Assignment-3 (Part-1)

1) $T(n) = \begin{cases} 1 & n = 1 \\ 2T(n/2) + n & n > 1 \end{cases}$

$T(n) = 2T(n/2) + n$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2}/2\right) + \frac{n}{2}$

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{n}{2}$

$T\left(\frac{n}{2}\right) = \frac{T(n) - n}{2}$

$\frac{T(n) - n}{2} = 2T\left(\frac{n}{4}\right) + \frac{n}{2}$

$T(n) - n = 4T\left(\frac{n}{4}\right) + n$

$T(n) = 4T\left(\frac{n}{4}\right) + 2n$

$T(n) = 4T\left(\frac{n}{4}\right) + 2n$

$T(n) = 2^2 T\left(\frac{n}{2^2}\right) + 2n$

$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3n$

$\vdots$

$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$

$\frac{n}{2^k} = 1 \Rightarrow n = 2^k$

$\log_2 n = \log_2 2^k$

$\log_2 n = k$

$$k = \log_2 n$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$$

$$T(n) = 2^{\log_2 n} T(1) + \log_2 n \cdot n$$

$$= n^{\log_2 2} \cdot 1 + n \log_2 n$$

$$= n + n \log_2 n$$

$$\boxed{O(n \log n)}$$

2) $T(n) = \begin{cases} 1 & n = 1 \\ 8T\left(\frac{n}{2}\right) + n^2 & n > 1 \end{cases}$

$T(n) = 8T\left(\frac{n}{2}\right) + n^2$

$T\left(\frac{n}{2}\right) = 8T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^2$

$T\left(\frac{n}{2}\right) = 8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2$

$\frac{T(n) - n^2}{8} = T\left(\frac{n}{2}\right)$

$\frac{T(n) - n^2}{8} = 8T\left(\frac{n}{2^2}\right) + \left(\frac{n}{2}\right)^2$

$T(n) - n^2 = 8T\left(\frac{n}{2}\right) + \left(\frac{n}{2}\right)^2$

$T(n) - n^2 = 8 \cdot 8T\left(\frac{n}{2^2}\right) + 8\left(\frac{n}{2}\right)^2$

$T(n) = 8^2 T\left(\frac{n}{2^2}\right) + 8\left(\frac{n}{2}\right)^2 + n^2$

$T(n) = 8^2 T\left(\frac{n}{2^2}\right) + \frac{8 \cdot n^2}{4} + n^2$

$T(n) = 8^2 T\left(\frac{n}{2^2}\right) + 3n^2$

$T(n) = 8^k \left(T\left(\frac{n}{2^k}\right) + \left(2^k \cdot 1\right) n^2\right)$

$$T(n) = 8^k T\left(\frac{n}{2^k}\right) + (2^k - 1)n^2$$

$$\frac{n}{2^k} = 1 \Rightarrow k = \log n$$

$$T(n) = 8^{\log_2 n} T(1) + (2^{\log_2 n} - 1)n^2$$

$$= 2^{3\log_2 n} + (n^{\log_2 2} - 1)n^2$$

$$= n^3 + (n-1)n^2$$

$$\boxed{T(n) = O(n^3)}$$