Worksheet 4: Numerical solutions of ODEs

If you are using Julia or Python, we recommend using a jupyter notebook. In WeLearn, you need to submit this file. Please clearly indicate in the markup cells, the number of the question for which you are writing the program. Also, please remember to add documentation through comments in your program.

You may also use scripts and use REPL to evaluate them. In that case, please keep all your files for a particular worksheet in a folder and you may upload the compressed archive of that folder.

Please feel free to ask for help!

1. (8 points) Consider the equation
$$\frac{dx}{dt} = -xt$$
   (a) (4 points) Use Euler and Midpoint methods to solve the equations using $h = 0.01$ from $t_{\text{ini}} = 0$ to $t_{\text{final}} = 15$. The given initial condition is $x(0) = 1.0$. Plot the solutions along with the exact solution.
   (b) (2 points) Now choose $h = 10.0^n$ for $n = -4$ to $n = -2$ in steps of 0.2. For each h, keep t_{final} fixed and solve the above equations using the two methods. For each h, for each method, estimate the error of the position value of the final point (absolute deviation from the exact solution).
   (c) (2 points) Plot and fit $log_{10} h$ versus log_{10} of the error with a straight line to estimate the errors of each of the method.

2. (12 points) Two coupled first-order equations
$$\frac{dy}{dt} = p$$
$$\frac{dp}{dt} = -4\pi^2 y$$
   define simple harmonic motion with period 1.
   (a) (8 points) Use Euler and Midpoint methods to solve the equations using $h = 0.01$ from $t_{\text{ini}} = 0$ to $t_{\text{final}} = 15$. The given initial condition is $y(0) = 1.0$ and $p(0) = 0.0$. Plot the solutions along with the exact solution.
   (b) (2 points) Now choose $h = 10.0^n$ for $n = -4$ to $n = -2$ in steps of 0.2. For each h, keep t_{final} fixed and solve the above equations using the two methods. For each h, for each method, estimate the error of the position value of the final point (absolute deviation from the exact solution).
   (c) (2 points) Plot and fit $log_{10} h$ versus log_{10} of the error with a straight line to estimate the errors of each of the method.