Group Assignment 3
Int Elligence

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1 Section 2.1

Answer 2:

The given functional dependencies are possible for those values of n which are of the form $\frac{(k)(k+1)}{2}$ where k $\in \mathbb{N}$ as the relation R has k functional dependencies.
So, $\frac{(k-1)(k)}{2}$ + k = n $\rightarrow$ $n = \frac{(k)(k+1)}{2}$

Answer 3:

The relation R has k keys where k = $\frac{\sqrt{(1+8n)}-1}{2}$ which are $A_1, A_2A_3, A_4A_5A_6, ...$ as they are the values which can be obtained by forming the closure of $A_1A_2A_3...A_n$ i.e , $(A_1A_2A_3...A_n)^+$ and to find the keys of the set $\{A_1, A_2...A_n\}$ and then we obtain A_1 closure after reduction and as this value is on the R.H.S of the functional dependency and we can this replace by the corresponding functional dependency and hence obtain the other keys A_2A_3, $A_4A_5A_6$,... and so on.

Answer 4:

Checking 1 NF:

We say a given relation is in 1 NF if all the attributes have atomic value in their domain. Since the relation of the functional dependency defined in the question is $R(A_1, A_2, A_3...A_n)$ and hence we can see all the attributes are atomic.

Checking 2 NF:

We say that a given relation is in 2NF if the relation is in 1 NF and there is no partial dependency. Since the 1 NF condition is satisfied now checking for the partial dependency. As we can observe that there exist no possible extraneous attribute in L.H.S of any non-trivial Functional dependency of the given relation. Example : Say a $\rightarrow$ b is a non-trivial functional dependency and If a = A_2A_3 then we cannot reduce it to A_2 or A_3 on the basis of the given non-trivial relations and hence we can clearly state that there is no extraneous attribute and hence no partial dependency. Hence we can conclude that the given relation is in 2 NF.

Checking 3 NF:

We say that a given relation is in 3 NF if the relation is in 2 NF and there is no transitive dependency which means that there is no dependency between non-prime attributes and non prime attributes(i.e NPA $\rightarrow$ NPA does not exists in a relation) .
Say a $\rightarrow$ b is a non-trivial functional dependency from the given relation then either:
(1) 'a' should be super key —-(a) (2) 'b' is a prime attribute

Now coming to our given condition i.e. given non-trivial functional dependencies F | = X $\rightarrow$ Y,
X $\in$ { $A_1, A_2A_3, ...$} and Y = R-X ; where R is given relation
X only contains candidate keys which are after all super keys hence from (a) it satisfy third normal form. Hence we can conclude that the given relation is in 3 NF.

Checking BCNF:

We say a given relation is in BCNF if it is in 3 NF and for each non-trivial functional dependency X $\rightarrow$ Y , X must be a super key.
Now since the first condition is already satisfied , we check for second condition. Since in the above equations all the L.H.S of the equation (i.e 'X' from our above analogy in this particular BCNF case) all the keys are super keys. Hence we can conclude that the given relation is in BCNF. Since the given relation is already in BCNF, we do not need any further operations on it.

Answer 5:

We can obtain the minimal cover by following procedure:
i. R.H.S of the functional dependency should be simple.
ii. Remove the redundant property in the obtained functional dependency.
iii. We should try to reduce the L.H.S(remove extraneous attributes) the dependency to single attribute

Step 1:
In this step we need to decompose R.H.S to simple attribute. This is obtained as:
Let S be a set defined as $\mathbb{S} = \{1, 2, 3, ...,n\}$ and k = $\frac{\sqrt{(1+8n)}-1}{2}$
(i=1)

$$A_1 \rightarrow A_j \qquad \forall j \in \mathbb{S} - \{1\}$$

(i=2)

$$A_2A_3 \rightarrow A_j \qquad \forall j \in \mathbb{S} - \{2,3\}$$

(i=3)

$$A_4A_5A_6 \rightarrow A_j \qquad \forall j \in \mathbb{S} - \{4,5,6\}$$

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(i=k)

$$A_{n-k+1}A_{n-k+2}...A_n \rightarrow A_j \qquad \forall j \in \mathbb{S} - \{n-k+1, n-k+2, ..., n\}$$

Step 2:
Now we need to remove redundant properties.

(i=1) We can remove the functional dependencies $A_1 \rightarrow A_i; \forall i \in \{4,5,...,n\}$ since these $A_i \subseteq A_2A_3^+$
(i=2) We can remove the functional dependencies $A_2A_3 \rightarrow A_i; \forall i \in \{1,7,8,...,n\}$ since these $A_i \subseteq A_4A_5A_6^+$
(i=3) We can remove the functional dependencies $A_4A_5A_6 \rightarrow A_i; \forall i \in \{1,2,3,11,...,n\}$ since these $A_i \subseteq A_7A_8A_9A_{10}^+$
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(i=k) We can remove the attributes $A_{n-k+1}A_{n-k+2}...A_n \rightarrow A_i; \forall i \in \{2,3,4,...,k\}$ since these $A_i \subseteq A_1^+$

Step 3:
Now in the functional dependencies obtained from Step 2 all the attributes on L.H.S are candidate keys which means that they cannot be decomposed further into simpler attributes.
After these steps our minimal cover comes out to be :

$$A_1 \to A_2, \quad A_1 \to A_3$$

$$A_2A_3 \to A_4, \quad A_2A_3 \to A_5, \quad A_2A_3 \to A_6$$

$$A_4A_5A_6 \to A_7, \quad A_4A_5A_6 \to A_8, \quad A_4A_5A_6 \to A_9, \quad A_4A_5A_6 \to A_{10}$$

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$$A_{n-k+1}A_{n-k+2}...A_n \to A_1$$

2 Section 2.2

Answer 1:

There are in total n keys and they are $A_i\,(\forall\, i \in \{1,2,3,...,n\})$ because A_i^+ will have all $A_j\,(\forall\, j \in \{1,2,3,...,n\}\,)$. Also any key with more than one attributes is not possible since it's subset containing one attribute will be a key.

Answer 2:

Checking 1 NF:

We say a given relation is in 1 NF if all the attributes have atomic value in their domain. Since the relation of the functional dependency defined in the question is $R(A_1,A_2,A_3...A_n)$ and hence we can see all the attributes are atomic. Hence we can conclude that the given relation is in 1 NF.

Checking 2 NF:

We say that a given relation is in 2NF if the relation is in 1 NF and there is no partial dependency. Since the 1 NF condition is satisfied we just have to check for partial dependency. As we see that the relation has keys which consist of only one prime attribute , therefore no case of partial dependency possible.
Hence we can conclude that the given relation is in 2 NF.

Checking 3 NF:

We say that a given relation is in 3 NF if the relation is in 2 NF and there is no transitive dependency which means that there is no dependency between non-prime attributes and non-prime attributes (i.e NPA $\rightarrow$ NPA does not exists in a relation) .

Say a $\rightarrow$ b is a non-trivial functional dependency from the given relation then either:
(1) 'a' should be superkey (2) 'b' is a prime attribute

Now in our case here , we do not have any non-prime attributes i.e here all the attributes are prime and also all Functional Dependency of given relation has L.H.S which is superkey (basically candidate key).
Hence, we can conclude that the given relation is in 3 NF.

Checking BCNF:

We say a given relation is in BCNF if it is in 3 NF and for each non-trivial functional dependency X$\rightarrow$ Y , X must be a super key.
Now since the first condition is already satisfied , we check for second condition. Since in the above equations all the L.H.S of the equation (i.e 'X' from our above analogy in this particular BCNF case) all the keys are super keys. Hence we can conclude that the given relation is in BCNF. Since the given relation is already in BCNF, we do not need any further operations on it.

Answer 3:

We can obtain the minimal cover by following procedure:
i. R.H.S of the functional dependency should be simple.
ii. Remove the redundant property in the obtained functional dependency.
iii. We should try to reduce the L.H.S(remove extraneous attributes) of the dependency to single attribute

Steps to find the minimal cover for the relation defined below:-

$A_i \rightarrow$ A$_j$; where i $\in$ (A$_1$,A_2,A_3...A_n) and j $\in$ (A$_1$,A_2,A_3...A_n) -A_i

Step 1:
Since in the above functional dependency , R.H.S is already singular, we do not need to do anything in this step.

Step 2:
We can remove the functional dependencies defined as $A_1 \rightarrow A_j$; $\forall$ j $\in (3,4,...,n)$
We can remove the functional dependencies defined as $A_2 \rightarrow A_j$; $\forall$ j $\in (1,4,5,...,n)$
We can remove the functional dependencies defined as $A_3 \rightarrow A_j$; $\forall$ j $\in (1,2,5,...,n)$

Following the above sequence of removing to other A_i where i $\in \{4,5,...,n\}$
We will remain with relation as following :

$$A_1 \rightarrow A_2$$
$$A_2 \rightarrow A_3$$
$$A_3 \rightarrow A_4$$
$$A_4 \rightarrow A_5$$
$$.$$
$$.$$
$$.$$
$$A_{n-1} \rightarrow A_n$$
$$A_n \rightarrow A_1$$

Step 3:
Now in the above functional dependency , the L.H.S is already singular, and hence we do not need to do anything in this step.

Hence the required minimal cover is :

$$A_1 \rightarrow A_2$$
$$A_2 \rightarrow A_3$$
$$A_3 \rightarrow A_4$$
$$A_4 \rightarrow A_5$$
$$.$$
$$.$$
$$.$$
$$A_{n-1} \rightarrow A_n$$
$$A_n \rightarrow A_1$$