Probability and Statistics: Lecture-35

Monsoon-2020

by Dr. Pawan Kumar (IIIT, Hyderabad)
on November 4, 2020

» Solved Example 2

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Example (Solved Example)

Let X, Y be jointly continuous RVs with joint PDF

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$$f_{X,Y} = \begin{cases} 6e^{-(2x+3y)} & x, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

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- Find $P(X > Y)$

$\iint_D f_{X,Y}(x,y)$

» Answer to previous problem...

①
$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$$
$$= \int_0^{\infty} 6\cdot e^{-(2x+3y)}\,dy$$
$$= 6e^{-2x}\int_0^{\infty} e^{-3y}\,dy$$
$$= 6e^{-2x}\left[\frac{e^{-3y}}{-3}\right]_0^{\infty}$$
$$= -2e^{-2x}\left[e^{-3y}\right]_0^{\infty}$$
$$= -2e^{-2x}(-1) = 2e^{-2x}$$

Similarly,
$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$$
$$= \int_0^{\infty} 6e^{-(2x+3y)}\,dx$$
$$= 6e^{-3y}\int_0^{\infty} e^{-2x}\,dx$$
$$= 6e^{-3y}\left[\frac{e^{-2x}}{-2}\right]_0^{\infty}$$
$$= -3e^{-3y}\left[e^{-2x}\right]_0^{\infty}$$
$$= 3e^{-3y}$$

$\Rightarrow f_{XY}(x,y) = 6e^{-(2x+3y)} = f_X(x)\cdot f_Y(y)$

$\Rightarrow X, Y$ are independent

» Solved Example 3

② $E[Y \mid X > 2]$

$= E[Y]$ Since X, Y are ind.

We observe that

$$f_Y(y) = 3e^{-3y}, \quad y \geq 0$$

$\Rightarrow Y$ is $\exp(3)$.

$\Rightarrow E[Y] = \frac{1}{3}$. //

③ Try.

» Solved Example 3

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We are also know that given $X = x$, the RV Y is uniformly distributed on $[-x, x]$.

» Solved Example 3

$$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

$$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

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- Find the joint PDF $f_{XY}(x, y)$
- Find $P_Y(y)$
- Find $P(|Y| < X^3)$

$P((x,y) \in D)$

$$\iint_D f_{XY}(x,y)\,dx\,dy$$

Exercise

» Answer to previous problem...

» Answer to previous problem...

» Solved Example 4

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Let X, Y be two jointly continuous RVs with joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \leq x \leq 1,\ 0 \leq y \leq \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

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$$f_{X,Y}(x,y) = \begin{cases} 6xy & 0 \leq x \leq 1,\ 0 \leq y \leq \sqrt{x} \\ 0 & \text{otherwise} \end{cases}$$

$\Rightarrow \boxed{y^2 \leq x \leq 1}$

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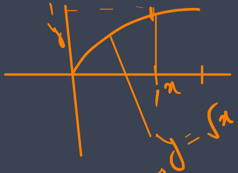
2. Find $f_X(x)$ and $f_Y(y)$
3. Are X and Y independent?
4. Find the conditional PDF of X given $Y = y$, $f_{X|Y}(x \mid y)$
5. Find $E[X \mid Y = y], 0 \leq y \leq 1$
6. Find $\mathrm{Var}(X \mid Y = y)$ for $0 \leq y \leq 1$

» Answer to previous problem...

① 

② $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy$

$= \int_0^{\sqrt{x}} 6xy\,dy = 6x\left[\frac{y^2}{2}\right]_0^{\sqrt{x}}$

$= 3x \cdot x = 3x^2$

$\Rightarrow f_X(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx$

$= \int_{y^2}^{1} 6xy\,dx = 6y\left[\frac{x^2}{2}\right]_{y^2}^{1}$

$= 3y(1-y^4)$

$f_Y(y) = \begin{cases} 3y(1-y^4) & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$

③ No, $f_{XY}(x,y) \neq f_X(x) f_Y(y)$

④

» Answer to previous problem...

(4) $f_{X|Y}(x|y) = \dfrac{f_{XY}(x,y)}{f_Y(y)}$

$$= \begin{cases} \dfrac{\cancel{6}^{2} x \cancel{y}}{\cancel{3}\cancel{y}(1-y^4)} & y^2 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(5) $E[X|Y=y] = \displaystyle\int_{-\infty}^{\infty} x \, f_{X|Y}(x|y)\, dx$

$$= \int_{y^2}^{1} x \cdot \frac{2x}{1-y^4}$$

$$\frac{2}{1-y^4} \int_{y^2}^{1} x^2 = \frac{2}{1-y^4}\left[\frac{x^3}{3}\right]_{y^2}^{1}$$

$$= \frac{2(1-y^6)}{3(1-y^4)}.$$

(6) $\mathrm{Var}(X|Y=y)$

$$= \underbrace{E[X^2|Y=y]}_{\uparrow \text{ do this}} - \underbrace{\left(E[X|Y=y]\right)}_{\uparrow \text{ found}}{}^2$$

$=$

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Let X and Y be two random variables. The covariance between X and Y is defined as

$$\mathrm{Cov}(X, Y) = E[(X - E[X])(Y - E[Y])] = \underbrace{E[XY] - (E[X])(E[Y])}$$

Derivation

$$E[XY - XE[Y] - E[X]Y + E[X]E[Y]]$$

$$= E[XY] - \cancel{E[Y]E[X]} - \cancel{E[X]E[Y]} + E[X]E[Y]$$

» Solved Example

$Y|X \sim \text{Exp}(X)$ L.O.I.E

Example (Solved Example)

Suppose $X \sim \text{Uniform}(1,2)$ and given $X = x$, Y is exponential with parameter $\lambda = x$. Find $\text{Cov}(X, Y)$.

$\text{Cov}(X,Y) = E[XY] - E[X]E[Y]$. We have $E[X] = \frac{3}{2}$.

We have

$$E[Y] = E[E[Y|X]] \quad \text{(by L.O.I.E)}$$
$$= E\left[\frac{1}{X}\right] = \int_1^2 \frac{1}{x} \cdot 1 = [\ln x]_1^2 = \ln 2$$

$$E[XY] = E[E[XY|X]] \quad \text{(L.O.I.E)}$$
$$= E[X\,E[Y|X]] = E\left[X \cdot \frac{1}{X}\right] = 1.$$

$Y|X \sim \text{Exp}(X)$

$$\boxed{\text{Cov}(X,Y) = 1 - \frac{3}{2} \cdot \ln 2}$$

Answer

» Properties of Covariance...

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$Cov(X, Y) = E[XY] - E[X]E[Y]$

If X, Y ind $\Rightarrow$ $E[XY] = E[X]E[Y]$

Properties of Covariance

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6. $\mathrm{Cov}(X + Y, Z) = \mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z)$

$$E[(X+Y)Z] - E[(X+Y)]E[Z]$$

$$= E[XZ] + E[YZ] - E[X]E[Z] - E[Y]E[Z]$$

$$\mathrm{Cov}(X, Z) + \mathrm{Cov}(Y, Z).$$

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$$\mathrm{Cov}\left(\sum_{i=1}^{m} a_i X_i, \sum_{j=1}^{n} b_j Y_j\right) = \sum_{i=1}^{m}\sum_{j=1}^{n} a_i b_j \,\mathrm{Cov}(X_i, Y_j)$$

inductively

Combining 4 & 6

» Solved Example...

Example (Solved example)

Let X and Y be two independent random variables following standard normal distribution, and

$$Z = 1 + X + XY^2$$
$$W = 1 + X$$

Find $\mathrm{Cov}(Z, W)$.

Solution

Prop 5.3

$$\mathrm{Cov}(Z, W) = \mathrm{Cov}(1+X+XY^2, 1+X) = \mathrm{Cov}(X+XY^2, X)$$

$$= \mathrm{Cov}(X, X) + \mathrm{Cov}(XY^2, X) \quad (\text{Prop } 6)$$

$$= \mathrm{Var}(X) + E[X^2Y^2] - E[XY^2]E[X] \quad (\text{Since } X, Y \text{ ind } \& \ X \sim N(0,1))$$

$$= 1 + E[X^2]E[Y^2] - E[X]^2 E[Y^2]$$

$$= 1 + 1 \cdot 1 - 0 \cdot 1 = 1 + 1 = \underline{\underline{2}}$$

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$$\mathrm{Var}(Z) = a^2\,\mathrm{Var}(X) + b^2\,\mathrm{Var}(Y) + 2ab\,\mathrm{Cov}(X, Y)$$

Try this

Solution

$\mathrm{Var}(Z) = \mathrm{Cov}(Z, Z) = \mathrm{Cov}(X+Y, X+Y)$

$= \mathrm{Cov}(X, X) + \mathrm{Cov}(X, Y) + \mathrm{Cov}(Y, X) + \mathrm{Cov}(Y, Y)$

$= \mathrm{Var}(X) + 2\,\mathrm{Cov}(X, Y) + \mathrm{Var}(Y)$ ✓

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$$\frac{X}{\rho_X} - \cancel{\frac{E[X]}{\rho_X}}$$

const

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4. $\rho(aX + b, cY + d) = \rho(X, Y)$ for $a, c > 0$

check

Exercise

» Answer to previous problem...

» Positive Correlation, Negative Correlation, Uncorrelation...

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Definition of positive, negative correlation

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Pairwise uncorrelation and Variance

If X and Y are uncorrelated, then

$$\mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) \quad + \cancelto{0}{\mathrm{Cov}(X,Y)}$$

More generally, if $X_1, X_2, \ldots, X_n$, are pairwise uncorrelated, i.e., $\rho(X_i, X_j) = 0$ when $i \neq j$, then

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$$\mathrm{Var}(X_1 + X_2 + \cdots + X_n) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \cdots + \mathrm{Var}(X_n)$$

$+ \mathrm{Cov}(X_1, X_2) + \mathrm{Cov}(X_1, X_3) + \cdots$

0

0