$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)\le D_0\\ 0 & \text{if } D(u,v)>D_0\end{cases}$$

$$D(u,v)=\left[(u-M/2)^2+(v-N/2)^2\right]^{1/2}$$

ii. Butterworth low-pass filter (BLPF)

The filter transfer function for the Butterworth low-pass filter is given by:

$$H(u,v)=\frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$

iii. Gaussian low-pass filter (GLPF)

The filter transfer function for the Gaussian low-pass filter is given by:

$$H(u,v)=e^{-D^2(u,v)/2D_0^2}$$

B. Image Sharpening (High-pass Frequency Domain Filters)

Sharpening of an image in the frequency domain can be achieved by high pass filtering process which attenuates (suppress) low frequency components without disturbing high frequency information in the Fourier transform of the image. The high-pass filter H_{hp} is often represented by its relationship to the low-pass filter (H_{lp}) as:

H_{hp} (u, v) =1- H_{lp} (u, v)

i. Ideal High-Pass Filter (IHPF)

The ideal high pass filter simply cuts off all the low frequencies lower than the specified cut-off frequency. The filter transfer function is given as:

$$H(u,v)=\begin{cases}0 & \text{if } D(u,v)\le D_0\\ 1 & \text{if } D(u,v)>D_0\end{cases}$$

ii. Butterworth High-pass Filter

The transfer function of Butterworth high-pass filter of order n and with a specified cut-off frequency is given by:

$$H(u,v)=1-e^{-D^2(u,v)/2D_0^2}$$

iii. Gaussian High Pass Filters

The transfer function of the Gaussian high-pass filter with cut-off frequency locus at a distance 0 D from the origin given by:

$$H(u,v)=\frac{1}{1+\left[D_0/D(u,v)\right]^{2n}}$$

In the above formulas, D_0 is cut-off frequency, a specified nonnegative number. *D(u, v)* is the distance from point *(u, v)* to the center of the filter.

Step-4 Compute Inverse Fourier Transform to get the enhanced image.

We then need to convert data back to real image to use in any applications. After the needed frequencies removed it is easy to return back to the spatial domain. Function represented by Fourier transform can be completely reconstructed by an inverse transform with no loss of information

For this the Inverse Fourier Transform of the filtered image is calculated by the following equation:

$$f(x,y)=\frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(ux/M+vy/N)}$$

4. RESULTS AND DISCUSSIONS

Fig. 4. Original Input Image

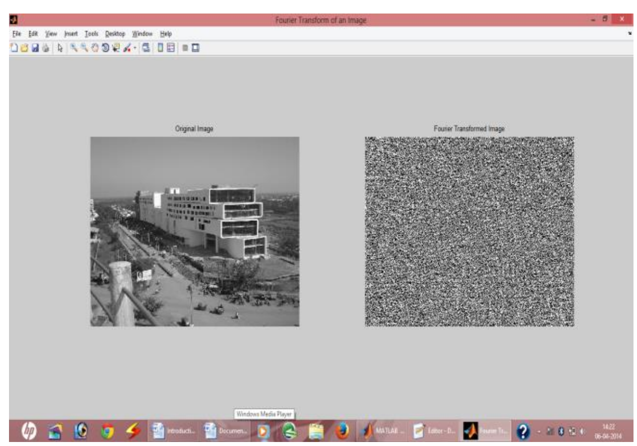

Fig. 5. Fourier Transform of an Image

Fig. 6. Ideal Low-Pass Filtered Image

In Fig. 6., filtering center component is responsible for blurring. The circular components are responsible for the ringing effects. The severe ringing effect in the blurred images is a characteristic of ideal filters.

Fig. 7. Butterworth Low-Pass Filtered Image

In Fig. 7., the BLPF with less number of orders does not have any ringing effect. As the order increases BLPF results in increasing ringing effects. Less ringing effect is due to the filter's smooth transition between low and high frequencies.

Fig. 8. Gaussian Low-Pass Filtered Image

In Fig. 8., there is no ringing effect of the GLPF. Ringing artifacts are not acceptable in fields like medical imaging. Hence Gaussian low-pass filter is used more instead of the ILPF/BLPF.

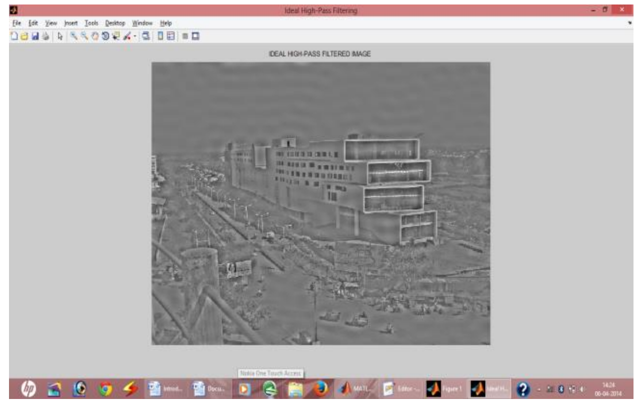

Fig. 9. Ideal High-Pass Filtered Image

The severe ringing effect in Fig. 9. is a characteristic of ideal filters. It is due to the discontinuity in the filter transfer function. Ringing effect in this filter is so severe that it produces distorted and thickened object boundaries.

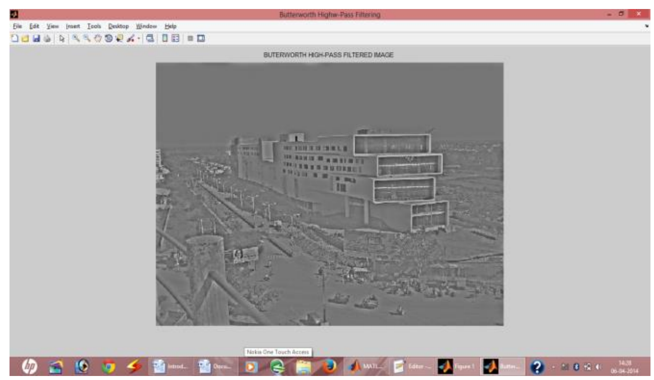

Fig. 10. Butterworth High-Pass Filtered Image

Boundaries in Fig. 10. are much less distorted compared to IHPF. This is more appropriate for image sharpening than the ideal HPF, since this not introduce ringing.

Fig. 11. Gaussian High-Pass Filtered Image