Problem Solving State-Space Search and Control Strategies

Chapter 2 (d)

Constrained Satisfaction

- Many AI problems can be viewed as problems of constrained satisfaction in which the goal is to solve some problem state that satisfies a given set of constraints.
- Example of such a problem are
  - **Crypt-Arithmetic** puzzles.
  - Many design tasks can also be viewed as constrained satisfaction problems.
  - N-Queen: Given the condition that no two queens on the same row/column/diagonal attack each other.
  - Map colouring: Given a map, colour three regions in blue, red and black, such that no two neighbouring regions have the same colour.
- Such problems do not require a new search methods.
  - They can be solved using any of the search strategies which can be augmented with the list of constraints that change as parts of the problem are solved.

Algorithm

- Until a complete solution is found or all paths have lead to dead ends

  {
  - Select an unexpanded node of the search graph.
  - Apply the constraint inference rules to the selected node to generate all possible new constraints.
  - If the set of constraints contain a contradiction, then report that this path is a dead end.
  - If the set of constraint describes a complete solution, then report success.
  - If neither a contradiction nor a complete solution has been found, then
    - apply the problem space rules to generate new partial solutions that are consistent with the current set of constraints.
    - Insert these partial solutions into the search graph.

}

Crypt-Arithmetic puzzle

- Problem Statement:
  - Solve the following puzzle by assigning numeral (0-9) in such a way that each letter is assigned unique digit which satisfy the following addition.
  - Constraints : No two letters have the same value. (The constraints of arithmetic).

	S	E	N	D
+	M	O	R	E
M	O	N	E	Y

- Initial Problem State
  - S = ? ; E = ? ;N = ? ; D = ? ; M = ? ;O = ? ; R = ? ;Y = ?

Carries:

$$C_4 = ?\ ; C_3 = ?\ ; C_2 = ?\ ; C_1 = ?$$

C_4	C_3	C_2	C_1	$\longleftarrow$	Carry
	S	E	N	D	
+	M	O	R	E	
M	O	N	E	Y	

Constraint equations:

$$Y = D + E \qquad C_1 \longrightarrow$$
$$E = N + R + C_1 \qquad C_2 \longrightarrow$$
$$N = E + O + C_2 \qquad C_3 \longrightarrow$$
$$O = S + M + C_3 \qquad C_4 \longrightarrow$$
$$M = C_4$$

- We can easily see that M has to be non zero digit, so the value of C4 =1

1. M = C4 ⇒ **M = 1**
2. O = S + M + C3 □ C4

For C4 =1, S + M + C3 > 9 ⇒

S + 1 + C3 > 9 ⇒ S+C3 > 8.

If C3 = 0, then S = 9 else if C3 = 1,

then S = 8 or 9.

- We see that for S = 9
  - C3 = 0 or 1
  - It can be easily seen that C3 = 1 is not possible as O = S + M + C3 ⇒ O = 11 ⇒ O has to be assigned digit 1 but 1 is already assigned to M, so not possible.
  - Therefore, only choice for C3 = 0, and thus O = 10. This implies that O is assigned 0 (zero) digit.
- **Therefore, O = 0**

M = 1, O = 0

C_4	C_3	C_2	C_1	←	Carry
	S	E	N	D	
+	M	O	R	E	
M	O	N	E	Y	

Y = D + E □ C1

E = N + R + C1 □ C2

N = E + O + C2 □ C3

O = S + M + C3 □ C4

M = C4

3. Since C3 = 0; N = E + O + C2 produces no carry.

- As O = 0, N = E + C2 .
- Since N ≠ E, therefore, C2 = 1.

Hence N = E + 1

- Now E can take value from 2 to 8 {0,1,9 already assigned so far }
  - If E = 2, then N = 3.
  - Since C2 = 1, from E = N + R + C1 , we get 12 = N + R + C1
    - If C1 = 0 then R = 9, which is not possible as we are on the path with S = 9
    - If C1 = 1 then R = 8, then
      - From Y = D + E , we get 10 + Y= D + 2 .
      - For no value of D, we can get Y.
  - Try similarly for E = 3, 4. We fail in each case.

C_4	C_3	C_2	C_1	←	Carry
	S	E	N	D	
+	M	O	R	E	
M	O	N	E	Y	

Y = D + E □ C1
E = N + R + C1 □ C2
N = E + O + C2 □ C3
O = S + M + C3 □ C4
M = C4

- If E = 5, then N = 6
  - Since C2 = 1, from E = N + R + C1 , we get 15 = N + R + C1 ,
  - If C1 = 0 then R = 9, which is not possible as we are on the path with S = 9.
  - If C1 = 1 then R = 8, then
    - From Y = D + E , we get 10 + Y= D + 5 i.e., 5 + Y = D.
    - If Y = 2 then D = 7. These values are possible.
- Hence we get the final solution as given below and on backtracking, we may find more solutions.

S = 9 ; E = 5 ; N = 6 ; D = 7 ;
M = 1 ; O = 0 ; R = 8 ;Y = 2

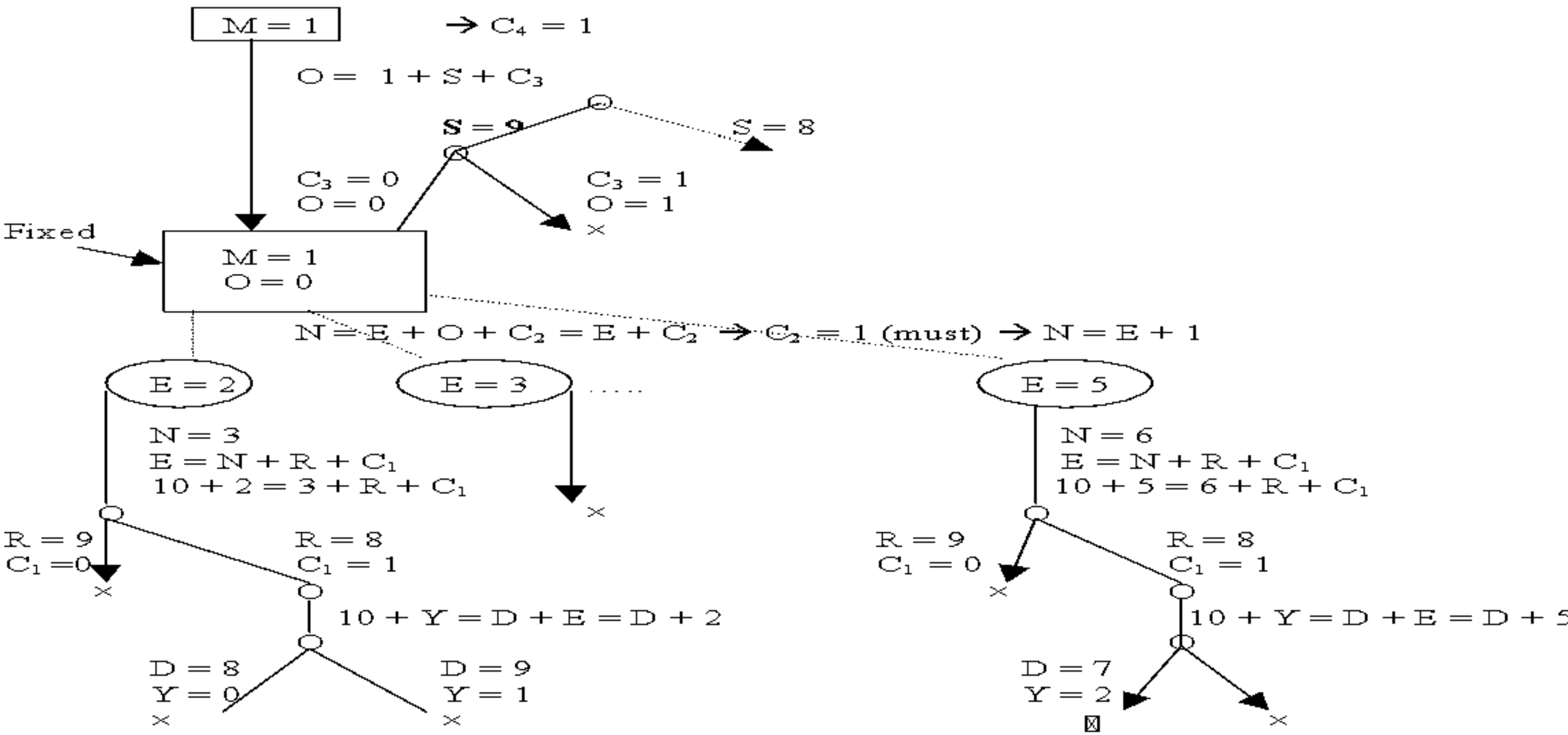
C_4	C_3	C_2	C_1	← Carry
	S	E	N	D
+	M	O	R	E
M	O	N	E	Y

Y = D + E □ C1
E = N + R + C1 □ C2
N = E + O + C2 □ C3
O = S + M + C3 □ C4
M = C4

Constraints:

$$Y = D + E \longrightarrow C_1$$
$$E = N + R + C_1 \longrightarrow C_2$$
$$N = E + O + C_2 \longrightarrow C_3$$
$$O = S + M + C_3 \longrightarrow C_4$$
$$M = C_4$$

Initial State



The first solution obtained is:

M = 1, O = 0, S = 9, E = 5, N = 6, R = 8, D = 7, Y = 2

C4	C3	C2	C1		← Carries
	B	A	S	E	
+	B	A	L	L	
G	A	M	E	S	

Constraints equations are:

E + L = S → C1

S + L + C1= E → C2

2A + C2 = M → C3

2B + C3 = A → C4

G = C4

Initial Problem State

G = ?; A = ?;M = ?; E = ?; S = ?; B = ?; L = ?

1. $G = C_4 \Rightarrow \mathbf{G = 1}$

2. $2B + C_3 = A \quad \rightarrow C_4$

   2.1 Since $C_4 = 1$, therefore, $2B + C_3 > 9 \Rightarrow B$ can take values from 5 to 9.

   2.2 Try the following steps for each value of B from 5 to 9 till we get a possible value of B.

- If $B = 5$
  - if $C_3 = 0 \Rightarrow A = 0 \Rightarrow M = 0$ for $C_2 = 0$ or $M = 1$ for $C_2 = 1$ $\times$
  - if $C_3 = 1 \Rightarrow A = 1$ $\times$ (as $G = 1$ already)
- For $B = 6$ we get similar contradiction while generating the search tree.
- If $\boxed{\mathbf{B = 7}}$, then for $C_3 = 0$, we get $\boxed{\mathbf{A = 4}}$ $\Rightarrow M = 8$ if $C_2 = 0$ that leads to contradiction, so this path is pruned. If $C_2 = 1$, then $\boxed{\mathbf{M = 9}}$.

3. Let us solve $S + L + C_1 = E$ and $E + L = S$

   - Using both equations, we get $2L + C_1 = 0 \Rightarrow \boxed{\mathbf{L = 5}}$ and $C_1 = 0$
   - Using $L = 5$, we get $S + 5 = E$ that should generate carry $C_2 = 1$ as shown above
   - So $S+5 > 9 \Rightarrow$ Possible values for E are {2, 3, 6, 8} (with carry bit $C_2 = 1$)
   - If $E = 2$ then $S + 5 = 12 \Rightarrow S = 7$ $\times$ (as $B = 7$ already)
   - If $E = 3$ then $S + 5 = 13 \Rightarrow S = 8$.
   - Therefore $\boxed{\mathbf{E = 3}}$ and $\boxed{\mathbf{S = 8}}$ are fixed up.

4. Hence we get the final solution as given below and on backtracking, we may find more solutions. In this case we get only one solution.

$\mathbf{G = 1;\ A = 4;\ M = 9;\ E = 3;\ S = 8; B = 7;\ L = 5}$