1 MATRICES

1. Find the value of k if $M = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - kM - I_2 = 0$

2. Using properties of determinants, prove that:

$$\begin{bmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{bmatrix} = (1+a^2+b^2)^3$$

3. Given two matrices A and B:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 1 \\ 1 & -3 & 2 \end{bmatrix} and \mathrm{B} = \begin{bmatrix} 11 & -5 & -14 \\ -1 & -1 & 2 \\ -7 & 1 & 6 \end{bmatrix}$$

Find AB and use this result to solve the following system of equations:
$x - 2y + 3z = 6, x + 4y + z = 12, x - 3y + 2z = 1$

2 CIRCLES

1. Find the equation of an ellipse whose latus rectum is 8 and eccentricity is $\frac{1}{3}$

2. Find the equation of the hyperbola whose foci are$(0, \pm\sqrt{10})$ and passing through the point $(2, 3)$.

3. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius $10cm$ is a square of side $10\sqrt{2}cm$.

4. Find the smaller area enclosed by the circle x^2+y^2 and the line $x+y = 2$.

3 TRIGONOMETRY

1. Solve: $\cos^{-1}(\sin\cos^{-1}x) = \frac{\pi}{6}$

4 INTEGRATION AND DIFFERENTIATION

1. Evaluate:

   $\int \frac{2y^2}{y^2+4} dy$

2. Evaluate: $\int_0^3 f(x), dx$, where $\quad f(x) = \begin{cases} \cos 2x & 0 \leq x \leq \frac{\pi}{2}, \\ 3, & \frac{\pi}{2} \leq x \leq 3. \end{cases}$

3. If $y = e^{m \cos^{-1} x}$, prove that:
   $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$

4. Evaluate:
   $\int \frac{\sec x}{1+\csc x} dx$

5. Solve the differential equation:

   $\frac{x}{e^y}\left(1 - \frac{x}{y}\right) + \left(1 + \frac{x}{e^y}\right)\frac{dx}{dy} = 0$ when $x = 0$, $y = 1$.

6. Solve the differential equation:
   $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

5 VECTOR GRAPHS

1. Using vectors, prove that angle in a semicircle is a right angle.

2. Find the volume of a parallelepiped whose edges are represented by the vectors: $\vec{a} = 2\hat{i} - 3\hat{j} - 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$.

3. Find the equation of the plane passing through the intersection of the planes: $x + y + z + 1 = 0$ and $2x - 3y + 5z - 2 = 0$ and the point $(-1, 2, 1)$.

4. Find the shortest distance between the line: $\vec{r} = \hat{i} + 2t\hat{j} + 3t\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = 2\hat{i} + 4t\hat{j} + 5t\hat{k} + \mu(4\hat{i} + 6\hat{j} + 8\hat{k})$

6 PROBABILITY

1. A card is drawn from a well shuffled pack of playing cards.What is the probability that it is either a spade or an ace or both?

2. An urn contains 2 white and 2 black balls. A ball is drawn at random. If it is white, it is not replaced into the urn.Otherwise, it is replaced with another ball of the same colour. The process is repeated. Find the probability that the third ball drawn is black.

3. Three persons A, B and C shoot to hit a target. If A hits the target four times in five trials, B hits it three times in four trials, and C hits it two times in three trials, find the probability that:

   i) Exactly two persons hit the target.
   ii) At least two persons hit the target.
   iii) None hit the target.

4. $BoxI$ contains two white and three black balls. $BoxII$ contains four white and one black balls, and $BoxIII$contains three white and four black balls. A dice having three red, two yellow and one green face, is thrown to select the box. If red face turns up,we pick up $BoxI$, if a yellow face turns up we pick $BoxII$, oth erwise we pick up $BoxIII$. Then, we draw a ball from the selected box. If the ball drawn is white, what is the probability that the dice had turned up with a red face?

5. Six dice are thrown simultaneously. If the occurrence of an odd number on a single dice is considered a success, find the probability of maximum three successes.

7 CORRELATION

1. In a contest the competitors are awarded marks out of 20 by two judges. The scores of the 10 competitors are given below. Calculate Spearman's rank correlation.

Competitors	A	B	C	D	E	F	G	H	I	J
Judge A	2	11	11	18	6	5	8	16	13	15
Judge B	6	11	11	9	14	20	4	3	13	17

2. The two lines of regressions are $4x + 2y - 3 = 0$ and $3x + 6y + 5 = 0$. Find the correlation co-efficient between x and y.

8 LINEAR EQUATIONS

1. If $1, \omega \quad and \omega^2$ are the cube roots of unity, prove that : $\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = \omega^2$

2. Solve the equation for x: $\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2}, \quad x \neq 0$

9 ALGEBRA

1. A, B, and C represent switches in 'on' position and A', B', and C' represent them in 'off' position. Construct a switching circuit representing the polynomial $ABC + AB'C' + A'B'C$. Using Boolean Algebra, prove that the given polynomial can be simplified to $C(A + B')$. Construct an equivalent switching circuit.

2. If $z = x + iy$,$w = \frac{2-iz}{2z-i}$, and $|\omega| = 1$, find the locus of z and illustrate it in the Argand Plane.

10 FUNCTIONS

1. Verify Lagrange's Mean Value Theorem for the following function:
   $f(x) = 2\sin x + \sin 2x \quad \text{on} \quad [0, \pi]$

11 REGRESSION

1. Given that the observations are:
   $(9, -4), (10, -3), (11, -1), (12, 0), (13, 1), (14, 3), (15, 5), (16, 8)$.
   Find the two lines of regression and estimate the value of y when $x = 13.5$.

12 COST AND INTEREST

1. Mr. Nirav borrowed ₹$50,000$ from the bank for 5 years. The rate of interest is 9% per annum compounded monthly. Find the payment he

makes monthly if he pays back at the beginning of each month.

2. A bill for ₹$7,650$ was drawn on $8^{\text{th}} March, 2013, at 7 months$. It was discounted on $18^{\text{th}} May, 2013$ and the holder of the bill received ₹$7,497$. What is the rate of interest charged by the bank?

3. The average cost function, AC for a commodity is given by $AC = x + 5 + \frac{36}{x}$ in terms of output x. Find:

   i The total cost, C and marginal cost, MC as a function of x.

   ii The outputs for which AC increases.

13 MIXTURES

1. A dietician wishes to mix two kinds of food $X$and Y in such away that the mixture contains at least $10 units$ of vitamin A, $12 units$ of vitamin B and $8 units$ of vitamin C. The vitamin contents of one kg of food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1 unit	2 units	3 units
Y	2 units	2 units	1 unit

14 PROFIT AND LOSS

1. Calculate the index number for the year 2014, with 2010 as the base year by the weighted aggregate method from the following data:

Commodity	Price in 2010(|)	Price in 2014(|)	Weight
A	2	4	8
B	5	6	10
C	4	5	14
D	2	2	19

2. The quarterly profits of a small scale industry (in thousands of rupees) are as follows:

Year	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2012	39	47	20	56
2013	68	59	66	72
2014	88	60	60	67

Calculate four quarterly moving averages. Display these and the original figures graphically on the same graph sheet.

15 LIMITS

1. Using L'Hospital's rule, evaluate: $\lim_{x\to 0} \frac{x-\sin x}{x^2 \sin x}$