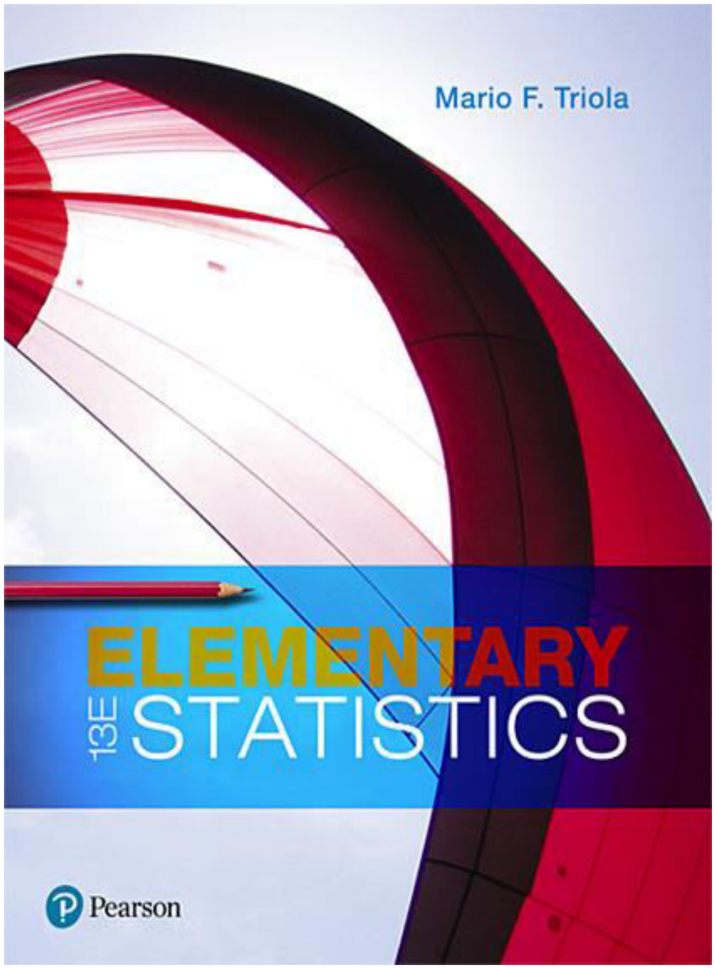
Elementary Statistics

Thirteenth Edition



Chapter 5

Probability Distributions

Probability Distributions

5-1 Probability Distributions

5-2 Binomial Probability Distributions

5-3 Poisson Probability Distributions

Key Concept

This section introduces the concept of a **random variable** and the concept of a **probability distribution.**

We illustrate how a **probability histogram** is a graph that visually depicts a probability distribution.

We show how to find the important parameters of mean, standard deviation, and variance for a probability distribution.

Most importantly, we describe how to determine whether outcomes are **significant** (significantly low or significantly high).

Basic Concepts of Probability Distribution (1 of 4)

- Random Variable
  - A **random variable** is a variable (typically represented by x) that has a single numerical value, determined by chance, for each outcome of a procedure.

Basic Concepts of Probability Distribution (2 of 4)

- Probability Distribution
  - A **probability distribution** is a description that gives the probability for each value of the random variable. It is often expressed in the format of a table, formula, or graph.

Basic Concepts of Probability Distribution (3 of 4)

- Discrete Random Variable
  - A **discrete random variable** has a collection of values that is finite or countable. (If there are infinitely many values, the number of values is countable if it is possible to count them individually, such as the number of tosses of a coin before getting heads.)

Basic Concepts of Probability Distribution (4 of 4)

- Continuous Random Variable
  - A **continuous random variable** has infinitely many values, and the collection of values is not countable. (That is, it is impossible to count the individual items because at least some of them are on a continuous scale, such as body temperatures.)

Probability Distribution Requirements (1 of 2)

Every probability distribution must satisfy each of the following three requirements.

1. There is a **numerical** (not categorical) random variable *x*, and its number values are associated with corresponding probabilities.
2. $\sum P(x) = 1$ where *x* assumes all possible values. (The sum of all probabilities must be 1, but sums such as 0.999 or 1.001 are acceptable because they result from rounding errors.)

Probability Distribution Requirements (2 of 2)

3. $0 \le P(x) \le 1$ for every individual value of the random variable x. (That is, each probability value must be between 0 and 1 inclusive.)

Example: Coin Toss (1 of 3)

Let's consider tossing two coins, with the following random variable:

x = number of heads when two coins are tossed

The above x is a random variable because its numerical values depend on chance.

***x*: Number of Heads When Two Coins Are Tossed**	***P*(*x*)**
0	0.25
1	0.50
2	0.25

Example: Coin Toss (2 of 3)

With two coins tossed, the number of heads can be 0, 1, or 2, and the table is a probability distribution because it gives the probability for each value of the random variable *x* and it satisfies the three requirements listed earlier:

1. The variable x is a **numerical** random variable, and its values are associated with probabilities.
2. $\sum P(x) = 0.25 + 0.50 + 0.25 = 1$
3. Each value of $P(x)$ is between 0 and 1.

Example: Coin Toss (3 of 3)

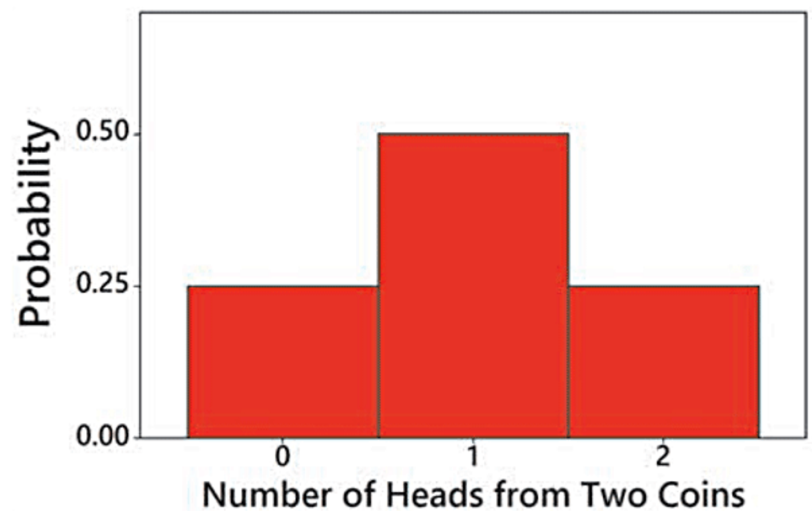
The random variable *x* in the table is a discrete random variable, because it has three possible values (0, 1, 2), and three is a finite number, so this satisfies the requirement of being finite.

***x*: Number of Heads When Two Coins Are Tossed**	***P*(*x*)**
0	0.25
1	0.50
2	0.25

Probability Histogram: Graph of a Probability Distribution

A probability histogram is similar to a relative frequency histogram, but the vertical scale shows **probabilities** instead of relative frequencies based on actual sample results.

Probability Histogram for Number of Heads When Two Coins Are Tossed



Probability Formula

A probability distribution could also be in the form of a formula. Consider the formula

$$P(x) = \frac{1}{2(2-x)!x!} \quad \text{(where } x \text{ can be 0, 1 or 2).}$$

We find that $P(0) = 0.25$, $P(1) = 0.50$, and $P(2) = 0.25$. The probabilities found using this formula are the same as those in the table.

Example: Job Interview Mistakes (1 of 2)

Hiring managers were asked to identify the biggest mistakes that job applicants make during an interview, and the table below is based on their responses (based on data from an Adecco survey). Does the table below describe a probability distribution?

x	$P(x)$
Inappropriate attire	0.50
Being late	0.44
Lack of eye contact	0.33
Checking phone or texting	0.30
Total	**1.57**

Example: Job Interview Mistakes (2 of 2)

Solution

The table violates the first requirement because *x* is not a **numerical** random variable. The "values" of *x* are categorical data, not numbers.

The table also violates the second requirement because the sum of the probabilities is 1.57, but that sum should be 1.

Because the three requirements are not all satisfied, we conclude that the table does **not** describe a probability distribution.

Parameters of a Probability Distribution (1 of 3)

Remember that with a probability distribution, we have a description of a **population** instead of a sample, so the values of the mean, standard deviation, and variance are **parameters,** not statistics.

The mean, variance, and standard deviation of a discrete probability distribution can be found with the following formulas:

Parameters of a Probability Distribution (2 of 3)

- **Mean, *μ*,** for a probability distribution

$$\mu = \sum [x \cdot P(x)]$$

- **Variance**, ***σ*²**, for a probability distribution

$$\sigma^2 = \sum \left[(x-\mu)^2 \cdot P(x) \right]$$ (This format is easier to understand.)

- **Variance**, ***σ*²**, for a probability distribution

$$\sigma^2 = \sum \left[x^2 \cdot P(x) \right] - \mu^2$$ (This format is easier for manual calculations.)

Parameters of a Probability Distribution (3 of 3)

- **Standard deviation**, ***σ***, for a probability distribution

$$\sigma = \sqrt{\sum \left[x^2 \cdot P(x) \right] - \mu^2}$$

Expected Value (1 of 2)

- Expected Value
  - The **expected value** of a discrete random variable x is denoted by E, and it is the mean value of the outcomes, so $E = \mu$ and E can also be found by evaluating $\sum [x \cdot P(x)]$.

Example: Finding the Mean, Variance, and Standard Deviation (1 of 5)

The table describes the probability distribution for the number of heads when two coins are tossed. Find the mean, variance, and standard deviation for the probability distribution described.

x: Number of Heads When Two Coins Are Tossed	**$P(x)$**
0	0.25
1	0.50
2	0.25

Example: Finding the Mean, Variance, and Standard Deviation (2 of 5)

x	$P(x)$	$x \cdot P(x)$	$(x - \mu)^2 \cdot P(x)$
0	0.25	$0 \cdot 0.25 = 0.00$	$(0 - 1.0)^2 \cdot 0.25 = 0.25$
1	0.50	$1 \cdot 0.50 = 0.50$	$(1 - 1.0)^2 \cdot 0.50 = 0.00$
2	0.25	$2 \cdot 0.25 = 0.50$	$(2 - 1.0)^2 \cdot 0.25 = 0.25$
Total		1.00 ↑ $\mu = \sum [x \cdot P(x)]$	0.50 ↑ $\sigma^2 = \sum [(x - \mu)^2 \cdot P(x)]$

Solution

The two columns at the left describe the probability distribution. The two columns at the right are for the purposes of the calculations required.

Example: Finding the Mean, Variance, and Standard Deviation (3 of 5)

x	$P(x)$	$x \cdot P(x)$	$(x-\mu)^2 \cdot P(x)$
0	0.25	$0 \cdot 0.25 = 0.00$	$(0-1.0)^2 \cdot 0.25 = 0.25$
1	0.50	$1 \cdot 0.50 = 0.50$	$(1-1.0)^2 \cdot 0.50 = 0.00$
2	0.25	$2 \cdot 0.25 = 0.50$	$(2-1.0)^2 \cdot 0.25 = 0.25$
Total		1.00 ↑ $\mu = \sum[x \cdot P(x)]$	0.50 ↑ $\sigma^2 = \sum[(x-\mu)^2 \cdot P(x)]$

Solution

Mean: $\sum[x \cdot P(x)] = 1.0$

Variance: $\sigma^2 = \sum\left[(x-\mu)^2 \cdot P(x)\right] = 0.5$

Example: Finding the Mean, Variance, and Standard Deviation (4 of 5)

x	$P(x)$	$x \cdot P(x)$	$(x - \mu)^2 \cdot P(x)$
0	0.25	$0 \cdot 0.25 = 0.00$	$(0 - 1.0)^2 \cdot 0.25 = 0.25$
1	0.50	$1 \cdot 0.50 = 0.50$	$(1 - 1.0)^2 \cdot 0.50 = 0.00$
2	0.25	$2 \cdot 0.25 = 0.50$	$(2 - 1.0)^2 \cdot 0.25 = 0.25$
Total		1.00 $\uparrow$ $\mu = \sum [x \cdot P(x)]$	0.50 $\uparrow$ $\sigma^2 = \sum [(x - \mu)^2 \cdot P(x)]$

Solution

The standard deviation is the square root of the variance, so

$$\text{Standard deviation: } \sigma = \sqrt{0.5} = 0.707107 = 0.7$$

Example: Finding the Mean, Variance, and Standard Deviation (5 of 5)

Interpretation

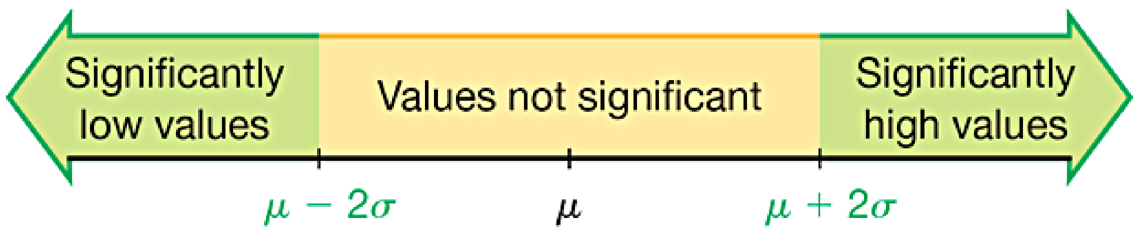
When tossing two coins, the mean number of heads is 1.0 head, the variance is 0.50 heads^2, and the standard deviation is 0.7 head.

Also, the expected value for the number of heads when two coins are tossed is 1.0 head, which is the same value as the mean. If we were to collect data on a large number of trials with two coins tossed in each trial, we expect to get a mean of 1.0 head.

Identifying Significant Results with the Range Rule of Thumb

Range Rule of Thumb for Identifying Significant Values

- **Significantly low** values are $(\mu - 2\sigma)$ or lower.
- **Significantly high** values are $(\mu + 2\sigma)$ or higher.
- **Values not significant**: Between $(\mu - 2\sigma)$ and $(\mu + 2\sigma)$.



Pearson

Example: Identifying Significant Results with the Range Rule of Thumb (1 of 3)

We found that when tossing two coins, the mean number of heads is $\mu = 1.0$ head and the standard deviation is $\sigma = 0.7$ head. Use those results and the range rule of thumb to determine whether 2 heads is a significantly high number of heads.

Example: Identifying Significant Results with the Range Rule of Thumb (2 of 3)

Solution

Using the range rule of thumb, the outcome of 2 heads is significantly high if it is greater than or equal to $(\mu + 2\sigma)$. With $\mu = 1.0$ head $\sigma = 0.7$ head, we get

$$(\mu + 2\sigma) = 1 + 2(0.7) = 2.4 \text{ heads}$$

Significantly high numbers of heads are 2.4 and above.

Example: Identifying Significant Results with the Range Rule of Thumb (3 of 3)

Interpretation

Based on these results, we conclude that 2 heads is not a significantly high number of heads (because 2 is not greater than or equal to 2.4).

Identifying Significant Results with Probabilities: (1 of 2)

- Significantly high number of successes:
  - *x* successes among *n* trials is a **significantly high** number of successes if the probability of *x* or more successes is 0.05 or less. That is, *x* is a significantly high number of successes if *P*(*x* **or more**) ≤ 0.05.

The value 0.05 is not absolutely rigid. Other values, such as 0.01, could be used to distinguish between results that are significant and those that are not significant.

Identifying Significant Results with Probabilities: (2 of 2)

- Significantly low number of successes:
  - x successes among n trials is a **significantly low** number of successes if the probability of x or fewer successes is 0.05 or less. That is, x is a significantly low number of successes if P(x **or fewer**) ≤ 0.05.

The value 0.05 is not absolutely rigid. Other values, such as 0.01, could be used to distinguish between results that are significant and those that are not significant.

The Rare Event Rule for Inferential Statistics

If, under a given assumption, the probability of a particular outcome is very small and the outcome occurs **significantly less than or significantly greater than** what we expect with that assumption, we conclude that the assumption is probably not correct.

Expected Value (2 of 2)

The expected value of a random variable x is equal to the mean μ. We can therefore find the expected value by computing $\sum[x \cdot P(x)]$, just as we do for finding the value of μ.

Example: Be a Better Bettor (1 of 6)

You have $5 to place on a bet in the Golden Nugget casino in Las Vegas. You have narrowed your choice to one of two bets:

Roulette: Bet on the number 7 in roulette.

Craps: Bet on the "pass line" in the dice game of craps.

Example: Be a Better Bettor (2 of 6)

a. If you bet \$5 on the number 7 in roulette, the probability of losing \$5 is $\frac{37}{38}$ and the probability of making a net gain of \$175 is $\frac{1}{38}$. (The prize is \$180, including your \$5 bet, so the net gain is \$175.) Find your expected value if you bet \$5 on the number 7 in roulette.

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Example: Be a Better Bettor (3 of 6)

b. If you bet \$5 on the pass line in the dice game of craps, the probability of losing \$5 is $\frac{251}{495}$ and the probability of making a net gain of \$5 is $\frac{244}{495}$. (If you bet \$5 on the pass line and win, you are given \$10 that includes your bet, so the net gain is \$5.) Find your expected value if you bet \$5 on the pass line.

Which of the preceding two bets is better in the sense of producing higher expected value?

Example: Be a Better Bettor (4 of 6)

Solution

a. **Roulette** The probabilities and payoffs for betting $5 on the number 7 in roulette are summarized in the table. The table also shows that the expected value is $\sum[x \cdot P(x)]$ = –26¢. That is, for every $5 bet on the number 7, you can expect to **lose** an average of 26¢.

Event	x	$P(x)$	$x \cdot P(x)$
Lose	–$5	$\frac{37}{38}$	–$4.868421
Win (net gain)	$175	$\frac{1}{38}$	$4.605263
Total			–$0.26 (rounded) (or -26¢)

Pearson

Example: Be a Better Bettor (5 of 6)

Solution

b. **Craps Game** The probabilities and payoffs for betting $5 on the pass line in craps are summarized in the table. The table also shows that the expected value is $\sum[x \cdot P(x)] = -7¢$. That is, for every $5 bet on the pass line, you can expect to lose an average of 7¢.

Event	***x***	***P*(*x*)**	***x* · *P*(*x*)**
Lose	–$5	$\frac{251}{495}$	–$2.535353
Win (net gain)	$5	$\frac{244}{495}$	–$2.464646
Total			–$0.07 (rounded) (or –7¢)

Example: Be a Better Bettor (6 of 6)

Interpretation

The \$5 bet in roulette results in an expected value of −26¢ and the \$5 bet in craps results in an expected value of −7¢. Because you are better off losing 7¢ instead of losing 26¢, the craps game is better in the long run, even though the roulette game provides an opportunity for a larger payoff when playing the game once.

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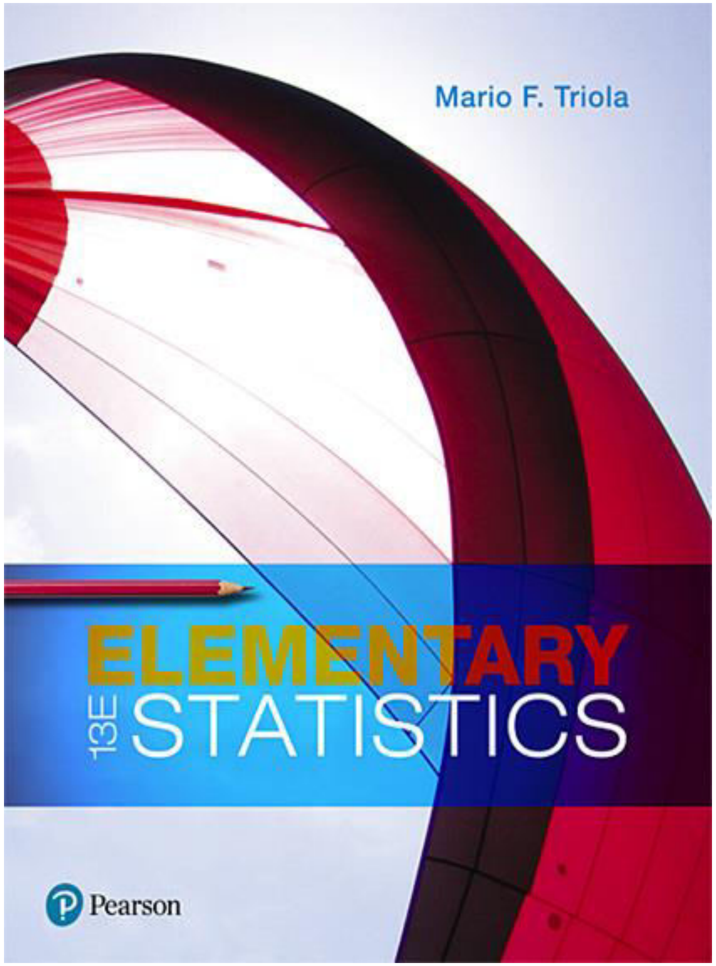


Chapter 6

Normal Probability Distributions

Normal Probability Distributions

6-1 The Standard Normal Distribution

6-2 Real Applications of Normal Distributions

6-3 Sampling Distributions and Estimators

6-4 The Central Limit Theorem

6-5 Assessing Normality

6-6 Normal as Approximation to Binomial

Key Concept

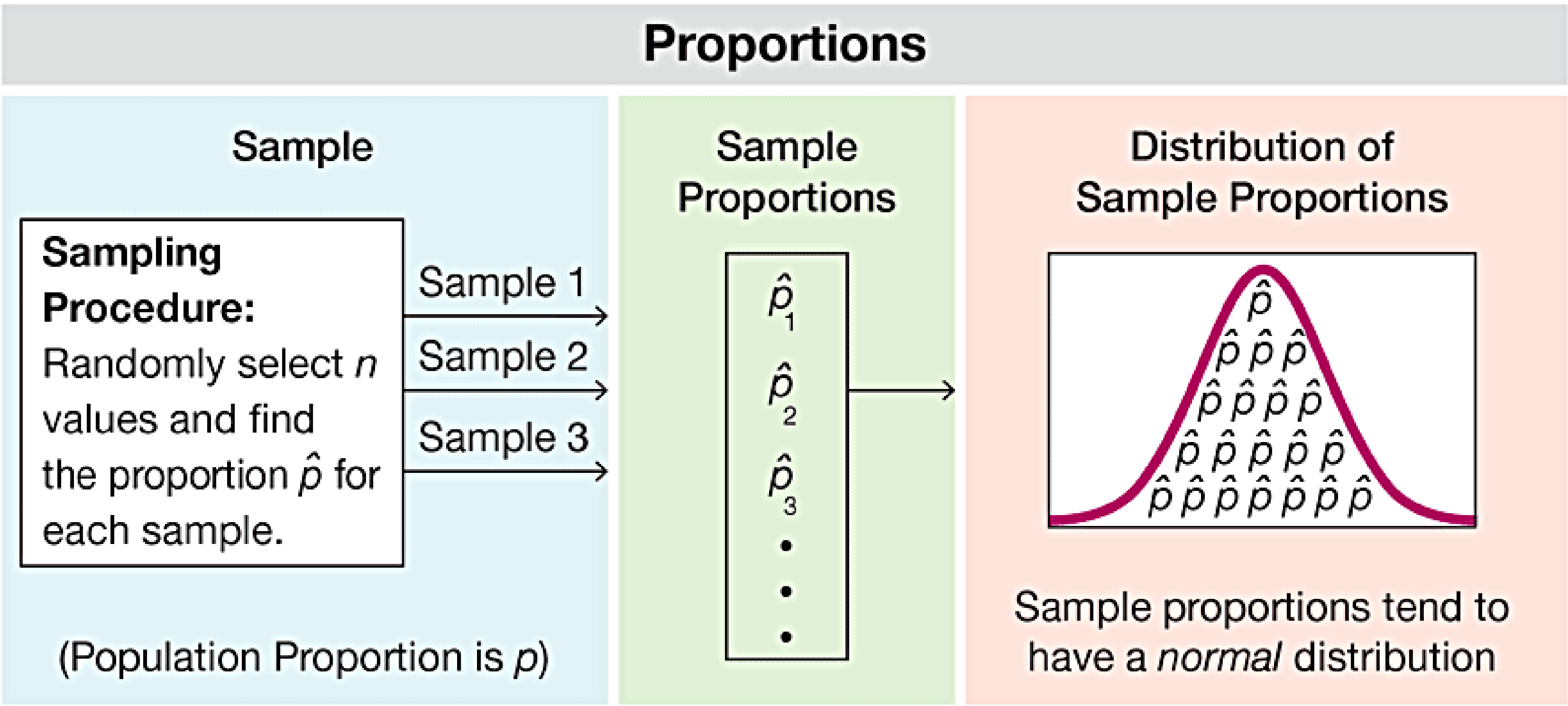
We now consider the concept of a **sampling distribution of a statistic.** Instead of working with values from the original population, we want to focus on the values of **statistics** (such as sample proportions or sample means) obtained from the population.

General Behavior of Sampling Distributions (1 of 4)

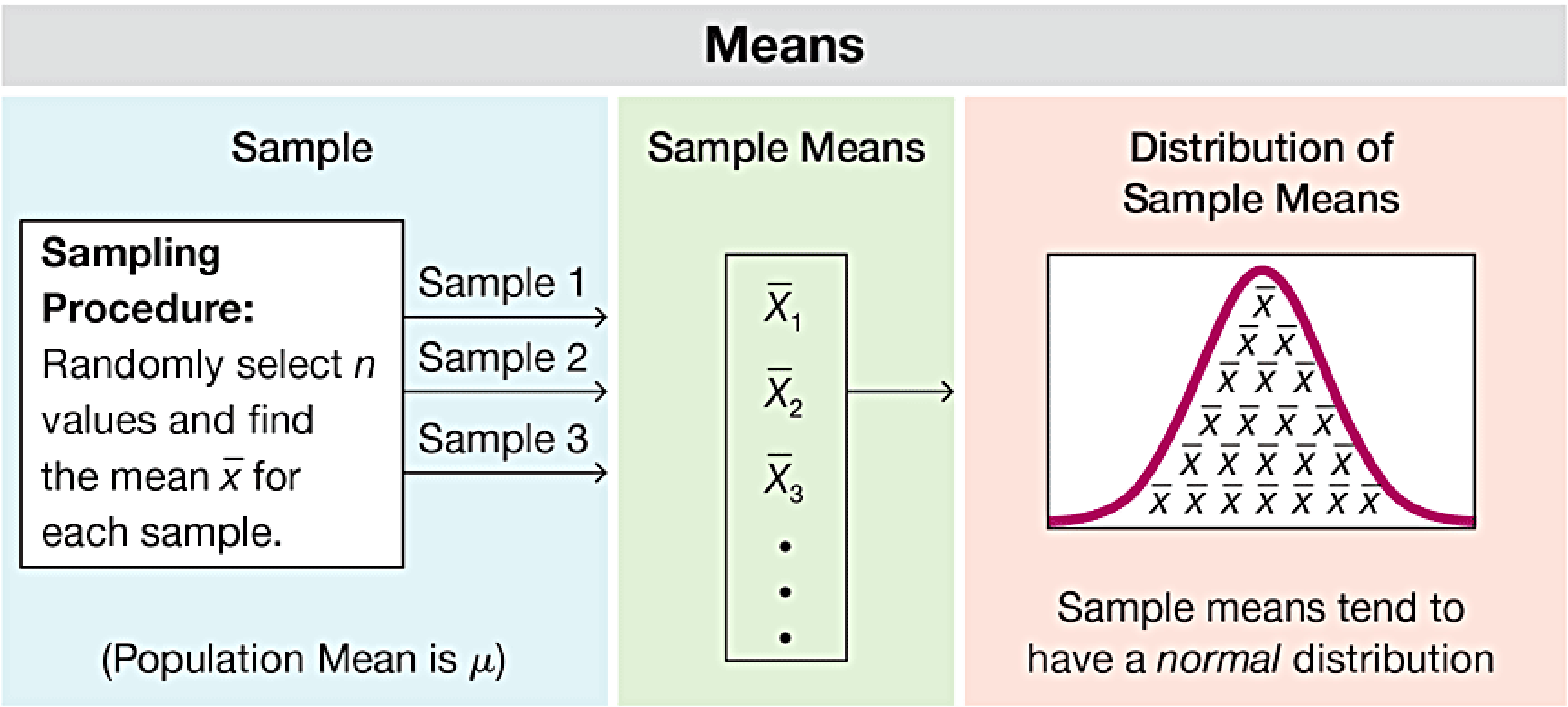
When samples of the same size are taken from the same population, the following two properties apply:

1. Sample proportions tend to be normally distributed.
2. The mean of sample proportions is the same as the population mean.

General Behavior of Sampling Distributions (2 of 4)



General Behavior of Sampling Distributions (3 of 4)



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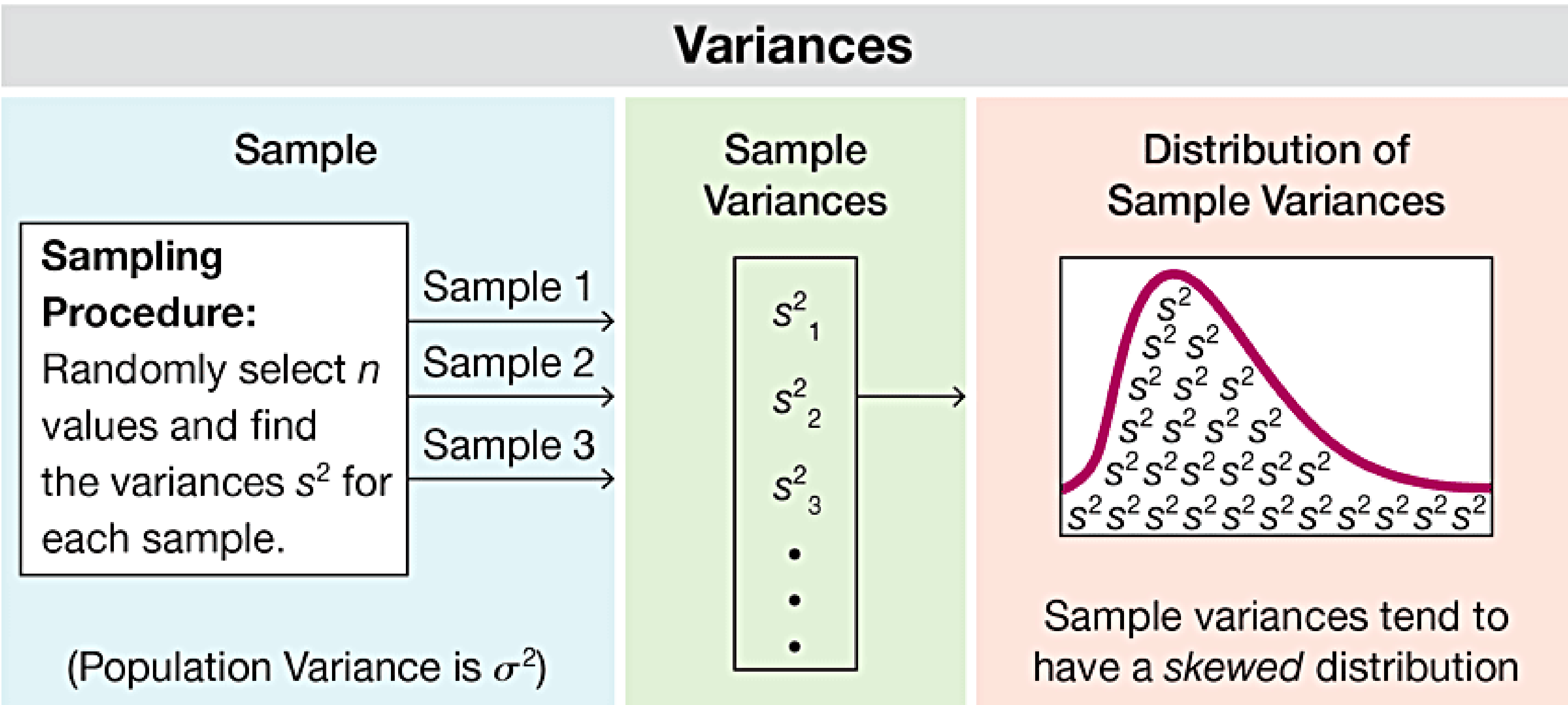
General Behavior of Sampling Distributions (4 of 4)



Sampling Distribution of a Statistic

- Sampling Distribution of a Statistic
  - The **sampling distribution of a statistic** (such as a sample proportion or sample mean) is the distribution of all values of the statistic when all possible samples of the same size *n* are taken from the same population. (The sampling distribution of a statistic is typically represented as a probability distribution in the format of a probability histogram, formula, or table.)

Sampling Distribution of the Sample Proportion

- Sampling Distribution of the Sample Proportion
  - The **sampling distribution of the sample proportion** is the distribution of sample proportions (or the distribution of the variable $\hat{p}$), with all samples having the same sample size n taken from the same population. (The sampling distribution of the sample proportion is typically represented as a probability distribution in the format of a probability histogram, formula, or table.)

Notations for Proportions

We need to distinguish between a population proportion p and some sample proportion:

p = **population** proportion

$\hat{p}$ = **sample** proportion

HINT $\hat{p}$ is pronounced "p-hat." When symbols are used above a letter, as in $\bar{x}$ and $\hat{p}$, they represent **statistics**, not parameters.

Behavior of Sample Proportions

1. The distribution of sample proportions tends to approximate a normal distribution.

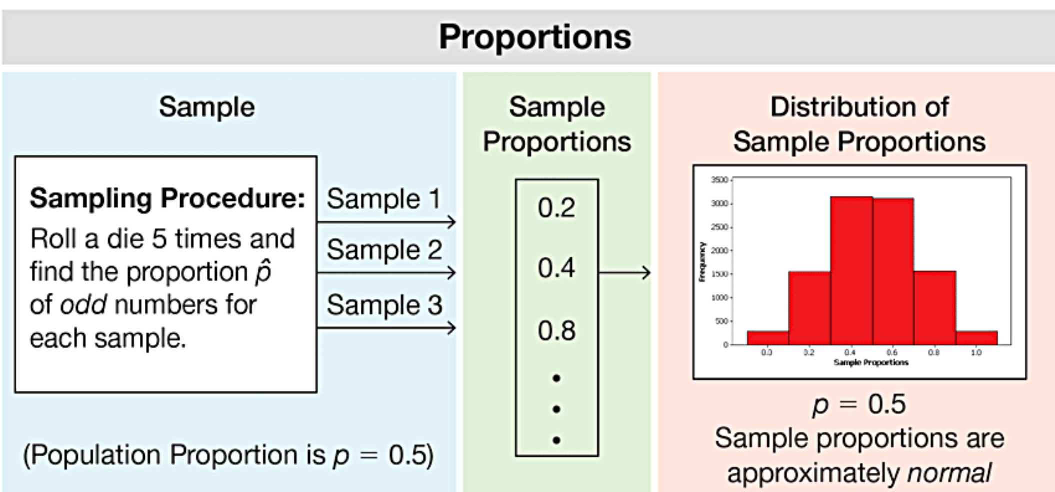
2. Sample proportions **target** the value of the population proportion in the sense that the mean of all of the sample proportions $\hat{p}$ is equal to the population proportion p; the expected value of the sample proportion is equal to the population proportion.

Example: Sampling Distributions of the Sample Proportion (1 of 3)

Consider repeating this process: Roll a die 5 times and find the proportion of **odd** numbers (1 or 3 or 5). What do we know about the behavior of all sample proportions that are generated as this process continues indefinitely?

Example: Sampling Distributions of the Sample Proportion (2 of 3)

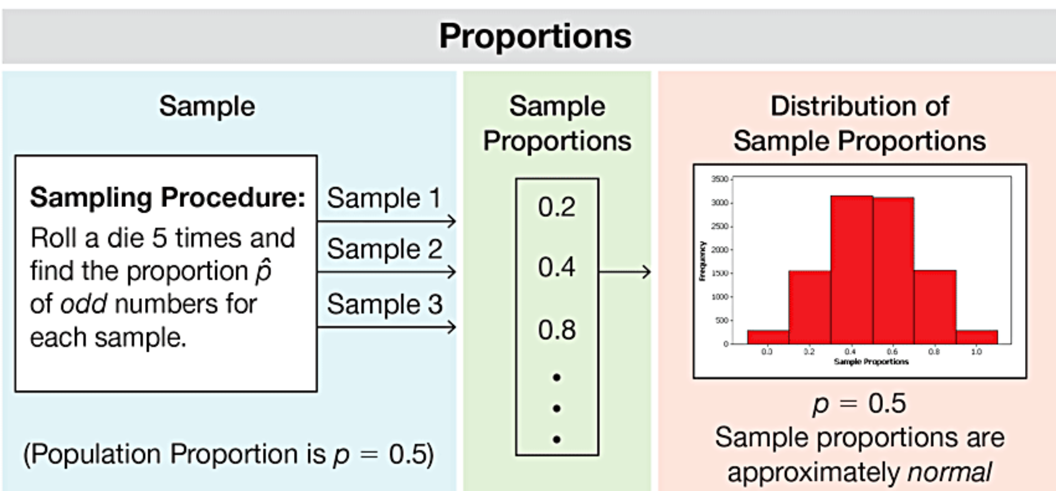
The figure illustrates a process of rolling a die 5 times and finding the proportion of odd numbers. (The figure shows results from repeating this process 10,000 times, but the true sampling distribution of the sample proportion involves repeating the process indefinitely.)



Sample Proportions from 10,000 Trials

Example: Sampling Distributions of the Sample Proportion (3 of 3)

The figure shows that the sample proportions are approximately normally distributed. (Because the values of 1, 2, 3, 4, 5, 6 are all equally likely, the proportion of odd numbers in the population is 0.5, and the figure shows that the sample proportions have a mean of 0.50.)



Sample Proportions from 10,000 Trials

Sampling Distribution of the Sample Mean

- Sampling Distribution of the Sample Mean
  - The **sampling distribution of the sample mean** is the distribution of all possible sample means (or the distribution of the variable $\bar{x}$), with all samples having the same sample size n taken from the same population. (The sampling distribution of the sample mean is typically represented as a probability distribution in the format of a probability histogram, formula, or table.)

Behavior of Sample Means

1. The distribution of sample means tends to be a normal distribution. (This will be discussed further in the following section, but the distribution tends to become closer to a normal distribution as the sample size increases.)
2. The sample means **target** the value of the population mean. (That is, the mean of the sample means is the population mean. The expected value of the sample mean is equal to the population mean.)

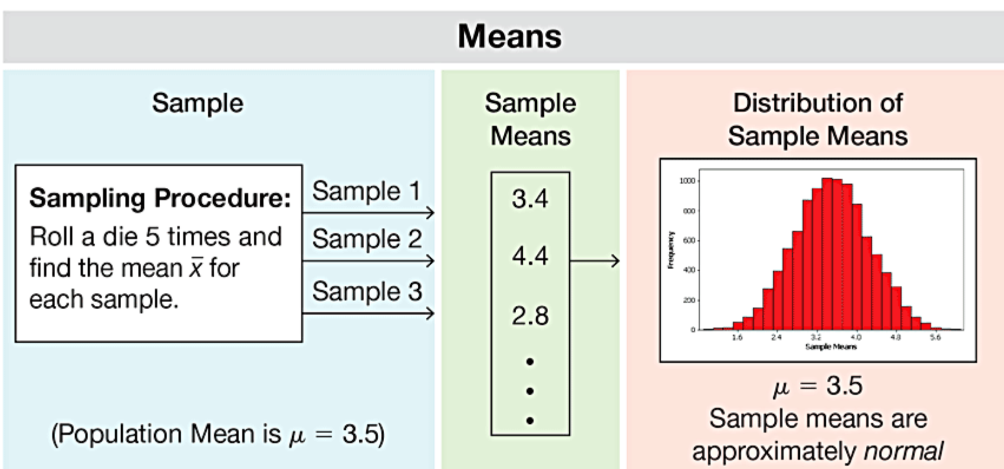
Example: Sampling Distribution of the Sample Mean (1 of 3)

Consider repeating this process: Roll a die 5 times to randomly select 5 values from the population {1, 2, 3, 4, 5, 6}, then find the mean $\bar{x}$ of the results.

What do we know about the behavior of all sample means that are generated as this process continues indefinitely?

Example: Sampling Distribution of the Sample Mean (2 of 3)

The figure illustrates a process of rolling a die 5 times and finding the mean of the results. The figure shows results from repeating this process 10,000 times, but the true sampling distribution of the mean involves repeating the process indefinitely.



Sample Means from 10,000 trials

Example: Sampling Distribution of the Sample Mean (3 of 3)

Because the values of 1, 2, 3, 4, 5, 6 are all equally likely, the population has a mean of $\mu = 3.5$. The 10,000 sample means included in the figure have a mean of 3.5. If the process is continued indefinitely, the mean of the sample means will be 3.5. Also, the figure shows that the distribution of the sample means is approximately a normal distribution.

Sampling Distribution of the Sample Variance

- Sampling Distribution of the Sample Variance
  - The **sampling distribution of the sample variance** is the distribution of sample variances (the variable s^2), with all samples having the same sample size n taken from the same population. (The sampling distribution of the sample variance is typically represented as a probability distribution in the format of a table, probability histogram, or formula.)

Population Standard Deviation and Population Variance

Population standard deviation:

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}}$$

Population variance:

$$\sigma^2 = \frac{\sum(x-\mu)^2}{N}$$

Behavior of Sample Variances

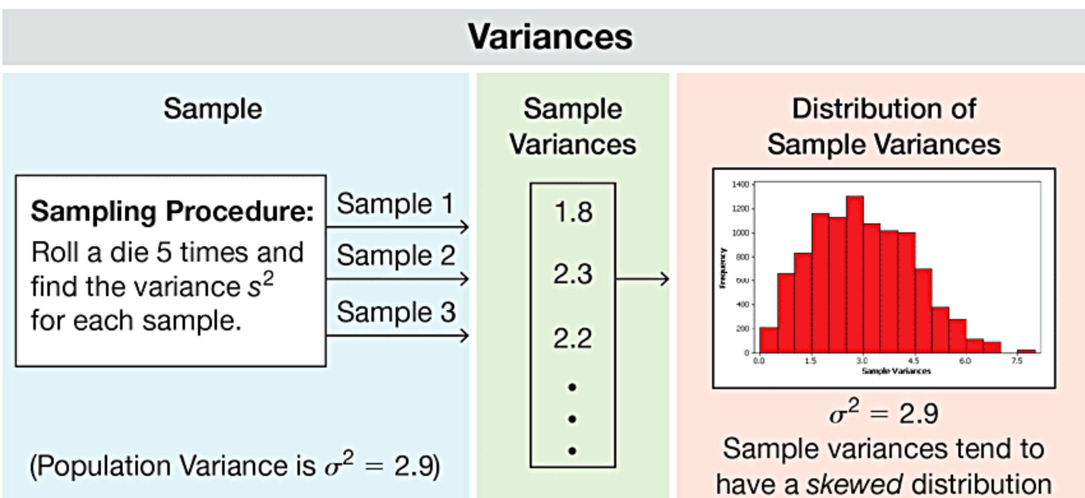
1. The distribution of sample variances tends to be a distribution skewed to the right.
2. The sample variances **target** the value of the population variance. (That is, the mean of the sample variances is the population variance. The expected value of the sample variance is equal to the population variance.)

Example: Sampling Distributions of the Sample Variances (1 of 4)

Consider repeating this process: Roll a die 5 times and find the variance s^2 of the results. What do we know about the behavior of all sample variances that are generated as this process continues indefinitely?

Example: Sampling Distributions of the Sample Variances (2 of 4)

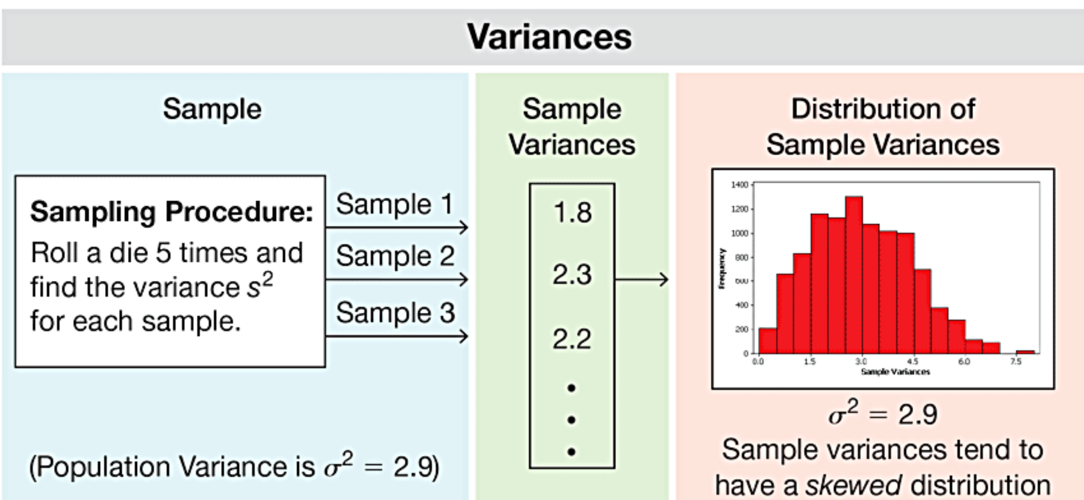
The figure illustrates a process of rolling a die 5 times and finding the variance of the results. The figure shows results from repeating this process 10,000 times, but the true sampling distribution of the sample variance involves repeating the process indefinitely.



Sample Variances from 10,000 trials

Example: Sampling Distributions of the Sample Variances (3 of 4)

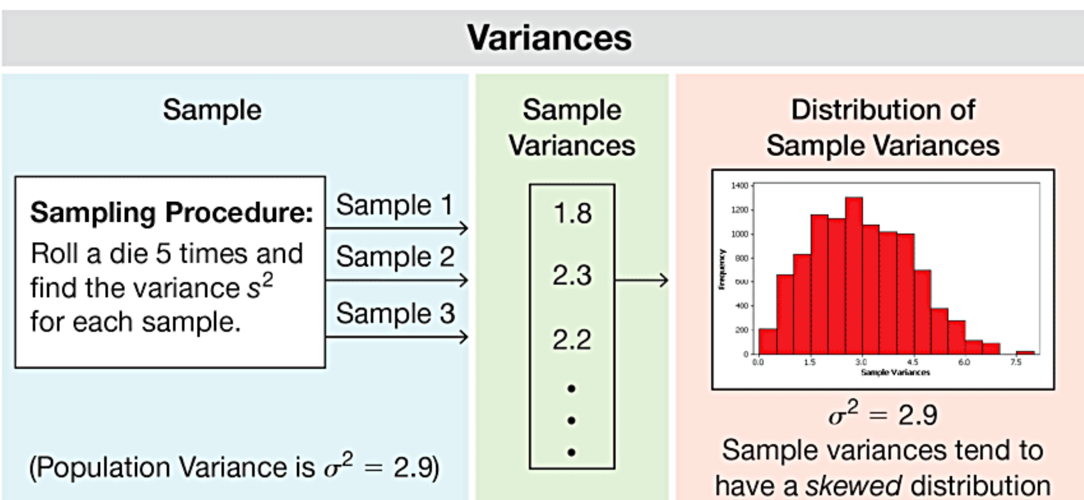
Because the values of 1, 2, 3, 4, 5, 6 are all equally likely, the population has a variance of $s^2 = 2.9$, and the 10,000 sample variances included in the figure have a mean of 2.9.



Sample Variances from 10,000 trials

Example: Sampling Distributions of the Sample Variances (4 of 4)

If the process is continued indefinitely, the mean of the sample variances will be 2.9. Also, the figure shows that the distribution of the sample variances is a skewed distribution, not a normal distribution with its characteristic bell shape.



Sample Variances from 10,000 trials

Estimator

- Estimator
  - An **estimator** is a statistic used to infer (or estimate) the value of a population parameter.

Unbiased Estimator

- Unbiased Estimator
  - An **unbiased estimator** is a statistic that targets the value of the corresponding population parameter in the sense that the sampling distribution of the statistic has a mean that is equal to the corresponding population parameter.

Estimators: Unbiased and Biased (1 of 2)

Unbiased Estimator

These statistics are unbiased estimators. That is, they each target the value of the corresponding population parameter (with a sampling distribution having a mean equal to the population parameter):

- Proportion $\hat{p}$
- Mean $\bar{x}$
- Variance s^2

Estimators: Unbiased and Biased (2 of 2)

Biased Estimator

These statistics are biased estimators. That is, they do **not** target the value of the corresponding population parameter:

- Median
- Range
- Standard deviation *s*

Why Sample with Replacement?

Sampling is conducted with replacement because of these two very important reasons:

1. When selecting a relatively small sample from a large population, it makes no significant difference whether we sample with replacement or without replacement.
2. Sampling with replacement results in **independent** events that are unaffected by previous outcomes, and independent events are easier to analyze and result in simpler calculations and formulas.

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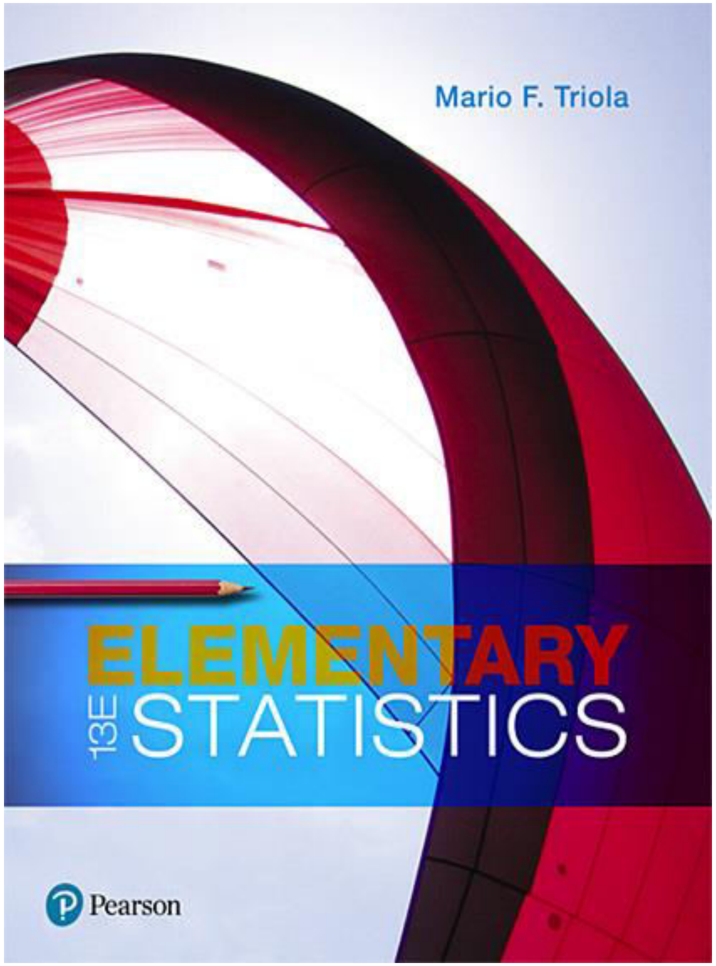


Chapter 6

Normal Probability Distributions

Normal Probability Distributions

6-1 The Standard Normal Distribution

6-2 Real Applications of Normal Distributions

6-3 Sampling Distributions and Estimators

6-4 The Central Limit Theorem

6-5 Assessing Normality

6-6 Normal as Approximation to Binomial

Key Concept

This section presents methods for working with normal distributions that are not standard. That is, the mean is not 0 or the standard deviation is not 1, or both.

The key is that we can use a simple conversion that allows us to “standardize” any normal distribution so that the same methods of the previous section can be used.

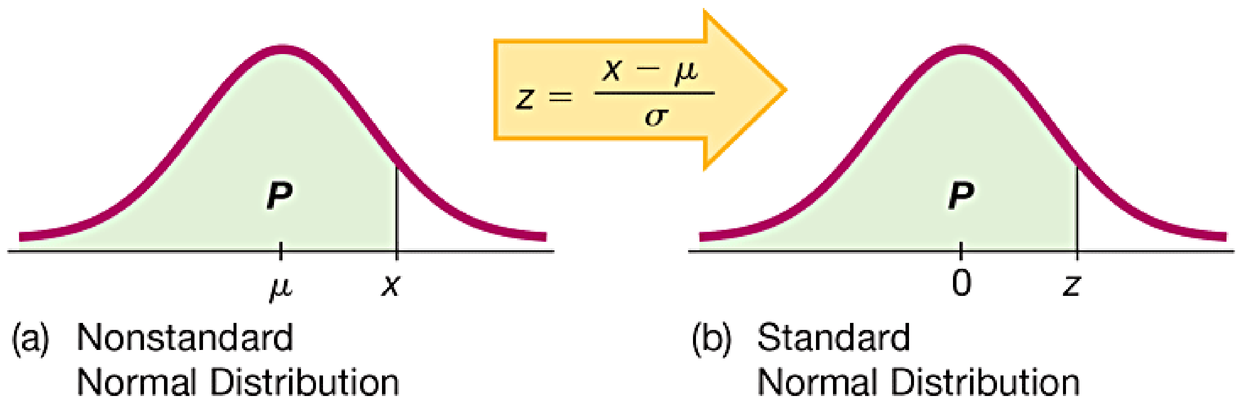
Conversion Formula

$$z = \frac{x - \mu}{\sigma}$$

(round *z* scores to 2 decimal places)

The formula allows us to “standardize” any normal distribution so that *x* values can be transformed to *z* scores.

Converting to a Standard Normal Distribution



The figures illustrate the conversion from a nonstandard to a standard normal distribution. The area in **any** normal distribution bounded by some score *x* (as in Figure a) is the **same** as the area bounded by the corresponding *z* score in the standard normal distribution (as in Figure b).

Procedure for Finding Areas with a Nonstandard Normal Distribution

1. Sketch a normal curve, label the mean and any specific *x* values, and then **shade** the region representing the desired probability.
2. For each relevant value *x* that is a boundary for the shaded region, use the formula
$$z = \frac{x - \mu}{\sigma}$$
to convert that value to the equivalent *z* score. (With many technologies, this step can be skipped.)
3. Use technology (software or a calculator) or Table A-2 to find the area of the shaded region. This area is the desired probability.

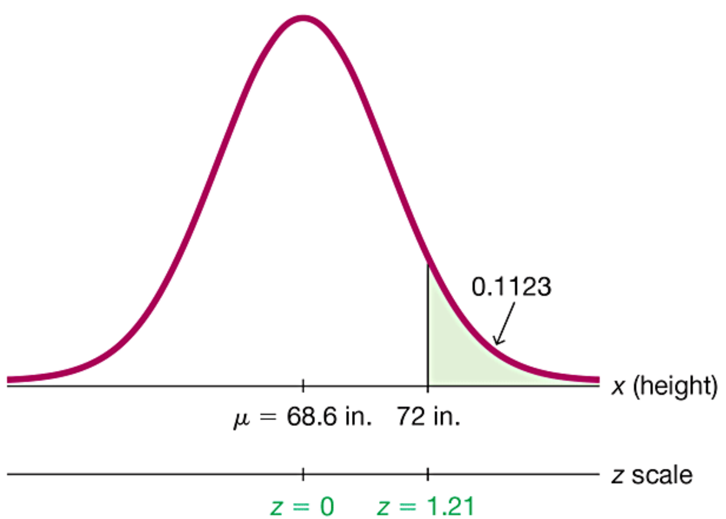
Example: What Proportion of Men Are Taller Than the 72 in. Height Requirement for Showerheads? (1 of 6)

Heights of men are normally distributed with a mean of 68.6 in. and a standard deviation of 2.8 in. Find the percentage of men who are taller than a showerhead at 72 in.

Example: What Proportion of Men Are Taller Than the 72 in. Height Requirement for Showerheads? (2 of 6)

Solution

Step 1: Men have heights that are normally distributed with a mean of 68.6 in. and a standard deviation of 2.8 in. The shaded region represents the men who are taller than the showerhead height of 72 in.



Example: What Proportion of Men Are Taller Than the 72 in. Height Requirement for Showerheads? (3 of 6)

Solution

Step 2: We can convert the showerhead height of 72 in. to the *z* score of 1.21 by using the conversion formula as follows:

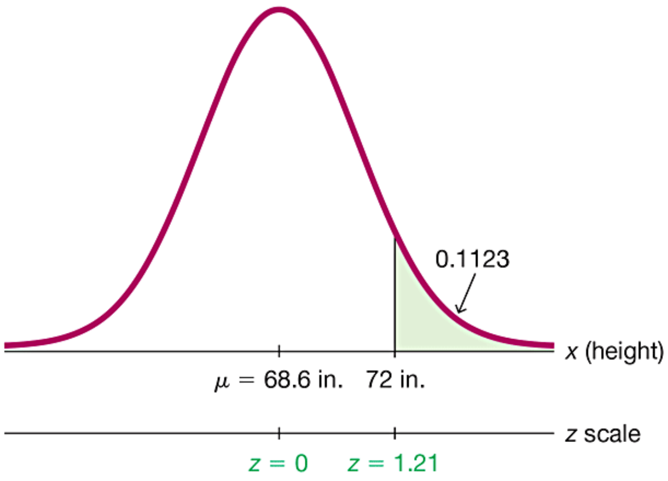
$$z = \frac{x - \mu}{\sigma} = \frac{72 - 68.6}{2.8}$$

$$= 1.21 \text{ (rounded to two decimal places)}$$

Example: What Proportion of Men Are Taller Than the 72 in. Height Requirement for Showerheads? (4 of 6)

Solution

Step 3: Technology: Technology can be used to find that the area to the right of 72 in. in the figure is 0.1123 rounded. (With many technologies, Step 2 can be skipped.) The result of 0.1123 from technology is more accurate than the result of 0.1131 found by using Table A-2.



Example: What Proportion of Men Are Taller Than the 72 in. Height Requirement for Showerheads? (5 of 6)

Solution

Table A-2: Use Table A-2 to find that the cumulative area to the **left** of $z = 1.21$ is 0.8869. (Remember, Table A-2 is designed so that all areas are cumulative areas from the **left**.) Because the total area under the curve is 1, it follows that the shaded area is $1 - 0.8869 = 0.1131$.

Example: What Proportion of Men Are Taller Than the 72 in. Height Requirement for Showerheads? (6 of 6)

Interpretation

The proportion of men taller than 72 in. is 0.1123, or 11.23%. About 11% of men may find the design to be unsuitable.

Finding Values From Known Areas (1 of 3)

Here are helpful hints for those cases in which the area (or probability or percentage) is known and we must find the relevant value(s):

1. Graphs are extremely helpful in visualizing, understanding, and successfully working with normal provability distributions, so they should always be used.

Finding Values From Known Areas (2 of 3)

2. **Don't confuse *z* scores and areas.** *z* scores are **distances** along the horizontal scale, but areas are **regions** under the normal curve. Table A-2 lists *z* scores in the left columns and across the top row, but areas are found in the body of the table.

3. **Choose the correct (right/left) side of the graph**. A value separating the **top** 10% from the others will be located on the right side of the graph, but a value separating the **bottom** 10% will be located on the left side of the graph.

Finding Values From Known Areas (3 of 3)

4. A z score must be **negative** whenever it is located in the left half of the normal distribution.
5. Areas (or probabilities) are always between 0 and 1, and they are never negative.

Procedure For Finding Values From Known Areas or Probabilities (1 of 2)

1. Sketch a normal distribution curve, enter the given probability or percentage in the appropriate region of the graph, and identify the x value(s) being sought.

2. If using technology, refer to the instructions at the end of this section. If using Table A-2, refer to the **body** of Table A-2 to find the area to the left of x, then identify the z score corresponding to that area.

Procedure For Finding Values From Known Areas or Probabilities (2 of 2)

3. If you know z and must convert to the equivalent *x* value, use the conversion formula by entering the values for μ, σ, and the *z* score found in step 2, and then solve for *x*. We can solve for *x* as follows:

$$x = \mu + (z \cdot \sigma)$$

(Another form of the conversion formula)

4. Refer to the sketch of the curve to verify that the solution makes sense in the context of the graph and in the context of the problem.

Example: Designing an Aircraft Cockpit (1 of 7)

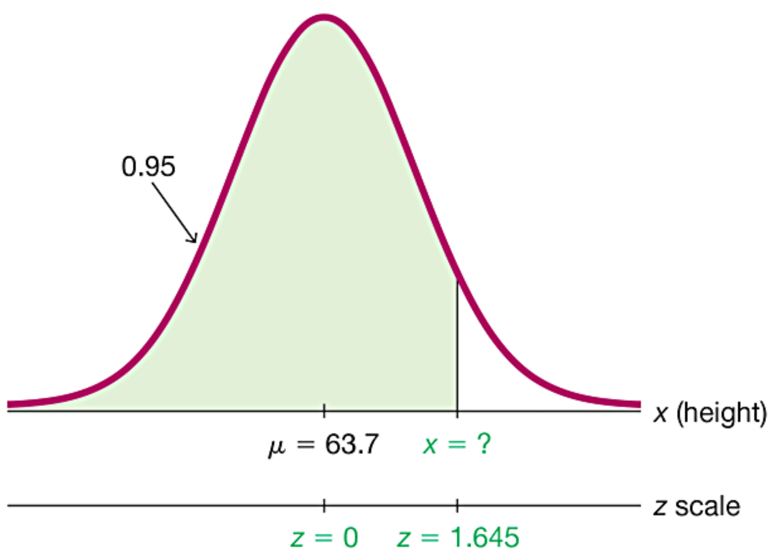
When designing equipment, one common criterion is to use a design that accommodates 95% of the population. We have seen that only 46% of women satisfy the height requirements for U.S. Air Force pilots. What would be the maximum acceptable height of a woman if the requirements were changed to allow the **shortest** 95% of women to be pilots? That is, find the 95th percentile of heights of women.

Assume that heights of women are normally distributed with a mean of 63.7 in. and a standard deviation of 2.9 in. In addition to the maximum allowable height, should there also be a minimum required height? Why?

Example: Designing an Aircraft Cockpit (2 of 7)

Solution

Step 1: The figure shows the normal distribution with the height *x* that we want to identify. The shaded area represents the shortest 95% of women.

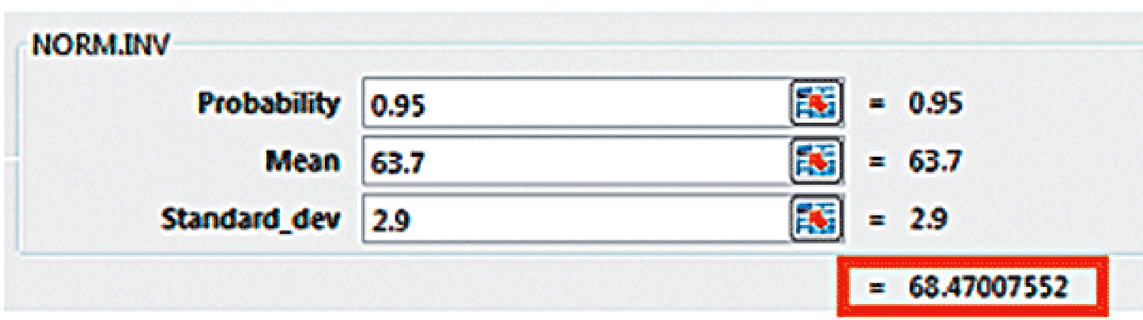


Example: Designing an Aircraft Cockpit (3 of 7)

Solution

Step 2: Technology: Technology will provide the value of *x*. For example, see the accompanying Excel display showing that *x* = 68.47007552 in., or 68.5 in. when rounded.

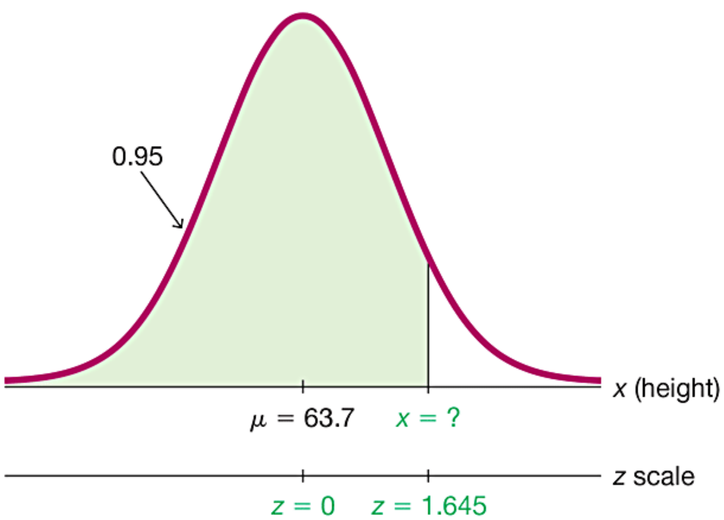
Excel



Example: Designing an Aircraft Cockpit (4 of 7)

Solution

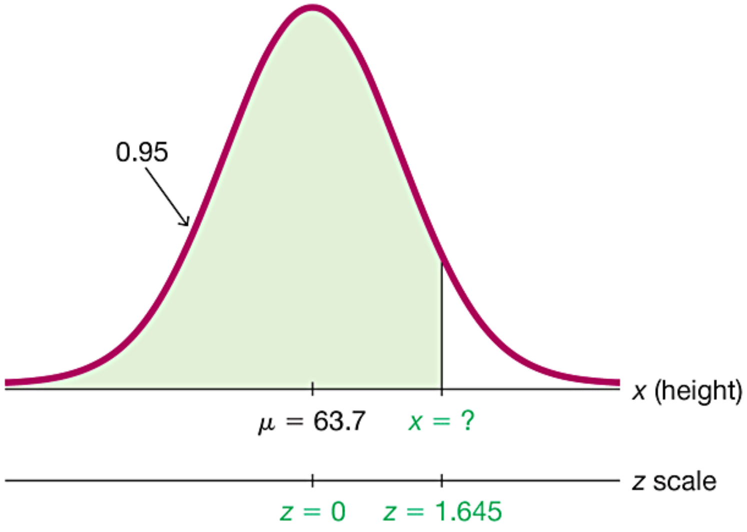
Table A-2: If using Table A-2, search for an area of 0.9500 **in the body** of the table. (The area of 0.9500 shown in the figure is a cumulative area from the left, and that is exactly the type of area listed in Table A-2.)



Example: Designing an Aircraft Cockpit (5 of 7)

Solution

The area of 0.9500 is between the Table A-2 areas of 0.9495 and 0.9505, but there is an asterisk and footnote indicating that an area of 0.9500 corresponds to z = 1.645.



Example: Designing an Aircraft Cockpit (6 of 7)

Solution

Step 3: With z = 1.645, μ = 63.7 in., and σ = 2.9 in., we can solve for x by using the conversion formula:

$$z = \frac{x - \mu}{\sigma} \text{ becomes } 1.645 = \frac{x - 63.7}{2.9}$$

The result of x = 68.4705 in. can be found directly or by using the following version of the conversion formula:

$$x = \mu + (z \cdot \sigma) = 63.7 + (1.645 \cdot 2.9) = 68.4705 \text{ in.}$$

Example: Designing an Aircraft Cockpit (7 of 7)

Solution

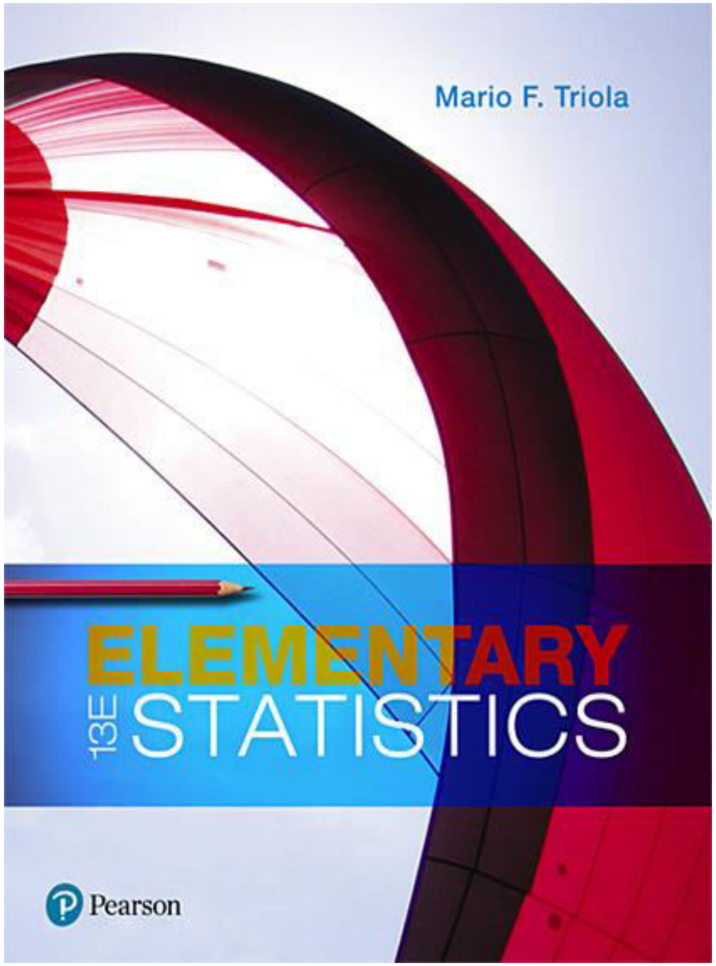
Step 4: The solution of x = 68.5 in. is reasonable because it is greater than the mean of 63.7 in.

Interpretation

A requirement of a height less than 68.5 in. would allow 95% of women to be eligible as U.S. Air Force pilots. There should be a **minimum** height requirement so that the pilot can easily reach all controls.

Elementary Statistics

Thirteenth Edition



Chapter 6

Normal Probability Distributions

Normal Probability Distributions

6-1 The Standard Normal Distribution

6-2 Real Applications of Normal Distributions

6-3 Sampling Distributions and Estimators

6-4 The Central Limit Theorem

6-5 Assessing Normality

6-6 Normal as Approximation to Binomial

Key Concept

In this section we present the **standard normal distribution,** which is a specific normal distribution having the following three properties:

1. Bell-shaped: The graph of the standard normal distribution is bell-shaped.
2. $\mu = 0$: The standard normal distribution has a mean equal to 0.
3. $\sigma = 1$: The standard normal distribution has a standard deviation equal to 1.

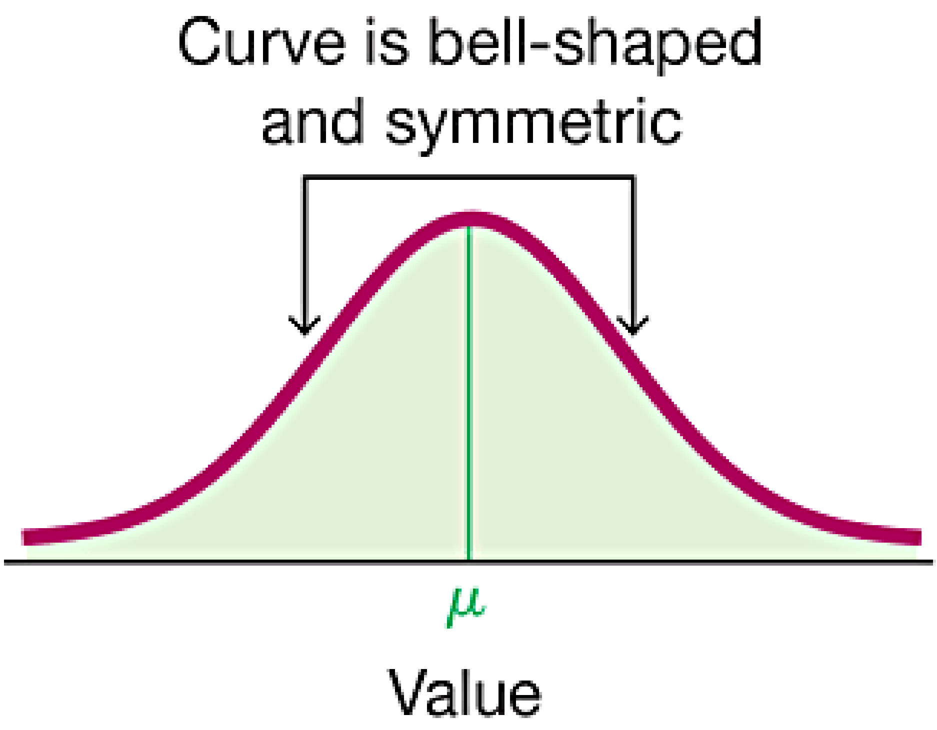
In this section we develop the skill to find areas (or probabilities or relative frequencies) corresponding to various regions under the graph of the standard normal distribution. In addition, we find *z* scores that correspond to areas under the graph.

Normal Distribution (1 of 2)

- Normal Distribution
  - If a continuous random variable has a distribution with a graph that is symmetric and bell-shaped, we say that it has a **normal distribution.**

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

Normal Distribution (2 of 2)



Uniform Distribution (1 of 2)

Properties of uniform distribution:

1. The area under the graph of a continuous probability distribution is equal to 1.
2. There is a correspondence between area and probability, so probabilities can be found by identifying the corresponding areas in the graph using this formula for the area of a rectangle:
   Area = height × width

Uniform Distribution (2 of 2)

- Uniform Distribution
  - A continuous random variable has a **uniform distribution** if its values are spread **evenly** over the range of possibilities. The graph of a uniform distribution results in a rectangular shape.

Density Curve

- Density Curve
  - The graph of any continuous probability distribution is called a **density curve,** and any density curve must satisfy the requirement that the total area under the curve is exactly 1.

Because the total area under any density curve is equal to 1, there is a correspondence between area and probability.

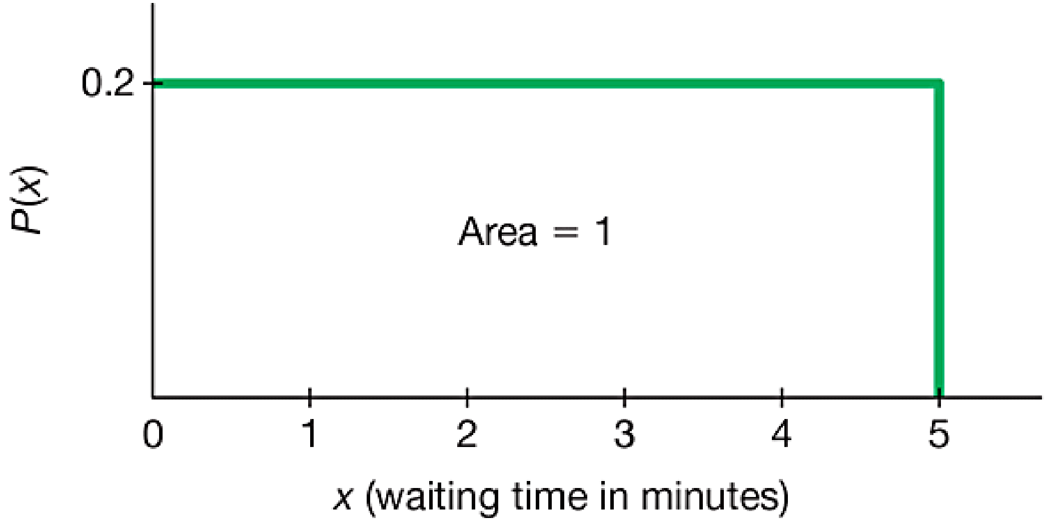
Example: Waiting Times for Airport Security (1 of 7)

During certain time periods at JFK airport in New York City, passengers arriving at the security checkpoint have waiting times that are uniformly distributed between 0 minutes and 5 minutes, as illustrated in the figure on the next page.

Example: Waiting Times for Airport Security (2 of 7)

Refer to the figure to see these properties:

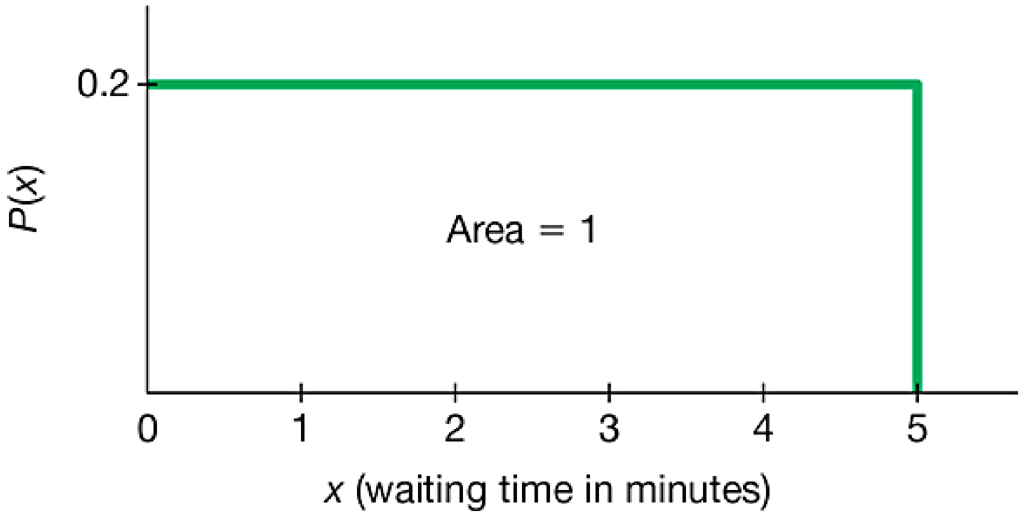
- All of the different possible waiting times are **equally likely.**



Example: Waiting Times for Airport Security (3 of 7)

Refer to the figure to see these properties:

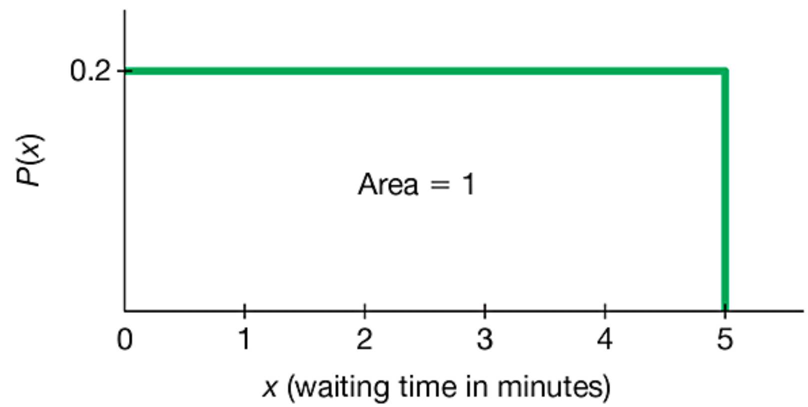
- Waiting times can be **any** value between 0 min and 5 min, so it is possible to have a waiting time of 1.234567 min.



Example: Waiting Times for Airport Security (4 of 7)

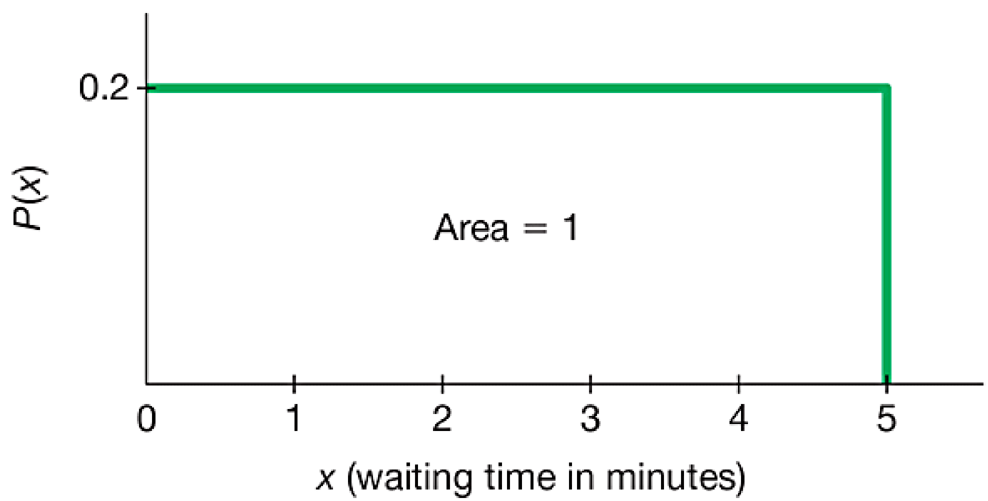
Refer to the figure to see these properties:

- By assigning the probability of 0.2 to the height of the vertical line in the figure, the **enclosed area is exactly 1.**
  (In general, we should make the height of the vertical line in a uniform distribution equal to $\frac{1}{\text{range}}$.)



Example: Waiting Times for Airport Security (5 of 7)

Given the uniform distribution illustrated in the figure, find the probability that a randomly selected passenger has a waiting time of at least 2 minutes.



Pearson

Example: Waiting Times for Airport Security (6 of 7)

Solution

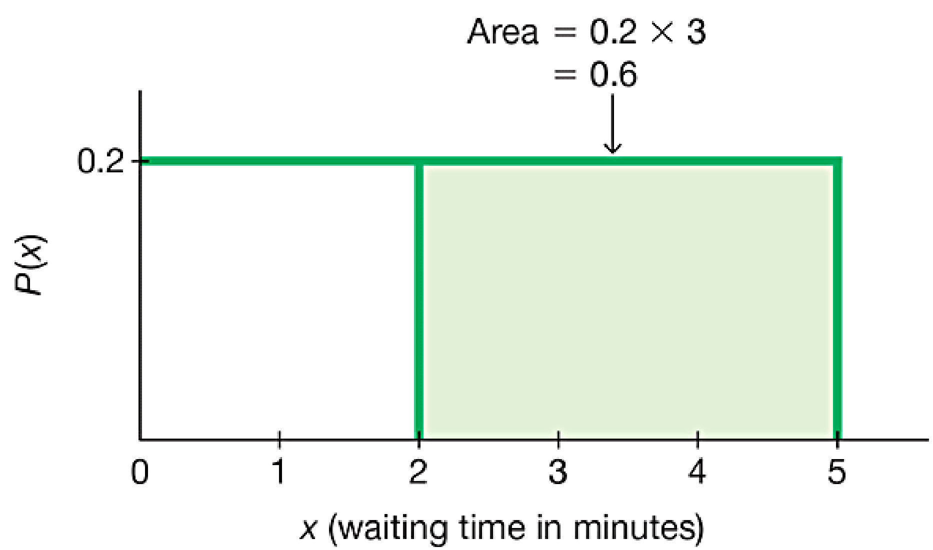
The shaded area represents waiting times of at least 2 minutes. Because the total area under the density curve is equal to 1, there is a correspondence between area and probability. We can easily find the desired **probability** by using **areas.**

Example: Waiting Times for Airport Security (7 of 7)

Solution

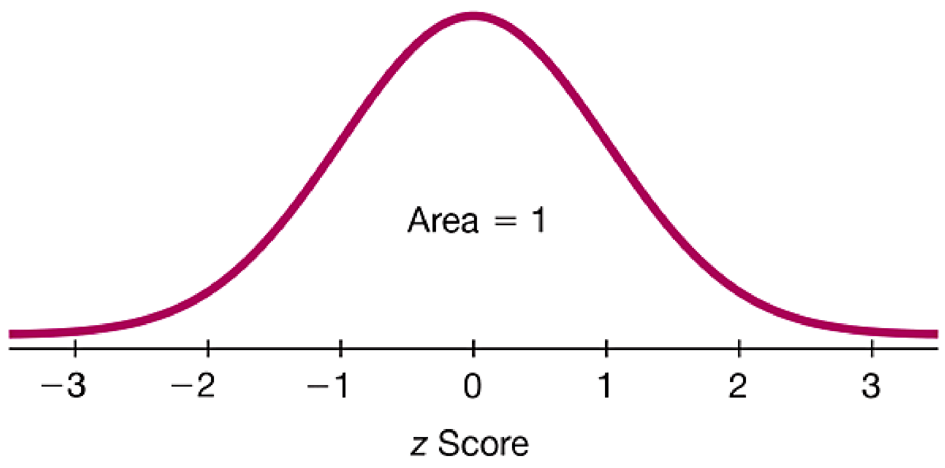
P(wait time of at least 2 min) = height × width of shaded area in the figure = 0.2 × 3 = 0.6

The probability of randomly selecting a passenger with a waiting time of at least 2 minutes is 0.6.



Standard Normal Distribution

- Standard Normal Distribution
  - The **standard normal distribution** is a normal distribution with the parameters of $\mu = 0$ and $\sigma = 1$. The total area under its density curve is equal to 1.



Finding Probabilities When Given z Scores (1 of 3)

- We can find areas (probabilities) for different regions under a normal model using technology or Table A-2.
- Technology is strongly recommended.

Because calculators and software generally give more accurate results than Table A-2, we **strongly** recommend using technology.

Finding Probabilities When Given z Scores (2 of 3)

If using Table A-2, it is essential to understand these points:

1. Table A-2 is designed only for the **standard** normal distribution, which is a normal distribution with a mean of 0 and a standard deviation of 1.
2. Table A-2 is on two pages, with the left page for **negative** *z* scores and the right page for **positive** *z* scores.
3. Each value in the body of the table is a **cumulative area from the left** up to a vertical boundary above a specific *z* score.

Finding Probabilities When Given z Scores (3 of 3)

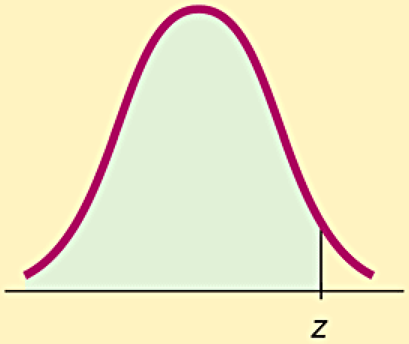
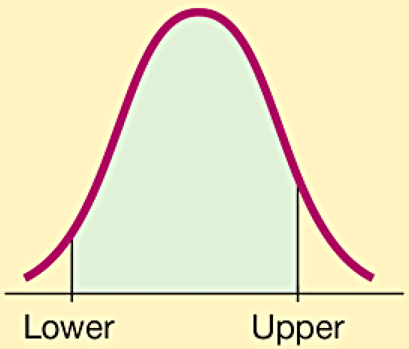
4. When working with a graph, avoid confusion between *z* scores and areas.

***z* score: Distance along the horizontal scale of the standard normal distribution (corresponding to the number of standard deviations above or below the mean); refer to the leftmost column and top row of Table A-2.**

Area: Region under the curve; refer to the values in the body of Table A-2.

5. The part of the *z* score denoting hundredths is found across the top row of Table A-2.

Formats Used for Finding Normal Distribution Areas

Cumulative Area from the Left The following provide the *cumulative area from the left* up to a vertical line above a specific value of *z*: • **Table A-2** • **Statdisk** • **Minitab** • **Excel** • **StatCrunch**	 **Cumulative Left Region**
Area Between Two Boundaries The following provide the area bounded on the left and bounded on the right by vertical lines above specific values. • **TI-83/84 Plus calculator** • **StatCrunch**	 **Area Between Two Boundaries**

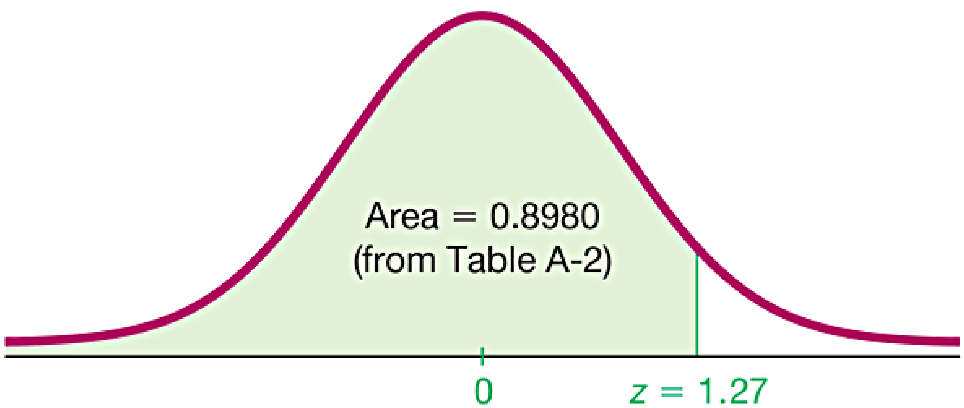
Example: Bone Density Test (1 of 7)

A bone mineral density test can be helpful in identifying the presence or likelihood of osteoporosis. The result of a bone density test is commonly measured as a *z* score. The population of *z* scores is normally distributed with a mean of 0 and a standard deviation of 1, so these test results meet the requirements of a standard normal distribution.

Example: Bone Density Test (2 of 7)

The graph of the bone density test scores is as shown in the figure.

A randomly selected adult undergoes a bone density test. Find the probability that this person has a bone density test score less than 1.27.

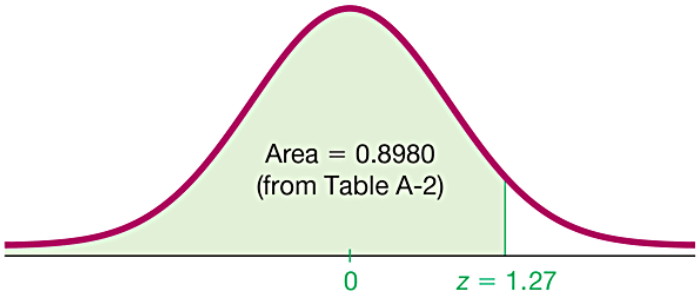


Example: Bone Density Test (3 of 7)

Solution

Note that the following are the **same** (because of the aforementioned correspondence between probability and area):

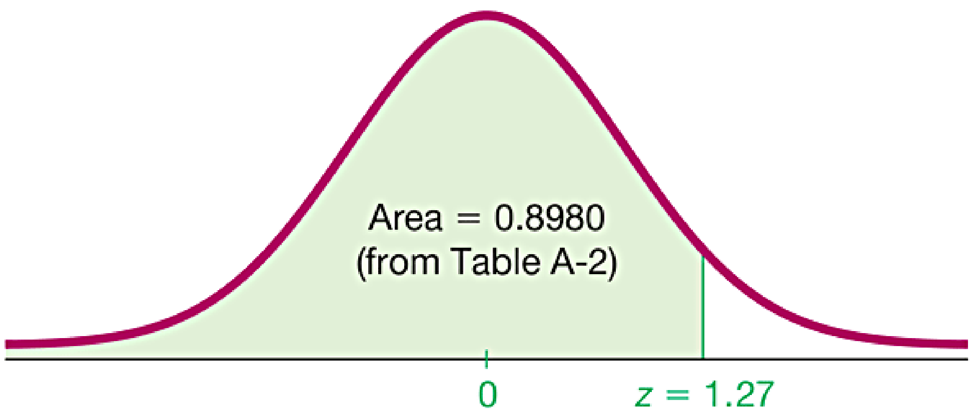
- **Probability** that the bone density test score is less than 1.27
- Shaded **area** shown in the figure



Example: Bone Density Test (4 of 7)

Solution

So we need to find the area in the figure to the left of $z = 1.27$.



Example: Bone Density Test (5 of 7)

Solution

Using Table A-2, begin with the *z* score of 1.27 by locating 1.2 in the left column; next find the value in the adjoining row of probabilities that is directly below 0.07, as shown:

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319

Example: Bone Density Test (6 of 7)

Solution

Table A-2 shows that there is an area of 0.8980 corresponding to $z = 1.27$. We want the area **below** 1.27, and Table A-2 gives the cumulative area from the left, so the desired area is 0.8980.

Because of the correspondence between area and probability, we know that the probability of a z score below 1.27 is 0.8980.

Example: Bone Density Test (7 of 7)

Interpretation

The **probability** that a randomly selected person has a bone density test result below 1.27 is 0.8980, shown as the shaded region. Another way to interpret this result is to conclude that 89.80% of people have bone density levels below 1.27.

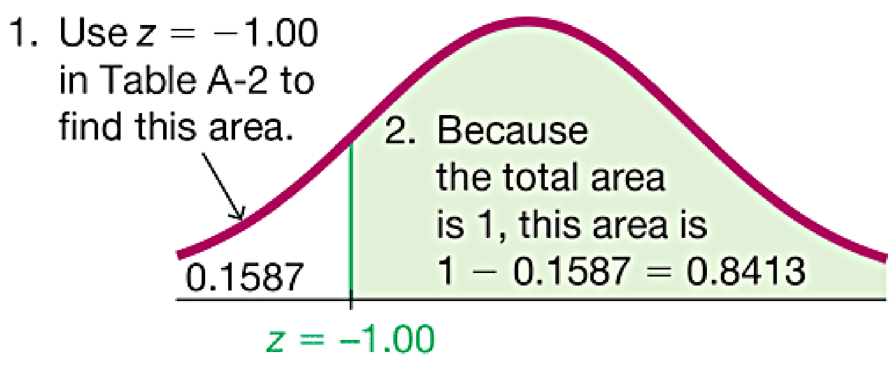
Example: Bone Density Test: Finding the Area to the Right of a Value (1 of 4)

Using the same bone density test, find the probability that a randomly selected person has a result above −1.00 (which is considered to be in the "normal" range of bone density readings).

Example: Bone Density Test: Finding the Area to the Right of a Value (2 of 4)

Solution

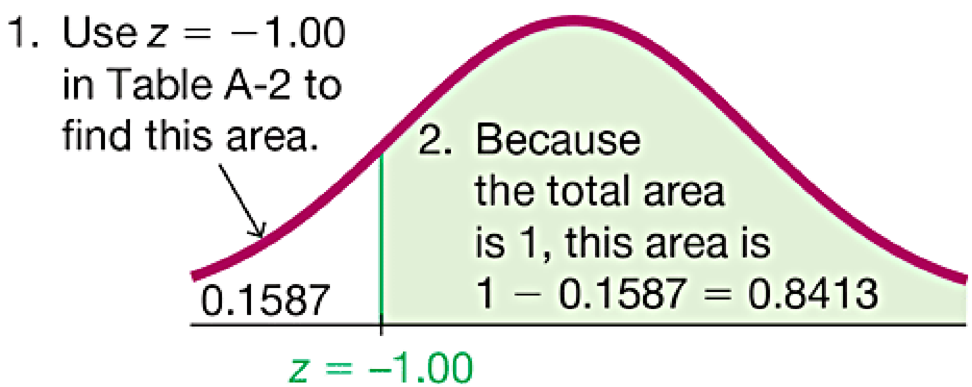
If we use Table A-2, we should know that it is designed to apply only to cumulative areas from the **left.** Referring to the page with **negative** z scores, we find that the cumulative area from the left up to $z = -1.00$ is 0.1587, as shown in the figure.



Example: Bone Density Test: Finding the Area to the Right of a Value (3 of 4)

Solution

Because the total area under the curve is 1, we can find the shaded area by subtracting 0.1587 from 1. The result is 0.8413.



Example: Bone Density Test: Finding the Area to the Right of a Value (4 of 4)

Interpretation

Because of the correspondence between probability and area, we conclude that the **probability** of randomly selecting someone with a bone density reading above −1 is 0.8413 (which is the **area** to the right of $z = -1.00$). We could also say that 84.13% of people have bone density levels above −1.00.

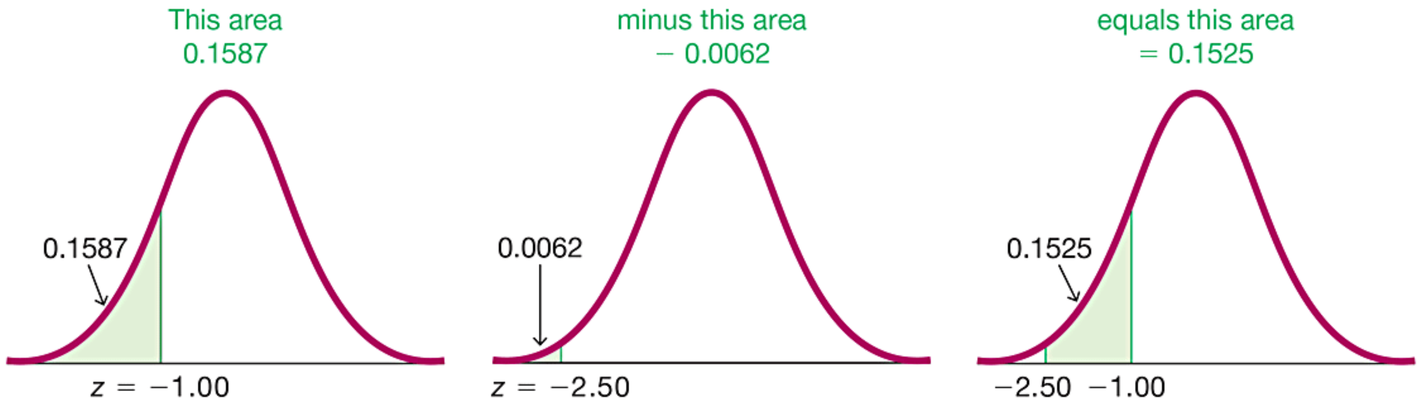
Example: Bone Density Test: Finding the Area Between Two Values (1 of 3)

A bone density reading between −1.00 and −2.50 indicates the subject has osteopenia, which is some bone loss. Find the probability that a randomly selected subject has a reading between −1.00 and −2.50.

Example: Bone Density Test: Finding the Area Between Two Values (2 of 3)

Solution

1. The area to the left of $z = -1.00$ is 0.1587.
2. The area to the left of $z = -2.50$ is 0.0062.
3. The area between $z = -1.00$ and $z = -2.50$ is the difference between the areas found above.



Example: Bone Density Test: Finding the Area Between Two Values (3 of 3)

Interpretation

Using the correspondence between probability and area, we conclude that there is a probability of 0.1525 that a randomly selected subject has a bone density reading between −1.00 and −2.50.

Another way to interpret this result is to state that 15.25% of people have osteopenia, with bone density readings between −1.00 and −2.50.

Generalized Rule

The area corresponding to the region **between** two *z* scores can be found by finding the difference between the two areas found in Table A-2.

Don’t try to memorize a rule or formula for this case. Focus on **understanding** by using a graph. Draw a graph, shade the desired area, and then get creative to think of a way to find the desired area by working with cumulative areas from the left.

Notation

- $P(a < z < b)$ denotes the probability that the z score is between a and b.
- $P(z > a)$ denotes the probability that the z score is greater than a.
- $P(z < a)$ denotes the probability that the z score is less than a.

Finding *z* Scores from Known Areas

1. Draw a bell-shaped curve and identify the region under the curve that corresponds to the given probability. If that region is not a cumulative region from the left, work instead with a known region that is a cumulative region from the left.
2. Use technology or Table A-2 to find the z score. With Table A-2, use the cumulative area from the left, locate the closest probability in the **body** of the table, and identify the corresponding z score.

Critical Value (1 of 2)

- Critical Value
  - For the standard normal distribution, a **critical value** is a z score on the borderline separating those z scores that are **significantly low** or **significantly high.**

Critical Value (2 of 2)

Notation

The expression z_α denotes the z score with an area of α to its right.

Example: Finding the Critical Value

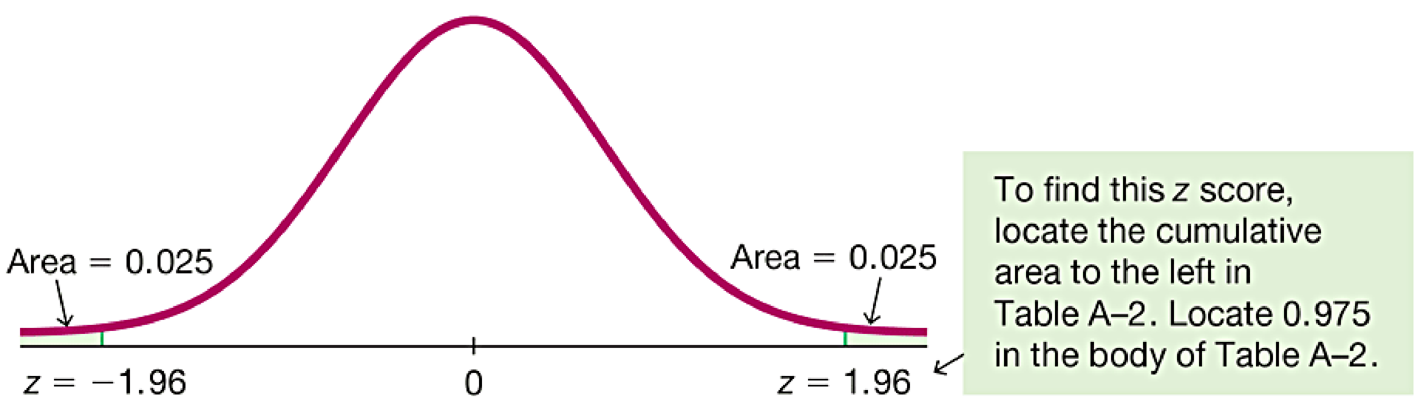
z_α (1 of 3)

Find the value of $z_{0.025}$. (Let $\alpha = 0.025$ in the expression z_α.)

Example: Finding the Critical Value z_α (2 of 3)

Solution

The notation of $z_{0.025}$ is used to represent the z score with an area of 0.025 to its **right.** Refer to the figure and note that the value of z = 1.96 has an area of 0.025 to its right, so $z_{0.025}$ = 1.96. Note that $z_{0.025}$ corresponds to a cumulative left area of 0.975.



Pearson

Example: Finding the Critical Value

z_{α} (3 of 3)

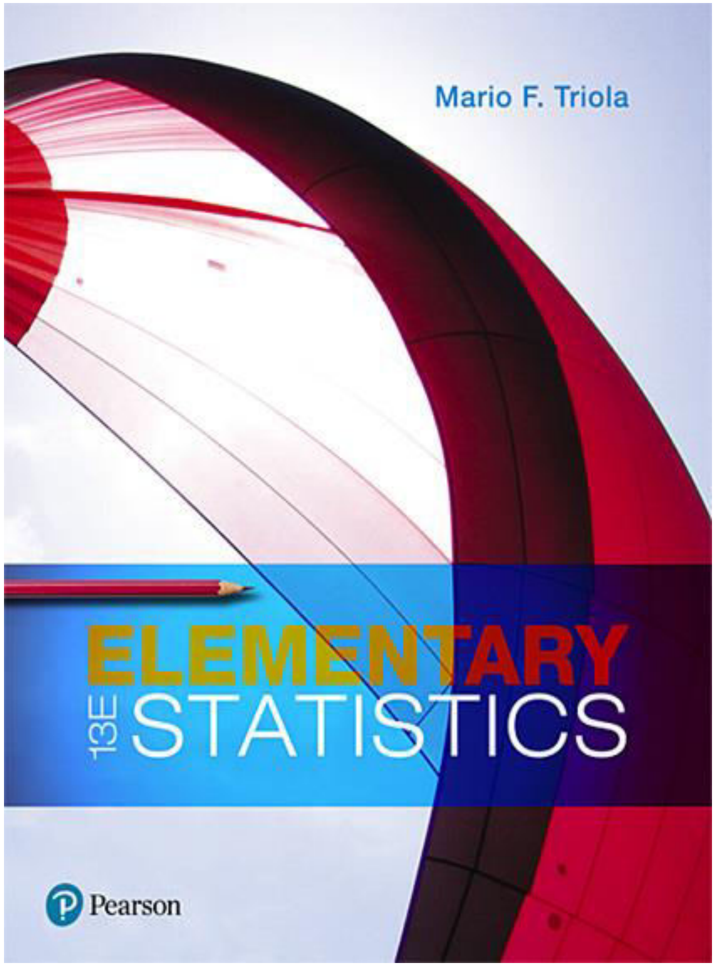
CAUTION

When finding a value of z_{α} for a particular value of α, note that α is the area to the **right** of z_{α}, but Table A-2 and some technologies give cumulative areas to the **left** of a given z score.

To find the value of z_{α}, resolve that conflict by using the value of $1 - \alpha$. For example, to find $z_{0.1}$, refer to the z score with an area of 0.9 to its left.

Elementary Statistics

Thirteenth Edition



Chapter 5

Probability Distributions

Probability Distributions

5-1 Probability Distributions

5-2 Binomial Probability Distributions

5-3 Poisson Probability Distributions

Key Concept

The focus of this section is the **binomial probability distribution** and methods for finding probabilities.

Easy methods for finding the mean and standard deviation of a binomial distribution are also presented.

As in other sections, we stress the importance of **interpreting** probability values to determine whether events are **significantly low** or **significantly high.**

Binomial Probability Distribution (1 of 2)

- Binomial Probability Distribution
  - A **binomial probability distribution** results from a procedure that meets these four requirements:
    1. The procedure has a **fixed number of trials.** (A trial is a single observation.)
    2. The trials must be **independent,** meaning that the outcome of any individual trial doesn’t affect the probabilities in the other trials.

Binomial Probability Distribution (2 of 2)

- Binomial Probability Distribution
  - A **binomial probability distribution** results from a procedure that meets these four requirements:
    3. Each trial must have all outcomes classified into exactly **two categories,** commonly referred to as **success** and **failure.**
    4. The probability of a success remains the same in all trials.

Notation for Binomial Probability Distributions (1 of 3)

S and F (success and failure) denote the two possible categories of all outcomes.

$P(S) = p(p$ = probability of a success)

$P(F) = 1 - p = q(q$ = probability of a failure)

n the fixed number of trials

Notation for Binomial Probability Distributions (2 of 3)

x - a specific number of successes in *n* trials, so *x* can be any whole number between 0 and *n*, inclusive

p - probability of **success** in **one** of the *n* trials

q - probability of **failure** in **one** of the *n* trials

P(*x*) - probability of getting exactly *x* successes among the *n* trials

Notation for Binomial Probability Distributions (3 of 3)

The word **success** as used here is arbitrary and does not necessarily represent something good. Either of the two possible categories may be called the success *S* as long as its probability is identified as *p*.

CAUTION When using a binomial probability distribution, always be sure that *x* and *p* are **consistent** in the sense that they both refer to the **same** category being called a success.

Example: Twitter (1 of 8)

When an adult is randomly selected (with replacement), there is a 0.85 probability that this person knows what Twitter is (based on results from a Pew Research Center survey). Suppose that we want to find the probability that exactly three of five randomly selected adults know what Twitter is.

a. Does this procedure result in a binomial distribution?

b. If this procedure does result in a binomial distribution, identify the values of n, x, p, and q.

Example: Twitter (2 of 8)

Solution

a. This procedure does satisfy the requirements for a binomial distribution, as shown below.

1. The number of trials (5) is fixed.
2. The 5 trials are independent because the probability of any adult knowing Twitter is not affected by results from other selected adults.

Example: Twitter (3 of 8)

Solution

3. Each of the 5 trials has two categories of outcomes: The selected person knows what Twitter is or that person does not know what Twitter is.
4. For each randomly selected adult, there is a 0.85 probability that this person knows what Twitter is, and that probability remains the same for each of the five selected people.

Example: Twitter (4 of 8)

Solution

b. Having concluded that the given procedure does result in a binomial distribution, we now proceed to identify the values of n, x, p, and q.

1. With five randomly selected adults, we have $n = 5$.
2. We want the probability of exactly three who know what Twitter is, so $x = 3$.

Example: Twitter (5 of 8)

Solution

3. The probability of success (getting a person who knows what Twitter is) for one selection is 0.85, so $p = 0.85$.
4. The probability of failure (not getting someone who knows what Twitter is) is 0.15, so $q = 0.15$.

Example: Twitter (6 of 8)

Solution

Again, it is very important to be sure that x and p both refer to the same concept of “success.” In this example, we use x to count the number of people who know what Twitter is, so p must be the probability that the selected person knows what Twitter is. Therefore, x and p do use the same concept of success: knowing what Twitter is.

Methods for Finding Binomial Probabilities (1 of 4)

Method 1: Binomial Probability Formula

$$P(x) = \frac{n!}{(x-n)!x!} \cdot p^x \cdot q^{n-x} \quad \text{for } x = 0, 1, 2, 3, \ldots, n$$

where

n = number of trials

x = number of successes among n trials

p = probability of success in any one trial

q = probability of failure in any one trial ($q = 1 - p$)

Example: Twitter (7 of 8)

Given that there is a 0.85 probability that a randomly selected adult knows what Twitter is, use the binomial probability formula to find the probability that when five adults are randomly selected, exactly three of them know what Twitter is. That is, apply the previous formula to find $P(3)$ given that $n = 5$, $x = 3$, $p = 0.85$, and $q = 0.15$.

Example: Twitter (8 of 8)

Solution

Using the given values of *n*, *x*, *p*, and *q* in the binomial probability formula, we get

$$P(3) = \frac{5!}{(5-3)!3!} \cdot 0.85^3 \cdot 0.15^{5-3}$$

$$= \frac{5!}{2!3!} \cdot 0.614125 \cdot 0.0225$$

$$= (10)(0.614125)(0.0225)$$

$$= 0.138178$$

$$= 0.138 \text{ (round to three significant digits)}$$

The probability of getting exactly three adults who know Twitter among five randomly selected adults is 0.138.

Methods for Finding Binomial Probabilities (2 of 4)

Method 2: Using Technology

Technologies can be used to find binomial probabilities. The screen displays on the next slide list binomial probabilities for n = 5 and p = 0.85, as in the previous example. Notice that in each display, the probability distribution is given as a table.

Methods for Finding Binomial Probabilities (3 of 4)

Method 2: Using Technology

Statdisk

Binomial Probability

Number of Trials, n: 5

Success Prob, p: 0.85

Evaluate

Results for all values of x are provided unless you enter a specific value for x here:

Mean: 4.2500
Standard Deviation: 0.7984
Variance: 0.6375

x	P(x)	P(x or fewer)	P(x or greater)
0	0.0000759	0.0000759	1.0000000
1	0.0021516	0.0022275	0.9999241
2	0.0243844	0.0266119	0.9977725
3	0.1381781	0.1647900	0.9733881
4	0.3915047	0.5562947	0.8352100
5	0.4437053	1.0000000	0.4437053

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TI-83/84 Plus CE

NORMAL FLOAT AUTO REAL RADIAN MP

L1	L2	L3	L4	L5
0	7.6E-5	------	------	------
1	.00215			
2	.02438			
3	.13818			
4	.3915			
5	.44371			
------	------			

Excel

	A	B
1	x	P(x)
2	0	7.594E-05
3	1	0.0021516
4	2	0.0243844
5	3	0.1381781
6	4	0.3915047
7	5	0.4437053

Minitab

x	P(X = x)
0	0.000076
1	0.002152
2	0.024384
3	0.138178
4	0.391505
5	0.443705

Example: Overtime Rule in Football (1 of 4)

We previously noted that between 1974 and 2011, there were 460 NFL football games decided in overtime, and 252 of them were won by the team that won the overtime coin toss. Is the result of 252 wins in the 460 games equivalent to random chance, or is 252 wins **significantly high?** We can answer that question by finding the probability of 252 wins or more in 460 games, assuming that wins and losses are equally likely.

Example: Overtime Rule in Football (2 of 4)

Solution

Using the notation for binomial probabilities, we have n = 460, p = 0.5, q = 0.5, and we want to find the sum of all probabilities for each value of x from 252 through 460. The formula is not practical here, because we would need to apply it 209 times—we don’t want to go there. Table A-1 (Binomial Probabilities) doesn’t apply because n = 460, which is way beyond the scope of that table. Instead, we wisely choose to use technology.

Example: Overtime Rule in Football (3 of 4)

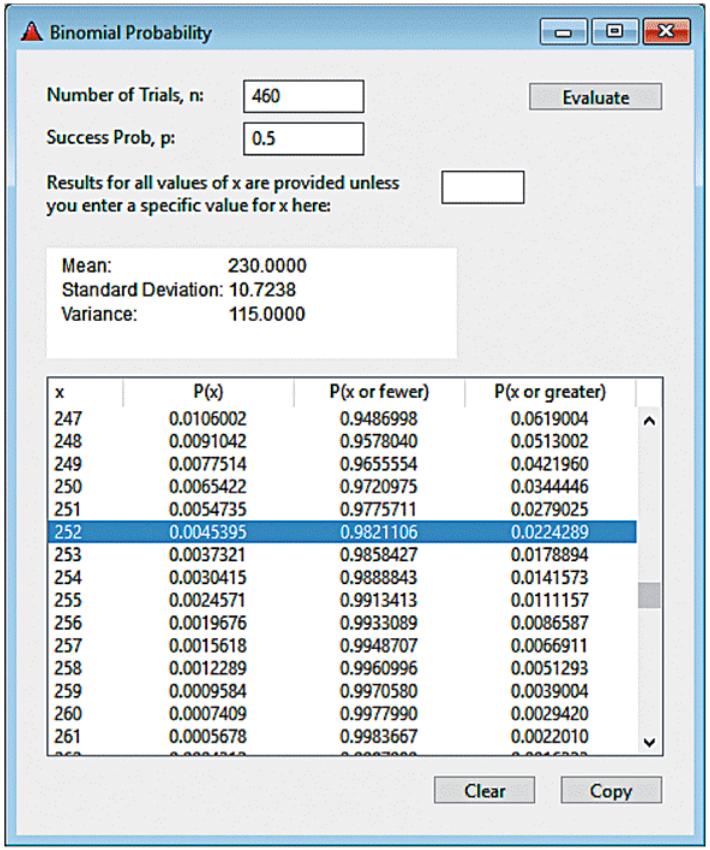
Solution

The Statdisk display on the next page shows that the probability of 252 or more wins in 460 overtime games is 0.0224 (rounded), which is low (such as less than 0.05). This shows that it is unlikely that we would get 252 or more wins by chance. If we effectively rule out chance, we are left with the more reasonable explanation that the team winning the overtime coin toss has a better chance of winning the game.

Example: Overtime Rule in Football (4 of 4)

Solution

Statdisk



Binomial Probability

Number of Trials, n: 460

Success Prob, p: 0.5

Evaluate

Results for all values of x are provided unless you enter a specific value for x here:

Mean: 230.0000
Standard Deviation: 10.7238
Variance: 115.0000

x	P(x)	P(x or fewer)	P(x or greater)
247	0.0106002	0.9486998	0.0619004
248	0.0091042	0.9578040	0.0513002
249	0.0077514	0.9655554	0.0421960
250	0.0065422	0.9720975	0.0344446
251	0.0054735	0.9775711	0.0279025
252	0.0045395	0.9821106	0.0224289
253	0.0037321	0.9858427	0.0178894
254	0.0030415	0.9888843	0.0141573
255	0.0024571	0.9913413	0.0111157
256	0.0019676	0.9933089	0.0086587
257	0.0015618	0.9948707	0.0066911
258	0.0012289	0.9960996	0.0051293
259	0.0009584	0.9970580	0.0039004
260	0.0007409	0.9977990	0.0029420
261	0.0005678	0.9983667	0.0022010

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Pearson

Methods for Finding Binomial Probabilities (4 of 4)

Method 3: Using Table A-1 in Appendix A

This method can be skipped if technology is available. Table A-1 in Appendix A lists binomial probabilities for select values of n and p. It cannot be used if $n > 8$ or if the probability p is not one of the 13 values included in the table.

To use the table of binomial probabilities, we must first locate n and the desired corresponding value of x. At this stage, one row of numbers should be isolated. Now align that row with the desired probability of p by using the column across the top. The isolated number represents the desired probability. A very small probability, such as 0.000064, is indicated by 0+.

Example: Devil of a Problem (1 of 3)

Based on a Harris poll, 60% of adults believe in the devil. Assuming that we randomly select five adults, use Table A-1 to find the following:

a. The probability that exactly three of the five adults believe in the devil

b. The probability that the number of adults who believe in the devil is at least two

Example: Devil of a Problem (2 of 3)

Solution

a. The following excerpt from the table shows that when $n = 5$ and $p = 0.6$, the probability for $x = 3$ is given by $P(3) = 0.346$.

TABLE A-1

Binomial Probabilities

n	x	.01
5	0	.951
	1	.048
	2	.001
	3	0+
	4	0+
	5	0+

	p	
.50	.60	.70
.031	.010	.002
.156	.077	.028
.313	.230	.132
.313	.346	.309
.156	.259	.360
.031	.078	.168

x	$P(x)$
0	.010
1	.077
2	.230
3	.346
4	.259
5	.078

Example: Devil of a Problem (3 of 3)

Solution

b. The phrase “at least two” successes means that the number of successes is 2 or 3 or 4 or 5.

$$\begin{aligned} P(\text{at least 2 believe in the devil}) &= P(2 \text{ or } 3 \text{ or } 4 \text{ or } 5) \\ &= P(2) + P(3) + P(4) + P(5) \\ &= 0.230 + 0.346 + 0.259 + 0.078 \\ &= 0.913 \end{aligned}$$

Using Mean and Standard Deviation for Critical Thinking

For Binomial Distributions

Mean: $\mu = np$

Variance: $\sigma^2 = npq$

Standard Deviation: $\sigma = \sqrt{npq}$

Range Rule of Thumb

Significantly low values ≤ $(\mu - 2\sigma)$

Significantly high values ≥ $(\mu + 2\sigma)$

Values not significant: Between $(\mu - 2\sigma)$ **and** $(\mu + 2\sigma)$

Example: Using Parameters to Determine Significance (1 of 4)

A previous example involved n = 460 overtime wins in NFL football games. We get p = 0.5 and q = 0.5 by assuming that winning the overtime coin toss does not provide an advantage, so both teams have the same 0.5 chance of winning the game in overtime.

a. Find the mean and standard deviation for the number of wins in groups of 460 games.

b. Use the range rule of thumb to find the values separating the numbers of wins that are significantly low or significantly high.

c. Is the result of 252 overtime wins in 460 games significantly high?

Example: Using Parameters to Determine Significance (2 of 4)

Solution

a. With n = 460, p = 0.5, and q = 0.5, previous formulas can be applied as follows:

$$\mu = np = (460)(0.5) = 230.0 \text{ games}$$

$$\sigma = \sqrt{npq} = \sqrt{(460)(0.5)(0.5)} = 10.7 \text{ games}$$

For random groups of 460 overtime games, the mean number of wins is 230.0 games, and the standard deviation is 10.7 games.

Example: Using Parameters to Determine Significance (3 of 4)

Solution

b. The values separating numbers of wins that are significantly low or significantly high are the values that are two standard deviations away from the mean. With $\mu = 230.0$ games and $\sigma = 10.7$ games, we get

$$(\mu - 2\sigma) = 230.0 - 2(10.7) = 208.6 \text{ games}$$

$$(\mu + 2\sigma) = 230.0 + 2(10.7) = 251.4 \text{ games}$$

Example: Using Parameters to Determine Significance (4 of 4)

Solution

Significantly low numbers of wins are 208.6 games or fewer, significantly high numbers of wins are 251.4 games or greater, and values not significant are between 208.6 games and 251.4 games.

c. The result of 252 wins is significantly high because it is greater than the value of 251.4 games found in part (b).

Using Probabilities to Determine When Results Are Significantly High or Low

Significantly high number of successes:

x successes among *n* trials is **significantly high** if the probability of *x* or more successes is 0.05 or less. That is, *x* is a significantly high number of successes if $P(x \text{ or more}) \leq 0.05$.

Significantly low number of successes:

x successes among *n* trials is **significantly low** if the probability of *x* or fewer successes is 0.05 or less. That is, *x* is a significantly low number of successes if $P(x \text{ or fewer}) \leq 0.05$.

Rationale for the Binomial Probability Formula

$$P(x) = \overbrace{\frac{n!}{(x-n)!x!}}^{\text{The number of outcomes with exactly } x \text{ successes among } n \text{ trials}} \cdot \overbrace{p^x \cdot q^{n-x}}^{\text{The probability of } x \text{ successes among } n \text{ trials for any one particular order}}$$

Pearson