Sentinels
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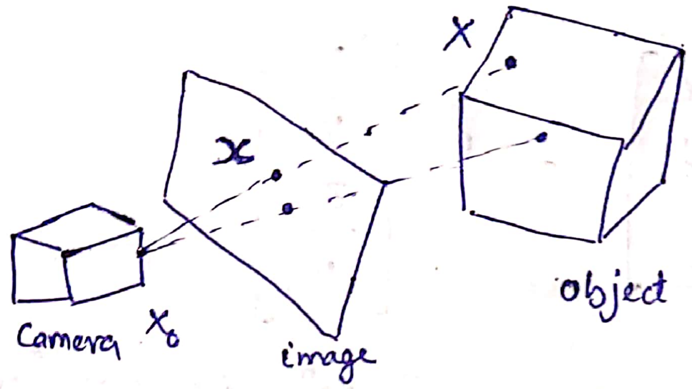
Sol 1 Direct Linear Transform

DLT maps any object X to image point x.



It estimates extrinsic and intrinsic parameters of camera, given the coordinates of object points (or the control points)

We are given a worldpoint "X" and the corresponding image point "x" and they satisfy

$$x = P X$$

$$x_{3\times1} = K_{3\times3} \, R_{3\times3} \underbrace{\left[\underset{3\times3}{I_3} \mid \underset{3\times1}{-X_0} \right]}_{3\times4} X_{4\times1}$$

Intrinsics : Camera - initial parameters
Given through K

Extrinsics : Pose parameters of camera
Given through X_0 and R

So the projection matrix P, contains both intrinsic and extrinsic parameters.

P is a 3x4 matrix, but has 11 unknowns that are 3 rotation, 3 translation, f_x, f_y, c, S and m. (its homogenous hence leaving us 11 unknown out of 12)

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = P \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Each point gives us 2 eq^n i.e one for x & other for y hence we need a minimum of 6 points for uncalibrated camera. and assuming the model of affine camera.

The two observation eq^n one for each image coordinate will look like

$$x = \frac{P_{11} X + P_{12} Y + P_{13} Z + P_{14}}{P_{31} X + P_{32} Y + P_{33} Z + P_{34}}$$

$$y = \frac{P_{21} X + P_{22} Y + P_{23} Z + P_{24}}{P_{31} X + P_{32} Y + P_{33} Z + P_{34}}$$

We stack them

$$x_i = P_{3\times 4} X_i$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = x_i = \begin{bmatrix} \boxed{p_{11} \quad p_{12} \quad p_{13} \quad p_{14}} \\ \boxed{p_{21} \quad p_{22} \quad p_{23} \quad p_{24}} \\ \boxed{p_{31} \quad p_{32} \quad p_{33} \quad p_{34}} \end{bmatrix} X_i = \begin{bmatrix} A^T \\ B^T \\ C^T \end{bmatrix} X_i$$

$$x_i = \frac{u_i}{w_i} = \frac{A^T X_i}{C^T X_i} \quad \text{and} \quad y_i = \frac{v_i}{w_i} = \frac{B^T X_i}{C^T X_i}$$

Leads to a system of eqⁿ which is linear in parameters A, B and C

$$-X_i^T A + 0 . B + x_i X_i^T C = 0 \quad — (1)$$

$$0 A - X_i^T B + y_i X_i^T C = 0 \quad — (2)$$

Collecting elements of P within parameter vector p;

$$p = (p_k) = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \text{vec}(P^T)$$

↑

row of P as column-vectors one below the other (12x1)

Rewriting eq (1) & (2) as

$$a_{x_i}^T p = 0 \quad — (4)$$

$$a_{y_i}^T p = 0 \quad — (5)$$

with $a_{x_i}^T = (-X_i^T, 0^T, x_i X_i^T)$

$$= (-X_i, -Y_i, -Z_i, -1, 0, 0, 0, 0, x_i X_i, x_i Y_i, x_i Z_i, x_i)$$

and

$$a_{y_i}^T = (0^T, -X_i^T, y_i X_i^T)$$

$$= (0, 0, 0, 0, -X_i, -Y_i, -Z_i, -1, +y_i X_i, +y_i Y_i, +y_i Z_i, +y_i)$$

(for ~~a~~ 6 or more points)

We combine the eq ④ & ⑤ to obtain

$$A_{2n \times 12} \, P_{12 \times 1} = 0 \qquad \left(A = \begin{bmatrix} a_{x_i}^T \\ a_{y_i}^T \end{bmatrix} \right)$$

Now due to noise, "a" would most likely be a full rank and have non-existent null space.

(no P would satisfy for all points in A)

hence we minimize

$$\|Ap\|_2^2 \quad \text{such that} \quad \|p\| = 1$$

Thus we can apply SVD to solve $Mp \neq 0$

as

$Mp = w$

finding p such that it minimizes

$\Omega = w^T w$

$\Rightarrow \hat{p} = \underset{p}{\arg\min}\ w^T w$

So the objective of DLT is

$$p = \underset{p}{\arg\min}\ w^T w$$

$$p = \underset{p}{\arg\min}\ p^T M^T M p$$

with $\|p\|_2 = \sum_{ij} p_{ij}^2 = \|p\| = 1$

Singular value decomposition

$$M_{2I \times 12} = U_{2I \times 12} \; S_{12 \times 12} \; V^T_{12 \times 12} = \sum_{i=1}^{12} S_i u_i v_i^T$$

- Choosing $p = v_{12}$ (the singular vector belonging to smallest singular value S_{12}) minimises Ω

The proof for why this is so was done in ~~cats'~~ class.

for last point above we stated, our problem now reduces to Constrained least - sq. minimization; i.e finding the h that minimizes $\|Ah\|$ subjected to $\|h\| = 1$

or $1 - h^T h = 0$ (from paper given to read on moodle)

to find an extreme (the sought h) we solve

$$\frac{d\left(h^T A^T A h + \lambda(1 - h^T h)\right)}{dh} = 0$$

$$\Rightarrow \left(A^T h - \lambda I\right) h = 0$$

This is the characteristic eq^n and we can say that h is eigen vector of $A^T A$ and λ is eigen value.

The least - square error is

$$e = h^T A^T A h = h^T \lambda h$$

This error will be minimum for

$\lambda = \min_i \lambda_i$ and the sought sol^n is

eigen vector of matrix $(A^T A)$ corresponding to smallest eigenvalue.

$$\text{SDV of } A = \underset{\substack{m\times n \\ \downarrow \\ n\times n}}{U} S \underset{n\times n}{V^T}$$

From orthonormality of U, V; it follows that

$$\|USV^T h\| = \|SV^T h\| \text{ and } \|V^T h\| = \|h\|$$

Substitute $y = V^T h$, now we minimize $\|Sy\|$ s.t. $\|y\| = 1$

Since S is diagonal and elements are sorted descendingly

it is clear that $y = [0, 0 \ldots 1]^T$

From substitution, we know that $h = Vy$ from which follows that sought h is the last column

of the matrix V.

Sol 2

DLT fails when:

a) if control points lie on a plane.

We have M of rank 11 if

→ no. of points ≥ 6

→ no gross error is assumed

$$M = \begin{bmatrix} \cdots \\ a_{x_i}^T \\ a_{y_i}^T \\ \cdots \end{bmatrix}$$

eg. assume all $Z=0$

$$= \begin{bmatrix} -X_i & -Y_i & \boxed{-Z_i} & -1 & 0 & 0 & \boxed{0} & 0 & x_i X_i & x_i Y_i & \boxed{x_i Z_i} & x_i \\ 0 & 0 & \boxed{0} & 0 & -X_i & -Y_i & \boxed{-Z_i} & -1 & y_i X_i & y_i Y_i & \boxed{y_i Z_i} & y_i \end{bmatrix}$$

leads to rank deficiency

b) All points X_i are the projected center X_o and located on a twisted cube curve

c) SDV dosen't deal well with outliners, that are irremovable

d) in majority of scenario, hence DLT fails for the case.

RANSAC (RANdom SAmple Consensus) is used to remove effect of outliners