

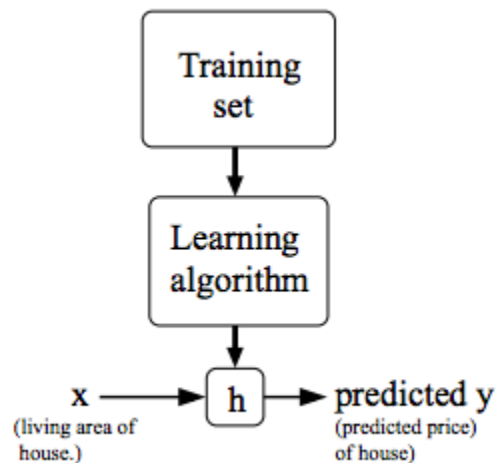
Linear Regression using Single Variable / Univariate Linear Regression :

In this we will have data set (training set) to build a model that find the relation between our input variable, X and output Variable y, so that our model predicts the value of y for unseen value of X.

Motivation Example : To predict housing price based on the size in sq. Ft.

Size in sq. Ft (X)	Price(\$) in 1000's (y)
2104	460
1416	232
1534	315
852	178
.....
.....

Model Representation:



Notations :

$X \rightarrow$ training data set

$Y \rightarrow$ output

$m \rightarrow$ size of our training data set.

$(x^{(i)}, y^{(i)}) \rightarrow$ i^{th} training example.

$h \rightarrow$ hypothesis ($h : X \rightarrow y$).

Here hypothesis is the function that show how X and y values are related to each other.

Representation of h: $h_{\theta}(x) = \theta_0 + \theta_1 * x$.

Cost Function (J) :

In the above equation of h, the θ_0, θ_1 are parameters (learning parameters), now the problem is how to choose these parameters.

Idea : Our goal is to have values of the θ_0, θ_1 such that our $h_{\theta}(x)$ value should be close to the y value (we can check that while we are training).

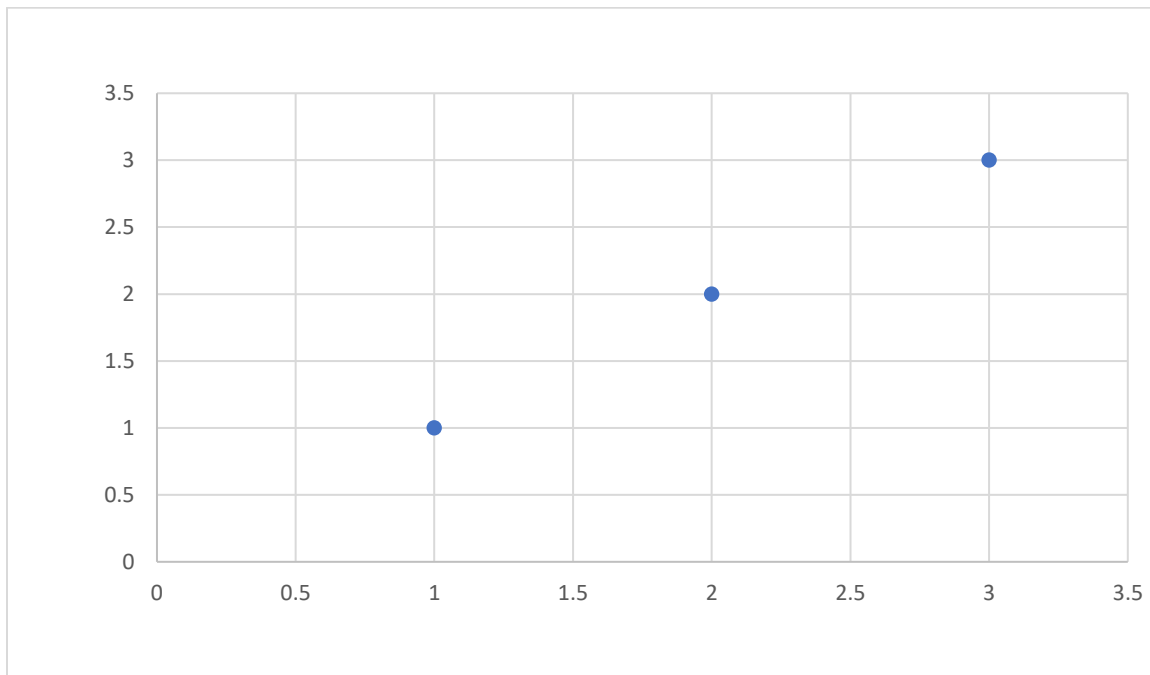
So, the difference $\rightarrow h_{\theta}(x) - y$ should be minimum, to achieve this we can use Error functions like Mean Square Error, Root Mean Square error,, etc.

$$\text{Mean Square Error} = J(\theta_0, \theta_1) = \frac{1}{2m} * \sum_{i=1}^m (h_{\theta}(x^i) - y^i)^2$$

Intuition:

For better understanding let us consider $\theta_0 = 0$, so our hypothesis would be $h_{\theta}(x) = \theta_1 * x$.

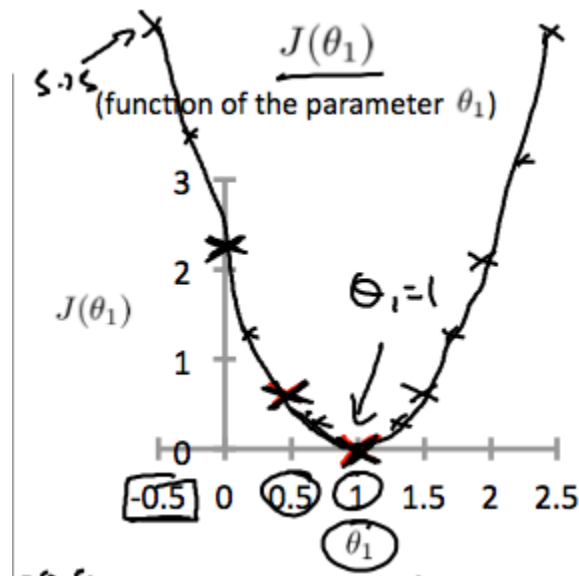
Consider the following data set.



Now calculate $h_{\theta}(x)$ for different values of θ_1 .

- For $\theta_1 = 0$, $h_{\theta}(x) = 0$ and cost function, $J(\theta_1) = 14/6$.
- For $\theta_1 = 1$, $h_{\theta}(x) = x$ and cost function, $J(\theta_1) = 0$.
- For $\theta_1 = 2$, $h_{\theta}(x) = 2x$ and cost function, $J(\theta_1) = 0.68$.

Now if we plot a graph between θ_1 and $J(\theta_1)$ then we can get a graph look like the following.



From the above graph, we can notice that the cost function is minimum at $\theta_1 = 1$.

In the similar way we can even include θ_0 in the minimization, but for visualization we need to use a 3-D plotting, or we can use contour plots.

Reference : <https://coursera.org/share/ca1bff16540cbeee4e7eb5fffd80d73e>

Instead of plotting the cost function over the parameters and manually figuring out what values of parameter will yield minimum cost function, but if we have multiple parameters the manual process will be difficult, so we need an efficient algorithm to automatically figuring out the right parameter values.

Gradient descent : It is Optimization algorithm used in machine learning to tweak the parameters iteratively to minimize the given cost function (differentiable function) to its local minimum.

Steps :

1. Start with some random values parameters.
2. Keep changing the parameters until we end up at a minimum value of cost function.

Algorithm :

Repeat until converges {

$$\theta_j = \theta_j - \alpha \frac{\partial J}{\partial \theta} J(\theta_0, \theta_1, \dots, \theta_n)$$

}

We must simultaneously update the parameter values. Suppose we have 2 parameters θ_0, θ_1 then we must update the parameters as follows :

Correct: Simultaneous update

- $\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$
- $\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$
- $\theta_0 := \text{temp0}$
- $\theta_1 := \text{temp1}$

Incorrect:

- $\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$
- $\theta_0 := \text{temp0}$
- $\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$
- $\theta_1 := \text{temp1}$

In the incorrect one the updated θ_0 will be used in updating the θ_1 which is an error, so we follow simultaneous update in gradient descent algorithm.