Image Interpolation

Interpolation is the process of using known data to estimate values at unknown locations.

Nearest Neighbour Interpolation: Assigns new value to the location by using nearest neighbor in the original image.

- May lead to undesirable artifacts, such as severe distortion of straight edges

Bilinear Interpolation: Use four nearest neighbours to estimate the intensity at a given location.

Bicubic Interpolation: Involves 16 nearest neighbours of a point

★ Some basic Relationships between pixels

Neighbors of a pixel

A pixel p at coordinates (x, y) has two horizontal and two vertical neighbors with co-ordinates:

$$(x+1, y), (x-1, y), (x, y+1), (x, y-1)$$

This set of pixels, called 4-neighbors of p, is denoted $N_4(p)$.

The four diagonal neighbors of p have coordinates

$$(x+1, y+1), (x+1, y-1), (x-1, y+1), (x-1, y-1)$$

and are denoted by $N_D(p)$.

$N_4(p)$ and $N_D(p)$ together are called $N_8(p)$.

This is called neighborhood of p.

It is closed if it contains p, open if it doesnt.

Adjacency, connectivity, regions and boundaries

Connectivity/Adjacency: Two pixels that are neighbors and have the same gray-level/intensity level are called adjacent.

(a) 4-adjacency - Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

Binary Image

0 1 0 1
0 0 $\boxed{1}$ 0
0 0 1 0
1 0 0 0

$V = \{1\}$

8-adjacency: 2 pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

8-adjacency

54	10	100	8
81	150	2	34
201	200	3	45
7	70	147	56

$V = \{1, 2, 3, \ldots, 10\}$

4-adjacency

Not

m-adjacency (mixed adjacency)

2 pixels p and q are m-adjacent if

(i) q is in $N_4(p)$ or

(ii) q is in $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

Its a modification of 8-adjacency introduced to eliminate ambiguities resulting from 8-adjacency.

Eg.

8 path

0	1	1
0	1	0
0	0	1

3 pixels at the top show multiple 8-adjacency.

$V = \{1\}$

8-adjacency

m path

0	1	1
0	1	0
0	0	1

Digital path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates $(x_0, y_0), (x_1, y_1), \ldots (x_n, y_n)$.

where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \leq i \leq n$.

Here n, is the length of path.

If $(x_0, y_0) = (x_n, y_n)$, The path is closed.

We can define 4-, 8- and m- paths depending on the type of adjacency.

Connected Set: Let S represent a subset of pixels in an image. Two pixels p and q are said to be connected in S if there exists a path between them consisting entirely of pixels in S.

For any pixel p in S, set of pixels that are connected to it in S is called a connected component.

If it has one component and that component is connected, then S is called a connected set.

Let R represent a sub-set of pixels of an image. R is a region if R is a connected set.

2 Regions R_i and R_j are said to be adjacent if their unions form a connected set.

Non-adjacent regions are called disjoint.

Eg

$$\left.\begin{matrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{matrix}\right\} R_i$$

$$\left.\begin{matrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}\right\} R_j$$

2 regions with 8-adjacency

Distance Measures

For pixels p, q and s with coordinates (x,y), (u,v) and (w,z) respectively, D is a distance function or metric if.

(a) $D(p,q) \geq 0$ $\quad (D(p,q) = 0$ iff $p=q)$,

(b) $D(p,q) = D(q,p)$ and

(c) $D(p,s) \leq D(p,q) + D(q,s)$

Euclidean Distance between p and q is defined as –

$$D_e(p,q) = \left[(x-u)^2 + (y-v)^2\right]^{1/2}$$

For this distance measure, pixels having a distance <= some value r from (x,y) are the points contained in a disc of radius r centered at (x,y)

D_4 distance: City-block distance between p and q is defined as

$$D_4(p,q) = |x-u| + |y-v|$$

D_4 distance from (x,y) <= some distance d form a diamond centered at (x,y).

eg D_4 distance ≤ 2 from (x,y) form

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

$D_4 = 1$ and 4 neighbors of (x,y).

D_8 distance: chessboard distance between p and q is defined as

$$D_8(p,q) = \max(|x-u|, |y-v|)$$

A square centered at (x,y)

D_8 distance ≤ 2 forms -

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

$D_8 = 1$ are the 8-neighbors of pixel at (x,y)

- D_4 and D_8 ~~path~~ distances are independent of any paths that might exist between these points. ∵ it involves only coordinates
- D_m between 2 points is defined as the shortest m-path b/w the points

p_3 p_4 $V=\{1\}$ p, p_2, p_4 are 1. p_1 & p_3 may be 0 or 1.

p_1 p_2

p If p_1 and p_3 are 0.

then length of shortest m-path = 2 b/w p and p_4

$p\,p_2\,p_4$.

If $p_1 = 1$

shortest m-path = 3 $p\,p_1\,p_2\,p_4$

If $p_3 = 1$

shortest m-path = 3 $p\,p_2\,p_3\,p_4$

If $p_1, p_3 = 1$

shortest m-path = 4 $p\,p_1\,p_2\,p_3\,p_4$.

INTRODUCTION TO BASIC MATHEMATICAL TOOLS USED IN DIP.

Elementwise V/s Matrix Operations.

- 2x2 images

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

elementwise product ($\odot$ or $\otimes$)

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \odot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

matrix product

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Linear Vs Non-linear operations

Consider a general operator $\mathcal{H}$ that produces an output image $g(x,y)$ from the given input image $f(x,y)$:

$$\mathcal{H}[f(x,y)] = g(x,y)$$

given 2 arbitrary constants a and b, & arbitrary images $f_1(x,y)$ and $f_2(x,y)$,

$\mathcal{H}$ is said to be a linear operator if

$\mathcal{H}[af_1(x,y) + bf_2(x,y)] = a\mathcal{H}[f_1(x,y)] + b\mathcal{H}[f_2(x,y)]$

$= ag_1(x,y) + bg_2(x,y)$ ← linearity

Eg1 $\Sigma[af_1(x,y) + bf_2(x,y)] = \Sigma af_1(x,y) + \Sigma bf_2(x,y)$ { summation is distributive

$= a\Sigma f_1(x,y) + b\Sigma f_2(x,y)$

$= ag_1(x,y) + bg_2(x,y)$

$\therefore \Sigma$ is a linear operator.

Eg2 $f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}$ $f_2 \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$, $a=1, b=-1$

LHS $\max\left\{ 1\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1)\begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix} \right\} = \max\left\{ \begin{bmatrix} -6 & -3 \\ -2 & -4 \end{bmatrix} \right\} = -2$

RHS

$(1) \max \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1) \max \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}$

$= 3 + (-1)7 = -4$.

$\therefore$ max is non-linear operator.

Arithmetic Operations

$s(x,y) = f(x,y) + g(x,y)$

$d(x,y) = f(x,y) - g(x,y)$

$p(x,y) = f(x,y) \times g(x,y)$

$v(x,y) = f(x,y) \div g(x,y)$

These are element-wise operator

Set & Logical operators

Set operations

A set is a collection of distinct objects

If a is an element of A, we write | if not

$a \in A$ | $a \notin A$

$A \subseteq B$

$C = A \cup B$

$D = A \cap B$

disjoint sets $A \cap B = \emptyset$

$A^c = \{w \mid w \notin A\}$

$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$

Example images

Relation: a relation on a set A is a collection of ordered pairs of elements from A.

universal set

(a) Reflexive: for any $a \in S$; aRa.

(b) transitive: aRb, bRc implies aRc.

(c) antisymmetric: aRb and bRa implies $a = b$.

Logical operations

Deal with TRUE and FALSE variables and expressions.

AND, OR, NOT.

Spatial operations

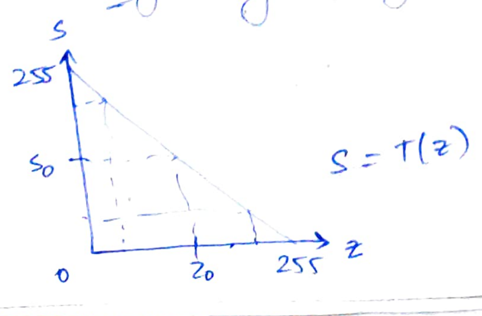
These are performed directly on pixels.

1. single-pixel operations
2. Neighborhood operations
3. Geometric spatial transformations.

(i) Single pixel operations

$s = T(z)$ where z is the intensity of a pixel in original image, s is the mapped intensity of corresponding pixel in processed image.

Eg negative of a binary image



Neighborhood operations

For a pixel centered at coordinate (x, y), its value is determined by operation on its neighborhood

Eg
$$g(x,y) = \frac{1}{mn} \sum_{(r,c) \in S_{xy}} f(r,c)$$

where r and c are the row & column coordinates of the pixels whose coordinates are in set S_{xy}. Image g is created by varying (x,y) so that the centre of neighborhood moves from pixel to pixel in image f.

$m \times n$ is the rectangular neighborhood centered on (x, y).

Eg Averaging

Geometric Transformations

Modifies the spatial arrangement of pixels in an image

This includes

① spatial transformation of coordinates.

② Intensity interpolation that assigns intensity values to the spatially transformed pixels.

This transformation can scale, rotate, translate and sheer an image.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = A \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Mapping → Forward mapping
Mapping → Inverse mapping

$(x', y') = A(x, y)$

Forward mapping → scanning the pixels in input image and at each location (x,y), computing spatial coordinates location (x', y') of the corresponding pixel in the output image.

- 2 or more pixels may be mapped onto same location in O/P image
- Some output locations may not be assigned any values

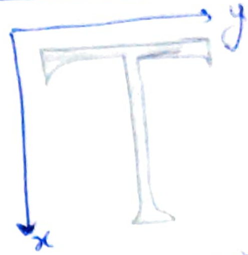
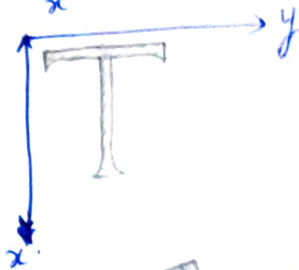
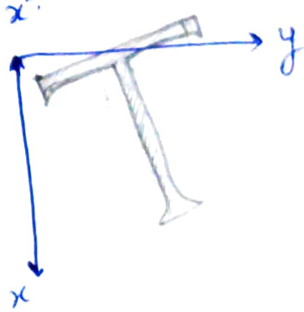
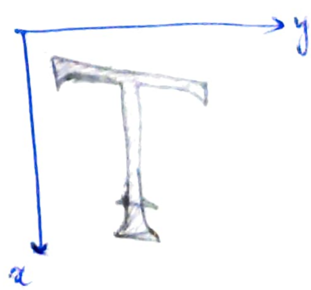
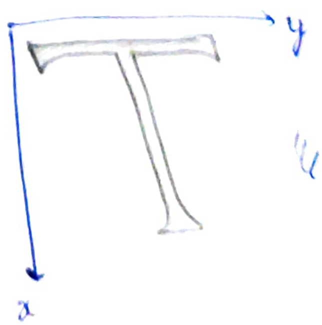
Inverse mapping

scans the O/P pixel location pixel-by-pixel at each location (x',y') and computes corresponding location in the input image using

$$(x,y) = A^{-1}(x',y').$$

It then interpolates to determine the intensity.

MATLAB uses this approach.

TRANSFORMATION NAME	Affine Matrix	Coordinate equations	Example
• Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = y$	
• Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = c_x x$ $y' = c_y y$	
• Rotation (about the origin)	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x\cos\theta - y\sin\theta$ $y' = x\sin\theta + y\cos\theta$	
• Shear (vertical)	$\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + s_v y$ $y = y$	
• shear (horizontal)	$\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = s_h x + y$	

Q1 Consider 2 image subsets S1 and S2. For $V=\{1\}$, determine whether these 2 are (a) 4-adjacent (b) 8-adjacent (c) m-adjacent connected.

S1				S2			
0	0	0	0	0	0	1	1
0	0	1	0	0	1	0	0
0	0	1	0	1 q	1	0	0
0	1	1	1 p	0	0	0	0

Sol^n

(a) S1 and S2 are not 4-connected ∵ q is not in $N_4(p)$.

(b) S1 and S2 are 8-connected ∵ q is in $N_8(p)$.

(c) S1 & S2 are m-connected because

(i) q is in $N_D(p)$ and $N_4(p) \cap N_4(q)$ is empty

Q2. Consider the image segment shown: Let $V=\{0,1\}$. compute the shortest 4-, 8- and m-path between p & q.

	3	1	2	1 q
	2	2	0	2
	1	2	1	1
p	1	0	1	2

4-path : Not possible

	3	1	2	1 q
	2	2	0	2
	1	2	1	1
p	1	0	1	2

8-path

3	1	2	1
2	2	0	2
1	2	1	1
1	0	1	2

shortest length = 4

m-path

3	1	2	1
2	2	0	2
1	2	1	1
1	0	1	2

shortest path = 5