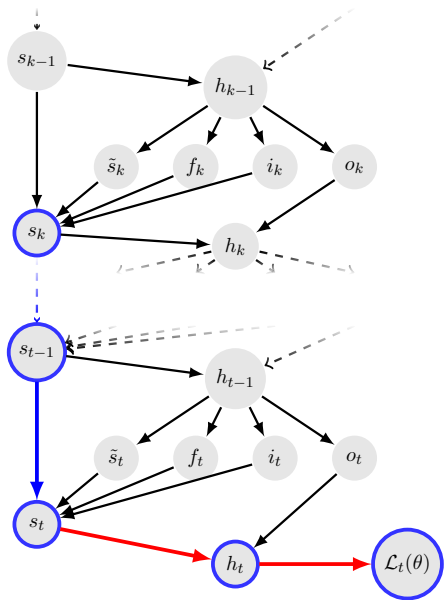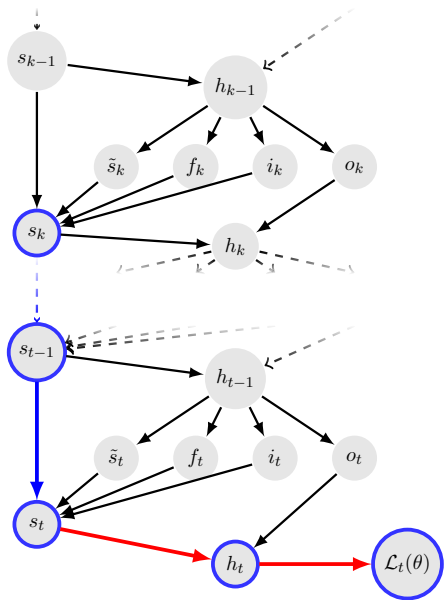- We will represent this diagonal matrix by $\mathcal{D}(o_t \odot \sigma'(s_t))$

- Now let us consider $\frac{\partial s_t}{\partial s_{t-1}}$

- Now let us consider $\frac{\partial s_t}{\partial s_{t-1}}$
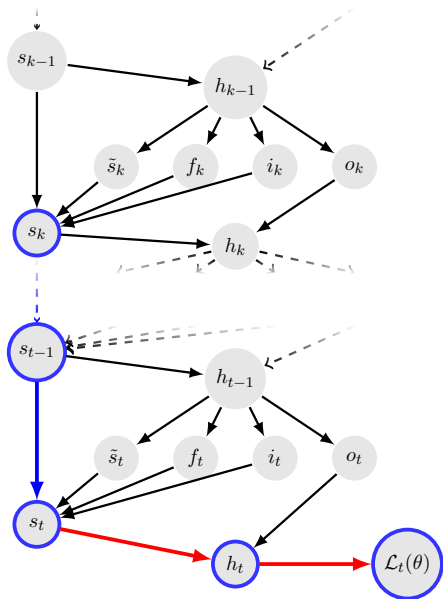- Recall that

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$

- Now let us consider $\frac{\partial s_t}{\partial s_{t-1}}$
- Recall that

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$

- Notice that $\tilde{s}_t$ also depends on s_{t-1} so we cannot treat it as a constant

- Now let us consider $\frac{\partial s_t}{\partial s_{t-1}}$
- Recall that

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$

- Notice that $\tilde{s}_t$ also depends on s_{t-1} so we cannot treat it as a constant
- So once again we are dealing with an ordered network and thus $\frac{\partial s_t}{\partial s_{t-1}}$ will be a sum of an explicit term and an implicit term (see slide 37 from Lecture 14)

- Now let us consider $\frac{\partial s_t}{\partial s_{t-1}}$
- Recall that

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$

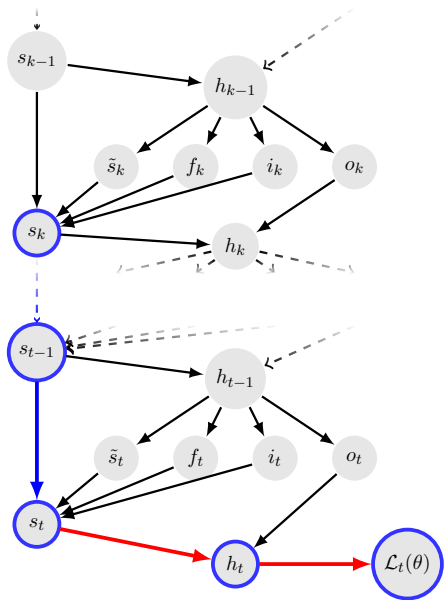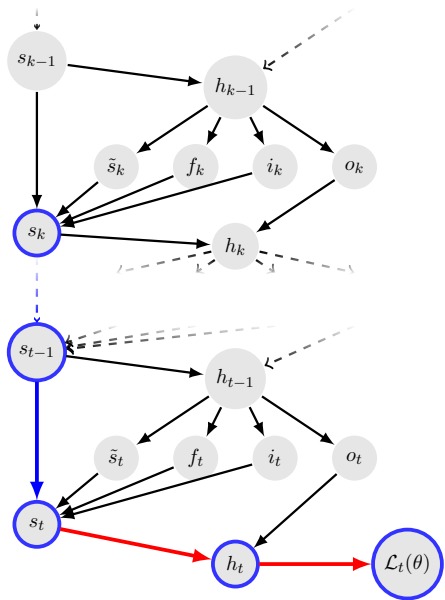- Notice that $\tilde{s}_t$ also depends on s_{t-1} so we cannot treat it as a constant
- So once again we are dealing with an ordered network and thus $\frac{\partial s_t}{\partial s_{t-1}}$ will be a sum of an explicit term and an implicit term (see slide 37 from Lecture 14)
- For simplicity, let us assume that the gradient from the implicit term vanishes (we are assuming a worst case scenario)

- Now let us consider $\frac{\partial s_t}{\partial s_{t-1}}$
- Recall that

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$

- Notice that $\tilde{s}_t$ also depends on s_{t-1} so we cannot treat it as a constant
- So once again we are dealing with an ordered network and thus $\frac{\partial s_t}{\partial s_{t-1}}$ will be a sum of an explicit term and an implicit term (see slide 37 from Lecture 14)
- For simplicity, let us assume that the gradient from the implicit term vanishes (we are assuming a worst case scenario)
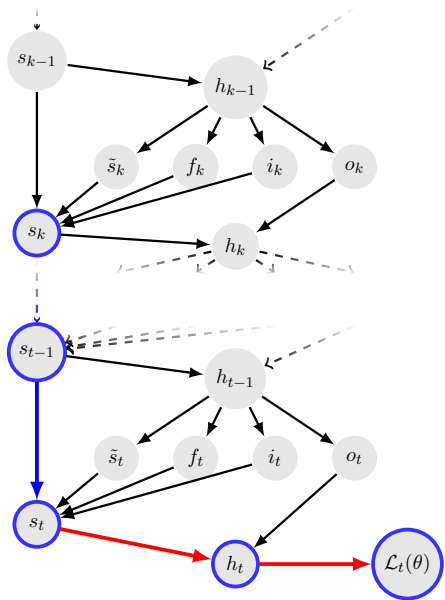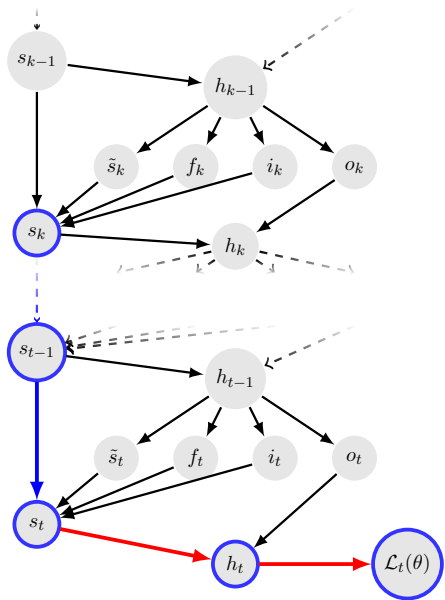- And the gradient from the explicit term (treating $\tilde{s}_t$ as a constant) is given by $\mathcal{D}(f_t)$

- We now return back to our full expression for t_0:

- We now return back to our full expression for t_0:

$$t_0 = \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k}$$

- We now return back to our full expression for t_0:

$$t_0 = \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k}$$

$$= \mathcal{L}'_t(h_t).\mathcal{D}(o_t \odot \sigma'(s_t))\mathcal{D}(f_t)\ldots\mathcal{D}(f_{k+1})$$
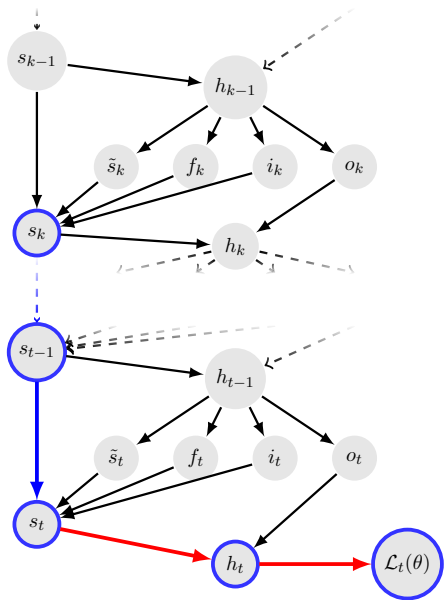
- We now return back to our full expression for t_0:

$$t_0 = \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k}$$

$$= \mathcal{L}'_t(h_t) . \mathcal{D}(o_t \odot \sigma'(s_t)) \mathcal{D}(f_t) \ldots \mathcal{D}(f_{k+1})$$

$$= \mathcal{L}'_t(h_t) . \mathcal{D}(o_t \odot \sigma'(s_t)) \mathcal{D}(f_t \odot \ldots \odot f_{k+1})$$

- We now return back to our full expression for t_0:

$$t_0 = \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k}$$
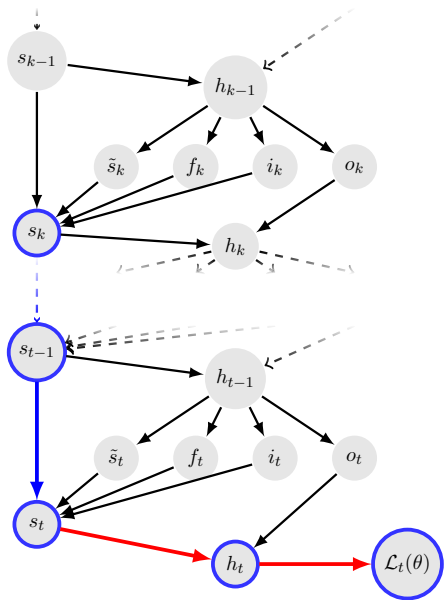
$$= \mathcal{L}'_t(h_t).\mathcal{D}(o_t \odot \sigma'(s_t))\mathcal{D}(f_t)\ldots\mathcal{D}(f_{k+1})$$

$$= \mathcal{L}'_t(h_t).\mathcal{D}(o_t \odot \sigma'(s_t))\mathcal{D}(f_t \odot \ldots \odot f_{k+1})$$

$$= \mathcal{L}'_t(h_t).\mathcal{D}(o_t \odot \sigma'(s_t))\mathcal{D}(\odot_{i=k+1}^{t} f_i)$$
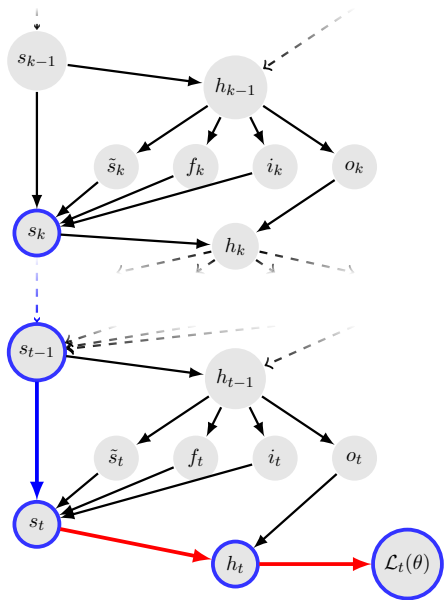
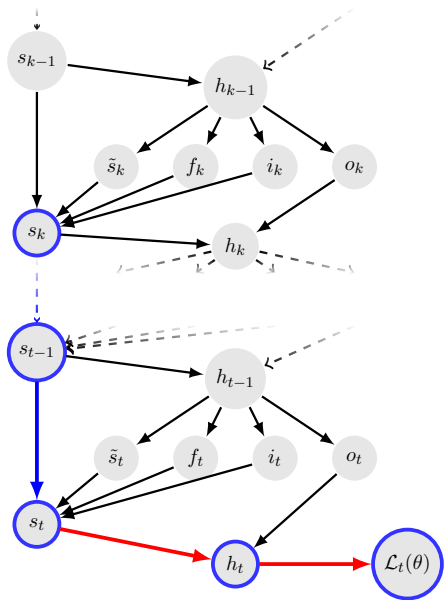- We now return back to our full expression for t_0:

$$
\begin{aligned}
t_0 &= \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k} \\
&= \mathcal{L}_t'(h_t) . \mathcal{D}(o_t \odot \sigma'(s_t)) \mathcal{D}(f_t) \ldots \mathcal{D}(f_{k+1}) \\
&= \mathcal{L}_t'(h_t) . \mathcal{D}(o_t \odot \sigma'(s_t)) \mathcal{D}(f_t \odot \ldots \odot f_{k+1}) \\
&= \mathcal{L}_t'(h_t) . \mathcal{D}(o_t \odot \sigma'(s_t)) \mathcal{D}(\odot_{i=k+1}^{t} f_i)
\end{aligned}
$$

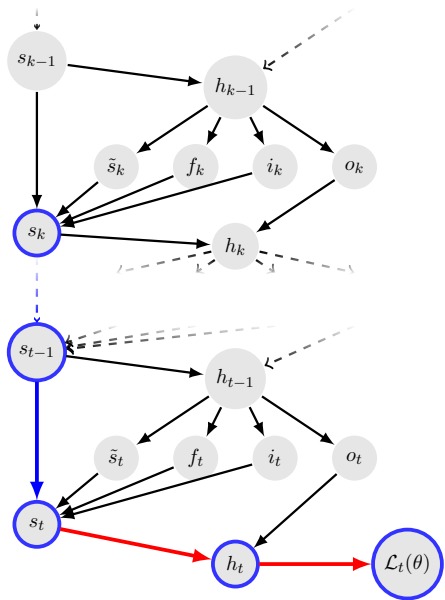- The red terms don't vanish and the blue terms contain a multiplication of the forget gates
- The forget gates thus regulate the gradient flow depending on the explicit contribution of a state (s_t) to the next state s_{t+1}

- If during forward pass s_t did not contribute much to s_{t+1} (because $f_t \rightarrow 0$) then during backpropagation also the gradient will not reach s_t

- If during forward pass s_t did not contribute much to s_{t+1} (because $f_t \to 0$) then during backpropagation also the gradient will not reach s_t
- This is fine because if s_t did not contribute much to s_{t+1} then there is no reason to hold it responsible during backpropagation

- If during forward pass s_t did not contribute much to s_{t+1} (because $f_t \to 0$) then during backpropgation also the gradient will not reach s_t
- This is fine because if s_t did not contribute much to s_{t+1} then there is no reason to hold it responsible during backpropgation (f_t does the same regulation during forward pass and backward pass which is fair)

- If during forward pass s_t did not contribute much to s_{t+1} (because $f_t \to 0$) then during backpropgation also the gradient will not reach s_t

- This is fine because if s_t did not contribute much to s_{t+1} then there is no reason to hold it responsible during backpropgation (f_t does the same regulation during forward pass and backward pass which is fair)

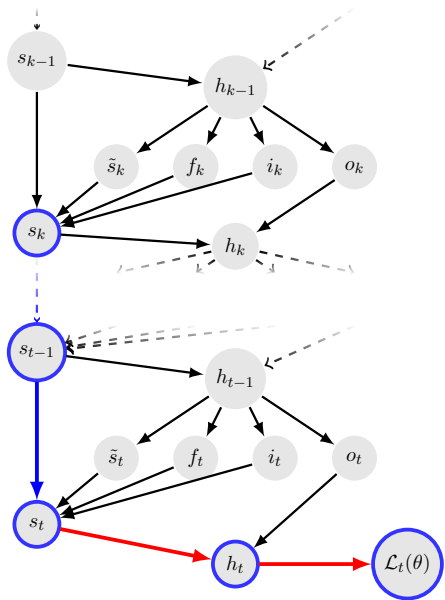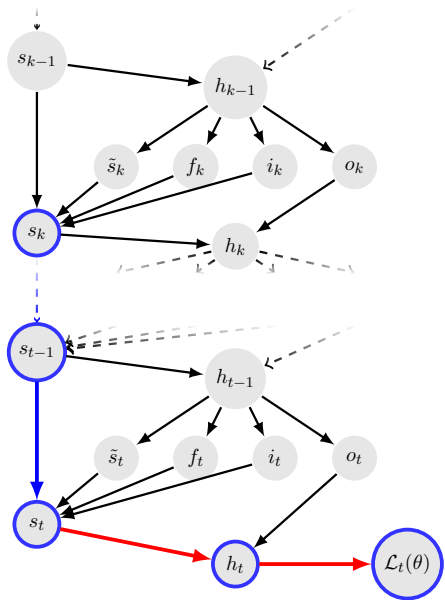- Thus there exists this one path along which the gradient doesn't vanish when it shouldn't

- If during forward pass s_t did not contribute much to s_{t+1} (because $f_t \to 0$) then during backpropgation also the gradient will not reach s_t
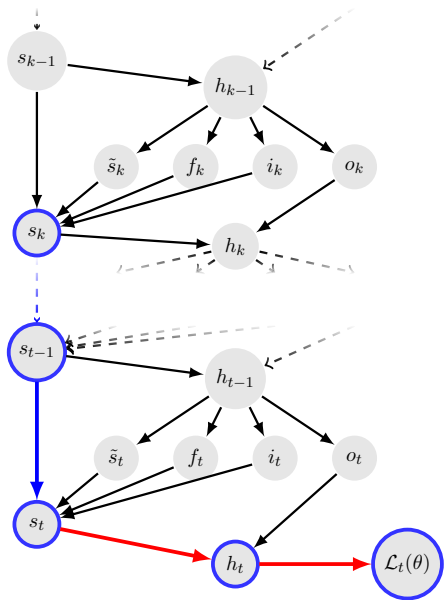- This is fine because if s_t did not contribute much to s_{t+1} then there is no reason to hold it responsible during backpropgation (f_t does the same regulation during forward pass and backward pass which is fair)
- Thus there exists this one path along which the gradient doesn't vanish when it shouldn't
- And as argued as long as the gradient flows back to W_f through one of the paths (t_0) through s_k we are fine !

- If during forward pass s_t did not contribute much to s_{t+1} (because $f_t \to 0$) then during backpropgation also the gradient will not reach s_t
- This is fine because if s_t did not contribute much to s_{t+1} then there is no reason to hold it responsible during backpropgation (f_t does the same regulation during forward pass and backward pass which is fair)
- Thus there exists this one path along which the gradient doesn't vanish when it shouldn't
- And as argued as long as the gradient flows back to W_f through one of the paths (t_0) through s_k we are fine !
- Of course the gradient flows back only when required as regulated by f_i's (but let me just say it one last time that *this is fair*)

- Now we will see why LSTMs do not solve the problem of exploding gradients
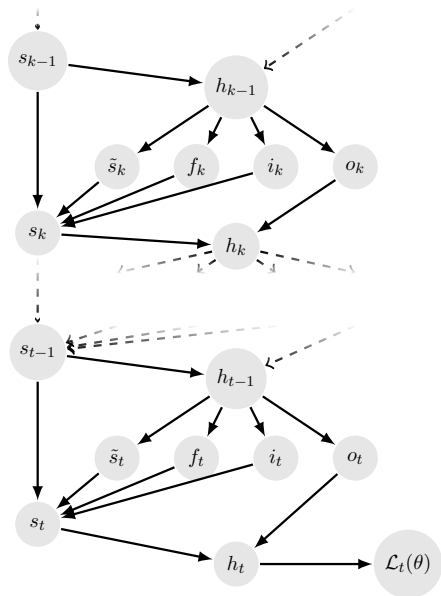
- Now we will see why LSTMs do not solve the problem of exploding gradients
- We will show a path through which the gradient can explode
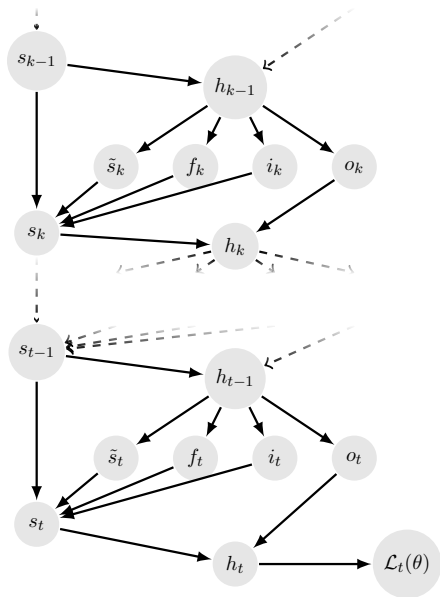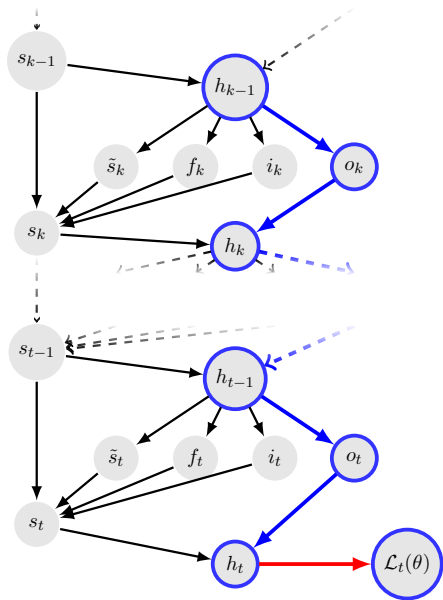
- Now we will see why LSTMs do not solve the problem of exploding gradients
- We will show a path through which the gradient can explode
- Let us compute one term (say t_1) of $\frac{\partial \mathcal{L}_t(\theta)}{\partial h_{k-1}}$ corresponding to the highlighted path

- Now we will see why LSTMs do not solve the problem of exploding gradients

- We will show a path through which the gradient can explode

- Let us compute one term (say t_1) of $\frac{\partial \mathcal{L}_t(\theta)}{\partial h_{k-1}}$ corresponding to the highlighted path

$$t_1 = \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \left(\frac{\partial h_t}{\partial o_t} \frac{\partial o_t}{\partial h_{t-1}} \right) \cdots \left(\frac{\partial h_k}{\partial o_k} \frac{\partial o_k}{\partial h_{k-1}} \right)$$
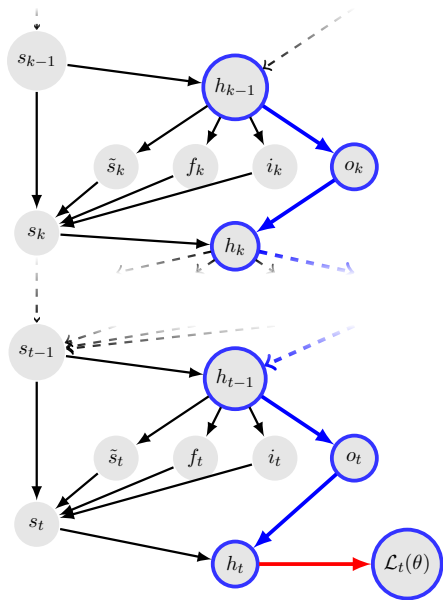
- Now we will see why LSTMs do not solve the problem of exploding gradients
- We will show a path through which the gradient can explode
- Let us compute one term (say t_1) of $\frac{\partial \mathcal{L}_t(\theta)}{\partial h_{k-1}}$ corresponding to the highlighted path

$$
\begin{aligned}
t_1 =& \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \left(\frac{\partial h_t}{\partial o_t} \frac{\partial o_t}{\partial h_{t-1}} \right) \cdots \left(\frac{\partial h_k}{\partial o_k} \frac{\partial o_k}{\partial h_{k-1}} \right) \\
=& \mathcal{L}'_t(h_t) \left(\mathcal{D}(\sigma(s_t) \odot o'_t).W_o \right) \cdots \\
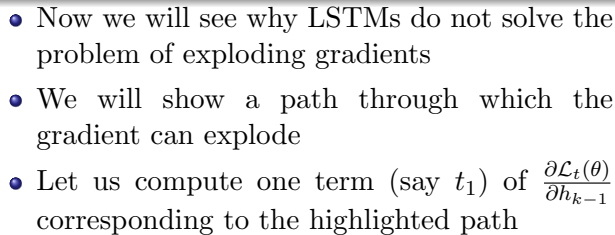& \qquad \left(\mathcal{D}(\sigma(s_k) \odot o'_k).W_o \right)
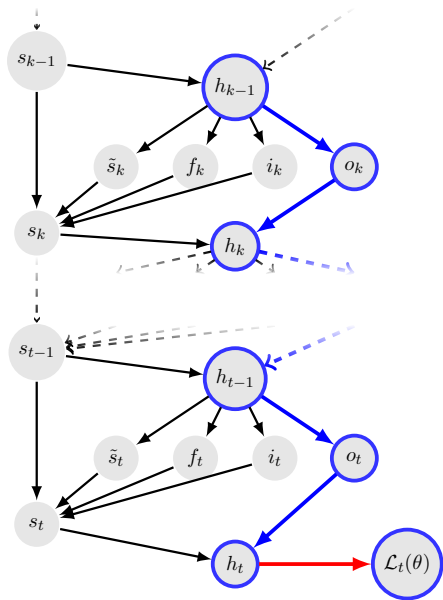\end{aligned}
$$

- Now we will see why LSTMs do not solve the problem of exploding gradients
- We will show a path through which the gradient can explode
- Let us compute one term (say t_1) of $\frac{\partial \mathcal{L}_t(\theta)}{\partial h_{k-1}}$ corresponding to the highlighted path

$$t_1 = \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \left(\frac{\partial h_t}{\partial o_t} \frac{\partial o_t}{\partial h_{t-1}} \right) \cdots \left(\frac{\partial h_k}{\partial o_k} \frac{\partial o_k}{\partial h_{k-1}} \right)$$
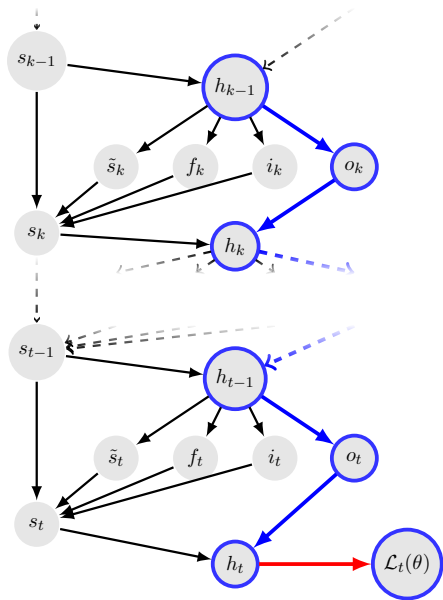
$$= \mathcal{L}'_t(h_t) \left(\mathcal{D}(\sigma(s_t) \odot o'_t).W_o \right) \cdots$$
$$\left(\mathcal{D}(\sigma(s_k) \odot o'_k).W_o \right)$$

$$\|t_1\| \leq \|\mathcal{L}'_t(h_t)\| \left(\|K\| \|W_o\| \right)^{t-k+1}$$

- Now we will see why LSTMs do not solve the problem of exploding gradients
- We will show a path through which the gradient can explode
- Let us compute one term (say t_1) of $\frac{\partial \mathcal{L}_t(\theta)}{\partial h_{k-1}}$ corresponding to the highlighted path

$$
\begin{aligned}
t_1 =& \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \left(\frac{\partial h_t}{\partial o_t} \frac{\partial o_t}{\partial h_{t-1}} \right) \cdots \left(\frac{\partial h_k}{\partial o_k} \frac{\partial o_k}{\partial h_{k-1}} \right) \\
=& \mathcal{L}_t'(h_t) \left(\mathcal{D}(\sigma(s_t) \odot o_t').W_o \right) \cdots \\
& \qquad \left(\mathcal{D}(\sigma(s_k) \odot o_k').W_o \right) \\
\|t_1\| \leq& \|\mathcal{L}_t'(h_t)\| \left(\|K\| \|W_o\| \right)^{t-k+1}
\end{aligned}
$$

- Depending on the norm of matrix W_o, the gradient $\frac{\partial \mathcal{L}_t(\theta)}{\partial h_{k-1}}$ may explode
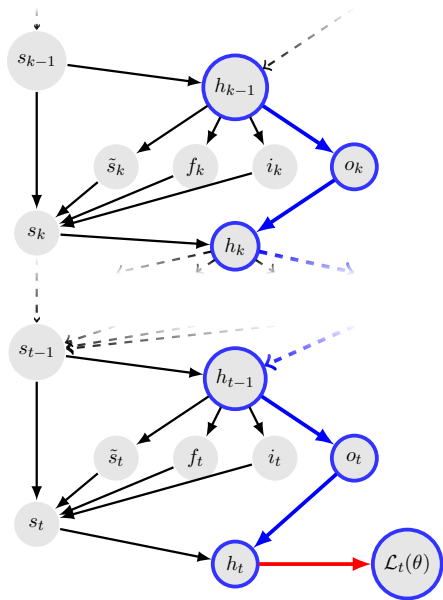
- Now we will see why LSTMs do not solve the problem of exploding gradients
- We will show a path through which the gradient can explode
- Let us compute one term (say t_1) of $\frac{\partial \mathcal{L}_t(\theta)}{\partial h_{k-1}}$ corresponding to the highlighted path

$$t_1 = \frac{\partial \mathcal{L}_t(\theta)}{\partial h_t} \left(\frac{\partial h_t}{\partial o_t} \frac{\partial o_t}{\partial h_{t-1}} \right) \cdots \left(\frac{\partial h_k}{\partial o_k} \frac{\partial o_k}{\partial h_{k-1}} \right)$$

$$= \mathcal{L}'_t(h_t) \left(\mathcal{D}(\sigma(s_t) \odot o'_t).W_o \right) \cdots$$
$$\left(\mathcal{D}(\sigma(s_k) \odot o'_k).W_o \right)$$

$$\|t_1\| \leq \|\mathcal{L}'_t(h_t)\| \left(\|K\| \|W_o\| \right)^{t-k+1}$$

- Depending on the norm of matrix W_o, the gradient $\frac{\partial \mathcal{L}_t(\theta)}{\partial h_{k-1}}$ may explode
- Similarly, W_i, W_f and W can also cause the gradients to explode

- So how do we deal with the problem of exploding gradients ?
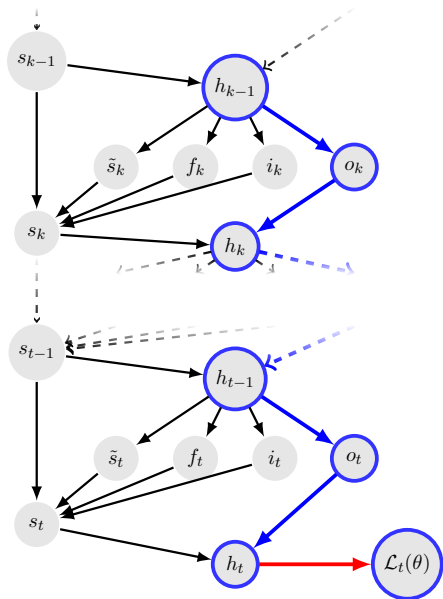
---

[*]Pascanu, Razvan, Tomas Mikolov, and Yoshua Bengio. "On the difficulty of training recurrent neural networks." ICML(3)28(2013):1310-1318
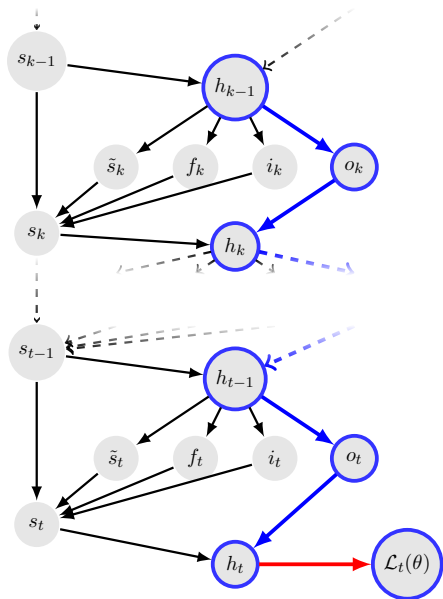
- So how do we deal with the problem of exploding gradients ?
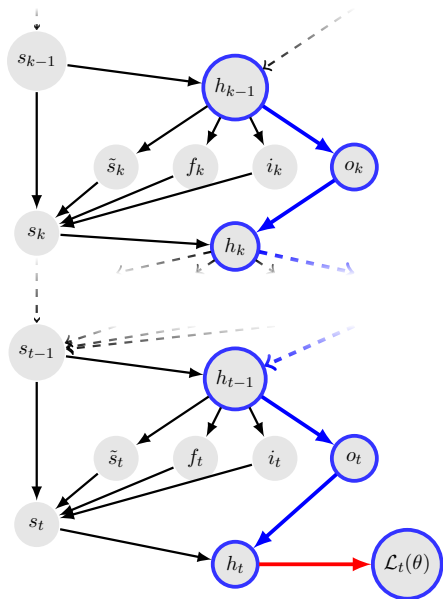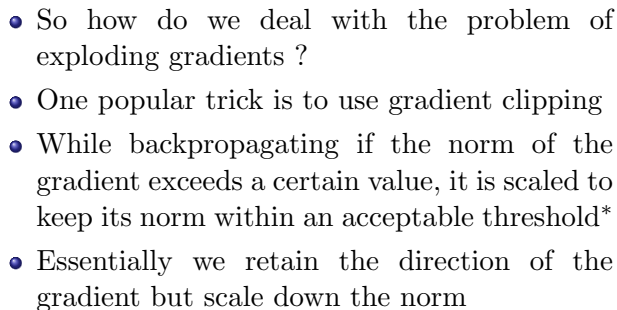- One popular trick is to use gradient clipping

---

[*]Pascanu, Razvan, Tomas Mikolov, and Yoshua Bengio. "On the difficulty of training recurrent neural networks." ICML(3)28(2013):1310-1318

- So how do we deal with the problem of exploding gradients ?
- One popular trick is to use gradient clipping
- While backpropagating if the norm of the gradient exceeds a certain value, it is scaled to keep its norm within an acceptable threshold[*]

---

[*]Pascanu, Razvan, Tomas Mikolov, and Yoshua Bengio. "On the difficulty of training recurrent neural networks." ICML(3)28(2013):1310-1318

- So how do we deal with the problem of exploding gradients ?
- One popular trick is to use gradient clipping
- While backpropagating if the norm of the gradient exceeds a certain value, it is scaled to keep its norm within an acceptable threshold[*]
- Essentially we retain the direction of the gradient but scale down the norm

---

[*]Pascanu, Razvan, Tomas Mikolov, and Yoshua Bengio. "On the difficulty of training recurrent neural networks." ICML(3)28(2013):1310-1318