Assignment - 4

1) $T(n) = 2T\left(\frac{n}{2}\right) + n$

Masters theorem $\qquad f(n) = \Theta(n^k \log^p n)$

$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$

$a = 2, \quad b = 2, \quad k = 1, \quad p = 0$

$\log_b^a = \log_2^2 = 1$

Case 2: $p > -1 \Rightarrow \Theta(n^k \log^{p+1} n)$

$$\boxed{\Rightarrow \Theta(n \log n)}$$

2) $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$f(n) = \Theta\left(n^k \log^p n\right)$

$a = 2, b = 2, k = 1, p = 1$

$\log_b^a = 1 = k$

Case 2: $p > -1$

$\Theta\left(n^k \log^{p+1} n\right)$

$\Theta\left(n \log^{1+1} n\right)$

$\Theta\left(n \log^2 n\right)$

3) $T(n) = 2T\left(\frac{n}{2}\right) + n^2$

$a = 2, b = 2, k = 2, p = 0$

$\log_b^a = \log_2^2 = 1 < k$

Case 3: $\log_b^a < k$

$p \geq 0 \Rightarrow n^k \log^p n$

$\Rightarrow n^2 \log n$

$\Theta(n^2 \log n)$

A) $T(n) = 8T(n/2) + n^2$

$a = 8, b = 2, K = 2, P = 0$

$\log_b^a = \log_2^8 = 3 \log_2^2 = 3 > K$

Case 1: $\log_b^a > K$

$\Theta(n^{\log_b^a})$

$\Theta(n^{\log_2^{2^3}})$

$\boxed{\Theta(n^3)}$