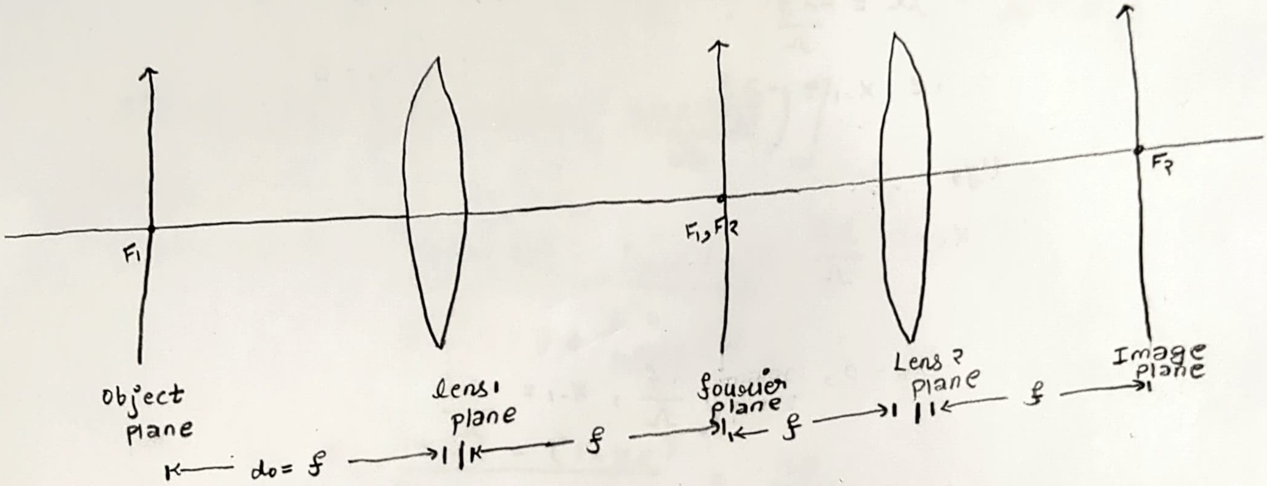
Optics In Imaging

Homework - 8

4f phase Contrast Optical System

Q.1 phase contrast Imaging with a 4-f system: Analytical



a. Electric Field in fourier plane:

$$E'(x') = FT[E_0(x)]$$

$$= FT[e^{i\beta(x)}]$$

$$= FT[e^{i\epsilon\cos(2\pi x/\Lambda)}]$$

$$e^{i\epsilon\cos\left(\frac{2\pi x}{\Lambda}\right)} = 1 + i\epsilon\cos\left(\frac{2\pi x}{\Lambda}\right) + \left[\frac{1}{2}\left(i\epsilon\cos\frac{2\pi x}{\Lambda}\right)^2\right] + \dots$$

$$= 1 + i\epsilon\cos\left(\frac{2\pi x}{\Lambda}\right) \quad \{\text{since } \epsilon \lll 1\}$$

$$= FT\left[1 + i\epsilon\cos\left(\frac{2\pi x}{\Lambda}\right)\right] \quad \{x \rightarrow k_x\}$$

$$= 2\pi\delta(k_x) + FT\left[i\epsilon\cos\left(\frac{2\pi x}{\Lambda}\right)\right]$$

$$= 2\pi\delta(k_x) + 2\pi * \frac{i\epsilon}{2}\delta\left(k_x - \frac{2\pi}{\Lambda}\right) + \frac{i\epsilon}{2} * 2\pi\,\delta\left(k_x + \frac{2\pi}{\Lambda}\right)$$

$$\therefore E'(x') = 2\pi\delta\left(\frac{2\pi}{\lambda}\frac{x'}{f}\right) + \left\{\frac{i\epsilon}{2}\delta\left(\frac{2\pi}{\lambda}\frac{x'}{f} - \frac{2\pi}{\Lambda}\right) + \frac{i\epsilon}{2}\delta\left(\frac{2\pi}{\lambda}\frac{x'}{f} + \frac{2\pi}{\Lambda}\right)\right\} * 2\pi$$

Here, $k_x = \frac{2\pi}{\lambda}\frac{x'}{f}$

$$\Rightarrow E'(x') = \lambda f\delta(x') + \frac{i\epsilon\pi}{2\pi}\lambda f\delta\left(x' - \frac{\lambda f}{\Lambda}\right) + \frac{i\epsilon\pi}{2\pi}\lambda f\delta\left(x' + \frac{\lambda f}{\Lambda}\right)$$

i.e $E'(x') = \lambda f \delta(x') + \frac{i\epsilon\lambda f}{2}\left\{\delta\left(x' - \frac{\lambda f}{\Lambda}\right) + \delta\left(x' + \frac{\lambda f}{\Lambda}\right)\right\}$

b. Here,

for x_{-1}:

$$k_x + \frac{2\pi}{\Lambda} = 0$$

$$\Rightarrow \frac{2\pi}{\lambda} \cdot \frac{x'}{f} = -\frac{2\pi}{\Lambda}$$

$$\Rightarrow x' = -\frac{\lambda f}{\Lambda}$$

i.e $x_{-1} = -\frac{\lambda f}{\Lambda}$

for, x_0:

$$\frac{2\pi}{\lambda} \cdot \frac{x'}{f} = 0$$

$$\Rightarrow x' = 0$$

i.e $x_0 = 0$

Ily,

$$x_{+1} = \frac{\lambda f}{\Lambda}$$

$\therefore$ $x_0 = 0$, $x_{+1} = \frac{\lambda f}{\Lambda}$, $x_{-1} = -\frac{\lambda f}{\Lambda}$

c. Here,

$$E''(x'') = FT\left[t(x')E'(x')\right]$$

$$= FT\left\{e^{i\pi/2}\lambda f \delta(x')\right\} + FT\left\{1 \cdot \frac{i\epsilon\lambda f}{2}\left\{\delta\left(x' - \frac{\lambda f}{\Lambda}\right) + \delta\left(x' + \frac{\lambda f}{\Lambda}\right)\right\}\right\}$$

$$= i\lambda f\, FT\{\delta(x')\} + \frac{i\epsilon\lambda f}{2} FT\left\{\delta\left(x' - \frac{\lambda f}{\Lambda}\right) + \delta\left(x' + \frac{\lambda f}{\Lambda}\right)\right\}$$

$$= i\lambda f . 1[x''] + \frac{i\epsilon\lambda f}{2}\left\{1[x'']e^{i2\pi x'' \cdot \frac{\lambda f}{\Lambda}} + 1[x'']e^{-i2\pi x'' \frac{\lambda f}{\Lambda}}\right\}$$

$$= i\lambda f + i\lambda f \epsilon . \cos\left[\frac{2\pi x'' \lambda f}{\Lambda}\right]$$

$$\therefore E''(x'') = i\lambda f\left[1 + \epsilon\cos\left(\frac{2\pi x'' \lambda f}{\Lambda}\right)\right]$$

Now, $I''(x'') = |E''(x'')|^2 = E''(x'') * E^{*''}(x'')$

$$= i\lambda f\left[1 + \epsilon\cos\left(\frac{2\pi x'' f\lambda}{\Lambda}\right)\right] * -i\lambda f\left[1 + \epsilon\cos\left(\frac{2\pi x'' f\lambda}{\Lambda}\right)\right]$$

$$\therefore I''(x'') = (\lambda f)^2\left[1 + \epsilon\cos\left(\frac{2\pi x'' f\lambda}{\Lambda}\right)\right]^2$$

d. we have:

$$I''(x'') = (\lambda f)^2 \left[1 + \epsilon \cos\left(\frac{2\pi x'' f\lambda}{\Lambda}\right)\right]^2$$

$$= (\lambda f)^2 \left[1 + 2\epsilon \cos\left(\frac{2\pi x'' \lambda f}{\Lambda}\right) + \cancel{\epsilon^2 \cos^2\left(\frac{2\pi x'' f\lambda}{\Lambda}\right)}^{\to 0}\right]$$

$$\therefore I''(x'') = (\lambda f)^2 \left[1 + 2\epsilon \cos\left(\frac{2\pi x'' \lambda f}{\Lambda}\right)\right]$$

Now,

$$\text{contrast} = \frac{I''_{max}(x'') - I''_{min}(x'')}{I''_{max}(x'') + I''_{min}(x'')}$$

$$= \frac{1+2\epsilon - (1-2\epsilon)}{(1+2\epsilon) + (1+2\epsilon)} = \frac{4\epsilon}{2}$$

$$\therefore \text{contrast} = 2\epsilon$$

Note: when phase mask is not used:

$$E''(x') = FT[E'(x')]$$

$$= FT\left[\lambda f \delta(x') + \frac{i\epsilon\lambda f}{2}\left\{\delta\left(x' - \frac{\lambda f}{\Lambda}\right) + \delta\left(x' + \frac{\lambda f}{\Lambda}\right)\right\}\right]$$

$$= \lambda f\, 1[x''] + i\epsilon\lambda f \cos\left[\frac{2\pi x'' \lambda f}{\Lambda}\right]$$

$$\therefore E''(x') = \lambda f\left[1 + i\epsilon \cos\left[\frac{2\pi x'' \lambda f}{\Lambda}\right]\right]$$

Now, irradiance: $I''(x') = (\lambda f)^2 * \left[1^2 + \cancel{\epsilon^2 \cos^2\left[\frac{2\pi x'' \lambda f}{\Lambda}\right]}^{\to 0}\right]$ since $\epsilon << 1$

so, contrast $= 0$

so, on adding phase mask $t(x')$, the contrast increases. so, the transparent object having an E-field profile can be made visible by using phase masks