The best transformation that will transform points from frame P to frame Q will be the one corresponding to which minimizes the following error:

$$\| Q - (RP + t) \|_2^2$$

Finding value of t in terms of R at minima of ~~following~~ given error

$$F(t) = \sum_{i=1}^{n} \| RP_i + t - Q_i \|^2$$

$$\frac{\partial F(t)}{\partial t} = 2 \sum (RP_i + t - Q_i) = 0$$

$$\Rightarrow \quad t = \frac{1}{n} \sum_{i=1}^{n} Q_i - \frac{R}{n} \sum_{i=1}^{n} P_i$$

$$\Rightarrow \quad t = \bar{Q} - R\bar{P}$$

Hence optimal $R = \underset{R \in SO(3)}{\arg\min} \; \| R(P_i - \bar{P}) - (Q_i - \bar{Q}) \|^2$

Let $X = (P_i - \bar{P})$, $Y = (Q_i - \bar{Q})$

$$Z = RX.$$

By using properties of trace of a matrix.

$$\sum_{i=1}^{n} \| Z_i - Y_i \|^2 = Tr\left((Z-Y)^T (Z-Y) \right)$$

$$Tr((Z-Y)^T(Z-Y)) = Tr(Z^TZ) + Tr(Y^TY) - 2\,Tr(Y^TZ)$$

Since R is orthogonal & $Z = RX$,

$Tr(Z^TZ)$ is independent of R

as $|Z|^2 = |X|^2$

Also $Tr(Y^TY)$ is independent of R.

Hence ~~of~~ optimal $R = \underset{R \in SO(3)}{\operatorname{argmax}}\ Tr(Y^TRX)$

$$Tr(Y^TRX) = Tr(XY^TR)$$

Using SVD on $XY^T = UDV^T$

$$Tr(XY^TR) = Tr(UDV^TR)$$

$$= Tr(DV^TRU)$$

$$= \sum_{i=1}^{3} d_i(v_i^TRu_i)$$

where $D = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$

$U = [u_1\ u_2\ u_3]$ & $V = [v_1\ v_2\ v_3]$

Let $M = V^T R U$.

$\Rightarrow Tr(Y^T X) = \sum_{i=1}^{3} d_i M_{ii} \leq \sum_{i=1}^{3} d_i$

Because, U, V & R are orthagonatnal
Hence M is orthognormal Hence
$\|M\| = 1$
$\Rightarrow |M_{ii}| \leq 1$

To maximize the trace, we need all M_{ii} to be maximized.
i.e. $M = I_3$

$\Rightarrow V^T R U = I$
$\Rightarrow R = V U^T$

Now, ~~if~~ to ensure $R \in SO(3)$, $\det(R)$ needs to be 1.
If the above formula gives us $\det(R) = -1$, then we need to find M corresponding to next largest value which would be
$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, because $d_1 \geq d_2 \geq d_3$

Hence, optimal $R = V C U^T$

where $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \det(VU^T) \end{bmatrix}$