Probability and Statistics: Lecture-28

Monsoon-2020

by Dr. Pawan Kumar (IIIT, Hyderabad)
on October 16, 2020

» Joint Cumulative Distribution Function...

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Definition of Joint Cumulative Distribution Function

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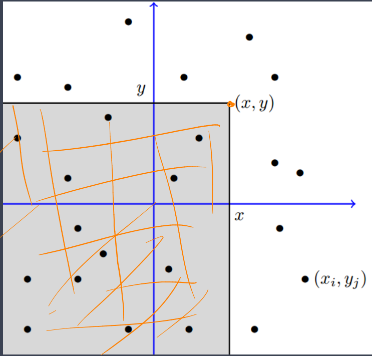

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» Marginal CDF...

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$$F_{XY}(\underbrace{\infty, \infty}_{\text{Sample space}}) = 1,$$

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Moreover, we have the following properties

$$F_{XY}(\infty,\infty) = 1, \quad F_{XY}(-\infty, y) = 0,$$

$P(X \le -\infty, Y \le y)$

$P(\underbrace{\{X \le -\infty\}}_{\phi} \cap \{Y \le y\})$

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empty

» Example of Joint PMF and Joint CDF...

Solved Example on Joint PDF and Joint CDF

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Let $X \sim \text{Bernoulli}(p)$ and $Y \sim \text{Bernoulli}(q)$ be independent, where $0 < p, q < 1$.

» Example of Joint PMF and Joint CDF...

$P_{X,Y}(X = x, Y = y) = $

$X = \{0, 1\}$, $Y = \{0, 1\}$

Solved Example on Joint PDF and Joint CDF

Let $X \sim \text{Bernoulli}(p)$ and $Y \sim \text{Bernoulli}(q)$ be independent, where $0 < p, q < 1$. Find the joint PMF and joint CDF for X and Y.

» Answer to previous problem...

The joint range of RVs X and Y is given by

$$R_{XY} = \{(0,0), (0,1), (1,0), (1,1)\}$$

[ϕ = Head, 0 = Tail]

$$P_{XY}(i,j) = P_X(i)\,P_Y(j)$$

(Since X, Y are ind.)

for $i, j = 0, 1$

$$P_{XY}(0,0) = P_X(0)\,P_Y(0) = (1-p)(1-q)$$

$$P_{XY}(0,1) = P_X(0)\,P_Y(1) = (1-p)\,q$$

$$P_{XY}(1,0) = p(1-q)$$

$$P_{XY}(1,1) = pq.$$

Joint CDF

$$F_{XY}(x,y) = P_{XY}(X \le x, Y \le y)$$

1. $F_{XY}(x,y) = 0$ if $x < 0$
2. $F_{XY}(x,y) = 0$ if $y < 0$
3. $F_{XY}(x,y) = 1$ if $x \geq 1$ & $y \geq 1$

$0 \le x < 1$ and $y \geq 1$

$$F_{XY}(x,y) = P_{XY}(X \le x, Y \le y)$$

$$= P(X = 0, \underbrace{Y \le 1})$$

added all y

$$= P(X = 0)$$

$$= 1 - p$$

marginal CDF w.r.t X.

» Answer to previous problem...

For $0 \le y < 1$ and $x \geq 1$

$$F_{XY}(x, y) = 1 - q \quad \text{(Check as for } X\text{)}$$

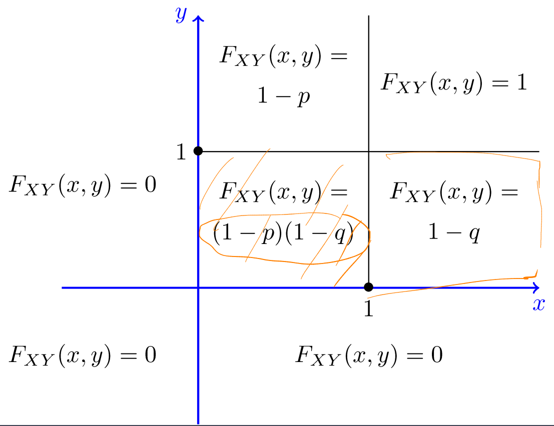
Finally for $0 \le x < 1$ and $0 \le y < 1$

$$F_{XY}(x,y) = P(X \le x, Y \le y)$$

$$= P(X = 0, Y = 0)$$

$$= (1-p)(1-q)$$

» Plot of Joint CDF



- Figure shows the values of $F_{XY}(x, y)$ in different regions

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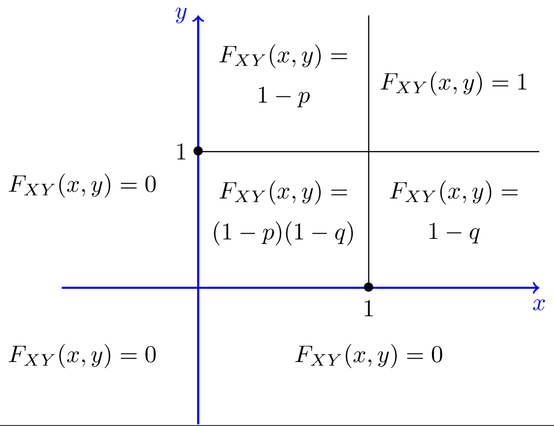
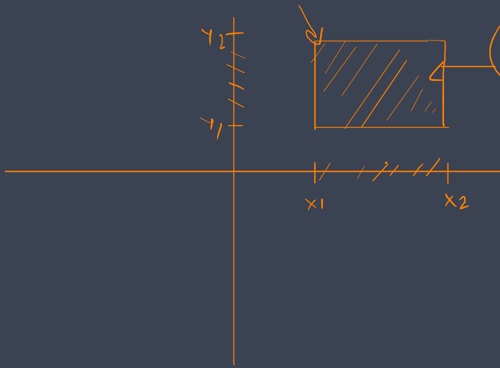


- Figure shows the values of $F_{XY}(x, y)$ in different regions
- Note that in general we need three dimensional graph to show a joint CDF of two random variables

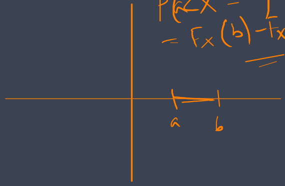
A result

$$P(x_1 < X \le x_2, y_1 < Y \le y_2) = F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1) + F_{XY}(x_1, y_1)$$



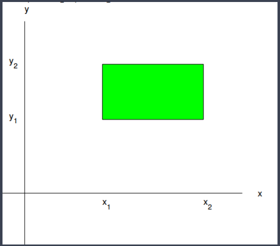
$(x_1 < X \le x_2,\ y_1 < Y \le y_2)$

$P(a < X \le b) = F_X(b) - F_X(a)$



A result

$$P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1) + F_{XY}(x_1, y_1)$$



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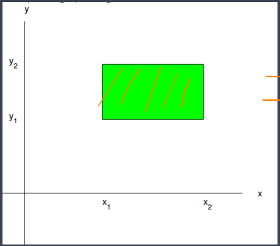

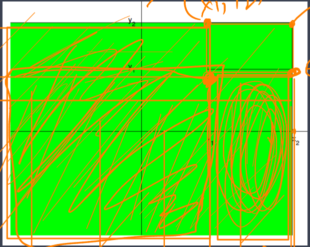

(x_1, y_2)

$F_{XY}(x_2, y_2)$

(x_2, y_1)

$P(X \leq x_2, Y \leq y_2)$

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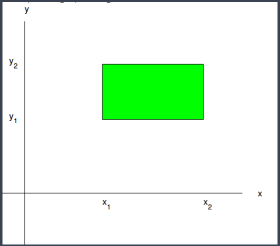

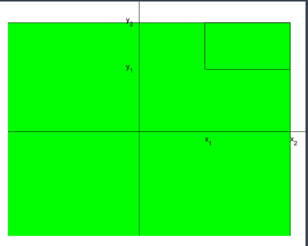

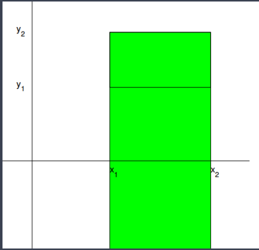

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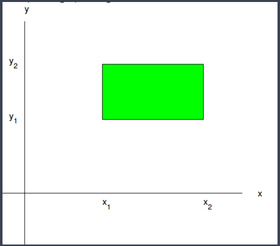

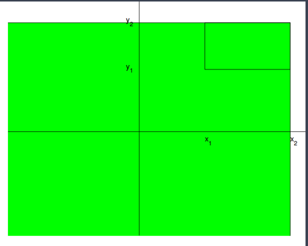

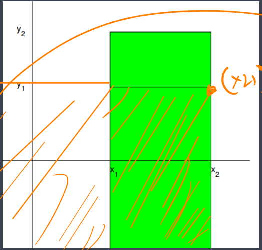

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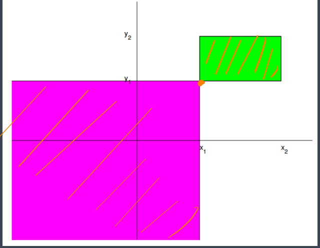

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Example Motivation for Conditional PMF and CDF

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I roll a fair die. Let X be the observed number.

Example Motivation for Conditional PMF and CDF

I roll a fair die. Let X be the observed number. Find the conditional PMF of X given that we know the observed number was less than 5.

Solution:

$R_X = \{1, 2, 3, 4, 5, 6\}$ $A = \{X < 5\}$

$P_{X|A}(1) = P(X=1 \mid \underbrace{X<5}_{A}) = \dfrac{P(X=1 \text{ and } X<5)}{P(X<5)} = \dfrac{P(X=1)}{P(X<5)} = \dfrac{1}{4}.$

Similarly $P_{X|A}(2) = P_{X|A}(3) = P_{X|A}(4) = \dfrac{1}{4}$

$P_{X|A}(5) = P_{X|A}(6) = 0$

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$$\begin{aligned} P_{X|A}(x_i) &= P(X = x_i | A) \\ &= \frac{P(X = x_i \text{ and } A)}{P(A)}, \quad \text{for any } x_i \in R_X \end{aligned}$$

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$$F_{X|A}(x) = P(X \leq x \mid A).$$

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$$P_{X|Y}(x_i, y_j) = \frac{P_{XY}(x_i, y_j)}{P_Y(y_j)}$$

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for any $x_i \in R_X$ and $y_j \in R_Y$.

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* If X and Y are independent if

$$P_{X|Y}(x_i|y_j) = P(X = x_i \mid Y = y_j) = \frac{P_{XY}(x_i, y_j)}{P_Y(y_j)} = \frac{P_X(x_i)\cancel{P_Y(y_j)}}{\cancel{P_Y(y_j)}} = P_X(x_i)$$

» Example of Joint and Marginal PDF and Conditional PMF...

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Example

Consider the set of points in set G defined as follows

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$$G = \{(x, y) \mid x, y \in \mathbb{Z}, \quad |x| + |y| \leq 2\}.$$

$X = \{0, 1, -1, 2, -2\}$

$Y = \{0, 1, -1, 2, -2\}$

$G = \{(0,0), (0,1), (0,2), (1,0), (0,-2), (0,-1), (0, \ldots\}$

13

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1. Find the joint and marginal PMFs of X and Y.
2. Find the conditional PMF of X given $Y = 1$.

» Example of Joint and Marginal PDF and Conditional PMF...

$P_{XY}(X=x_i, Y=y_i) = \_\_\_$

$P_X(x_i) = \sum_{y_i} P_{XY}(x_i, y_i)$

Example

Consider the set of points in set G defined as follows

$$G = \{(x, y) \mid x, y \in \mathbb{Z}, \quad |x| + |y| \leq 2\}.$$

$P_{X|Y}(X=x_i \mid Y=1) = \frac{P_{XY}(x_i, 1)}{P_Y(1)} = \_\_\_$

$= P_X(x_i)$

If we pick a point (X, Y) from this grid at random, then the probability of choosing a point is $1/13$.

1. Find the joint and marginal PMFs of X and Y.
2. Find the conditional PMF of X given $Y = 1$.
3. Are X and Y independent?

» Answer to previous problem...

① Joint PMF

$$P_{XY}(x,y) = \begin{cases} \frac{1}{13} & (x,y) \in G \\ 0 & \text{otherwise} \end{cases}$$

$$P_Y(y) = \sum_{x_i \in R_X} P_{XY}(x_i, y)$$

$$P_X(x) = \sum_{y_i \in R_Y} P_{XY}(x, y_i)$$

$$P_X(\underset{x}{-2}) = P_{XY}(-2, 0) = \frac{1}{13}$$

$$P_X(-1) = P_{XY}(-1,-1) + P_{XY}(-1,0) + P_{XY}(-1,1)$$

$$= \frac{3}{13}.$$

$$P_X(0) = P_{XY}(0,-2) + P_{XY}(0,-1) + P_{XY}(0,0) + P_{XY}(0,1) + P_{XY}(0,2)$$

$$= \frac{5}{13}$$

$$P_X(i) = P_Y(i) = \begin{cases} \frac{1}{13} & \text{for } i = 2, -2 \\ \frac{3}{13} & \text{for } i = -1, 1 \\ \frac{5}{13} & \text{for } i = 0 \\ 0 & \text{otherwise} \end{cases}$$

» Answer to previous problem...

(2) $Y=1$ Possible X : $-1, 0, 1$

$$P_{X|Y}(i,1) = \frac{P_{X,Y}(i,1)}{P_Y(1)} = \frac{1/13}{3/13} = \frac{1}{3}$$

for $-1, 0, 1$

$$\underbrace{P_{X|Y}(i|1)} = \begin{cases} \frac{1}{3} & \text{for } i=-1,0,1 \\ 0 & \text{otherwise} \end{cases}$$

$\Rightarrow \underbrace{X|Y=1}$ is uniformly distr. over the set $\{-1, 0, 1\}$

(3) Is X, Y ind.

Is $P_{X|Y}(i,1) \neq P_X(i)$

$\forall i$

$\Rightarrow X$ & Y are not ind.

» Definition of Conditional Expectation...

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$E[X|Y] = \sum x_i P_{X|Y}$

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$$E[X \mid A] = \sum_{x_i \in R_X} x_i P_{X|A}(x_i)$$

$$E[X \mid Y = j_j] = \sum_{x_i \in R_X} x_i P_{X|Y}(x_i \mid y_j)$$

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$P_{X|Y} = \begin{cases} \frac{1}{3}, & x = -1, 0, 1 \\ 0 \end{cases}$

$E[X|Y=1] = (-1)\frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3}$

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1. Find $E[X \mid Y = 1]$

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$P_{X|Y}(x_i \mid -1 < Y < 2)$

$= \text{——}$

$\sum x_i P_{X|Y}$

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$\sum_i a_i P_{X|Y}$

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3. Find $E[|X| \mid -1 < Y < 2]$