IMGS-633: HW-7: Beam Propagation Method

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1. Point Spread Function of a Circular Aperture
Write a computer program using Python, Matlab, or similar to perform a two-dimensional ($N \times N$ pixels) "Fast Fourier Transform" (FFT) operation on the function $E_1(x, y) = \exp(-((x^2 + y^2)/w^2)^{50})$ where $x = (i_x - n_{cen})\Delta x$ and $y = (i_y - n_{cen})\Delta x$, $w = \sqrt{N/\pi}\,\Delta x$, $n_{cen} = 1 + N/2$, $i_x = 1, 2, 3, ..., N$ and $i_y = 1, 2, 3, ..., N$. Let $N = 1024$ so that $w = 18\Delta x$ and let $\Delta x = 10\lambda$.

Verify that E_1 appears as a circular function centered on the numerical grid. You can do this by rendering the data as a grayscale image.

You must read the help page documentation about the 2-D FFT algorithm for your programming language. Pay attention to the section that discusses the *fftshift* algorithm for a 2-D array. After defining E_1 you will do the following operations on the data array: $E_1' = \text{fftshift}(E_1)$, $\quad E_1'' = \text{fft}(E_1')$, $\quad E_1''' = \text{fftshift}(E_1'')$.

(A) Render grayscale images of absolute values (moduli): $|E_1|$, $|E_1'|$, $|E_1''|$, and $|E_1'''|$. Feel free to crop the images your turn in for $|E_1|$ and $|E_1'''|$ to remove uninteresting regions, but please report the number of pixels in the cropped image (e.g., 100 x 100).

(B) The function $|E_1'''|^2$ is called the point spread function of the aperture. Examine your rendered image and determine the pixel distance between its peak and its first minimum. You will need this distance in Problem 2. If you crop this image for better viewing, you must indicate the x and y range of the cropped image. (Note: Theoretically that distance corresponds to $1.22\lambda f^{\#}$ for a focusing lens).

2. Resolving Two Point Objects
Repeat the above FFT operations for a tilted plane wave at the aperture, represented by the function $E_2(x, y) = \left(\exp(-((x^2 + y^2)/w^2)^{50})\right)\exp(i\tilde{k}_x x)$ where $\tilde{k}_x = (2\pi/\lambda)\sin\phi$. Assume $\lambda = 1\mu m$

(A) Adjust the value of ϕ until the peak of $|E_2'''|^2$ coincides with the first zero of $|E_1'''|^2$ calculated in Problem 2. This is the Rayleigh resolution criterion for two mutually incoherent point objects. If you crop the images for better viewing, you must indicate the x and y range of the cropped image.

(B) Report the value of ϕ that satisfies the Rayleigh criterion, and render as a grayscale image the sum of the intensities: $|E_1'''|^2 + |E_2'''|^2$.

3. Beam Propagation
Use the Beam Propagation Method instructions (BPM4b.pdf) to write a computer code to propagate the initial electric field described in Problem 1 in the near-field regime ($z < z_d$), where $z_d = (1/2)kw_0^2$. Assume $\lambda = 1\mu m$, $w_0 = 100\mu m$. You will propagate the beam to five different distances: $z = 0.2z_d$, $z = 0.4z_d$, $z = 0.6z_d$, $z = 0.8z_d$, $z = 1.0z_d$.

Hand in computer-generated images of $|E(x, y, 0)|$ as well the field magnitude for the five propagation distances above. Also hand in the phase images (using the hsv color map) for $z = 0$ and the five other distances. Note that $\Phi(x, y, 0)$ should be a constant value. The phase may be found by taking the arctangent of the ratio $\text{Im}[E]/\text{Re}[E]$ by use of the function atan2.

4. Spot of Arago Let $E(x, y, 0) = 1 - \exp(-(r^2/w_0^2)^\eta)$. This corresponds to an annulus of light (a disk with a central black hole). Vary the propagation distance (i.e., the value of z) until you find the so-called "Poisson spot" or "Spot of Arago" – namely, a central bright white spot inside the shadow region of the black hole. Relate your discovered value of z to $z_d = \pi w_0^2/\lambda$. This spot has an interesting history that helped cement the fact that light has a wave nature.