Matched Filtering

- Data $\boldsymbol{y}$ with a perfectly known covariance matrix C We have a model $\boldsymbol{y} = A\boldsymbol{m}$, which is linear in one parameter, the amplitude A, and has a fixed shape $\boldsymbol{m}$.

- If indeed $\boldsymbol{y} = A\boldsymbol{m}$ **+ Gaussian Noise (as per C),** there exists an estimator $\hat{A}$ such that:

$$\hat{A} = \frac{(C^{-1}m) \cdot y}{(C^{-1}m) \cdot m}$$

Obtained from a maximum likelihood estimate (derivation included separately)

- With a variance given as (assuming Gaussian noise propagation):

$$\sigma^2_{\hat{A}} = \frac{(C^{-1}m)^T C (C^{-1}m)}{[(C^{-1}m) \cdot m]^2} = \frac{1}{(C^{-1}m) \cdot m}$$

- Therefore, we can derive the SNR as:

$$S/N = A\sqrt{(C^{-1}m) \cdot m}$$

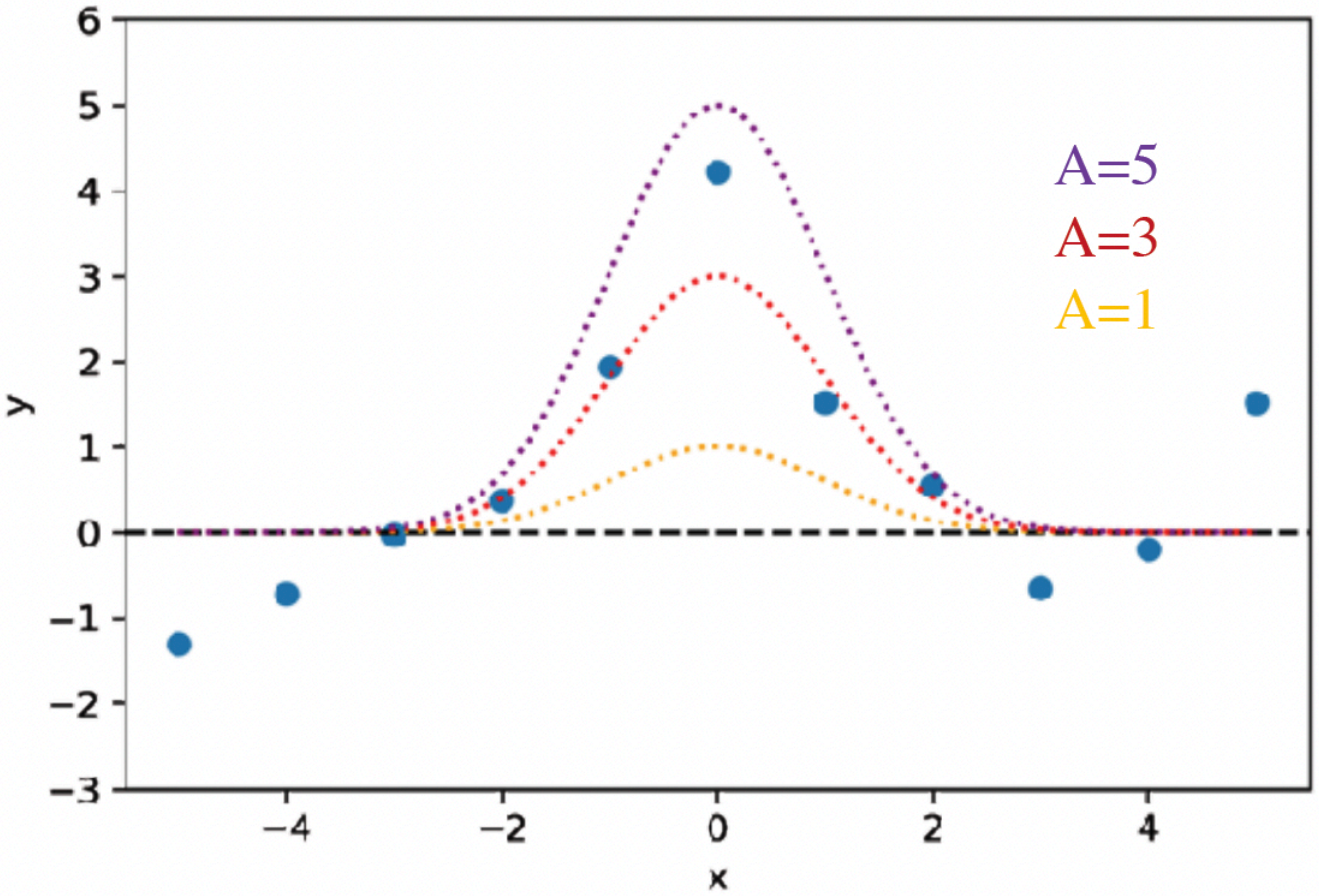

Image credit: Daniel Gruen's lecture slides

From SNR to chi-square values

- Replacing ***m*** with ***y*** in A and σ_A gives:

$$S/N = 1.\sqrt{(C^{-1}y) \cdot y}$$

- Squaring, we get:

$$(S/N)^2 = (C^{-1}y) \cdot y = \chi^2$$

However, we lose information about m here, which is crucial to the maximum likelihood estimate, and in turn, the values of A and σ_A. We also encode the Gaussian noise directly into ***y*** instead of having the explicitly encoded signal shape in ***m***.In this case, we can only have an agnostic prediction about the presence of a signal in the data instead of the optimal significance.