IMGS-633: HW-6: Exact Interference of Spherical Waves

Submit to MyCourses by 2:00pm, Thursday 9 March 2023

Exact Interference of Spherical Waves:
The net electric field from multiple monochromatic spherical waves, all emitted from the plane $z = 0$ may be expressed

$$
\begin{aligned}
E(x, y, z) &= \sum_{j}^{N} \frac{A_j}{r_j} e^{ikr_j} e^{i\phi_j} \text{ where } r_j = ((x - x_j)^2 + (y - y_j)^2 + z^2)^{1/2} \\
&\equiv |E| \exp^{i\Phi}
\end{aligned}
$$

Assume $N = 3$ with all having the same amplitude and same phase. Write a computer program to compute and render the amplitude and phase distributions at the distance $z = 1000\lambda$: $|E(x, y, z)|$ and $\Phi(x, y, z)$ for the following cases. Allow your images to span 1024×1024 pixels with the "center" at the point $(513, 513)$. Set the pixel sizes to $\Delta x = \Delta y = \lambda$. Let $\lambda = 1$ $[\mu m]$.

(a) $x_j = a\cos(2\pi j/N)$, $\;y_j = a\sin(2\pi j/N)$ where the radial separation parameter has the value $a = 10\lambda$.

(b) $x_j = a\cos(2\pi j/N)$, $\;y_j = a\sin(2\pi j/N)$ where the radial separation parameter has the value $a = 30\lambda$.

(c) Describe your observations for each image. Provide a reason why the outer region of the irradiance plots differ from the central region. Compare the two irradiance plots and also compare the two phase plots. Provide a reason why the patterns become more compact in the central region as the value of a increases.

(d) Let $N = 2$ and set $x_1 = -a$ and $x_2 = a$ with $y_{1,2} = 0$. Let the separation parameter have the value $a = 10\lambda$. As above, compute and render the amplitude and phase distributions at the distance $z = 1000\lambda$.

(e) Repeat (d) for the case $a = 30\lambda$.

(f) Describe your observations for the rendered images in (d) and (e). Provide a reason why the outer region of the irradiance plots differ from the central region. Compare the two irradiance plots and also compare the two phase plots. Provide a reason why the patterns become more compact in the central region as the value of a increases.

(g) Use your code and your innate scientific curiosity to explore how the amplitude distribution $|E(x, y, z)|$ changes at different values of z, say from the near field $z < \pi a^2/\lambda$ to the far field $z > \pi a^2/\lambda$. Examples are shown below. Write a few comments about your findings.

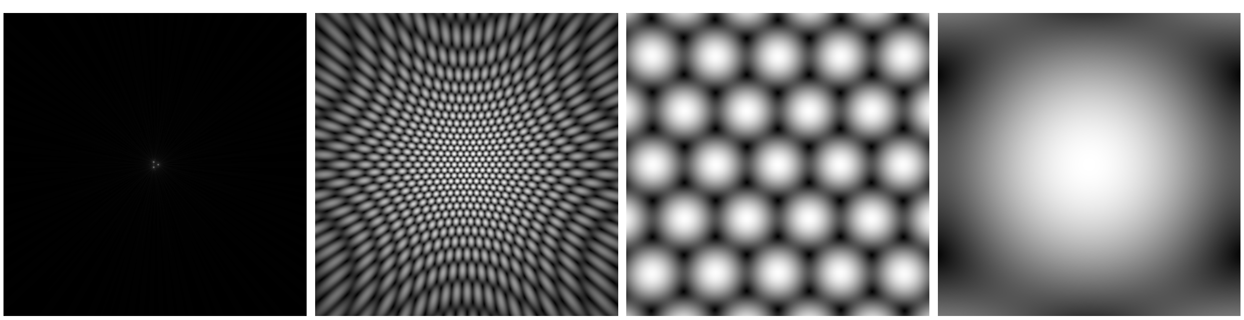

Figure 1: Electric field amplitudes $|E|$ from three point sources. Left to right: Extreme near-field, intermediate, far-field, and extreme far-field zones. Each image spans the transverse (x, y) coordinate range: $1024\lambda \times 1024\lambda$.

(h) Which of the images in Fig. 1 may be represented by a summation of only three equal amplitude plane waves:

$$
|E| \sim \left| \sum_{j=1}^{3} \exp(ik_{x,j}x + ik_{y,j}y) \right|, \text{ where } k_{x,j} = (2\pi/\lambda)(x_j/z), \text{ and } k_{y,j} = (2\pi/\lambda)(y_j/z)
$$