

## Assignment Questions

**Date: 22/07/2019**

**Q.1.** Consider the following decision problem:

K-Color: Given a graph  $G$ , can we use at most  $K$ -Colors to color  $G$  such that no two adjacent vertices have the same color.

- Show that 2-Color is in P.[Hint: Bipartite Graph]
- Show that 3-Color is in NP-Complete. [Hint: Show that 3-SAT can be reduced to 3-Color in Polynomial time.]

**Q.2.** Consider the “Sum of Subset” problem.

- Show that “Sum of Subset” is NP-Complete.  
[Hint: Show that 3-SAT can be reduced to “Sum of Subset” in Polynomial time. Take an instance of 3-SAT and convert it to Sum of Subset instance. Solve Sum of Subset problem and obtain the truth assignments for 3-SAT.]

**Q.3.** Show that “HAM-CYCLE” is NP-Complete. [Hint: Reduce 3-SAT to HAM-CYCLE.]

**HAM-CYCLE: Given a graph  $G$ , does there exist Hamiltonian cycle in  $G$ ?**

**Q.4.** Define: 3-Dimensional Matching problem. Compare it with Bipartite Matching problem. Classify both of the problems i.e. whether they are in P or NP.

**Q.5.** Consider the following problem. You are managing a Communication Network, modeled by a directed graph  $G = \langle V, E \rangle$ . There are  $C$  users who are interested in making use of this Network. User  $i$  (for each  $i = 1, 2, \dots, c$ ) issues a request to reserve a specific path  $P_i$  in  $G$  on which to transmit data. You are interested in accepting as many of the path requests as possible, subject to the following restriction: if you accept  $P_i$  and  $P_j$ , then  $P_i$  and  $P_j$  cannot share any nodes.

**Thus the Path selection problem asks:** Given a directed graph  $G = \langle V, E \rangle$ , a set of requests  $P_1, P_2, \dots, P_c$ - Each of which must be a Path in  $G$ - and a number  $K$ , is it possible to select at least  $K$  of the paths so that no two of the selected paths share any nodes? **Prove that Path Selection is NP-Complete.**

**Q.6.** The set partition problem takes as input a set  $S$  of numbers. The question is whether the numbers can be partitioned into two sets  $A$  and  $B$  such that  $\sum A = \sum B$ . Show that the Set Partition Problem is NP-Complete.

**Q.7.** Given an integer  $M \times N$  matrix  $A$  and an integer  $m$ -vector  $b$ , **the 0-1 integer programming problem** asks whether there exists an integer  $n$ -vector  $x$  with elements in the set  $\{0, 1\}$  such that  $Ax = b$ . Prove that 0-1 integer Programming is NP-Complete. [Hint: Reduce from 3-CNF-SAT.]