ASSIGNMENT-1

August 31, 2024

Questions

1 VECTOR GRAPHS

1. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides linesegment PQ in the ratio $3:1$ and S is the mid-point of line segment PR. The position vector of S is :

   (a) $\frac{\vec{p}+3\vec{q}}{4}$

   (b) $\frac{\vec{p}+3\vec{q}}{8}$

   (c) $\frac{5\vec{p}+3\vec{q}}{4}$

   (d) $\frac{5\vec{p}+3\vec{q}}{8}$

2. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of $Y-axis$ is :

   (a) $\frac{5\pi}{6}$

   (b) $\frac{3\pi}{4}$

   (c) $\frac{5\pi}{4}$

   (d) $\frac{7\pi}{4}$

3. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line
   $\vec{r} = (2+\lambda)\,\hat{i} + \lambda\hat{j} + (2\lambda - 1)\,\hat{k}$ is

   (a) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$

   (b) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$

   (c) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$

   (d) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

2 INTEGRALS

1. $\int \frac{1}{x(\log x)^2} dx$ is equal to :

    (a) $2 \log (\log x) + c$

    (b) $-\frac{1}{\log x} + c$

    (c) $\frac{(\log x)^3}{3} + c$

    (d) $\frac{3}{(\log x)^3} + c$

2. The value of $\int_{-1}^{1} x|x|dx$ is :

    (a) $\frac{1}{6}$

    (b) $\frac{1}{3}$

    (c) $-\frac{1}{6}$

    (d) 0

3 AREA AND CURVES

1. Area of the region bounded by curve $y^2 = 4x$ and the $X-axis$ between $x = 0$ and $x = 1$ is :

    (a) $\frac{2}{3}$

    (b) $\frac{8}{3}$

    (c) 3

    (d) $\frac{4}{3}$

2. Given a curve $y = 7x - x^3$ and x increases at the rate of $2 units per second$. The rate at which the slope of the curve is changing, when $x = 5$ is :

    (a) $-60 units/sec$

    (b) $60 units/sec$

    (c) $-70 units/sec$

    (d) $-140 units/sec$

4 MATRIX

1. If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew symmetric-matrix, then the value of $2a - (b + c)$ is :

    (a) 0

(b) 1

(c) −10

(d) 10

2. If A is a square matrix of order 3 such that the value of $|adj.A|= 8$,then the value of $|A^T|$ is :

(a) $\sqrt{2}$

(b) $-\sqrt{2}$

(c) 8

(d) $2\sqrt{2}$

3. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :

(a) −4

(b) 1

(c) 3

(d) 4

4. If $\begin{bmatrix} x & 2 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is :

(a) −1

(b) 0

(c) 1

(d) 2

5. Find the matrix A^2 , where $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is a 2×2 matrix whose elements are given by $a_{ij} = $ maximum (i, j) - minimum (i, j) :

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

5 PROBABILITY

1. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :

   (a) $A \subset B, but A \neq B$
   (b) $A = B$
   (c) $A \cap B = \phi$
   (d) $P(A) = P(B)$

6 FUNCTIONS

1. A function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :

   (a) injective but not surjective
   (b) surjective but not injective
   (c) both injective and surjective
   (d) neither injective nor surjective

2. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to:

   (a) 2
   (b) 1
   (c) 0
   (d) -2

7 DIFFERENTIAL EQUATIONS

1. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :

   (a) -1
   (b) 1
   (c) $-e$
   (d) $-\frac{1}{e}$

2. Derivative of $e^{sin^2 x}$ with respect to $\cos x$ is :

   (a) $\sin x e^{\sin^2 x}$
   (b) $\cos x e^{\sin^2 x}$
   (c) $-2 \cos x e^{\sin^2 x}$
   (d) $-2 \sin^2 x cosx e^{\sin^2 x}$

3. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left[\frac{d^2y}{dx^2}\right] = 5$ is

   (a) 4

   (b) 3

   (c) 2

   (d) not defined

8   ASSERTION

1. Assertion-Reason Based Questions

   Direction: In questions numbers 19 and 20, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the following options:

      (a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

      (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).

      (c) Assertion (A) is true, but Reason (R) is false.

      (d) Assertion (A) is false, but Reason (R) is true.

   Assertion (A): Domain of $y = \cos^{-1}(x)$ is $\begin{bmatrix} -1, & 1 \end{bmatrix}$.

   Reason (R) : The range of the principal value branch of $y = \cos^{-1}(x)$ is $\begin{bmatrix} 0, & \pi \end{bmatrix} - \left\{\frac{\pi}{2}\right\}$.