Question - 2 (b)

→ For univariate generalized linear model the parameter estimation can be obtained by iteratively reweighted least square.

→ A similar Algorithm can be used even for the non-linear model with scale parameter

→ The contribution from a single multinomial observation $(n_1, \ldots, n_k)$ to the likelihood function is $\pi_1^{n_1} \ldots \pi_k^{n_k}$ where π_i, $1 \le i \le k$ is probability of i^{th} category

→ Since we are dealing with cummulative probabilities, we define

$$R_1 = n_1 \qquad z_1 = R_1/n$$

$$R_2 = n_2 \qquad z_2 = R_2/n$$

$$\vdots \qquad\qquad \vdots$$

$$R_k = \sum n_j = n \qquad z_k = n/n = 1$$

In terms of parameter of comulative transformation, the likelihood can be written as product of $(k-1)$ quantities,

$$\left\{ \left(\frac{r_1}{r_2}\right)^{R_1} \left(\frac{r_2 - r_1}{r_2}\right)^{R_2 - R_1} \right\} \left\{ \left(\frac{r_2}{r_3}\right)^{R_2} \left(\frac{r_3 - r_2}{r_3}\right)^{R_3 - R_2} \right\} \ldots$$

$$\ldots \left\{ \left(\frac{r_{k-1}}{r_k}\right)^{R_{k-1}} \left(\frac{r_k - r_{k-1}}{r_k}\right)^{R_k - R_{k-1}} \right\}$$

where,

$$r_j(x) = \pi_1(x) + \pi_2(x) + \ldots + \pi_j(x)$$

It is convenient to define,

$$\phi_j = \log\left(\frac{r_j}{r_{j+1} - r_j}\right) = \text{logit}\left(\frac{r_j}{r_{j+1}}\right)$$

$$g(\phi) = \log(1 + \exp(\phi))$$

$$= \log\left(\frac{r_{j+1}}{r_{j+1} - r_j}\right)$$

Whence the likelihood is,

$$l = n\left[\{z_1\phi_1 - z_2 g(\phi_1)\} + \{z_2\phi_2 - z_3 g(\phi_2)\} + \{z_{k-1}\phi_{k-1} - g(\phi_{k-1})\}\right]$$

→ The Netwon-Raphson method with Fisher Scoring converges rapidly even when initial estimates are poor. Generally about 4 to 5 cycles are required to produce accuracy to four significant digits in all paramete.

For univariate generalized linear model

A univariate generalized linear model contains only one of the above component, The following relationships follow form:

$$E(z_j) = r_j$$
$$E(z_j \mid z_{j+1}) = z_{j+1} g'(\phi_j) = z_{j+1}(r_j / r_{j+1})$$

$$\text{var}(z_j) = r_j(1 - r_j)/n$$
$$\text{var}(z_j \mid z_{j+1}) = z_{j+1} g''(\phi_j)/n$$