Analysis of Computer Algorithms
Homework Assignment 1

Due: 2/2/2024 on Canvas

(100 points)

Question 1 (15 points) Answer the following questions. All running times refer to the worst-case analysis.

1. John came up with an algorithm for some problem that runs in time $\Theta(n^2 \log(n))$, and Bill came up with an algorithm for the same problem that runs in time $o(n^2 \log(n))$. Based on this information, which one would you choose? Why?

2. John tells you that a certain algorithm runs in time $O(n^3 + 200n)$, and Bill tells you that the same algorithm runs in time $\Omega(n^3)$. Can both John and Bill be correct? Why?

3. John tells you that a certain algorithm runs in time $\Omega(n^2 + 200n)$, and Bill tells you that the same algorithm runs in time $\Omega(n^3)$. Assume that both statements are correct, which one is more informative, i.e., gives you a better estimation of the running time? Why?

Question 2 (20 points) Show that $\frac{n^2}{2} - 2n = \Theta(n^2)$. Show your work and give specific values for c_1, c_2, and n_0.

Question 3 (20 points) Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Show that $f(n) + g(n) = O(max(f(n), g(n)))$. Show your work and give specific values for c and n_0.
Hint: For a given n, $max(f(n), g(n))$ is $f(n)$ if $f(n) \geq g(n)$, otherwise it is $g(n)$.

Question 4 (10 points) Show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$. Do not use the Master Theorem.

Question 5 (15 points) Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 3T(n/3) + n$. Use the substitution method to verify your answer.

Question 6 (20 points) Solve the following recurrences using the Master Theorem.

- $T(n) = 2T(n/4) + 1$
- $T(n) = 2T(n/4) + \sqrt{n}$
- $T(n) = 2T(n/4) + n$
- $T(n) = 2T(n/4) + n^2$