

MNIST Digits Classification using SVM & PCA

Methodology:

- 1) Normalisation of Data : Finding min,max value in column and apply this formula to normalise data between -1 and 1.
$$\text{Norm}(X) = ((X-\text{min})/(\text{max}-\text{min}))*2 - 1$$
- 2) PCA : Finding the eigenvectors of Covariance Matrix of input X . Storing those vectors based on eigenvalues larger to smaller. Pick first K-eigenvectors and transform input X into k dimensions by **dot_product(X,K-comp)**
- 3) SupportVectorModel :

As we know soft SVM Primal Problem is

$$\min_{w,b,\epsilon} F(w,b,\epsilon) = \frac{\|w\|^2}{2} + C \sum_{n=1}^N \epsilon_n$$
$$\text{s.t. } y_n(w^T x_n + b) \geq 1 - \epsilon_n$$
$$\epsilon_n \geq 0$$

so, substituting ϵ_n in the objective function gives the new unconstrained objective.

$$\min_{w,b} f(w,b) = \frac{\|w\|^2}{2} + C \sum_{n=1}^N \max\{0, 1 - y_n(w^T x_n + b)\}$$

If we find gradients of $f(w,b)$ w.r.t w, b then we can get update equation.

$$\text{Condition} \Rightarrow y_i * [(x_i \cdot w_t) + b_t] \geq 1$$
$$dw_t \Rightarrow \begin{cases} w_t & \text{if condition} = \text{true} \\ w_t - C \cdot y_i \cdot x_i & \text{else} \end{cases}$$
$$db_t \Rightarrow \begin{cases} 0 & \text{if condition} = \text{true} \\ -C \cdot y_i & \text{else} \end{cases}$$

$$\therefore w_{t+1} = w_t - \text{learning rate} * dw_t$$
$$b_{t+1} = b_t - \text{learning rate} * db_t$$

Using this update equations we will update the weights for **num_iterations** times .

- 4) MultiClassSVM : Fit SupportVectorModel for all 10 classes pass the data by making y values as 1 for that class and -1 for remaining classes. Find these metric values for each class .

True Positive(i) = $\#(y==y_pred==i)$.

False Positive(i) = $\#(y!=i \ \&\& \ y_pred==i)$

False Negative(i) = $\#(y==i \ \&\& \ y_pred!=i)$

Precision(i) = $\text{True Positive}(i) / (\text{True Positive}(i) + \text{False Positive}(i))$

Recall(i) = $\text{True Positive}(i) / (\text{True Positive}(i) + \text{False Negative}(i))$

F1_Score(i) = $2 * \text{Precision}(i) * \text{Recall}(i) / (\text{Precision}(i) + \text{Recall}(i))$

Precision = Avg(Precision(i) for all i)

Recall = Avg(Recall(i) for all i)

F1_Score = Avg(F1_Score(i) for all i)

Plot the results with respect to K value.

Results

Learning_rate : 0.001

num_iters : 100000

k=5, accuracy=0.5008, precision=0.4827, recall=0.4842, f1_score=0.4281

k=10, accuracy=0.6993, precision=0.6654, recall=0.6803, f1_score=0.6535

k=20, accuracy=0.7744, precision=0.7854, recall=0.7534, f1_score=0.7478

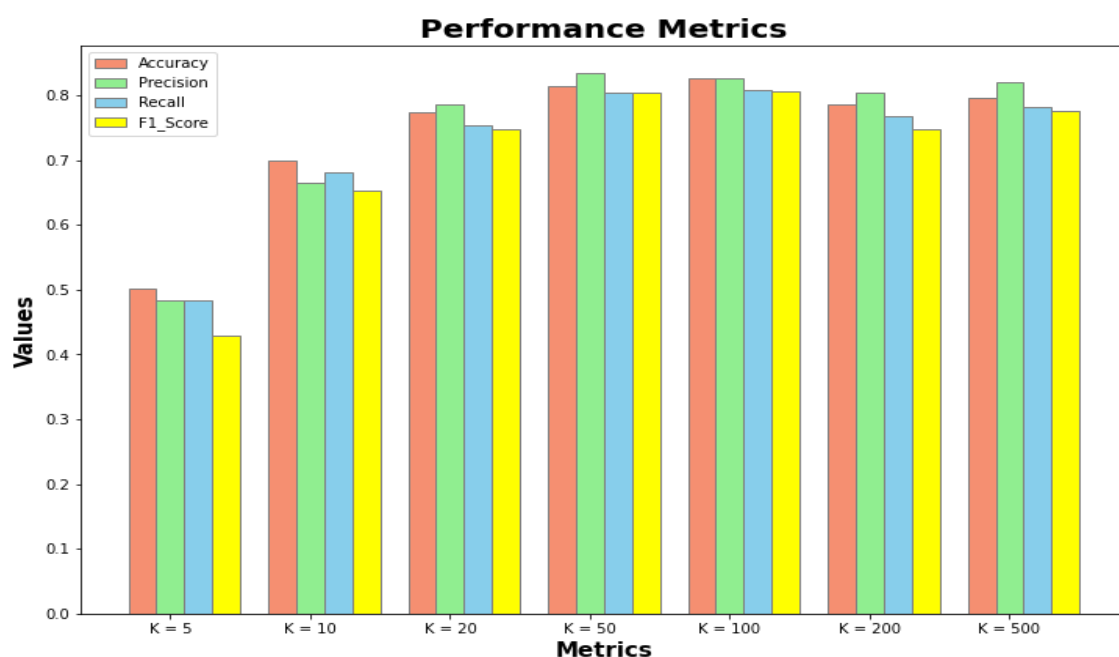
k=50, accuracy=0.8136, precision=0.835, recall=0.8037, f1_score=0.8033

k=100, accuracy=0.826, precision=0.8262, recall=0.8083, f1_score=0.8053

k=200, accuracy=0.785, precision=0.8032, recall=0.7684, f1_score=0.7477

k=500, accuracy=0.7951, precision=0.8193, recall=0.7817, f1_score=0.7759

Plot:



Analysis:

In my code hyper_parameter values are

learning_rate = 0.001

num_iters = 100000

C = 1.0

Here accuracy depends on num_iterations upto some number after that it will go to overfit so that it doesn't perform well on new samples .In data set training samples are 60000 so I took num_iterations = 100000 . We are performing stochastic gradient descent on the model so it will update the weights on every feature vector sample.In my results I got high accuracy in 100 components 82.6% . Here I am using a learning rate as 0.001 due to accuracy. I tried with 0.01 but it's highest accuracy among all components is 63.6%.These are my observations in this experiment