Projectile Motion

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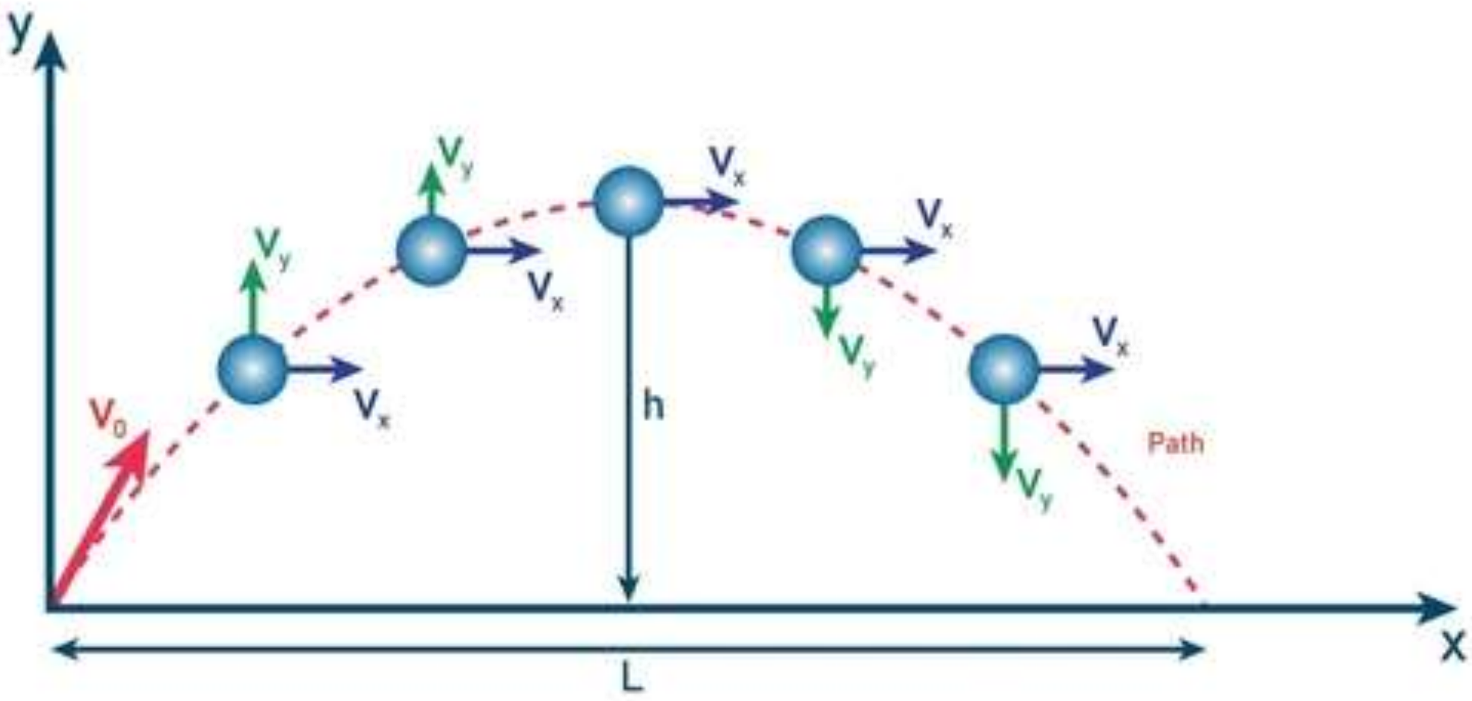
Connectomic Theory of Knowledge

- All these lessons will be handled in the light of the Connectomic Theory of Knowledge that I developed.
- The key principles are:
  - From Known to Unknown
  - Knowledge should be built up from scratch
  - No numerous unconnected formulas
- Inputs and Outputs
- From Real life applications to Engineering applications

Throwing a stone upwards

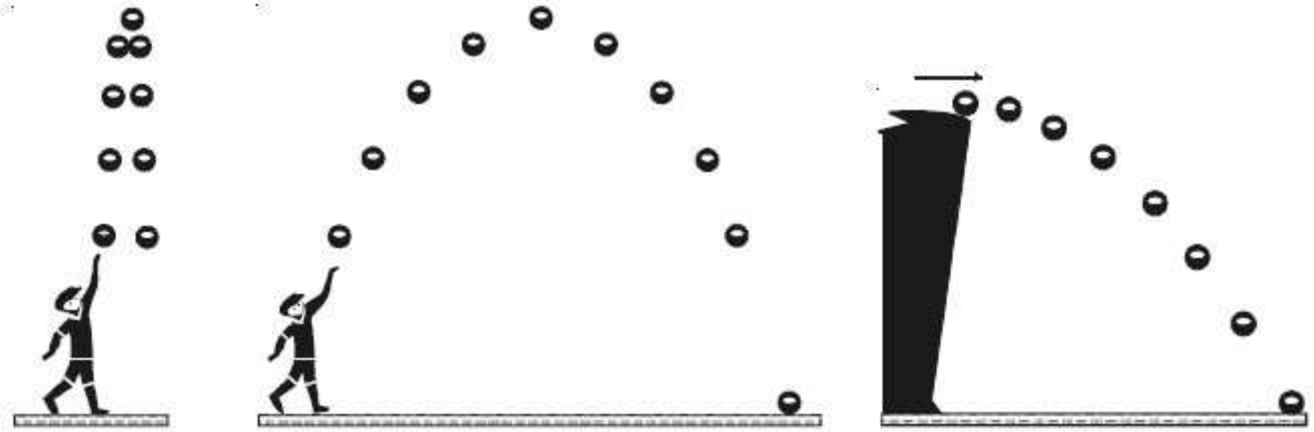
- If you throw a stone exactly 90 to the ground, it would be a motion in a single dimension.
- If you throw it at an angle the stone follows a parabolic path.
- It is called Motion in Two dimensions or motion in a Plane.
- In fact, it is motion in three dimensions, but for the sake of simplicity we discuss it in two dimensions.
- The Engineering applications of this are projecting a bullet or a cannon ball to exactly hit a target or projecting a rocket to reach the moon or to dock with a satellite, etc.



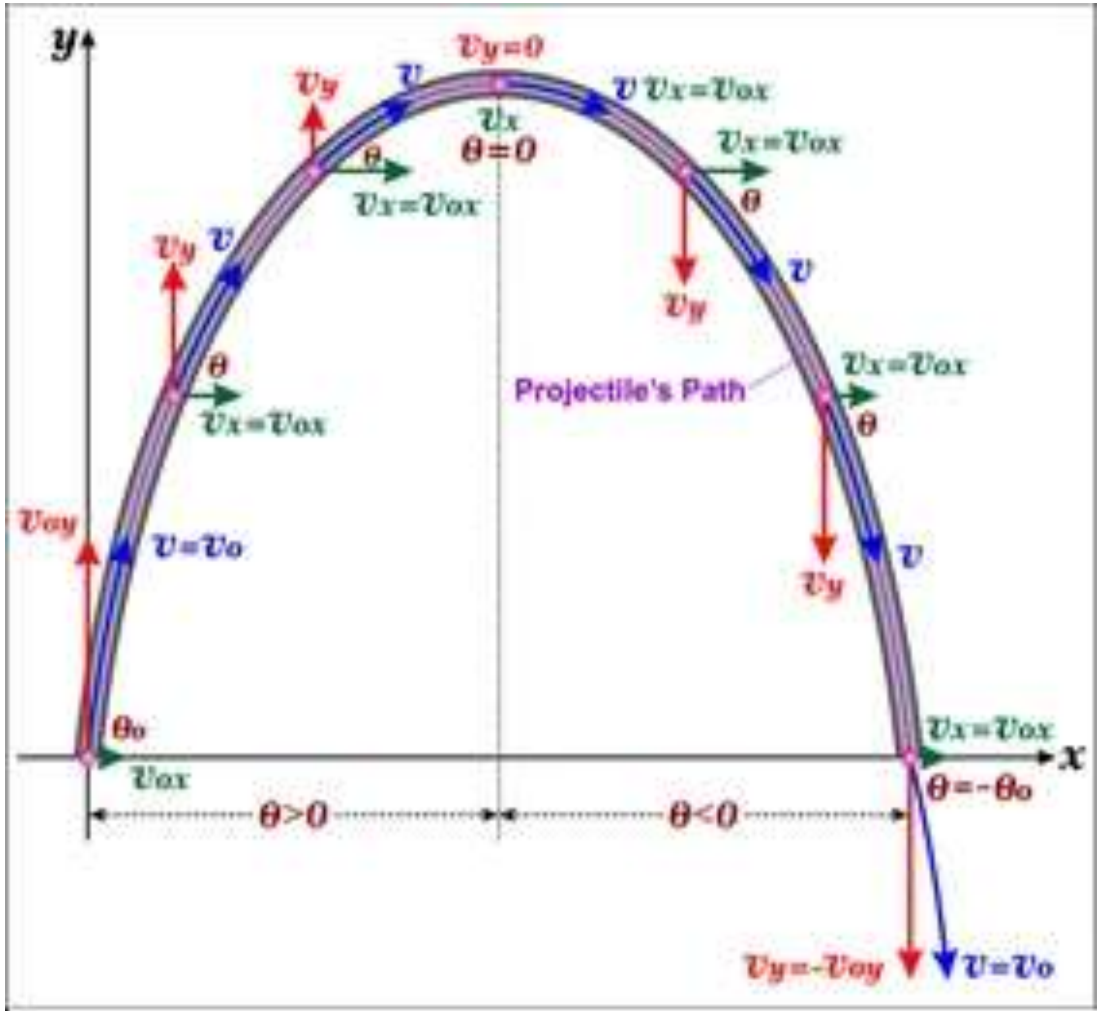
y
V_y
V_y
V_x
V_x
V_x
V_x
V_x
V_0
h
V_y
V_y
Path
x
L

Projectile Motion

- The height it reaches depends not only on the initial speed but also on the angle of projection.
- We need to deduce the maximum height, the horizontal range and the time taken to reach a particular point in space or to reach the ground.
- We also discuss the projection from the ground, from a height horizontally or again at an angle.



Different types of projectiles



y
x
vy=0
v
v vx=vox
vy
vx
θ=0
vx=vox
θ
vx=vox
θ
vy
v
vy
v
θ
vx=vox
vx=vox
Projectile's Path
θ
voy
v=vo
vy
v
θo
vox
vx=vox
θ=-θo
θ>0
θ<0
vy=-voy
v=vo

Inputs and Outputs

- The inputs to this topic are the time, displacement, velocity and acceleration.
- We also assume that the student has knowledge of some trigonometry and some basic calculus.
- Recall, that in an XY coordinate system cosθ is x and sin θ is y.
- Recall that any point in this universe can be represented by the three coordinates - x, y and z. These are represented, though unnecessarily, by the unit vectors - i, j, and k.
- Recall that we use the triple method of trigonometry in place but will use the standard notation also so that our students will not be out of place.

Central Point in Projectile Motion

- Projectile motion is influenced by gravity alone.
- Hence the vertical component of velocity decreases from the initial velocity to zero at the highest point.
- The horizontal component of velocity is a constant, dependent on the initial velocity and the angle only.
- The vertical component of acceleration g is a constant, but the horizontal component is zero.
- The time taken in either a free fall or in a projection is the same to reach the ground from the same height. Only the distance increases because of the initial velocity.

Variables in Two Dimensional Motion

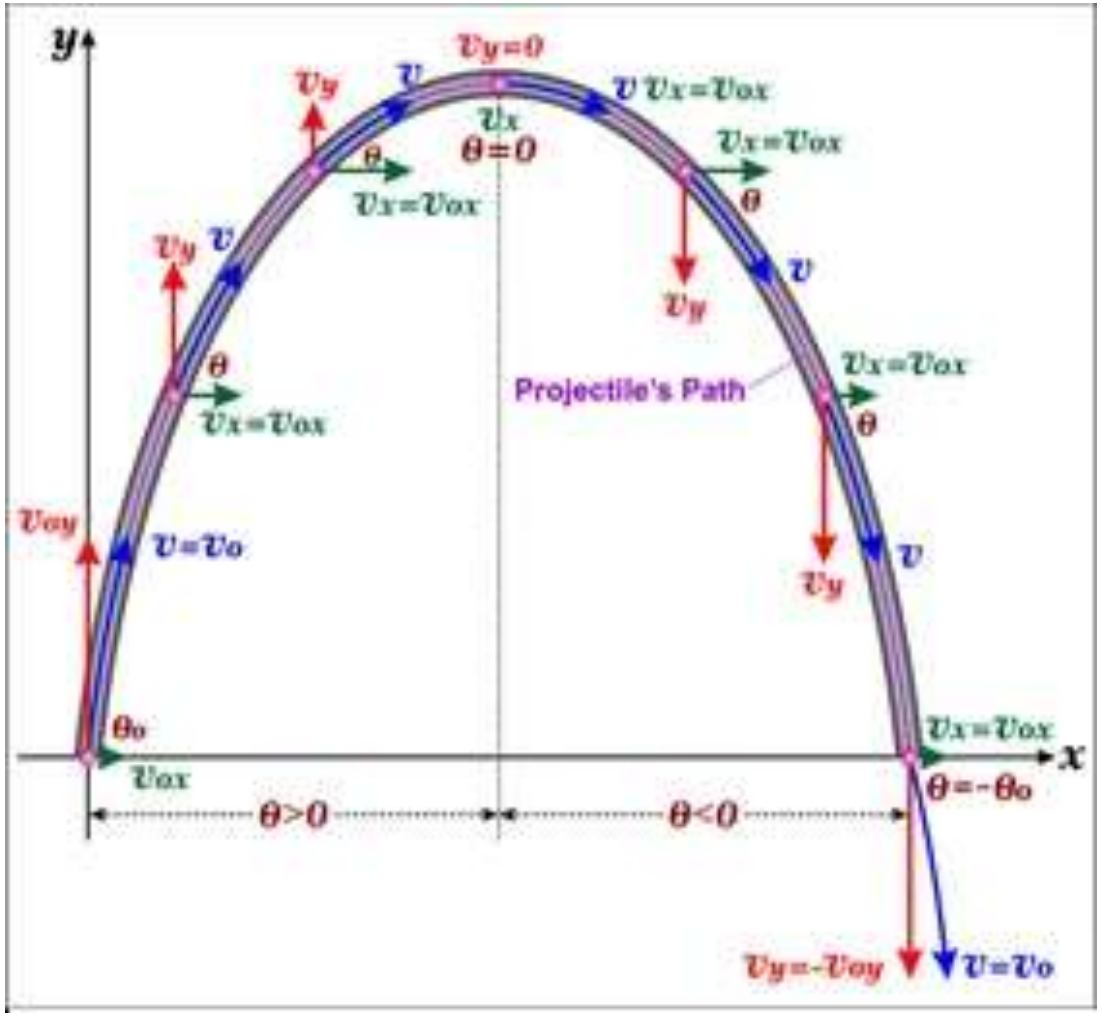
- The variables in One D Motion are time t, displacement s, velocity v and acceleration a. (t, s, v, a)
- Their counterparts in 2D motion are Total time of flight (tx) and Time to reach maximum height (t_y); Range of motion (S_x) and maximum height (S_y); horizontal acceleration (a_x) and Vertical acceleration (a_y or g).
- In summary t_x and t_y; s_x and s_y; a_x and a_y.
- Total time of flight $t_x=2t_y$

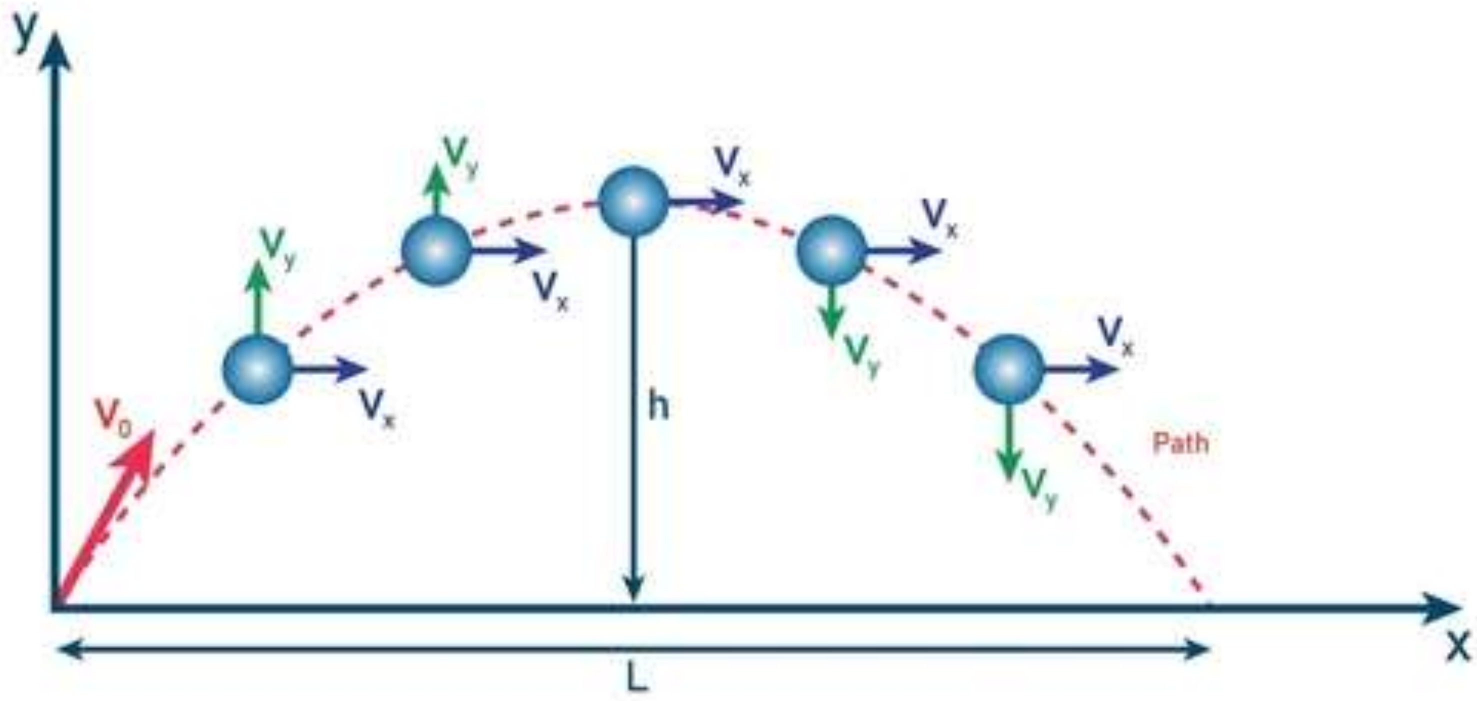
Known and Unknown

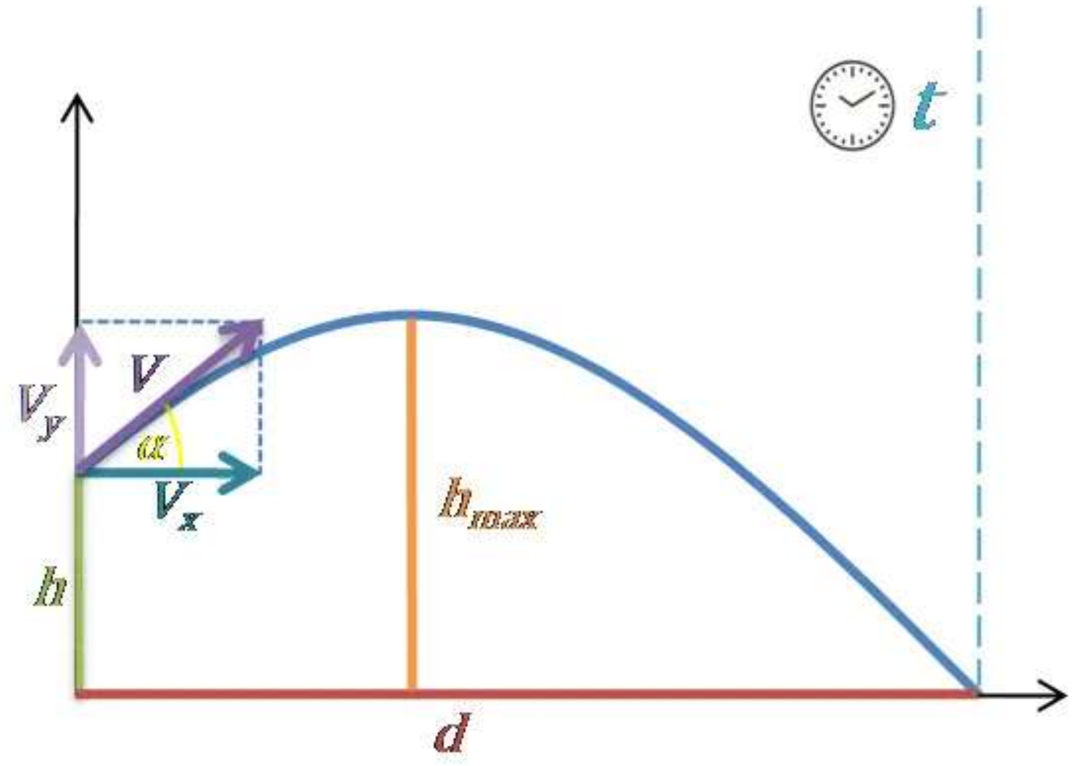
- If you know two variables you can calculate the other two.
- If you know the initial velocity and angle, you can calculate the time and distance in any axis.
- If you know the time and distance you can calculate the initial velocity and angle.
- Note.
  - The velocity would be the same at the same height, whether going up or down.
  - The time taken would be the same in a similar manner.
  - This is because the whole motion is under the effect of the same gravity.

Formulas

- in 1D motion: $v=\frac{s}{t}$, $s = \frac{\mathrm{j}}{\mathrm{k}} gt^2$, $s = v.t$, $t^2=\frac{\mathrm{k}s}{g}$ or $\frac{\mathrm{k}h}{g}$
- in 2D motion:
  - $s_x = v_x.t$ and $s_y = v_y.t$; $v_x = \frac{s_x}{t}$ and $v_y = \frac{s_y}{t}$, $t_y = \frac{v_y}{g}$, $t_y{}^2 = \frac{\mathrm{k}s_y}{g}$
- T = 2t
- The same thing in standard notation:
  - Range = Vcosθ.t = $\frac{v^{\mathrm{k}}\ sin\mathrm{k}\theta}{g}$
  - Max. height = Vsinθ.t = $\frac{v^{\mathrm{k}}\ sin^{\mathrm{k}}\theta}{\mathrm{k}g}$
  - time to max. height = $\sqrt{\frac{\mathrm{k}g}{h}}$
  - Total time of flight = 2t


y
x
$v_y=0$
v
v_y
v_x
$\theta=0$
θ
$v_x=v_{ox}$
v $v_x=v_{ox}$
v_y
v
Projectile's Path
v_{oy}
$v=v_o$
θ_o
v_{ox}
$\theta>0$
$\theta<0$
$\theta=-\theta_o$
$v_y=-v_{oy}$
$v=v_o$


y
Vy
Vy
Vx
Vx
Vx
Vx
Vx
Vy
Vy
V0
h
Path
x
L



V – velocity

V_x – horizontal velocity

V_y – vertical velocity

α – angle of launch

h – initial height

t – time of flight

d – distance (range)

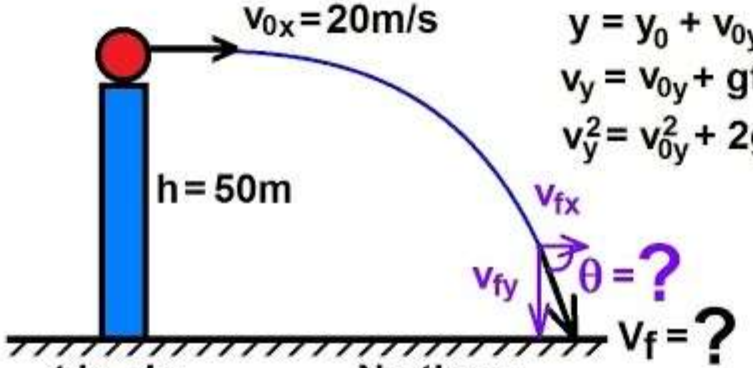
h_{max} – maximum height

2-D Motion - Final Velocity = ?

$$y = y_0 + v_{0y}t + \frac{1}{2}gt^2$$

$$v_y = v_{0y} + gt$$

$$v_y^2 = v_{0y}^2 + 2g\,\Delta y$$



t in air:

$0 = 50 + 0 - 4.9t^2$

No time:

$v_y^2 = 0 + 2(-9.8)(-50)$

Kinematic Equations for Constant Acceleration in Two Dimensions

$$v_x = v_{x0} + a_x t \qquad v_y = v_{y0} + a_y t$$

$$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2 \qquad y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

$$v_x^2 = v_{x0}^{+}2a_x(x - x_0) \qquad v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$$

Kinematic Equations for Projectile Motion

$$v_x = v_{x0} \qquad v_y = v_{y0} - gt$$

$$x = x_0 + v_{x0}t \qquad y = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

Range Equation

$$R = \frac{v_0^2 \sin(2\theta_0)}{g}$$

The subscript 0 means "at $t = 0$."

TABLE 3–1 General Kinematic Equations for Constant Acceleration in Two Dimensions

***x* component (horizontal)**		***y* component (vertical)**
$v_x = v_{x0} + a_x t$	(Eq. 2–11a)	$v_y = v_{y0} + a_y t$
$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$	(Eq. 2–11b)	$y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

We can simplify Eqs. 2–11 to use for projectile motion because we can set $a_x = 0$. See Table 3–2, which assumes y is positive upward, so $a_y = -g = -9.80\ \mathrm{m/s^2}$.

TABLE 3–2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0$, $a_y = -g = -9.80\ \mathrm{m/s^2}$)

Horizontal Motion ($a_x = 0$, v_x = constant)		**Vertical Motion[†] ($a_y = -g$ = constant)**
$v_x = v_{x0}$	(Eq. 2–11a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0} t$	(Eq. 2–11b)	$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$
	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

[†] If y is taken positive downward, the minus ($-$) signs in front of g become + signs.

PROJECTILE MOTION:

VELOCITY COMPONENTS

ALONG X- AXIS (HORIZONTAL MOTION)

DISPLACEMENT : $s_x = u \cos\theta\, t$

VELOCITY : v_x = CONSTANT= $u \cos\theta$

ACCELERATION : a_x = ZERO

ALONG Y- AXIS (VERTICAL MOTION)

DISPLACEMENT : $s_y = u \sin\theta\, t - \frac{1}{2} g t^2$

VELOCITY : $v_y = u \sin\theta - gt$

ACCELERATION : $a_y = -g$

ALITER →

DISPLACEMENT : $s_x = U_0 t$

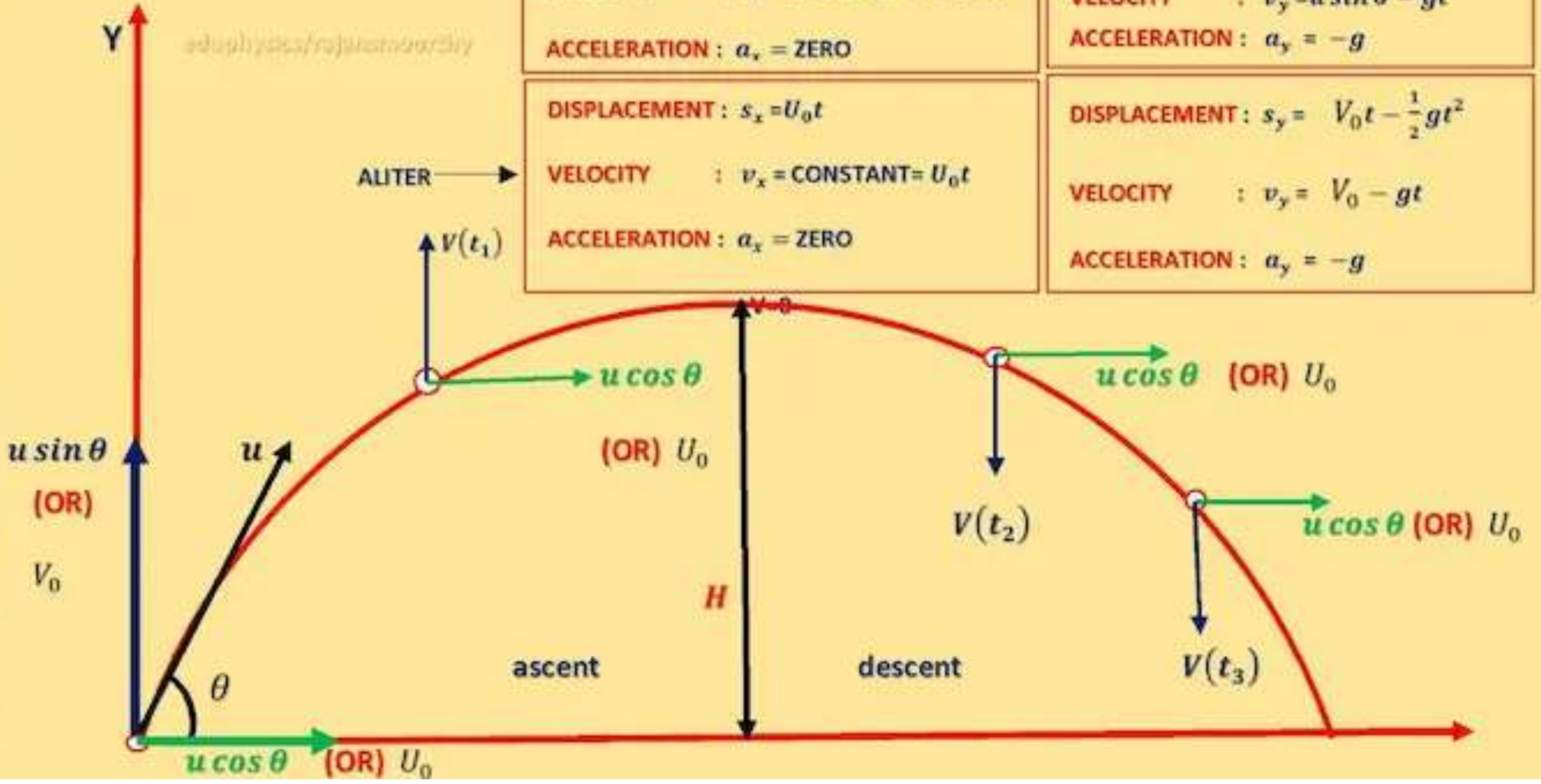
VELOCITY : v_x = CONSTANT= $U_0 t$

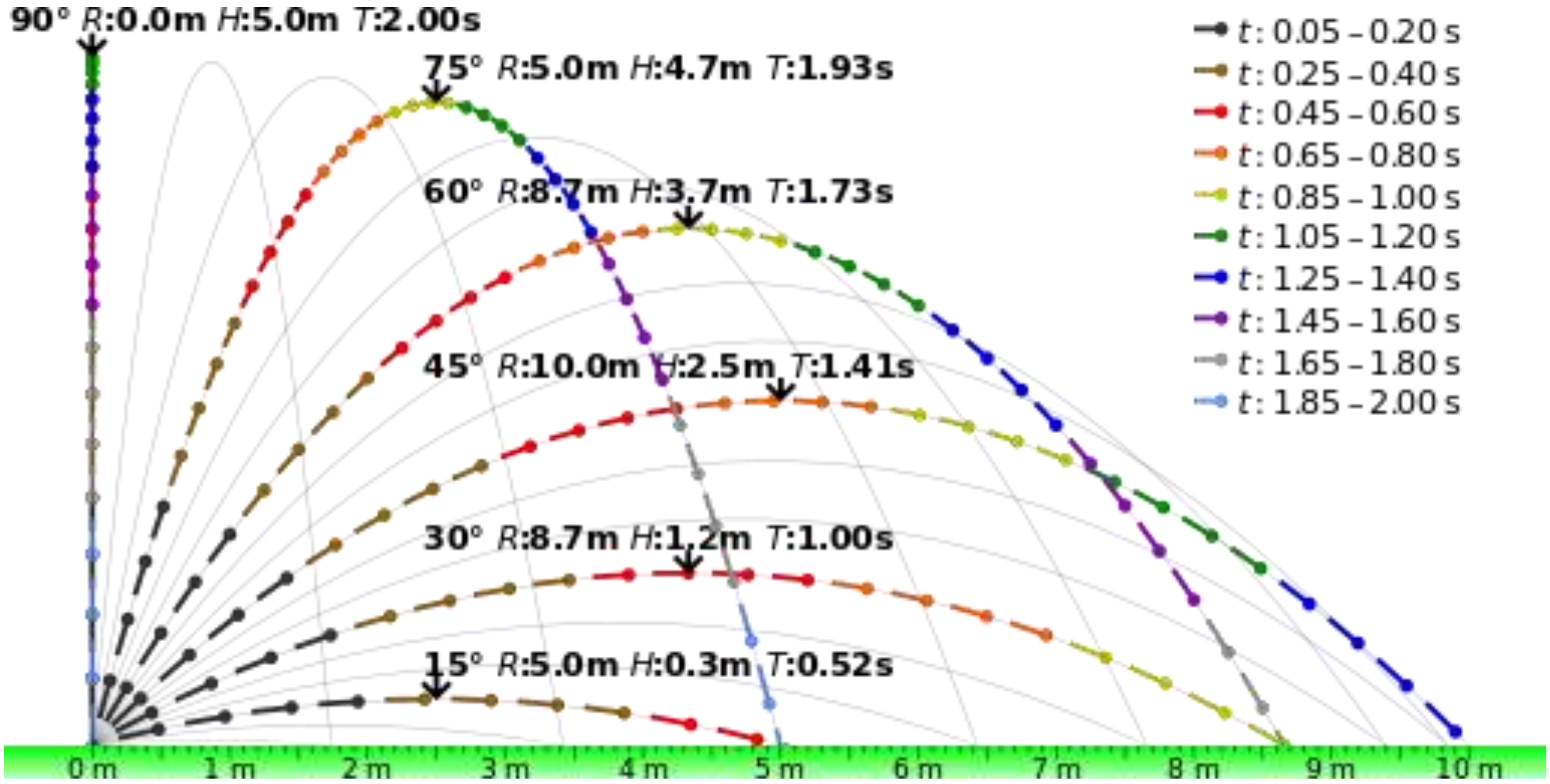
ACCELERATION : a_x = ZERO

DISPLACEMENT : $s_y = V_0 t - \frac{1}{2} g t^2$

VELOCITY : $v_y = V_0 - gt$

ACCELERATION : $a_y = -g$




90° R:0.0m H:5.0m T:2.00s
75° R:5.0m H:4.7m T:1.93s
60° R:8.7m H:3.7m T:1.73s
45° R:10.0m H:2.5m T:1.41s
30° R:8.7m H:1.2m T:1.00s
15° R:5.0m H:0.3m T:0.52s
t: 0.05 – 0.20 s
t: 0.25 – 0.40 s
t: 0.45 – 0.60 s
t: 0.65 – 0.80 s
t: 0.85 – 1.00 s
t: 1.05 – 1.20 s
t: 1.25 – 1.40 s
t: 1.45 – 1.60 s
t: 1.65 – 1.80 s
t: 1.85 – 2.00 s
0 m
1 m
2 m
3 m
4 m
5 m
6 m
7 m
8 m
9 m
10 m

Let’s Summarize

- v=s/t, a=v/t,s=v.t
- t=s/v = v/a=as/v^2
- In the context of 2D motion they transform into:
  - t=Vy/g and T=2Vy/g
  - Sy=Vy.t = Vy.Vy/g = Vy^2/g
  - Sx = Vx.t = Vx. Vy/g = V^2.2xy/g
- These are the only formulas you need to know.
- In standard form:
  - t = Vsinθ/g and T = 2Vsinθ/g
  - h = V^2sin^2θ/g
  - R = V^2sin2θ/g
  - V_t = Vsinθ − gt
  - θ = $\tan^{-1}\frac{V_y}{V_x}$

Calculate the time to reach Maximum height (t) and Total time of flight (T) for a cannon ball projected at 45 degrees at different velocities. $g=10m/s^2$

Initial Velocity	time to reach max. height t	Total time of flight T
100m/s		
200m/s		
300m/s		
400m/s		
500m/s		

Hint

Calculate the Maximum height and Range for a cannon ball projected at 45 degrees at different velocities. $g=10m/s^2$

Initial Velocity	Maximum Height	Range
100m/s		
200m/s		
300m/s		
400m/s		
500m/s		

Hint

Calculate the time to reach Maximum height (t) and Total time of flight (T) for a cannon ball projected at 100 m/s but at different angles. $g=10m/s^2$

Initial Velocity	time to reach max. height t	Total time of flight T
45		
90		
30		
60		

Hint

Calculate the Maximum height and Range for a cannon ball projected at 45 degrees at different velocities. $g=10m/s^2$

Initial Velocity	Maximum Height	Range
100m/s		
200m/s		
300m/s		
400m/s		
500m/s		

Hint

Calculate the angle of projection for a constant speed of 100 m/s given the Total time of flight for a cannon ball. $g=10m/s^2$

Total time of flight	Range	Angle of Projection

Hint

Calculate the Time of flight for a constant speed 100 m/s given the angle of projection for a cannon ball. $g=10m/s^2$

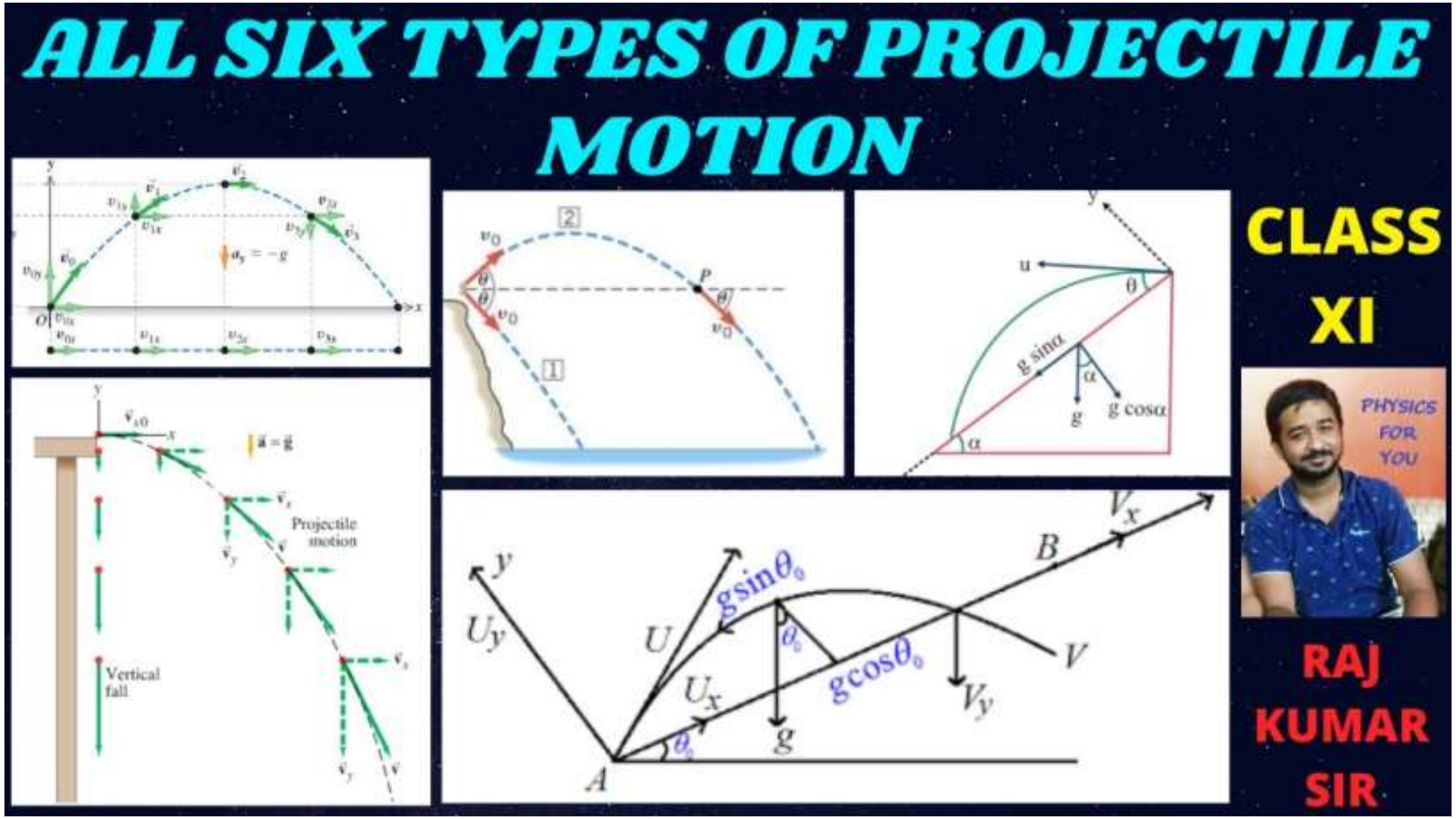
Angle of projection	Time of flight
90 degrees	
45	
30	
60	

Hint

Calculate the angle of projection for a constant speed 100 m/s given the Time of flight for a cannon ball. $g=10m/s^2$

Time of flight	Angle of Projection

Hint


ALL SIX TYPES OF PROJECTILE MOTION
CLASS XI
PHYSICS FOR YOU
RAJ KUMAR SIR
Projectile motion
Vertical fall
g sinα
g cosα
gsinθ₀
gcosθ₀

Horizontal Projection from a Height

- It is very similar to the second part of the projectile motion from the ground.
- S_y would be the height of the tower.
- t = $\sqrt{\frac{\mathrm{k}g}{h}}$
- S_x = V_x.t = V_x. $\sqrt{\frac{\mathrm{k}h}{g}}$
- V_y = $\sqrt{2gh}$

Calculate the time and range of flight when a ball is projected horizontally from a height of 100 metres at different velocities

Velocity	Time of flight	Range of flight
100 m/s		
200 m/s		
300 m/s		
400 m/s		

Calculate the velocity when a ball is projected horizontally from a height of 100 metres and reaches different ranges

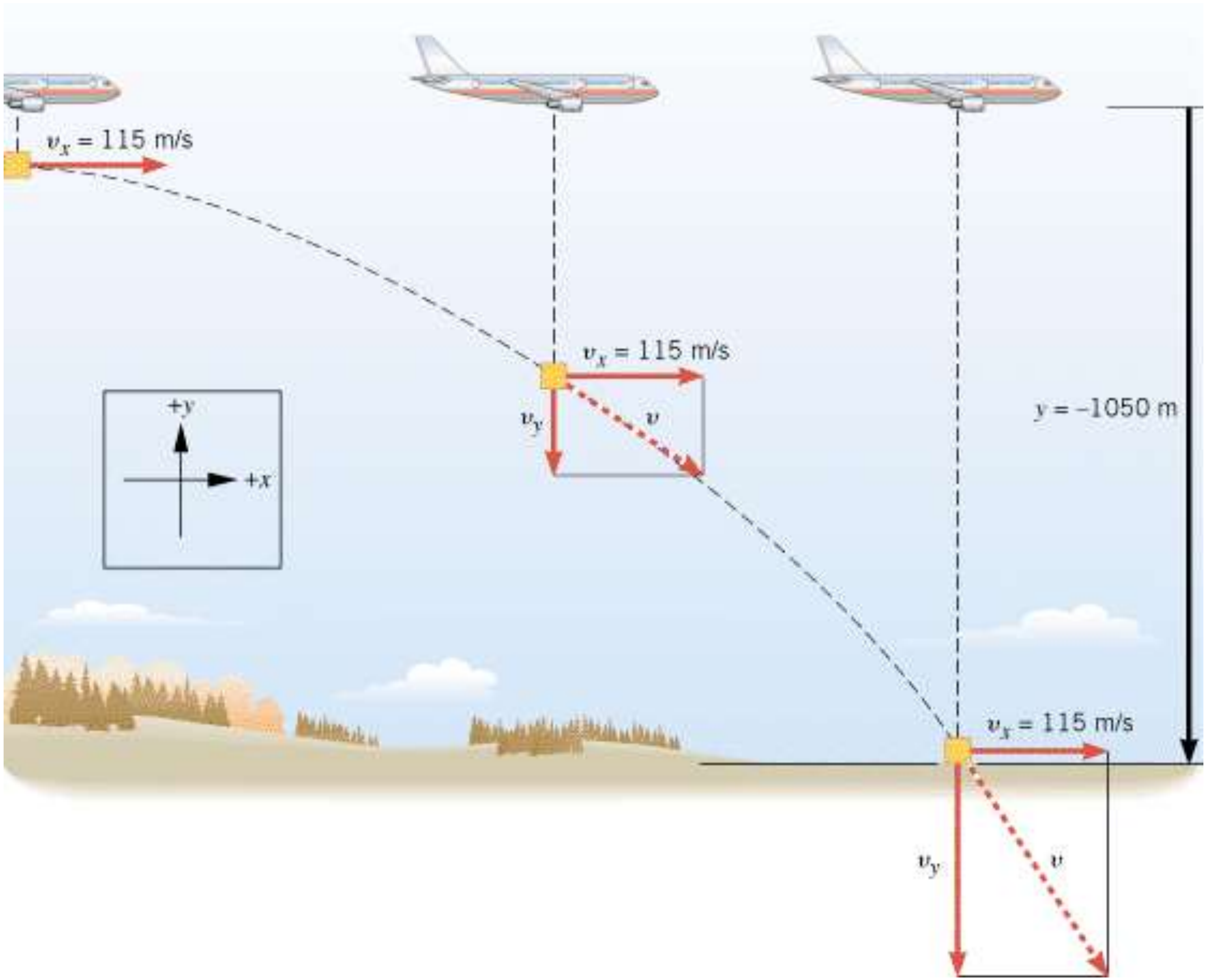
Range	Time of flight	Velocity
100 m		
200 m		
300 m		
400 m		

Calculate the height of projection when a ball is projected horizontally from at a velocity of 100 m/s and reaches different ranges.

Range	Time of flight	Height of projection
100 m		
200 m		
300 m		
400 m		

Inclined Projection from a Height

- It is very much similar to the previous discussion until it reaches the same horizontal level as of projection.
- But it has to travel an additional distance to reach the ground.
- This additional distance is the same as if it was a horizontal projection from a height. The V_x would be the same as the V_x of the initial projection.



v_x = 115 m/s
v_x = 115 m/s
v_y
v
+y
+x
y = −1050 m
v_x = 115 m/s
v_y
v

Inputs and Outputs

- Inputs
  - v = $\frac{s}{t}$
  - a = $\frac{v}{t}$
  - s = v.t
  - v = a.t
  - t = $\frac{s}{v} = \frac{v}{a} = \frac{v}{g} = \sqrt{\frac{\mathrm{k}h}{g}}$

- Outputs
- V = $\sqrt{{v_x}^{\mathrm{k}} + {v_y}^{\mathrm{k}}}$
- S_x = Vx.T = $\frac{v \sin \mathrm{k}\theta}{g}$
- S_y = Vy.t = $\frac{v^{\mathrm{k}}\, sin^{\mathrm{k}}\theta}{g}$
- t = $\frac{V_y}{g} = \sqrt{\frac{\mathrm{k}h}{g}}$
- θ = $\tan^{-\mathrm{j}} \frac{V_y}{V_x}$

Key Insights

- Gravity is the only force acting on a projectile.
- The vertical velocity changes from v to zero at the hgihest point and reaches the same v when it strikes the ground. The vertical acceleration is constant $\pm$g.
- The horizontal velocity is unaffected as it is uniformly affected by gravity. Horizontal acceleration is zero.
- The vertical velocity is the same at the same horizontal level.
- The time taken to travel downwards from a height is the same as it is in a free fall. But the distance increases as it is projected with some horizontal velocity.
- The angle of projection (during upward journey) is the same as the angle of incidence (downward journey).
- Calculating the time simplifies the matter and uses few formulas.