Assignment - 2

1) $f(n) = n - 10$

$g(n) = n + 10$

$f(n) = \Theta(g(n))$

$f(n) \geq c \cdot g(n)$

$n - 10 \geq c \cdot (n + 10)$

Consider $c = 1/2$

$n = 1000$

$1000 - 10 \geq 1/2 (1000 + 10)$

$990 \geq 1/2\ 1010$

Satisfied

$f(n) \leq g(n) \cdot c$

$c = 1$

$n - 10 \leq n + 10$ ✓

Satisfied

$f(n) = \Theta(g(n))$

2) $f(n) = n$

$g(n) = n$

$f(n) = \Theta(g(n))$

$f(n) \leq c \cdot g(n)$

$n \leq c \cdot n$

$c = 1/2$

$n \leq \frac{n}{2}$

$n = 1$

$1 \leq 1/2$

$f(n) \geq g(n) \cdot c$

$n \geq c \cdot n$

$c = 2$

$n = 1$

$1 \geq 2 \cdot 1$

$f(n) = \Theta(g(n))$

3) $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$

$f(n) \leq c \cdot g(n)$

$2^{6\log_2 n} \cdot 2^{5\log_2 n} = n^5$

$2^{(\log_2 n)} \cdot 2^{5\log_2 n} = n^5$

$n^{(\log_2 2)} \cdot n^{5\log_2 2} = n^5$

$n^6 \cdot n^5 = n^5$

$n^{11} > n^5$ ×

$f(n) \neq O(g(n))$

4) $\frac{4^n}{2^n} = O(2^n)$

$f(n) \leq c \cdot g(n)$

$\frac{4^n}{2^n} \leq c \cdot 2^n$

$2^{n-n} \leq c \cdot 2^n$

$2^n \leq c \cdot 2^n$

$c = 1$

$2^n < 1 \cdot 2^n$

satisfied

$f(n) = O(g(n))$

3) $128^{\log_2 n} \cdot n^2 = \Theta(n^9)$

$2^{7\log_2 n} \cdot n^2 = \Theta(n^9)$

$f(n) \leq c \cdot g(n)$

$n^7 \cdot n^2 \leq n^9 \cdot c$

$n^9 \leq c \cdot n^9$

$c = 1$ Satisfied

$f(n) \geq c \cdot g(n)$

$n^9 \geq 2 \cdot n^9$

$c = 2$.

Satisfied

$f(n) = \Theta(g(n))$