total grade: 5.4/6.0

[0.7/0.7]

```r
# Given sample
sample_data <- c(5, 1, 2, 3, 1, 2, 1, 1, 2, 2, 3, 2, 1, 1, 4, 4, 3, 2, 4, 4)
n <- length(sample_data)

# Calculating MLE
lambda_hat <- mean(sample_data)

# Number of bootstrap samples
num_bootstrap <- 10000

# Generating bootstrap samples
bootstrap_samples <- replicate(num_bootstrap, mean(sample(sample_data, replace = TRUE)))

# Calculating bootstrap statistics (MLE of lambda)
bootstrap_statistics <- bootstrap_samples

# Constructing 95% confidence interval by leaving 2.5% in each tail
lower_bound <- quantile(bootstrap_statistics, 0.025)
upper_bound <- quantile(bootstrap_statistics, 0.975)

cat("MLE of lambda:", lambda_hat, "\n")
```

✓

```
## MLE of lambda: 2.4
```

```r
cat("95% Confidence Interval for lambda:", lower_bound, "to", upper_bound, "\n")
```

```
## 95% Confidence Interval for lambda: 1.85 to 2.95
```

Date
Page

[0.7/0.8]

Q.6

a] $x_1, x_2, \ldots, x_n \overset{iid}{\sim} N(\mu, 1)$ be random sample

We know MLE of μ is

$$\hat{\mu}_n = \frac{1}{n}\sum_{i=1}^{n} x_i$$

The CDF of Normal distribution for $g(\mu)$ will be,

$$g(\mu) = P[x_i \leq 2] = \Phi\left(\frac{x-\mu}{\sigma^2}\right)$$ ... Φ is standard normal CDF

$$= \left(\frac{2-\mu}{1}\right)$$ ✓

$$U_n = \hat{g}(\hat{\mu}) = \Phi\left(\frac{2-\hat{\mu}}{1}\right)$$ ✓ ... μ replaced with its 'MLE'.

b] Using Delta method find the Asymptotic Distribution of U_n :-

If $g(x)$ is function of random variable x, then asymptotic variance of $g(\hat{\theta})$ ($\hat{\theta}$ is MLE) given by:-

$$Var(g(\hat{\theta})) \approx (\nabla g(\mu))^2 \, Var(\hat{\theta})$$

In this case $g(\mu) = \Phi\left(\frac{2-\mu}{1}\right)$ & $\hat{\theta}$ is $\hat{\mu}$

Talking Derivative of $g(\mu)$ with respecte to μ :-

$$\nabla g(\mu) = -\phi\left(\frac{2-\mu}{1}\right)$$ ✓ ... ϕ is standard normal PDF.

The asymptotic variance of $\hat{g}(\hat{\mu})$ is :-

$$Var(\hat{g}(\hat{\mu})) \approx \left(-\phi\left(\frac{2-\hat{\mu}}{1}\right)\right)^2 Var(\hat{\mu})$$

$\therefore$ The asymptotic distribution of U_n is normal with mean: $\Phi\left(\frac{2-\mu}{1}\right)$ & variance given by above expression.

R Notebook

Code

Q. 2 [0.7/0.7]

Hide

```
library(boot)

#vector of excess returns stored as acme$acme
acme_data <- acme$acme

#Median
median_func <- function(data, indices) {
  median(data[indices])
}

set.seed(123)

#Bootstrap resampling
boot_results <- boot(data = acme_data, statistic = median_func, R = 10000)

#Bootstrap confidence interval
bootstrap_ci <- boot.ci(boot_results, type = "basic")

#Confidence interval
print(bootstrap_ci)
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 10000 bootstrap replicates

CALL :
boot.ci(boot.out = boot_results, type = "basic")

Intervals :
Level      Basic
95%   (-0.1095, -0.0651 )
Calculations and Intervals on Original Scale
```

Hide

```
#Checking for 0
contains_zero <- bootstrap_ci$basic[4] <= 0 && bootstrap_ci$basic[5] >= 0
print(paste("Does the interval contain zero?", contains_zero))
```

```
[1] "Does the interval contain zero? FALSE"
```

Q. 3 [1.6/1.6]

Hide

```
set.seed(1234)

# Given data
power <- c(198, 184, 245, 223, 263, 246, 206, 216, 191, 237, 208, 244, 221, 209, 256, 276, 22
6, 208, 198, 207)
temp <- c(30, 25, 37, 38, 27, 36, 33, 29, 26, 34, 24, 35, 37, 28, 37, 36, 33, 31, 26, 34)

#Correlation coefficient
cor_func <- function(data, indices) {
  cor(data[indices, "Power"], data[indices, "Temp"])
}


data_df <- data.frame(Power = power, Temp = temp)

#Bootstrap resampling
boot_results <- boot(data = data_df, statistic = cor_func, R = 5000)

#Bonfidence intervals
bootstrap_ci_percentile <- boot.ci(boot_results, type = "perc")
bootstrap_ci_normal <- boot.ci(boot_results, type = "norm")


print("Bootstrap Confidence Interval (Percentile):")
```

```
[1] "Bootstrap Confidence Interval (Percentile):"
```

Hide

```
print(bootstrap_ci_percentile)
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 5000 bootstrap replicates

CALL :
boot.ci(boot.out = boot_results, type = "perc")

Intervals :
Level     Percentile
95%   ( 0.1959,  0.8578 )
Calculations and Intervals on Original Scale
```

Hide

```
cat("\n")
```

Hide

```
print("Bootstrap Confidence Interval (Normal):")
```

```
[1] "Bootstrap Confidence Interval (Normal):"
```

Hide

```
print(bootstrap_ci_normal)
```

```
BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 5000 bootstrap replicates

CALL :
boot.ci(boot.out = boot_results, type = "norm")

Intervals :
Level      Normal
95%   ( 0.2334,  0.9283 )
Calculations and Intervals on Original Scale
```

5 - (a)

[0.6/1.0]

Hide

```
#Posterior distribution
calculate_posterior <- function(pi, x, n) {
  likelihood <- choose(n, x) * pi^x * (1 - pi)^(n - x)
  prior <- 1  # Flat (uniform) prior
  posterior <- likelihood * prior
  return(posterior)
}

# Values
x <- 12  # Number of heads
n <- 20  # Total number of tosses

#Generating a sequence of values for pi
pi_values <- seq(0, 1, by = 0.01)

#Posterior for each value of pi
posterior_values <- sapply(pi_values, function(pi) calculate_posterior(pi, x, n))

#Normalizing
posterior_values <- posterior_values / sum(posterior_values)

par(mfrow=c(2,1))  # Set up a 2-row grid for plots

#Prior distribution
plot(pi_values, rep(1, length(pi_values)), type='l', col='blue', lwd=2,
     main='Prior Distribution', xlab='π', ylab='Density', ylim=c(0, 1))

#Posterior distribution
plot(pi_values, posterior_values, type='l', col='red', lwd=2,
     main='Posterior Distribution', xlab='π', ylab='Density', ylim=c(0, max(posterior_value
s)))
```

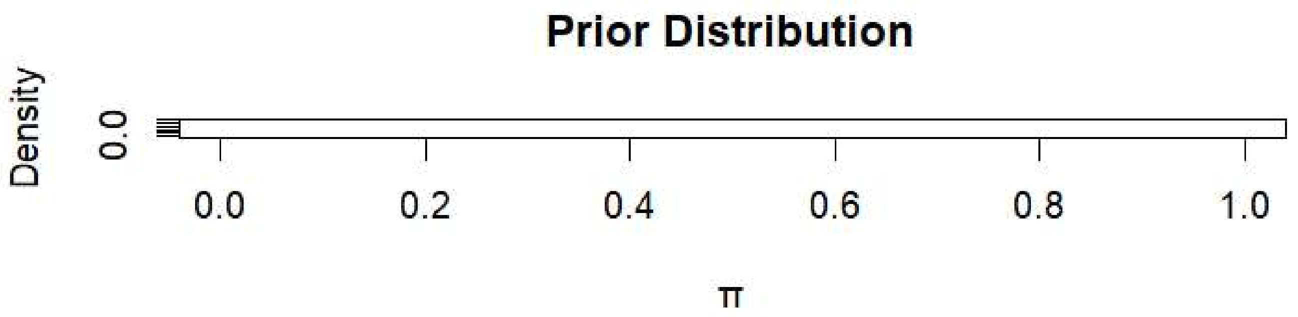
The code looks correct.
but
in this example, you can find the posterior analytically

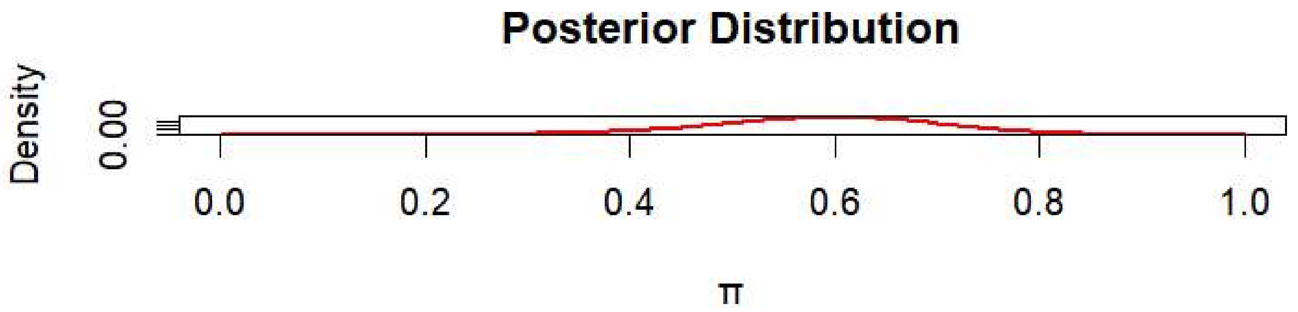
Do we have conjugate prior distributions here?

✓

Hide

```
par(mfrow=c(1,1))
```





one cannot get any idea from these plots!

Q4) a) Let $x_1, x_2, x_3 \ldots, x_n$.

Prior distribution for λ - Gamma (α, β)

[1.1/1.2]

$$\pi(\lambda | x_1, x_2, \ldots, x_n) \propto f(x_1, x_2, \ldots x_n | \lambda) \cdot \pi(\lambda)$$

$$f(x_1, x_2, x_3, \ldots x_n | \lambda) = \frac{e^{-n\lambda} \lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i !}$$

$$\pi(\lambda) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \lambda^{\alpha-1} e^{-\beta\lambda}$$ ✓

$$\text{Posterior} \propto e^{-n\lambda} \lambda^{\sum_{i=1}^{n} x_i} \cdot \lambda^{\alpha-1} e^{-\beta\lambda}$$

$$\propto e^{-(n+\beta)\lambda} \lambda^{\alpha + \sum_{i=1}^{n} x_i - 1}$$ ✓

$$\text{Gamma}\left(\alpha + \sum_{i=1}^{n} x_i, \; n+\beta\right)$$ ✓

b) The Poisson parameter λ is a Gamma distribution with α and β. and the likelihood is a Paison distribution the posterior distribution for λ, The Gamma distribution is a conjuagate prior. ✓

c) $$\lambda_n = \int_0^{\alpha} \lambda \cdot \pi(\lambda | x_1, x_2 \ldots, x_n) \, d\lambda$$

Gamma distribution with parameters $\alpha + \sum_{i=1}^{n} x_i$, $n+\beta$

$$\lambda_n = \int_0^{\alpha} \lambda \cdot \text{Gamma}\left(\lambda \mid \alpha + \sum_{i=1}^{n} x_i, \; n+\beta\right) d\lambda$$

$$\text{Posterior mean} = \frac{\alpha}{\beta}$$

No :

Date: / /

Therefore $\hat{\lambda}_n = \dfrac{\alpha + \sum_{i=1}^{n} x_i}{n+\beta}$ ✓

5) b) $(X) = 12$

$n = 20$

$\hat{\pi}_{ML} = \dfrac{X}{n} = \dfrac{12}{20} = 0.6$ ✓

$\hat{\mu}_{sam} = n\hat{\pi}_{MoM}$

$12 = 20 \cdot \hat{\pi}_{MoM}$

$\hat{\pi}_{mom} = \dfrac{12}{20} = 0.6$ ✓

$(\alpha = \beta = 1$ non informative uniform dist$)$ ✓

$\hat{\pi}_{MAP} = \dfrac{X + \alpha - 1}{n + \alpha + \beta - 2} = \dfrac{12 + \cancel{1} - \cancel{1}}{20 + \cancel{1} + \cancel{1} - \cancel{2}} = 0.6$

where does it come from???

~~MAP~~ here $\hat{\pi}_{ML} = \hat{\pi}_{MAP} = \hat{\pi}_{MoM}$

c) The posterior distribution for π is Beta $(\alpha_{post}, \beta_{post})$

$\alpha_{post} = X + \alpha_{prior}$

??

$\beta_{post} = \cancel{\beta} \; n - X + \beta_{prior}$

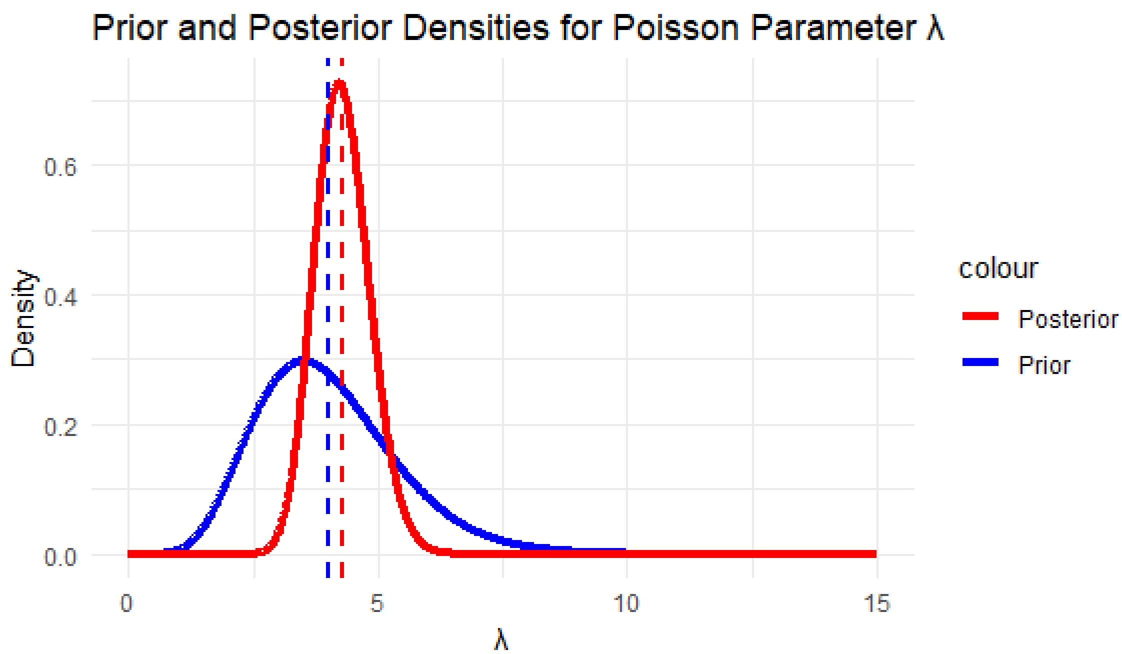
$\alpha_{post} = 12 + 1 = 13$

$\beta_{post} = 20 - 12 + 1 = 9$

$P(\pi_L < \pi < \pi_u \mid X) = 0.9$

? ?

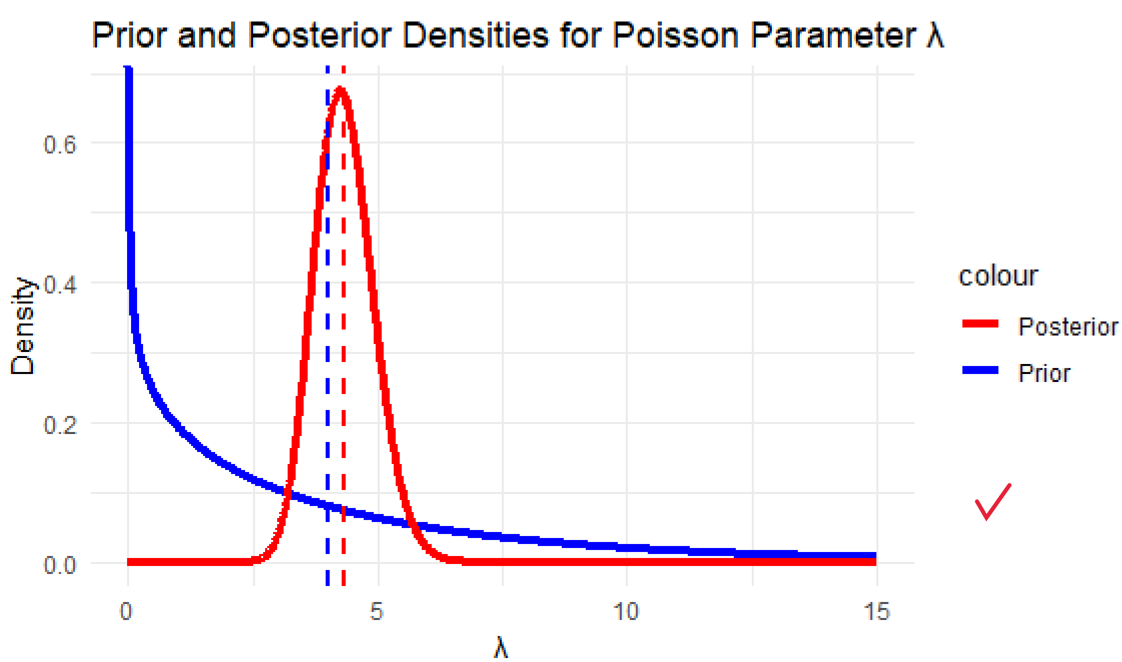
4 d)



✓

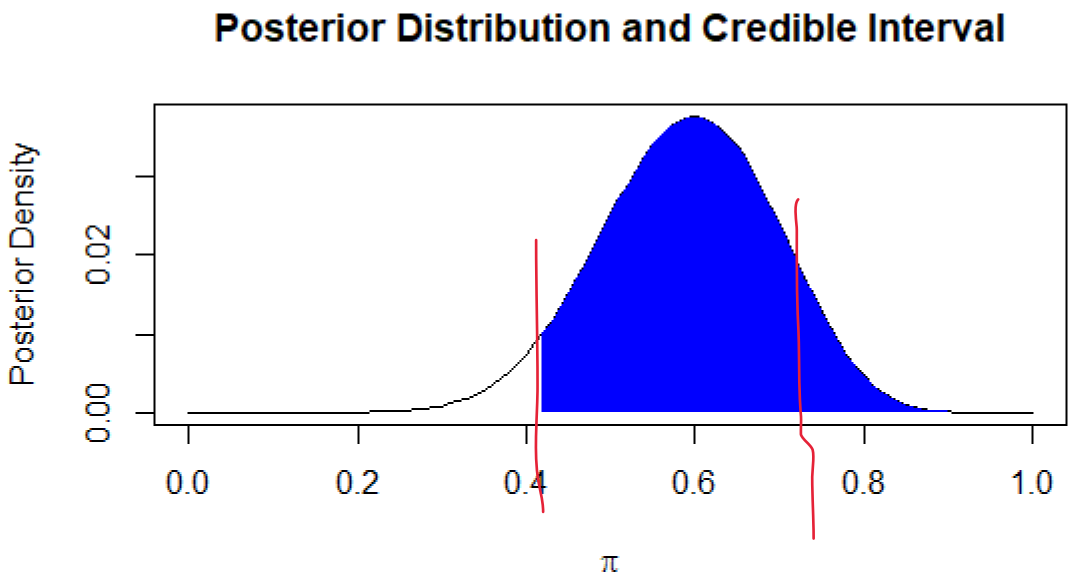
e)

cannot find the code?



✓

5 c)



Posterior Distribution and Credible Interval
Posterior Density
0.00
0.02
0.0
0.2
0.4
0.6
0.8
1.0
π