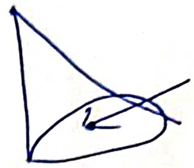
Scene Radiance & Image Irradiance



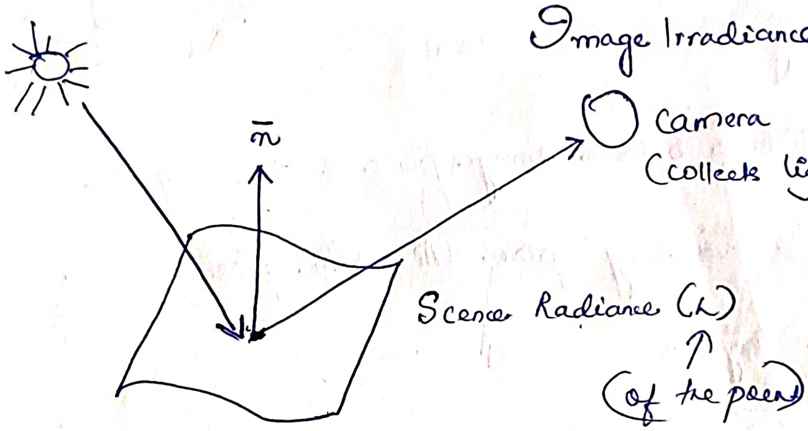
- Establish r/t b/t brightness on the point of the scene & its brightness in the image.

  " Scence Radiance → "brightness in the scene".

  " Image Irradiance is proportional to brightness in the Image "

  (flux falling per unit area in the Image)

Problem:



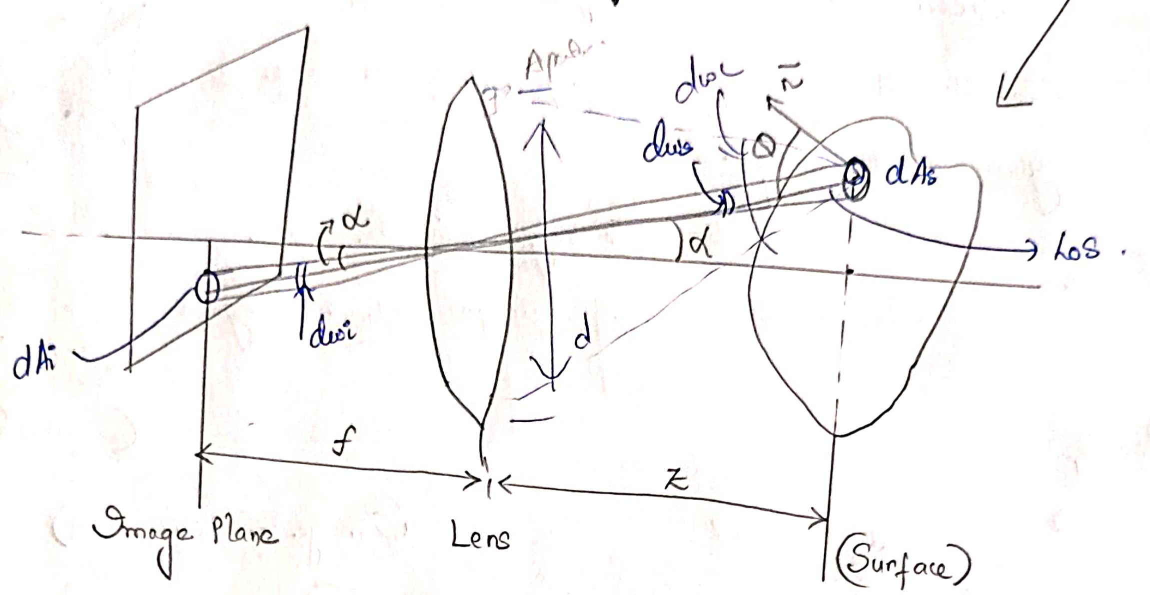
↳ Given a surface with a point where the light falls on, it has an orientation $\bar{n}$, This point is illuminated by the light source (Sunlight) & this produces a radiance in the direction of the camera. This scene Radiance is projected by the lens of the camera. The image plane produces, Image Irradiance / brightness (E).

Q) What is the Relationship between L & E? .

$L \rightarrow$ Scene Radiance of the point on the surface (3D Scene)

$E \rightarrow$ Brightness of image on the image plane. (2D).

Scene Radiance (L) & Image Irradiance:



$f \rightarrow$ distance between Image plane & Lens (effective focal length)

Intensity captured by pixel dA_i, This pixel observed dA_s (area) through a perspective projection

$dA_s \rightarrow$ has an orientation $\bar{n}$, and θ, wrt to line of sight.

→ LOS also makes angle 'α' wrt the optical lenses.

- $z \rightarrow$ depth of 'dA_s' wrt to lens.
- $d\omega_s \rightarrow$ Solid angle subtended by the patch (dA_s) wrt lens.

$d\omega_i \rightarrow$ " " " " " dA_i wrt lens.

Derivation

(Solid Angles are equal)

$$\boxed{d\omega_i = d\omega_s}$$

$$\boxed{\frac{dA_i \cdot \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cdot \cos\theta}{(z/\cos\alpha)^2}} \Rightarrow \boxed{\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2}$$

eqn (1)

$\frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2 \quad \frac{dA_s}{dA_i}$

↳ Solid angle subtended by the lens from the surface point (dA_s).

Let diameter of lens → d

$d\omega_L$ = angle subtended by the lens from the patch dA_s.

$r = \frac{d}{2}$

Mathematically,

$$\boxed{d\omega_L = \frac{\pi d^2}{4} \times \frac{\cos\alpha}{(z/\cos\alpha)^2}} \quad (2)$$

Area (circle) $= \pi r^2$

$r = \frac{d}{2}$

$= \pi\left(\frac{d^2}{4}\right)$

$\pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$

Energy Conservation:

Flux received by lens from dA_s = Flux projected onto dA_i

Scene Radiance:

$$\boxed{L = \frac{d^2\phi}{(dA_s \cdot \cos\theta)\, d\omega_L}}$$

$$\boxed{d^2\phi = L\,(dA_s \cdot \cos\theta)\, d\omega_L} \quad (3)$$

(flux received by lens from dA_s)

Image Irradiance:

$$\boxed{E = \frac{d\phi}{dA_i}}$$

$\left(\frac{\text{flux received by the pixel}}{\text{Area of the pixel}}\right)$

$$\boxed{d\phi = E \cdot dA_i} \quad (4)$$

(flux projected onto dA_i.

[Using 4 equations, we can derive the r/t b/t E & L]

(equation-1)

$$\boxed{\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2}$$

(equation-2)

$$\boxed{d\omega_L = \frac{\pi d^2}{4}\cdot\frac{\cos\alpha}{(z/\cos\alpha)^2}}$$

(equation-3)

$$\boxed{d^2\phi = L\,(dA_s\cos\theta)\,d\omega_L}$$

(equation-4)

$$\boxed{d\phi = E\cdot dA_i}$$

$$\boxed{E = L\cdot\frac{\pi}{4}\left(\frac{d}{f}\right)^2\cdot\cos^4\alpha}$$

$E \propto L$

(Image Irradiance)

(Scene Radiance)

(1/effective F-number)

(Since f is effective focal length).

- Image Irradiance is proportional to Scene Radiance.
- Image brightness falls off from Image center as $\cos^4\alpha$
- For small fields of view, effects of $\cos^4\alpha$ are small.

Q Does image brightness vary with scene depth?.. (No).

$$\boxed{E = L\,\frac{\pi}{4}\left(\frac{d}{f}\right)^2\cdot\cos^4\alpha} \leftarrow \text{(there is no } z\text{)}$$

pulling camera away from the scene, the brightness of scene point change

(lesser light collected) → (~~brightness reduces~~) z = d/s b/t lens and ~~image~~ point on the scene.