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Applied Optimization (WS 2023/2024)
Exercise Sheet No. 5

Upload Date: 07.12.2023.
Submission Deadline: In groups of three until **11.01.2024, 07:55 for the modelling and 18.01.2024, 07:55 for the review** to the Moodle.
Return date: 22.01.2024 and 25.01.2024 in the tutorials.

In case you have questions do not hesitate to ask your tutor or to contact the tutor team at apopt@techfak.uni-bielefeld.de.

Remark: When handing in your solution, do not only provide the source code, but also all the other information we ask for (e.g. the output of the optimization, the runtime, etc.). We will *not* execute your code!

Organizational Matters

The following exercise sheet consists of two parts. In the first part you are asked to do a modeling. You have until the 11$^{\text{th}}$ of January to complete this part. Hand in an anonymized version of your work via the Moodle. We will *redistribute the solutions to other groups for a review process* (See Exercise 3). You have one week to write and hand in the review.

We will grade both your solution *and* your review. Your review of another group will *not* have an impact on the points the other group receives.

Introduction

Dear students,
the following exercise is really difficult, and we do *not* expect you to come up with a "complete" solution! Please, notice that all we expect from you is problem orientated thinking and reasoning, rather than amazing programming skills or a tremendous knowledge about optimization theory or the topic at hand. The most important part is that you find a *reasonable and working* solution to the problem. Please note, that it is totally pointless if your algorithm produces "good results" if you cannot explain why those results are relevant, "good", or why your algorithm makes sense in the first place. Furthermore, it is also not helpful if you have a perfectly justified model, but you are unable to come up with an implementation that is able to solve the problem with a reasonable amount of resources (in particular time). It is perfectly fine (and common practice, by the way) to reduce a problems complexity by using approximations – for example to approximate a complicated expression by a simpler one if they are similar enough or splitting the problem into smaller problems that can be solved one after the other – as long as those choices are *explained and justified*.

What we *expect* from you is that you are able to understand and analyze a given problem, find a reasonable formalization which you can explain and justify, and that you are able to criticize your own ideas and thoughts.

1 Modelling

(*31 points*)

Imagine the following scenario:
Ms Chen is extensively renovating an old house she has inherited from her grandmother. The house has four stories and a cellar as well as an attic. Ms Chen wants to have a shop as well as three flats in the house. She has already made floor plans with an architect and knows where she needs the plumbers to install sinks, showers, bathtubs and so on. But she does not yet know how to heat the water for her house. The pipe system has become a bit complicated with multiple adjustments over the decades, too.
It is important for Ms Chen and future tenants that the water heating system will be as inexpensive as possible. Ms Chen has already decided that **the cost of either placing additional pipes or new water heaters is irrelevant** given the rapidly increasing energy costs.
The plumbers and architects Ms Chen has hired have already provided plans for the house - the flats will all have the same layout, and the attic will not need to be connected to the water network, so the following three plans illustrate the layout of the house.

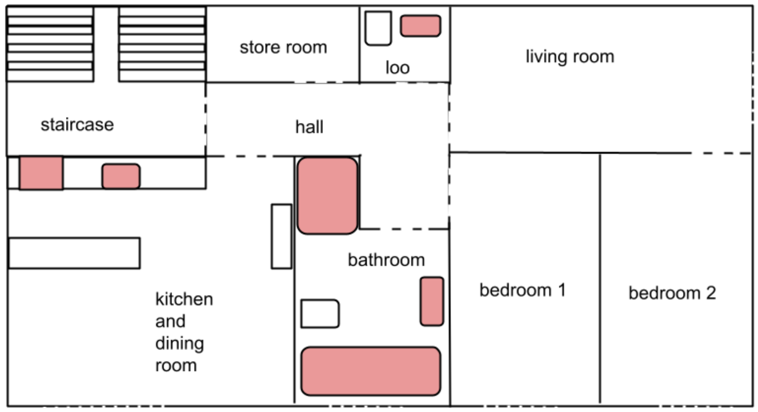

Figure 1: Flats - Floors 1 to 3

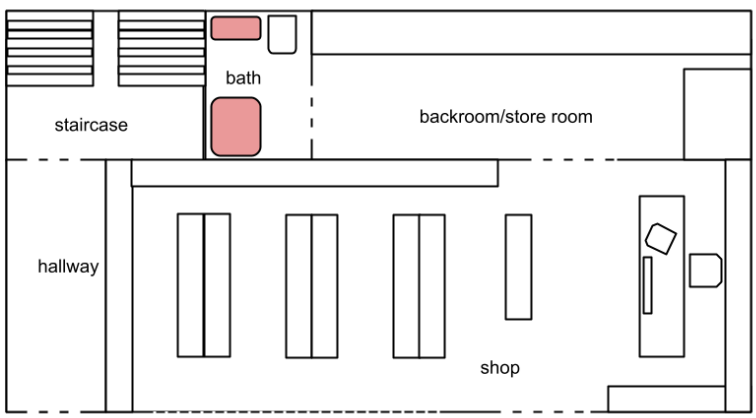

Figure 2: Shop and Entrance - Ground Floor

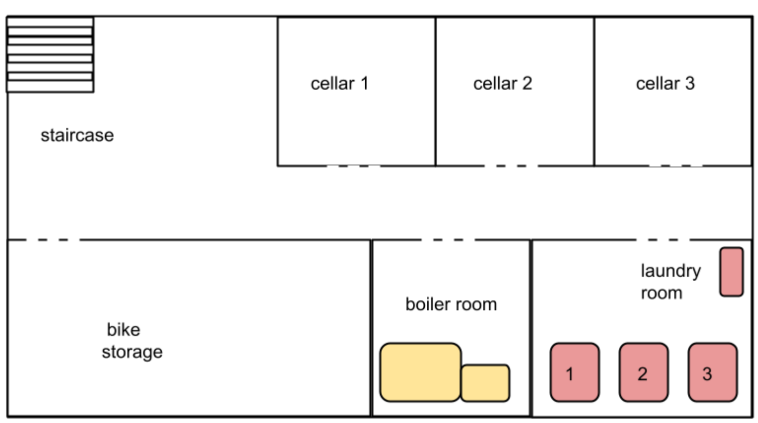

Figure 3: Cellar

As you can see, there are 24 taps/devices that need to be supplied with hot water distributed over the five floors. The central boiler is situated in the boiler room in the basement. The boiler is heating its water from 20°C to 60°C, which requires $160kJ/l$. It is a very big boiler, so you can assume that there is always enough hot water in it. For your model, you can assume that local heating units can be placed directly under any tap/device that may need them. They precisely heat the water up to energy level required. Please note that the cold water pipes have already been placed, so if a device gets its water heated locally, you would not need to connect that device to the central network. The local heating units have an efficiency of 25%. Of course you can assume that the first law of thermodynamics holds, though we assume that the energy loss per hour is constant.

Your task is to decide which devices will be supplied by the central boiler and which will use local heating units. The water should be heated as efficiently as possible to keep costs low. We are looking for a solution that specifies the method of heating for all 24 devices as well as which of the possible pipes we will actually need to built.
Keep in mind that a local heating device will be less efficient than using a central boiler, but also that the water will lose heat in the pipes.

You can find the location as well as the energy use of all 24 devices that use hot water in `project_locations_users.csv` (contained in `data.zip`) which you can load with pandas, for example. The possible pipe nodes (places, where the pipes can connect, as well as the outputs for the pipes for connection to devices) are specified in `pipe_nodes.csv` and their connection, diameter (in cm) and their energy loss per time and liter (in kilo Joule per hour and litre $kJ/(h \cdot l)$) is specified in `pipe_connections.csv` as `loss_of_energy`. The coordinates are x, y and z. Their unit is meters and they were derived by treating the floor plans as coordinate systems for x and y (with the origin in the bottom left corner), while the z coordinate indicates the height. Note that the height in the cellar is negative, as a z-coordinate of 0 indicates street level.

Hint: The volume of a cylinder is $V = \pi r^2 \cdot l$ with radius r and length l.

(a) *(12 pts.)* Formalize the problem described above. Notice, that not all aspects are formalized precisely. Explain your choices! Also discuss other possible choices and explain why you decided the way you did.
You may focus on the following aspects:

- What is the overall objective?
- How to model the overall situation?
- What are your assumptions? How can those be justified?
- If any, why are your approximations/simplifications appropriated?

Depending on the precise objective, the optimization problem which is to be solved may take on slightly different forms.

(b) *(3 pts.)* Turn your formalization into an optimization problem in standard form.

(c) *(6 pts.)* Implement your approach by writing a Python-script that finds an optimal solution to the problem.
Hint: You may speed up your implementation, if you make use of Linear Programming, Local Optimization or Markov approaches.

(d) *(10 pts.)* Discuss your results: Do you obtain a good solutions? What are strengths and weaknesses of your approach and your solution? What problems did you encounter? What caused those problems and how did you solve them? Analyze the obtained method with respect to the parameters; are your results stable under minimal changes of the input data or the initial state? How many resources were needed to obtain that result, i.e. computation time, memory, etc.

2 Review

(*9 points*)

Provide a review of the solution presented to you, pay particular attention to the following points:

(a) Please summarize the main idea of the approach in your own words (1-2 sentences or paragraphs).

(b) Describe the strengths and weaknesses of the work, with respect to the following criteria: soundness of the claims (theoretical grounding, empirical evaluation), and clarity of writing.

(c) Do the authors explicitly and clearly state the main assumptions and limitations of their approach? Are there hidden assumptions?

(d) Is the approach technically sound? The claims and conclusions are supported by flawless arguments. Proofs are correct, formulas are correct. Are the assumptions empirically checked and validated?

(e) Are the experiments well designed, sufficient, clearly described?

(f) Is the work well-organized and clearly written, should there be additional explanations or illustrations?