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Assignment - 8

CSCE - 5150

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The statement "CNF-SAT is polynomial-time reducible to DNF-SAT by distributing ∨ over ∧ and distributing ∧ over ∨" is false because transforming a CNF formula by distribution can result in an exponential increase in the size of the formula, not a polynomial time operation.

Example:
consider a CNF formula F with three variables x_1, x_2 and x_3:

$$F = (x_1 \vee x_2) \wedge (\neg x_1 \vee x_3) \wedge (x_2 \vee \neg x_3)$$

By distributing ∨ over ∧, we attempt to convert F into a DNF:

$$F_{DNF} = (x_1 \wedge \neg x_1 \wedge x_2) \vee (x_1 \wedge \neg x_1 \wedge \neg x_3) \vee (x_1 \wedge x_3 \wedge x_2) \vee \ldots \vee (\neg x_1 \wedge x_3 \wedge \neg x_3)$$

Notice that F_{DNF} is a disjunction of 8 clauses (not all listed), which is 2^3 for 3 variables.

For a CNF with n variables, this process can produce up to 2^n clauses in the worst case, which is clearly an expotential expansion.

if we could convert CNF to DNF in polynomial time, we would be able to solve CNF-SAT (an NP-complete problem) in polynomial time by converting it to DNF-SAT (which can be solved in polynomial time), thus providing P=NP.

∴ The statement that CNF-SAT is reducible to DNF-SAT in polynomial time is false. Converting CNF to DNF can cause an expontential increase in the formula's size, not adhering to polynomial-time complexity. Since CNF-SAT is NP-Complete and DNF-SAT is polynomially solvable, such a reduction would imply P=NP, which ~~contradicts~~ contradicts prevailing beliefs in computational complexity.