Probability and Statistics: Lecture-27

Monsoon-2020

by Dr. Pawan Kumar (IIIT, Hyderabad)
on October 14, 2020

» Mixed Random Variable...

Example of mixed random variable

Let X be a continuous random variable with the following PDF

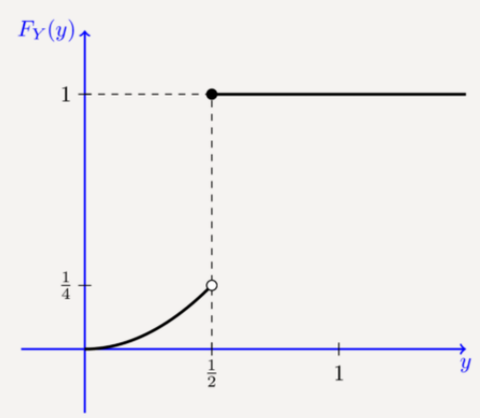
$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let

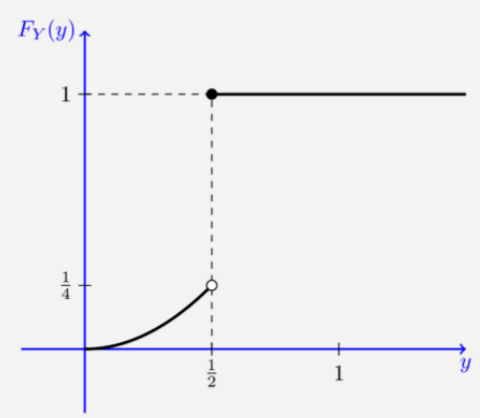
$$Y = g(X) = \begin{cases} X & 0 \leq X \leq \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

Find the CDF of Y.

» Plot of the Mixed Random Variable Example

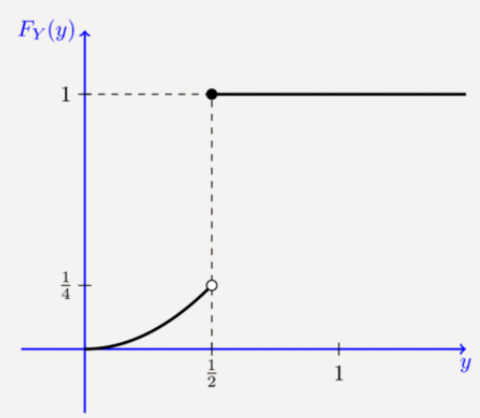


» Plot of the Mixed Random Variable Example



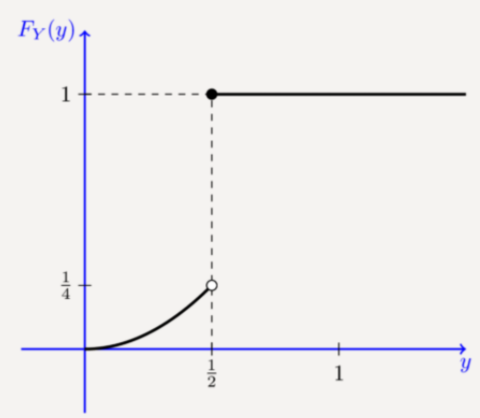
* the CDF is not continuous, so Y is not *a* continuous random variable

» Plot of the Mixed Random Variable Example



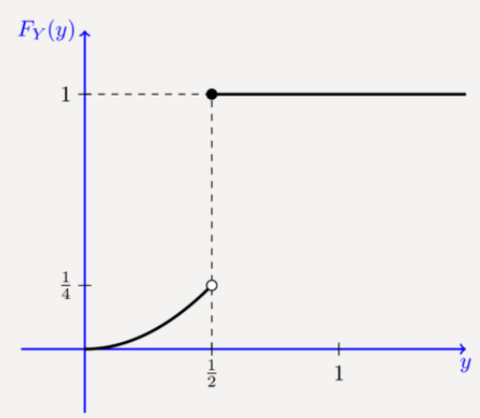
- the CDF is not continuous, so Y is not *a* continuous random variable
- the CDF is not in the staircase form, so it is not *a* discrete random variable either

» Plot of the Mixed Random Variable Example



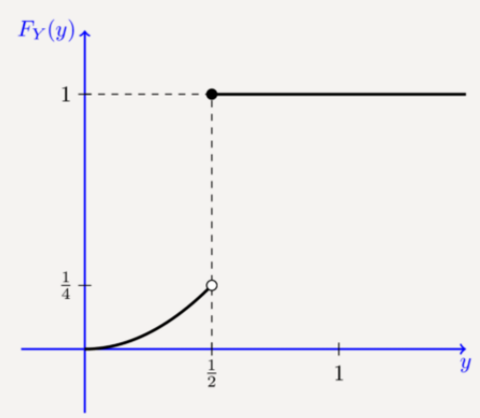
- the CDF is not continuous, so Y is not *a* continuous random variable
- the CDF is not in the staircase form, so it is not *a* discrete random variable either
- It is indeed *a* mixed random variable

» Plot of the Mixed Random Variable Example



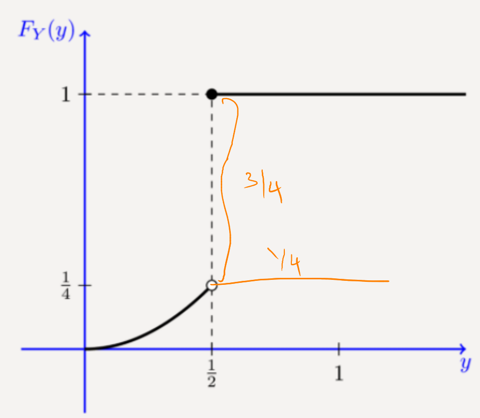
* the CDF is not continuous, so Y is not a continuous random variable
* the CDF is not in the staircase form, so it is not a discrete random variable either
* It is indeed a mixed random variable
* there is jump at $y = 1/2$

» Plot of the Mixed Random Variable Example



- the CDF is not continuous, so Y is not *a* continuous random variable
- the CDF is not in the staircase form, so it is not *a* discrete random variable either
- It is indeed *a* mixed random variable
- there is jump *at* $y = 1/2$
- *a*mount of jump is 1 - 1/4 = 3/4

» Plot of the Mixed Random Variable Example



- the CDF is not continuous, so Y is not a continuous random variable
- the CDF is not in the staircase form, so it is not a discrete random variable either
- It is indeed a mixed random variable
- there is jump at $y = 1/2$
- amount of jump is 1 - 1/4 = 3/4
- CDF is continuous at other points

» CDF of mixed RV as a sum of continuous and discrete RV...

» CDF of mixed RV as a sum of continuous and discrete RV...

- In the previous example, CDF of Y was

» CDF of mixed RV as a sum of continuous and discrete RV...

* In the previous example, CDF of Y was

$$F_Y(y) = \begin{cases} 1 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

» CDF of mixed RV as a sum of continuous and discrete RV...

- In the previous example, CDF of Y was

$$F_Y(y) = \begin{cases} 1 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

- The CDF can be written as a sum of continuous and discrete part

» CDF of mixed RV as a sum of continuous and discrete RV...

* In the previous example, CDF of Y was

$$F_Y(y) = \begin{cases} 1 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

* The CDF can be written as a sum of continuous and discrete part

$$F_Y(y) = C(y) + D(y),$$

» CDF of mixed RV as a sum of continuous and discrete RV...

* In the previous example, CDF of Y was

$$F_Y(y) = \begin{cases} 1 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

* The CDF can be written as a sum of continuous and discrete part

$$F_Y(y) = C(y) + D(y),$$

where the continuous part is

» CDF of mixed RV as a sum of continuous and discrete RV...

* In the previous example, CDF of Y was

$$F_Y(y) = \begin{cases} 1 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

* The CDF can be written as a sum of continuous and discrete part

$$F_Y(y) = C(y) + D(y),$$

where the continuous part is

$$C(y) = \begin{cases} 1/4 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & y < 0 \end{cases}$$

» CDF of mixed RV as a sum of continuous and discrete RV...

* In the previous example, CDF of Y was

$$F_Y(y) = \begin{cases} 1 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

* The CDF can be written as a sum of continuous and discrete part

$$F_Y(y) = C(y) + D(y),$$

where the continuous part is

$$C(y) = \begin{cases} 1/4 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & y < 0 \end{cases}$$

and the discrete part is

» CDF of mixed RV as a sum of continuous and discrete RV...

* In the previous example, CDF of Y was

$$F_Y(y) = \begin{cases} 1 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

* The CDF can be written as a sum of continuous and discrete part

$$F_Y(y) = C(y) + D(y),$$

where the continuous part is

$$C(y) = \begin{cases} 1/4 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & y < 0 \end{cases}$$

and the discrete part is

jump point

$$D(y) = \begin{cases} 3/4 & y \geq 1/2 \\ 0 & y < 1/2 \end{cases}$$

» CDF of a mixed RV as a sum of Continuous and Discrete CDF...

» CDF of a mixed RV as a sum of Continuous and Discrete CDF...

CDF of Mixed Random Variable

» CDF of a mixed RV as a sum of Continuous and Discrete CDF...

CDF of Mixed Random Variable

The CDF of mixed random variable Y can be written as a sum of continuous and discrete/staircase function

$$F_Y(y) = C(y) + D(y).$$

» CDF of a mixed RV as a sum of Continuous and Discrete CDF...

CDF of Mixed Random Variable

The CDF of mixed random variable Y can be written as a sum of continuous and discrete/staircase function

$$F_Y(y) = C(y) + D(y).$$

We differentiate the continuous part $c(y) = \frac{dC(y)}{dy}$, where ever $C(y)$ is differentiable.

additional assumption

CDF of a mixed RV as a sum of Continuous and Discrete CDF...

CDF of Mixed Random Variable

The CDF of mixed random variable Y can be written as a sum of continuous and discrete/staircase function

$$F_Y(y) = C(y) + D(y).$$

We differentiate the continuous part $c(y) = \frac{dC(y)}{dy}$, where ever $C(y)$ is differentiable.

Let $\{y_1, y_2, \dots, \}$ be the set of jump points of $D(y)$, i.e., the points for which $P(Y = y_k) > 0$.

» CDF of a mixed RV as a sum of Continuous and Discrete CDF...

CDF of Mixed Random Variable

The CDF of mixed random variable Y can be written as a sum of continuous and discrete/staircase function

$$F_Y(y) = C(y) + D(y).$$

$\int f_X(x) = 1$ $\quad \sum_i P_X(x_i) = 1$

We differentiate the continuous part $c(y) = \dfrac{dC(y)}{dy}$, where ever $C(y)$ is differentiable.

Let $\{y_1, y_2, \ldots, \}$ be the set of jump points of $D(y)$, i.e., the points for which $P(Y = y_k) > 0$. We have

$$\int_{-\infty}^{\infty} c(y)\, dy + \sum_{y_k} P(Y = y_k) = 1$$

PMF of D

PDF of C

» CDF of a mixed RV as a sum of Continuous and Discrete CDF...

CDF of Mixed Random Variable

The CDF of mixed random variable Y can be written as a sum of continuous and discrete/staircase function

$$F_Y(y) = C(y) + D(y).$$

We differentiate the continuous part $c(y) = \dfrac{dC(y)}{dy}$, where ever $C(y)$ is differentiable.

Let $\{y_1, y_2, \ldots,\}$ be the set of jump points of $D(y)$, i.e., the points for which $P(Y = y_k) > 0$. We have

$$\int_{-\infty}^{\infty} c(y)\, dy + \sum_{y_k} P(Y = y_k) = 1$$

The expected value of Y can be obtained as

» CDF of a mixed RV as a sum of Continuous and Discrete CDF...

CDF of Mixed Random Variable

The CDF of mixed random variable Y can be written as a sum of continuous and discrete/staircase function

$$F_Y(y) = C(y) + D(y).$$

We differentiate the continuous part $c(y) = \dfrac{dC(y)}{dy}$, where ever $C(y)$ is differentiable.

Let $\{y_1, y_2, \dots, \}$ be the set of jump points of $D(y)$, i.e., the points for which $P(Y = y_k) > 0$. We have

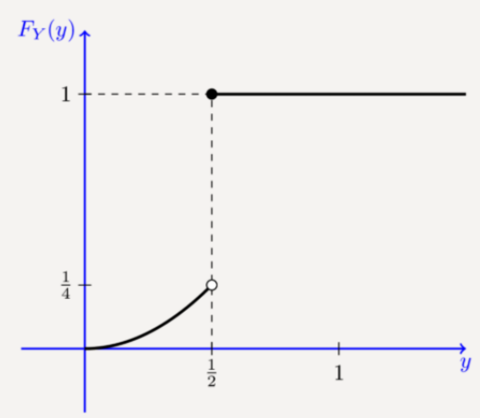
$$\int_{-\infty}^{\infty} c(y)\, dy + \sum_{y_k} P(Y = y_k) = 1$$

The expected value of Y can be obtained as

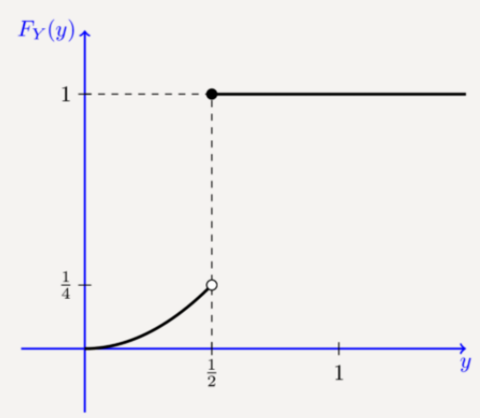
$$E[Y] = \int_{-\infty}^{\infty} y\, c(y)\, dy + \sum_{y_k} y_k\, P(Y = y_k)$$

» Check the Validity of CDF of Mixed RV...

» Check the Validity of CDF of Mixed RV...

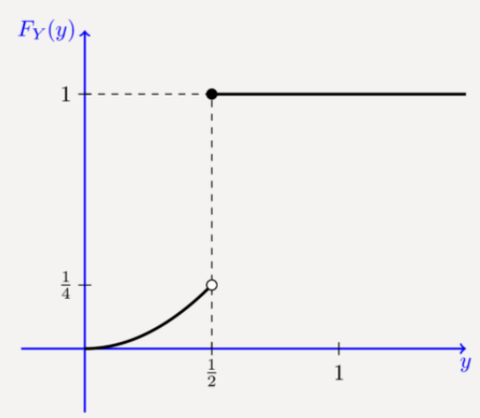


» Check the Validity of CDF of Mixed RV...



$$F_Y(y) = C(y) + D(y),$$

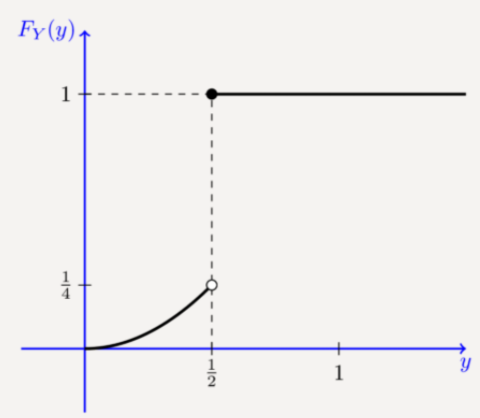
» Check the Validity of CDF of Mixed RV...



$$F_Y(y) = C(y) + D(y),$$

where the continuous part is

» Check the Validity of CDF of Mixed RV...

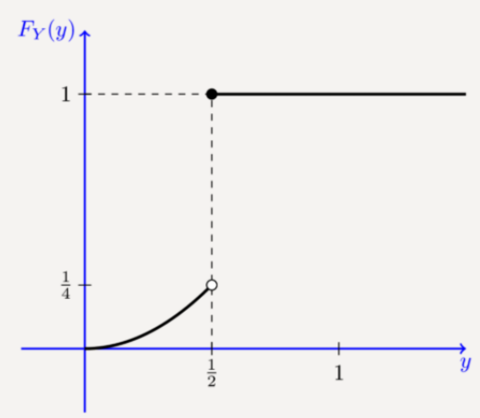


$$F_Y(y) = C(y) + D(y),$$

where the continuous part is ← last class

$$C(y) = \begin{cases} 1/4 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & y < 0 \end{cases}$$

» Check the Validity of CDF of Mixed RV...



$$F_Y(y) = C(y) + D(y),$$

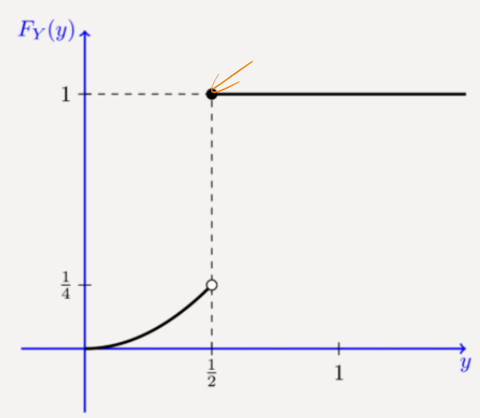
where the continuous part is

$$C(y) = \begin{cases} 1/4 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & y < 0 \end{cases}$$

and the discrete part is

» Check the Validity of CDF of Mixed RV...

PMF 3/4 PDF



$$F_Y(y) = C(y) + D(y),$$

where the continuous part is

$$c(y) = \begin{cases} 0 & y \geq 1/2 \\ 2y & 0 \leq y < 1/2 \\ 0 & y < 0 \end{cases}$$

$c(y) = 0$

$$C(y) = \begin{cases} 1/4 & y \geq 1/2 \\ y^2 & 0 \leq y < 1/2 \\ 0 & y < 0 \end{cases}$$

$c(y) = 2y$

$c(y) = 0$

and the discrete part is

$R_D(y) = \{\frac{1}{2}\}$

$$D(y) = \begin{cases} 3/4 & y \geq 1/2 \\ 0 & y < 1/2 \end{cases}$$

Check that

$$\int_{-\infty}^{\infty} c(y)\, dy + \sum_{y_k} P(Y = y_k) = 1$$

$\frac{1}{4} + \frac{3}{4} = 1$

$\int_{-\infty}^{\infty} c(y)\,dy = \int_{-\infty}^{\infty} = \int_0^{1/2} 2y\,dy = 2\left[\frac{y^2}{2}\right]_0^{1/2} = \frac{1}{4}$

» Computing Probabilities of Mixed Random Variable...

» Computing Probabilities of Mixed Random Variable...

Problem: Computing probabilities of mixed RV

Let X be a continuous random variable with the following PDF

» Computing Probabilities of Mixed Random Variable...

Problem: Computing probabilities of mixed RV

Let X be a continuous random variable with the following PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Problem: Computing probabilities of mixed RV

Let X be a continuous random variable with the following PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let

$$Y = g(X) = \begin{cases} X & 0 \leq X \leq \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

Problem: Computing probabilities of mixed RV

Let X be a continuous random variable with the following PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let

$$Y = g(X) = \begin{cases} X & 0 \leq X \leq \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

* Find $P(1/4 \leq Y \leq 3/8)$

» Computing Probabilities of Mixed Random Variable...

Problem: Computing probabilities of mixed RV

Let X be a continuous random variable with the following PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let

$$Y = g(X) = \begin{cases} X & 0 \leq X \leq \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

* Find $P(1/4 \leq Y \leq 3/8)$
* Find $P(Y \geq 1/4)$

Problem: Computing probabilities of mixed RV

Let X be a continuous random variable with the following PDF

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let

$$Y = g(X) = \begin{cases} X & 0 \leq X \leq \frac{1}{2} \\ \frac{1}{2} & X > \frac{1}{2} \end{cases}$$

* Find $P(1/4 \leq Y \leq 3/8)$
* Find $P(Y \geq 1/4)$
* Find $E[Y]$

» Answer to previous problem...

(a) $P(\frac{1}{4} \le Y \le \frac{3}{8}) = \underbrace{F_Y(\frac{3}{8}) - F_Y(\frac{1}{4})}_{P(\frac{1}{4} < Y \le \frac{3}{8})} + \underbrace{P_Y(\frac{1}{4})}_{\text{add back}}$

add back because disc. RV

$= \left(\frac{3}{8}\right)^2 - \left(\frac{1}{4}\right)^2 + 0$

Rk: For $P_Y(\frac{1}{4})$ we look into $D(y)$.

(b) $P(Y \ge \frac{1}{4}) = 1 - P(Y < \frac{1}{4})$

$= 1 - F_Y(\frac{1}{4}) + P(Y = \frac{1}{4}) \to 0$

$= 1 - \left(\frac{1}{4}\right)^2 = \frac{15}{16}$

(c) $E[Y]$ $\quad c(y) = \frac{dC(y)}{dy} = \begin{cases} 2y & 0 \le y \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

$E[Y] = \int_0^{1/2} y \cdot 2y \, dy + \frac{1}{2} \cdot P(Y = \frac{1}{2}) = \frac{2}{3} \cdot \frac{1}{8} + \frac{1}{2} \cdot \frac{3}{4} = \frac{1}{12} + \frac{3}{8} = \frac{11}{24}$

$F_Y(a) = P(Y \le a)$

Assuming a mixed R.V.

$F_Y(a) = P(Y < a) + P(Y = a)$

$\Rightarrow P(Y < a) = F_Y(a) - P(Y = a)$

[need to be careful in Disc.]

If X is mixed RV, then

$P(a < X \le b) \ne P(a \le X \le b)$

Recall: How was $F_Y(a)$

$\boxed{F_Y(a) = P(Y \le a)} \leftarrow P(Y < a) + P(Y = a)$

» Solved Problem 1

» Solved Problem 1

Problem 1

Let X be a random variable with CDF

» Solved Problem 1

Problem 1

Let X be a random variable with CDF

$$F_X(x) = \begin{cases} 1 & x \geq 1 \\ \frac{1}{2} + \frac{x}{2} & 0 \leq x < 1 \\ 0 & x < 0 \end{cases}$$

» Solved Problem 1

Problem 1

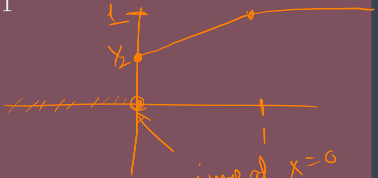
Let X be a random variable with CDF

$$F_X(x) = \begin{cases} 1 & x \geq 1 \\ \frac{1}{2} + \frac{x}{2} & 0 \leq x < 1 \\ 0 & x < 0 \end{cases}$$

1

$\frac{x+1}{2}$

0



1. Is the RV X discrete, continuous, or mixed type?

jump at $x=0$

$\Rightarrow$ X is mixed

» Solved Problem 1

Problem 1

Let X be a random variable with CDF

$$F_X(x) = \begin{cases} 1 & x \geq 1 \\ \frac{1}{2} + \frac{x}{2} & 0 \leq x < 1 \\ 0 & x < 0 \end{cases}$$

1. Is the RV X discrete, continuous, or mixed type?
2. Find the PDF of X, $f_X(x)$

Problem 1

Let X be a random variable with CDF

$$F_X(x) = \begin{cases} 1 & x \geq 1 \\ \frac{1}{2} + \frac{x}{2} & 0 \leq x < 1 \\ 0 & x < 0 \end{cases}$$

1. Is the RV X discrete, continuous, or mixed type?
2. Find the PDF of X, $f_X(x)$
3. Find $E(e^X)$

» Solved Problem 1

Problem 1

Let X be a random variable with CDF

$$F_X(x) = \begin{cases} 1 & x \geq 1 \\ \frac{1}{2} + \frac{x}{2} & 0 \leq x < 1 \\ 0 & x < 0 \end{cases}$$

1. Is the RV X discrete, continuous, or mixed type?
2. Find the PDF of X, $f_X(x)$
3. Find $E(e^X)$
4. Find $P(X = 0 \mid X \leq 0.5)$

» Answer to previous problem...

» Answer to previous problem...

» Solved Problem 2

Problem 2

Let $X \sim \text{Uniform}(-2, 2)$ be a continuous random variable. Let $Y = g(X)$ where

$$g(x) = \begin{cases} 1 & x > 1 \\ x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$x < 0$

Find the CDF of Y.

$\underbrace{R_Y = [0, 1]}$

$F_Y(0) = 0$ $\quad y < 0$

$F_Y(1) = 1$ $\quad y \geq 1$

» Answer to previous problem...

For $0 < y < 1$

$$F_Y(y) = P(Y \le y) = P(X \le y)$$

$$= F_X(y) = \int_{-2}^{y} \frac{1}{4}\, dx$$

$$= \frac{1}{4}\left[x\right]_{-2}^{y} = \frac{1}{4}(y+2)$$

CDF of Y

$$F_Y(y) = \begin{cases} \frac{1}{4}(y+2), & 0 \le y < 1 \\ 1, & y \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

Recall $X \sim U(a,b)$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b \\ 0 & x < a \text{ or } x > b \end{cases}$$

$$R_X = [\overset{a}{-2}, \overset{b}{2}]$$

$$\boxed{F_X(a) = \int_{-\infty}^{a} f_X(x)\, dx}$$

» Definition of Joint Probability Mass Function...

$P(X = x, Y = y)$

» Definition of Joint Probability Mass Function...

Definition of Joint Probability Mass Function

The joint probability mass function of two discrete random variables X and Y is defined as follows

Definition of Joint Probability Mass Function

The joint probability mass function of two discrete random variables X and Y is defined as follows

$$P_{XY}(x, y) = P(X = x, Y = y).$$

Definition of Joint Probability Mass Function

The joint probability mass function of two discrete random variables X and Y is defined as follows

$$P_{XY}(x, y) = P(X = x, Y = y).$$

* The joint range for X and Y is

Definition of Joint Probability Mass Function...

Definition of Joint Probability Mass Function

The joint probability mass function of two discrete random variables X and Y is defined as follows

$$P_{XY}(x, y) = P(X = x, Y = y).$$

* The joint range for X and Y is

$$R_{XY} = \{(x, y) \mid P_{XY} > 0\}$$

» Definition of Joint Probability Mass Function...

Definition of Joint Probability Mass Function

The joint probability mass function of two discrete random variables X and Y is defined as follows

$$P_{XY}(x, y) = P(X = x, Y = y).$$

* The joint range for X and Y is

$$R_{XY} = \{(x, y) \mid P_{XY} > 0\}$$

* In particular, if $R_X = \{x_1, x_2, \ldots\}$,

Definition of Joint Probability Mass Function

The joint probability mass function of two discrete random variables X and Y is defined as follows

$$P_{XY}(x, y) = P(X = x, Y = y).$$

* The joint range for X and Y is

$$R_{XY} = \{(x, y) \mid P_{XY} > 0\}$$

* In particular, if $R_X = \{x_1, x_2, \ldots\}$, $R_Y = \{y_1, y_2, \ldots\}$, then

» Definition of Joint Probability Mass Function...

Definition of Joint Probability Mass Function

The joint probability mass function of two discrete random variables X and Y is defined as follows

$$P_{XY}(x, y) = P(X = x, Y = y).$$

* The joint range for X and Y is

$$R_{XY} = \{(x, y) \mid P_{XY} > 0\}$$

* In particular, if $R_X = \{x_1, x_2, \ldots\}$, $R_Y = \{y_1, y_2, \ldots\}$, then

$$R_{XY} \subset \underline{R_X \times R_Y} = \{(x_i, y_j) \mid x_i \in R_X,\ y_j \in R_Y\}$$

* Sum of joint probabilities must sum to 1: $\sum_{(x_i, y_j) \in R_{XY}} P_{XY}(x_i, y_j) = 1$

» Computing Probabilities with Joint Distributions...

» Computing Probabilities with Joint Distributions...

Computing Joint Probability

For any $A \subset \mathbb{R}^2$, we have

» Computing Probabilities with Joint Distributions...

Computing Joint Probability

For any $A \subset \mathbb{R}^2$, we have

$$P((X, Y) \in A) = \sum_{(x_i, y_j) \in (A \cap R_{XY})} P_{XY}(x_i, y_j)$$

» Computing Probabilities with Joint Distributions...

Computing Joint Probability

For any $A \subset \mathbb{R}^2$, we have

$$P((X, Y) \in A) = \sum_{(x_i, y_j) \in (A \cap R_{XY})} P_{XY}(x_i, y_j)$$

Marginal PMF: Computing Individual Probabilities from Joint PMF

» Computing Probabilities with Joint Distributions...

Computing Joint Probability

For any $A \subset \mathbb{R}^2$, we have

$$P((X,Y) \in A) = \sum_{(x_i,y_j)\in(A\cap R_{XY})} P_{XY}(x_i,y_j)$$

Marginal PMF: Computing Individual Probabilities from Joint PMF

We can obtain PMF of X from its joint PMF with Y as follows

» Computing Probabilities with Joint Distributions...

Computing Joint Probability

For any $A \subset \mathbb{R}^2$, we have

$$P((X, Y) \in A) = \sum_{(x_i, y_j) \in (A \cap R_{XY})} P_{XY}(x_i, y_j)$$

Marginal PMF: Computing Individual Probabilities from Joint PMF

We can obtain PMF of X from its joint PMF with Y as follows

fixed

$$P_X(x) = P(X = x) = \sum_{y_j \in R_Y} P(X = x, Y = y_j) = \sum_{y_j \in R_Y} P_{XY}(x, y_j), \quad \text{for any } x \in R_X$$

» Computing Probabilities with Joint Distributions...

Computing Joint Probability

For any $A \subset \mathbb{R}^2$, we have

$$P((X, Y) \in A) = \sum_{(x_i, y_j) \in (A \cap R_{XY})} P_{XY}(x_i, y_j)$$

Marginal PMF: Computing Individual Probabilities from Joint PMF

We can obtain PMF of X from its joint PMF with Y as follows

$$P_X(x) = P(X = x) = \sum_{y_j \in R_Y} P(X = x, Y = y_j) = \sum_{y_j \in R_Y} P_{XY}(x, y_j), \quad \text{for any } x \in R_X$$

We call $P_X(x)$ the marginal PMF of X. Similarly, we have

» Computing Probabilities with Joint Distributions...

Computing Joint Probability

For any $A \subset \mathbb{R}^2$, we have

$$P((X,Y) \in A) = \sum_{(x_i,y_j)\in(A\cap R_{XY})} P_{XY}(x_i, y_j)$$

Marginal PMF: Computing Individual Probabilities from Joint PMF

We can obtain PMF of X from its joint PMF with Y as follows

$$P_X(x) = P(X = x) = \sum_{y_j\in R_Y} P(X = x, Y = y_j) = \sum_{y_j\in R_Y} P_{XY}(x, y_j), \quad \text{for any } x \in R_X$$

We call $P_X(x)$ the marginal PMF of X. Similarly, we have

$$P_Y(y) = \sum_{x_i\in R_X} P_{XY}(x_i, y), \quad \text{for any } y \in R_Y$$

sum over x_i

» Solved Example

Example

Let X and Y be two random variables with joint PMF as follows:

» Solved Example

Example

Let X and Y be two random variables with joint PMF as follows:

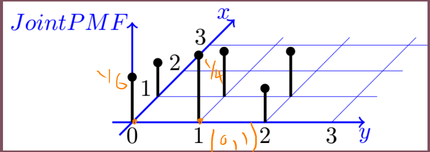
	Y=0	Y = 1	Y = 2
X = 0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
X = 1	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

» Solved Example

Example

Let X and Y be two random variables with joint PMF as follows:

	Y=0	Y = 1	Y = 2
X = 0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
X = 1	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$



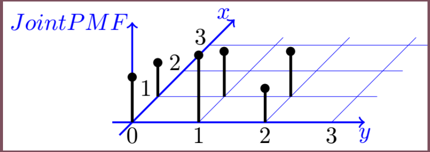
1. Find $P(X = 0, Y \leq 1)$

» Solved Example

Example

Let X and Y be two random variables with joint PMF as follows:

	Y=0	Y = 1	Y = 2
X = 0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
X = 1	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$



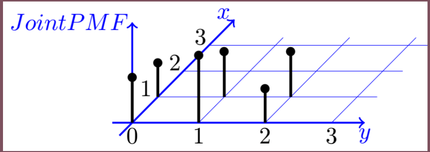
1. Find $P(X = 0, Y \leq 1)$
2. Find the marginal PMFs of X and Y

» Solved Example

Example

Let X and Y be two random variables with joint PMF as follows:

	Y=0	Y = 1	Y = 2
X = 0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
X = 1	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$



1. Find $P(X = 0, Y \leq 1)$
2. Find the marginal PMFs of X and Y
3. Find $P(Y = 1 \mid X = 0)$

» Solved Example

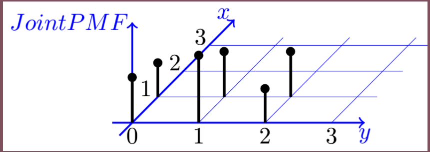
Example

Let X and Y be two random variables with joint PMF as follows:

	Y=0	Y = 1	Y = 2
X = 0	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{8}$
X = 1	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$

Sum this row to get $P_X(0)$

Sum to get $P_X(1)$



1. Find $P(X = 0, Y \leq 1)$
2. Find the marginal PMFs of X and Y
3. Find $P(Y = 1 \mid X = 0)$
4. Are X and Y independent?

» Answer to previous problem...

(1) $P(X=0, \underbrace{Y \leq 1}_{y \in \{0,1\}}) = P_{XY}(0,0) + P_{XY}(0,1)$

$= \frac{1}{6} + \frac{1}{4} = \frac{5}{12}$.

(2) Find $P_X(x)$ & $P_Y(y)$ (Marginals)

$R_X = \{0,1\}$, $R_Y = \{0,1,2\}$

$P_X(0) = P_{XY}(0,0) + P_{XY}(0,1) + P_{XY}(0,2)$

$= \frac{1}{6} + \frac{1}{4} + \frac{1}{8} = \_\_$

$P_X(1) = \frac{1}{8} + \frac{1}{6} + \frac{1}{6}$

(3) $P(Y=1 \mid X=0) = \frac{P(X=0, Y=1)}{P(X=0)}$

$= \frac{P_{XY}(0,1)}{P_X(\cancel{x}=0)} = \_\_$

(4) X & Y ind?

$P(Y=1 \mid X=0) = \frac{P_{XY}(0,1)}{P_X(0)}$

$= \frac{\frac{1}{4}}{\frac{1}{6}+\frac{1}{4}+\frac{1}{8}} = P(Y=1)$ check this

If they are equal, then try other conditional

Recall for X, Y ind

$P(Y=y_i \mid X=x_j) = P(Y=y_i)$

$\forall i, j$