IMGS-633: HW-8: 4f Phase Contrast Optical System

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1. Phase Contrast Imaging with a 4-f system: Analytical.
In 1953 Frits Zernike was awarded the Physics Nobel prize for the invention of a 4-f phase contrast imaging system, whereby a transparent object having an electric field profile, $E_o(x)$ in the x-plane, can be made visible by use of a phase mask $t(x')$ in the Fourier transform plane (x'-plane). A coherently illuminated object is placed in the front focal plane of lens-1, having a focal length f. The mask is placed in the back focal plane of lens-1, which is also the front focal plane of lens-2. Assume the lenses have the same focal lengths and have diameters of infinite extent.

$$\begin{aligned} &\text{Object: } E_o(x) = e^{i\beta(x)}, \text{ where } \beta = \epsilon\cos(2\pi x/\Lambda) \\ &\text{Fourier Plane: } E'(x') = FT[E_o(x)] \\ &\text{Image Plane: } E''(x'') = FT[t(x')E'(x')] \end{aligned} \tag{1}$$

where the period is much greater than the wavelength: $\Lambda >> \lambda$.

(a) Making use of the Taylor series expansion by assuming $\epsilon << 1$, determine the field in the Fourier plane, $E'(x')$.

(b) Determine a simple expression for the distance, x', of each diffracted peak, labeling these X_{-1}, X_0, and X_{+1}.

(c) Let the transmission function in the Fourier plane be given by

$$t(x') = \begin{cases} \exp(i\pi/2)\,, & |x'| < (1/2)\lambda f/\Lambda \\ 1\,, & |x'| \geq (1/2)\lambda f/\Lambda \end{cases} \tag{2}$$

Determine simplified expressions for both the field in the imaging plane, $E''(x'')$ and the corresponding irradiance $I''(x'') = |E''(x'')|^2$.

(d) Determine the contrast of the image $I''(x'')$ and compare it to the contrast in the case where the phase mask t is removed from the system.

2. Phase Contrast Imaging with a 4-f system: Numerical. Here you will numerically model the above system for a two-dimensional example. Instead of using the simple function for β in Eq. (1) above, use an interesting 8-bit digital photograph (i.e., each pixel has a value between 0 and 255). Let the function $g(x,y)$ represent the normalized photograph so that each pixel has a value between 0.00 and 1.00. Let $\beta(x,y) = \epsilon\, g(x,y)$ where $\epsilon = 0.10$. (Naturally, the curious student will also try the function $\beta = \epsilon\cos(2\pi x/\Lambda)$ to compare their numerical and analytical solutions.)

Crop the photograph to $N \times N$ pixels, where $N = 2^m$.

Mask the object field $E(x,y) = \exp(i\beta)$ with a hypergaussian aperture function $A(x,y)$ having a radius corresponding to $\sqrt{N/\pi}$ pixels. This latter step will provide real-world diffraction effects corresponding to a lens having a finite diameter. Hint: once your code is working for a modest value of N, change the value to N=4096 or larger if your computer has sufficient memory.

(a) Render grayscale images of both β (i.e., your digital photo), and the magnitude of the Fourier transform $|E'(x',y')| = |FT[A(x,y)\exp(i\beta(x,y))]|$.

(b) Applied a phase filter in the Fourier plane,

$$t(x',y') = \begin{cases} \exp(i\phi)\,, & |r'| < s \\ 1\,, & |r'| \geq s \end{cases} \tag{3}$$

where $r' = \sqrt{x'^2 + y'^2}$ and s is a value that produces the best high contrast image (see more below). Fourier transform the filtered image and render the magnitude of the result $|E''(x'',y'')|$ for the case $\phi = \pi/2$.

(c) Repeat part (b) for the case $\phi = 0$.

(d) Compute and compare the contrast within the central regions of interest of parts (b) and (c). Vary the value of s until achieve the highest contrast in part (b).

(e) Provide a brief narrative describing your findings and why your value of s produces the highest contrast.

3. Fourier Filtering with a 4-f system: Numerical. Repeat Problem 2, instead using a high-pass amplitude filter in the Fourier plane:

$$t(x', y') = \begin{cases} 0\,, & |r'| < s \\ 1\,, & |r'| \geq s \end{cases} \tag{4}$$

where $0 < s < \sqrt{N/\pi}$.

For $s = \sqrt{1/2}\sqrt{N/\pi}$ render grayscale images of
(a) $|E'(x', y')| = |FT[A(x, y)\exp(i\beta(x, y))]|$,
(b) $|t(x', y')E'(x', y')|$, and
(c) $|E''(x'', y'')| = |FT[t(x', y')E'(x', y')]|$.

(d) Provide a brief narrative describing the effect of increasing the value of s from zero to $\sqrt{N/\pi}$ pixels. Why do we call this a high pass filter?

Note: To receive credit ALL OF THE ABOVE rendered image must be accompanied by a figure caption and labels that clearly identify what is being represented and any other relevant information.