Final report
Stellar Astrophysics

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Contents

Abstract

The following report consists of a brief overview of the structure of stars, their evolution and formation. References used are "An Overview of Stellar Evolution", [1] and an article on star formation, [2]

1 Preliminaries

The period of inactivity of stars is very long because stars are highly stable objects. The self gravitational forces are balanced by steep internal pressure gradients caused due to high temperatures.

1.1 Hydrostatic Equilibrium

Let us consider a spherically symmetric, non rotating and a non magnetic star on which the net force is zero, implying no net accelerations. There may be internal motions, like those due to convection but we assume them to average out overall.We also assume that the stellar material is so constituted that internal stresses are isotropic.

The following are a few defined symbols and quantities that will be used throughout this text.

r : radial distance measured from the stellar centre (cm)

R : total stellar radius

$\rho(r)$: mass density at r (g cm^{-3})

$T(r)$: temperature at r (deg K)

$P(r)$: pressure at r (dyne cm^{-2})

M_r : mass of a sphere contained within radius r (g)

M : total stellar mass $M = M_R$

L_r : luminosity, rate of energy flow through a sphere of radius r (erg s^{-1})

L : total stellar luminosity $L = L_R$

$g(r)$: local acceleration due to gravity (cm s^{-2})

$G = 6.6726 \times 10^{-8} g^{-1} cm^3 s^{-2}$

For a spherical shell of infinitesimal thickness dr at r,

$$M_{r+dr} - M_r = dM_r = 4\pi r^2 \rho(r) dr \tag{1}$$

Mass within r

$$M_r = \int_0^r 4\pi r^2 \rho(r)\, dr \tag{2}$$

This is known as the *equation of mass conservation or mass equation.*

Consider a volume element of 1 cm^2 $\times$ dr at r,

Then the inward gravitational force,

$$\rho g dr = \rho \frac{GM_r}{r^2} dr \tag{3}$$

$P(r)$ pressure pushing against the inner shell must be greater than the pressure acting inward on the outer face. Net pressure outward

$$P(r) - P(r + dr) = -\frac{dP}{dr} dr. \tag{4}$$

On adding gravitational forces,

$$\rho \ddot{r} = -\frac{dP}{dr} - \frac{GM_r}{r^2} \rho \tag{5}$$

where $\ddot{r}$ is the acceleration

$$\ddot{r} = \frac{d^2 r}{dt^2}$$

Since we considered the $\ddot{r} = 0$, we obtain the *equation of hydrostatic equilibrium*

$$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2} = g\rho \tag{6}$$

This shows that pressure must decrease everywhere.

1.2 Energy Principle

Now, we see the global view where equilibrium is posed as an integral constraint on the entire star.

Total gravitation potential, Ω of a self gravitating body is defined as negative of total energy required to disperse all elements of the body to infinity.

Ω is the energy required to assemble the star. (represents the negative work done).

We can get to the dispersed state by successively peeling off spherical shells. Considering that we have already done so to a layer of $Mr + dr$, and we are removing the next shell of M_r.

Moving this shell from r' to $r' + dr'$ requires an energy of $\frac{GM_r}{r'^2} dM_r dr'$

$$d\Omega = -\int_r^\infty \frac{GM_r}{r'^2} dM_r \, dr' = -\frac{GM_r}{r} dM_r \tag{7}$$

$$\begin{aligned} \Omega &= -\int_0^M \frac{GM_r}{r} dM_r \\ &= -q\frac{GM^2}{R} \end{aligned} \tag{8}$$

where q is some constant. For a sphere of uniform density, q = 3/5.

If we neglect gross mass motions such as turbulence, then, total energy of star is the sum of Ω and the total internal energy arising from microscopic processes.
Let E be local specific internal energy

$$W = \int_M E dM_r + \Omega = U + \Omega \tag{9}$$

where total internal energy is

$$U = \int_M E dM_r$$

The equilibrium state of the star corresponds to a stationary point with respect to W. This implies that W for a star in hydrostatic equilibrium is an extremum.

To test this idea, we change the state of the system adiabatically in an infinitesimal fashion. If arbitrary, but small adiabatic changes result in no change in W, then the initial state is in hydrostatic equilibrium.

Thus, the stellar hydrostatic equilibrium state is that for which

$$(\delta W)_{ad} = 0$$

To show this, we have to look how U and Ω change when ρ, T, etc., are varied adiabatically.

$$(\delta W)_{ad} = (\delta u)_{ad} + (\delta \Omega)_{ad}$$

$$U + \delta U = U + \delta \int_M E\, dM_r = U + \int_M \delta E\, dM_r$$

For an infinitesimal and reversible change, the combined first and second laws of thermodynamics state that

$$dQ = dE + PdV_\rho = TdS$$

Here dQ is the heat added to the system, dE is the increase in internal specific energy, and PdV_ρ is the work done by the system on its surroundings if the "volume" changes by dV_ρ . This volume is the specific volume, with

$$V_\rho = \frac{1}{\rho}$$

and is that associated with a given gram of material.

Since Q and S are mass related quantities, they do not change. $dS = 0$

$$(\delta Q)_{ad} = -P\delta V_\rho \tag{10}$$

$$(\delta U)_{ad} = -\int_M P\delta V_\rho\, dM_r$$

. From the definition of specific volume, we have

$$v_\rho = \frac{1}{\rho} = \frac{4\pi r^2 dr}{dM_r} = \frac{d(4\pi r^3/3}{dM_r} \tag{11}$$

For simplicity, we consider spherical symmetry.

$$V_\rho + \delta V_\rho \; = \; \frac{d(4\pi(r+\delta r)^3/3}{dM_r} \; = \; V_\rho + \frac{d(4\pi r^2 dr)}{dM_r} \tag{12}$$

The variation in internal energy is then,

$$(\delta U)_{ad} = -\int_M P \frac{d(4\pi r^2 dr)}{dM_r} dM_r \tag{13}$$

We now introduce two boundary conditions. Firstly, we don't want the centre to move

$$\delta r(M_r = 0) = 0$$

The second is called the "zero boundary condition on pressure" and it requires that the pressure at the surface vanish.

$$P_s = P(M_r = M) = 0$$

Now, integrating equation 13 by parts and applying boundary conditions,

$$(\delta U)_{ad} = \int_M \frac{dP}{dM_r} 4\pi r^2 \delta M_r.$$

Using a similar analysis for $(\delta\Omega)_{ad}$ gives,

$$\Omega + \delta\Omega = -\int_M \frac{GM_r}{r+\delta r} \, dM_r = \Omega + \int_M \frac{GM_r}{r^2} \delta r \, dM_r$$

Putting it all together, we have

$$(\delta W)_{ad} = \int_M \left[\frac{dP}{dM_r} 4\pi r^2 \; + \; \frac{GM_r}{r^2} \right] \delta r \, dM_r$$

For equilibrium condition, we set this value to 0. We have,

$$\frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4} \tag{14}$$

Which is the same as what we obtained in the equation for hydrostatic equilibrium.

1.3 Virial Theorem and Applications

Virial theorem is used for making simple estimates of temperature, density and a few important stellar time scales.
Consider the scalar product $\sum \boldsymbol{p_i.r_i}$ where $\boldsymbol{p_i}$ is the vector momentum of a free particle of mass m_i located at $\boldsymbol{r_i}$ and the sum is over all the particles making the star.

$$\begin{aligned}\frac{d}{dt}\sum_i \boldsymbol{p_i.r_i} &= \frac{d}{dt}\sum m_i(\frac{dr_i}{dt})\boldsymbol{.r_i} \\ &= \frac{1}{2}\frac{d}{dt}\sum \frac{d}{dt}(m_i r_i^2) \\ &= \frac{1}{2}\frac{d^2 I}{dt^2}\end{aligned} \tag{15}$$

Derivative of original sum,

$$\frac{d}{dt}(\sum \boldsymbol{p_i.r_i}) = \sum_i \frac{\boldsymbol{dp_i}}{dt}\boldsymbol{.r_i} + \sum_i \frac{d\boldsymbol{r_i}}{dt}\boldsymbol{.p_i}$$

$$\sum_i \frac{d\boldsymbol{r_i}}{dt}\boldsymbol{.p_i} = \sum_i m_i v_i^2$$

$$\frac{d\boldsymbol{p_i}}{dt} = \boldsymbol{F_i}$$

$$\frac{1}{2}\frac{d^2 I}{dt^2} = 2K + \sum_i \boldsymbol{F_i.r_i} \tag{16}$$

$\sum_i F_i r_i$ is the mututal attraction of all the particles.

$$\boldsymbol{F_{ij} = -F_{ji}}$$

We know that

$$\boldsymbol{F_{ij}} = -\frac{Gm_i m_j}{r_{ij}^3}\boldsymbol{(r_i - r_j)} \tag{17}$$

$$\sum_i \boldsymbol{F_i.r_i} = \sum_{i,ji<j} \boldsymbol{(F_{ij}.r_i + F_{ji}.r_j)} = \sum \boldsymbol{F_{ij}.(r_i - r_j)}$$

$$\sum \boldsymbol{F_{ij}.(r_i - r_j)} = -\frac{Gm_i m_j}{r_{ij}} = virial \tag{18}$$

This is the negative of work required for dispersal to infinity

$$\text{Virial } = \Omega$$

Using equation(11) we have,

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2K + \Omega \tag{19}$$

This is known as the *Virial theorem* and is for the star as a whole.

If we had chosen a portion $r_s < R$, and volume V_s Then,

$$\frac{1}{2}\frac{d^2I}{dt^2} = 2K + \Omega - 3P_sV_s \tag{20}$$

Where P_s is the pressure at the surface.

We interpret what the energy K represents

$$2K = \sum_i m_i v_i^2 = \sum_i \boldsymbol{p_i}.\boldsymbol{v_i}$$

This measures the rate of momentum transfer and we can relate it to pressure as follows,

$$P = \frac{1}{3}\int n(\boldsymbol{p})\, \boldsymbol{p}.\boldsymbol{v}\, d^3\boldsymbol{p} \tag{21}$$

where $n(\boldsymbol{p})$ is the number density of particles with momentum $\boldsymbol{p}$

$$2K = 3\int_v P\, dV \tag{22}$$

Since $dM_r = \rho\, dV$

$$2K = 3\int_M \frac{P}{\rho}\, dM_r \tag{23}$$

Virial theorem becomes

$$\frac{1}{2}\frac{d^2I}{dt^2} = 3\int_M \frac{P}{\rho}\, dM_r + \Omega \tag{24}$$

1.3.1 Application: Global energies

Pressure and internal energy are related as

$$P = (\gamma - 1)\,\rho\,E \tag{25}$$

Since $2K = 3(\gamma - 1)\int E\,dM_r$; From (18)

$$K = \frac{3}{2}(\gamma - 1)\,U \tag{26}$$

The virial theorem now becomes

$$\frac{1}{2}\frac{d^2I}{dt^2} = 3(\gamma - 1)\,U + \Omega \tag{27}$$

If we write $W = U + \Omega$

$$\frac{1}{2}\frac{d^2I}{dt^2} = 3(\gamma - 1)\,W - (3\gamma - 4)\Omega \tag{28}$$

For Hydrostatic equilibrium,

$$\frac{1}{2}\frac{d^2I}{dt^2} = 0$$

$$\therefore\ W = \frac{3\gamma - 4}{3(\gamma - 1)}\Omega \tag{29}$$

The energy W is that which is available to do useful work. W must be less than 0. Else, the star would have enough energy to dissipate a part of itself.

1.3.2 The Kelvin-Helmholtz Time Scale

Usually, a star derives its energy from three sources: internal energy, thermonuclear fuel and gravitation contraction. If a star contracts very gradually, while maintaining sphericity and hydrostatic equilibrium, Ω and W may change.

We know,

$$W = \frac{3\gamma - 1}{3(\gamma - 1)}E$$

$$\Delta W = \frac{3\gamma - 1}{3(\gamma - 1)}\Delta E \tag{30}$$

Let $\mathcal{R}$ be the total stellar radius and $\Delta\mathcal{R}$ be the change in radius due to contraction. ($\Delta\mathcal{R} < 0$)

We assume $\gamma > \frac{4}{3}$

$$\Omega \ \propto -\frac{GM^2}{\mathcal{R}}$$

This implies that

$$\Delta W \ \propto \frac{GM^2}{\mathcal{R}^2}\Delta\mathcal{R} \tag{31}$$

$$\therefore \Delta W \ < 0 \text{ and } \Delta\Omega \ < 0$$

This shows that energy has been liberated in some form and thus the system on a whole has lost energy.

A part of it has been used to increase the internal energy of the star.

Equating equation (22) to 0, we get

$$\Delta U = -\frac{1}{3(\gamma - 1)}\Delta\Omega \tag{32}$$

$$[\Delta U > 0]$$

This is the part of the total energy used to heat up the star. We assume the rest is expended in the form of luminosity.
If $\gamma = \frac{5}{3}$,

$$\Delta U = -\Delta W = -\frac{\Delta\Omega}{2}$$

the split between the energy and time integrated luminosity is equal.

$$\Delta W = \frac{\Delta\Omega}{2} = \frac{q\,GM^2}{2\mathcal{R}^2}\Delta\mathcal{R} \tag{33}$$

$$L = -\frac{dW}{dt} = -\frac{q\,GM^2}{2\mathcal{R}^{\in}}\frac{d\mathcal{R}}{dt} \tag{34}$$

If $\mathcal{L}$ is constant, this equation defines a characteristic e-folding time for the radius decrease of

$$t_{kh} = \frac{q}{2}\frac{GM^2}{\mathcal{L}\mathcal{R}}$$

1.4 Constant Density Model

We now construct a stellar model by considering that the density is constant everywhere. If we set $\rho = \rho_c$ (ρ_c being the density at the centre), then the mass $M_r = \frac{4}{3}\pi r^3 \rho_c$

This is true upto the surface $r = R$ where $M_r = M$

$$M_r = \frac{r^3}{R^3} M$$

We know that,

$$\frac{dP}{dM_r} = -\frac{GM_r}{4\pi r^4}$$

The pressure gradient turns out to be

$$\frac{dP}{dM_r} = -\frac{GM_r}{4\pi R^4} \left(\frac{M_r}{M}\right)^{-1/3} \tag{35}$$

Taking the pressure at the boundary to be zero and integrating, we get

$$\begin{aligned} P &= P_c \, [1 - \left(\frac{M_r}{M}\right)^{2/3}] \\ &= P_c \, [1 - \left(\frac{r}{R}\right)^2] \end{aligned} \tag{36}$$

where P_c is the pressure at centre.

1.5 Energy Generation

Most stars heat up thermonuclear fuel to temperatures exceeding a million and contain it for a sufficiently long time. We now extend our notion of energy equilibrium to include energy generation and leakage.

Consider a spherically symmetric shell of mass dM_r and thickness dr.

Let ϵ be the power generated per gram. The total power generated in the shell

$$4\pi r^2 \rho \epsilon dr = \epsilon dM_r$$

To balance this power generated, we must have a net flux of energy leaving. Let this flux be $\mathcal{F}(r)$.

$$L_r = 4\pi r^2 \, \mathcal{F}(r)$$

is the total power entering the shell's inner surface.

$$L_{r+dr} = 4\pi r^2 \, \mathcal{F}(r + dr)$$

is the total power leaving the shell's outer surface.

The difference of these two terms is the net loss/gain of power.

$$L_{r+dr} - L_r = dL_r = 4\pi r^2 \rho \epsilon dr$$

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

$$\frac{dL_r}{dM_r} = \epsilon \tag{37}$$

This is known as **the energy equation**

Since we considered $\epsilon \geq 0$, L_r must be either a constant (if $\epsilon = 0$) or increase monotonically with r.
ϵ is usually a function of temperature. Ideally, ϵ should be largest in the inner stellar regions implying that L_r should increase rapidly from the centre starting from 0.
The power law expression for ϵ is of the form

$$\epsilon = \epsilon_0 \, \rho^{\lambda} \, T^{v} \tag{38}$$

where ϵ_0, λ and v depend on the conditions.
The total energy released in conversion of hydrogen to helium is approximately 6×10^{18} per gram of hydrogen consumed.

1.6 Evolutionary Lifetimes on Main Sequence

A main sequence star is a stellar body that is in hydrostatic and thermal equilibrium. Its main source of energy is conversion of hydrogen to helium in its core. About 90% of stars in the universe are main sequence stars and their mass can range from 1/10th to 200 times the mass of the sun.

Stars spend most of their active life converting hydrogen to helium. When approximately 10% of a star's hydrogen is converted to helium, the star undergoes structural transformation. This may cause a change in its radius and/or luminosity enough that it can no longer be called a main sequence star.

Thus, the main sequence lifetime depends on the rate at which fusion takes. To find this time t_{nuc}, we need to find the energy released by burning 10% of the star's available hydrogen and compare it to main sequence luminosity.

$$t_{nuc} \approx \frac{0.1 \times 0.7 \times M \times 6 \times 10^8}{L} \tag{39}$$

The factor 0.7 is the typical value of the hydrogen mass fraction X.

The main sequence if the sun is expected to be around 10^{10} years. More massive stars have shorter lifetimes since their rate of fuel consumption is higher to maintain their high luminosities.

2 Stellar Evolution

2.1 Young Stellar Objects

A Young Stellar Object (YSO) denotes a star in its early stages of evolution. These are mainly of two groups: protostars and pre-main sequence stars.

A protostar becomes a star when the energy released by thermonuclear fusion exceeds the energy released due to its gravitational contraction. The young stars are classified into three classes, 0, 1 and 2 based on observational qualities like ratio of infrared to visible light, amount of molecular gas and how it moves, etc.

The class 0 stars are still contracting and have short time scales. Class 1 and 2 stars are already using their nuclear energy and blowing material off their surfaces in bipolar outflows.

Figure 1: Bipolar outflow

Young Stellar Objects are characterized by variability in visible light, emission lines observed in theirs spectra. They also have more infrared luminosity and X-ray emission from the hot corona.

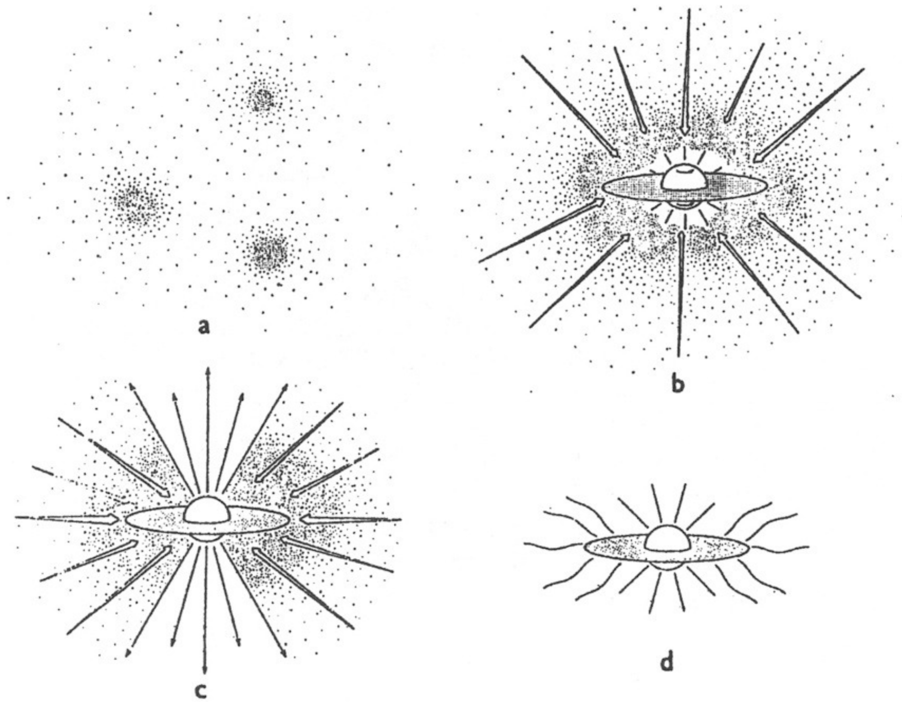

Figure 2: Four stages of Star Formation

2.2 Zero Main Age Sequence(ZAMS)

Recently formed stars are called ZAMS stars.“Zero-age” means that the star has changed so little in luminosity, radius, and T_{eff} since it first started hydrogen fusion that you cannot notice it. ZAMS is the time when the star first joins the main sequence by burning hydrogen in its core.

2.3 Main Sequence

All stars spend most of their active lives on the main sequence. The important property of a star is its mass. The mass of a star determines the temperature, luminosity and size. The source of energy in the main sequence is fusion of hydrogen to helium by two processes, proton-proton chain (pp-chain) and the CN or CNO cycle.

For stars with mass less than 1.5 $M_{\odot}$ the main energy source is pp-chain because the protons can get close enough. This chain starts in gas that is not hot enough for the CNO cycle.

In smaller stars, the energy is transported mostly due to convection whereas in more massive stars, radiation carries the power.

Most main sequence stars change very slowly in both interior and external structures. When four ionized hydrogen atoms fuse to one ionized helium atom, eight separate particles become only three. Since $P = nkT$ and a fixed central pressure is needed to balance gravity, the stellar core must slowly contract and heat up. This makes the nuclear reactions go faster, and the star gradually brightens. The surface temperature also goes down.

2.3.1 Brown Dwarfs

A gas mass that does not get hot enough to fuse hydrogen all the way to helium is called a brown dwarf. Their mass is usually below 0.085 $M_{\odot}$

The first certain brown dwarf, Gliese 229B, was discovered in 1995. One of the keys to its identification, was the presence of methane in its spectrum. Any "real" star is far too hot to allow methane to form in its atmosphere.

2.3.2 Leaving the main sequence

Stars with mass less than 0.3 $M_{\odot}$ keep fusing and mixing until all the hydrogen is converted to helium which takes approximately 10^{12} years.

Stars with mass more than 0.3 $M_{\odot}$ will eventually use up all the hydrogen in their centres while much remains in their outer envelopes. The star is still radiating and hence requires energy. The core cannot cool down or pressure wouldn't balance the gravity. Hence, the core contracts releasing gravitational energy to keep the star shining.

2.3.3 Early Evolution of Massive Stars

Not many massive stars are made in the course of star formation, but their high luminosities have an effect on the formation of their nearby, less massive stars.

The early stages of their evolution is not only governed by fusion of hydrogen but also how fast mass is driven from their surfaces due to radiation pressure.

2.4 Red Giant Phase

A red giant is a star which has used up the supply of hydrogen in its core and has begun fusion of hydrogen in a shell surrounding the core. They have radii 10 to 100 times that of the sun but the outer surface is of a comparatively lower temperature, giving them a reddish-orange hue.

Red-giant-branch stars have luminosities up to nearly three thousand times that of the Sun. In 5.4 billion years from now, the sun will enter the red giant phase of its evolution.

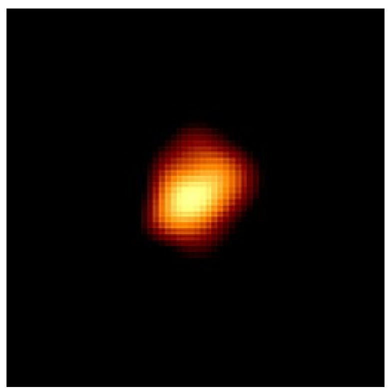

Figure 3: This is an image if Betelgeuse, in the red giant phase captured by Hubble

The fusion of hydrogen in the shell surrounding the core happens only through the CNO cycle. This is perhaps the only source of nitrogen in the universe.

Energy production in the thin, hot shell also drives some convection.

The red giant phase lasts, on average, about 10 % as long as the main sequence phase, because there is a comparable amount of hydrogen fuel available and the stars are ten times as bright.

The red giants were mainly recognised because they were much brighter than the stars of same temperature. The hydrogen burning shell works its way out through the star, so that the mass of inert helium gradually increases. Hence, both central density and central temperature increase.

2.5 Helium Flash or Fizzle

The star can now take two paths. Its central temperature can increase from about 10^7 to 10^8 or its central density can increase from its main sequence value of about $10^{2\pm1}$ to 10^6 gm cm^{-3}.

If density increases, helium gas becomes degenerate. Such gas can contract no further and can heat no further. Some envelope may leave in a wind, but the core will be left as a helium white dwarf. Stars of initial mass less than about 0.4 $M_\odot$ will meet this fate.

The universe is not old enough for any star to have met this fate. However, helium white dwarfs can be seen in binary star systems where rapid mass transfer takes place and an initial massive star has been stripped down to a helium core of mass less than 0.4 $M_\odot$.

The other path is that the central temperature reaches about 10^8 K while central density is less than 10^6 gm cm^{-3}. This happens in stars of more than about 1.5 $M_\odot$ which experience peaceful helium ignition and do not change their structure rapidly at this point in their lives.

More massive the star, the less it is affected by sequences of nuclear fusion. Stars with mass greater than 3 $M_\odot$ do not even get brighter on leaving the main sequence.

Stars with mass greater than 0.3 $M_\odot$ but less than 1.5 $M_\odot$ ignite fuel while helium is partly degenerate. The central density is around 10^6 gm cm^{-3} T_c is about 10^8 K. This is the temperature of barrier penetration which allows Helium nuclei to fuse and resulting in a nuclear explosion called "Helium Flash".

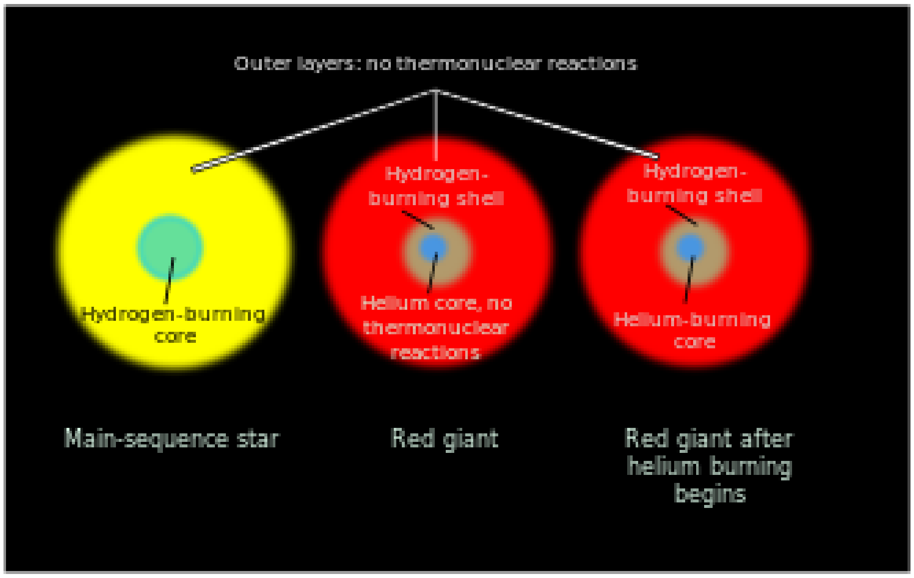

Figure 4: Helium Flash

Since the Pressure is a function of only density, the gas is degenerate and no expansion occurs due to the heating. The increased temperature results in more helium nuclei which in turn again increases the temperature and this goes on till the thermal pressure exceeds a certain "degenerate pressure". At this point the gas suddenly expands with vigor.

The central density reduces to about 10^3 - 10^4 gm cm^{-3}. The stellar core expands while the external envelope contracts. This is reflected in a rapid decrease of stellar luminosity and increase of temperature.

2.5.1 Helium Core Burning

Helium fusion can occur only when it is hot enough for two helium nuclei to move out of their mutual coulomb barrier to fuse together and dense enough for a third helium nuclei to fuse in the short time. At higher temperatures, the product ^{12}C can capture another helium nucleus to give ^{16}O.

$$3\ ^4He \longrightarrow {}^{12}C$$

In smaller stars, the yield of carbon is higher while in more massive stars. oxygen is higher. The temperatures for yielding ^{20}Ne are not appropriate and hence the fusion stops effectively with carbon and oxygen. The reaction is very slow unless temperatures are very high.

The hydrogen burning also continues in a thin shell around the helium core. The energy from hydrogen fusion is comparatively higher than that of helium fusion and this phase has a lifetime of a few percent of the main sequence lifetime of the star.

Stars that are not badly shaken up by helium flash continue to the red super giant phase and may later pool back to bluer colours. Less massive stars that were shaken up by helium ignition have colours and temperatures that are characteristic of the horizontal branch of the HR diagram.

2.5.2 Helium Core Exhaustion

There comes a point in the star's life where it has exhausted its central fuel supply and must readjust its structure, because energy is flowing outward but none is being generated by nuclear reactions at the centre. This adjustment is less drastic because the thin shell of CNO cycle continues to supply some energy.

2.5.3 Asymptotic Giant Branch

The centre of the star continues to contract and the hydrogen fusion still occurs in a thin shell around the core. When this temperature is hot enough, helium fusion occurs in this shell resulting in what is called a "double shell burning"

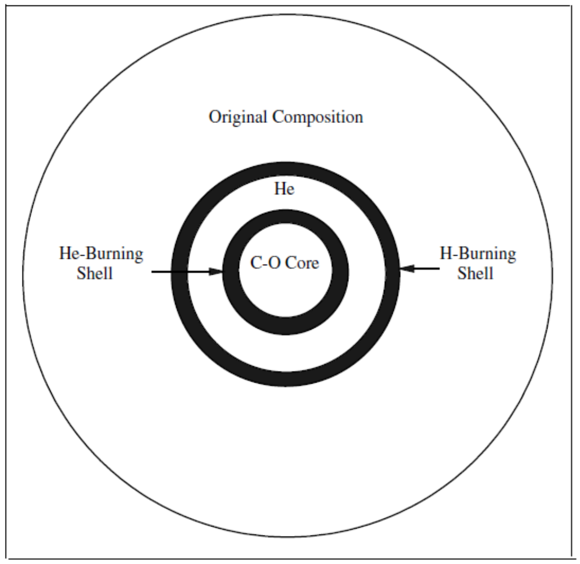

Figure 5: Double shell burning

In the case of red giants of small and moderate masses, core expansion and cooling happens along with core contraction. This phase is called the "Asymptotic Giant Branch" because in the HR diagram, these stars can be seen as an asymptote to the main Red Giant Branch.

The stars are very bright and this phase lasts less than 1% of the main sequence lifetime. Most of these stars are also unstable to pulsations which results in wind blowing off from the surface. The wind speeds are comparable to the escape velocities and the density is large enough that the star may lose 10-50% of its mass. Towards the end, the wind densities may go even higher resulting in what we call "superwind" and stars of mass 1 $M_\odot$ end up as white dwarfs of mass about 0.6 $M_\odot$.

2.6 Later Phases

(For initial masses lesser than 6-10 $M_\odot$)

When we say "initial mass", we need to consider the fact that wind has been removing envelope material for thousands of years (which is higher for more massive stars).In post AGB stars, enough material has been removed by superwind, pulsations and the last few helium flashes that the hotter layers are now exposed. The ejecta can harbor dust and OH molecular masers and the entities suffering all this are called OH/IR stars.

As more and more envelope blows off, hotter layers are uncovered. As a result, wind speeds and escape velocities increase as well. After about 10,000 years, photons are directly leaving from a layer of temperature greater than 50,000 K. These photons begin dissociating the molecules and ionizing the atoms in the ejecta. This ejecta radiates a line spectrum characterised by ordinary hydrogen lines and forbidden lines of O, N, and C, and other species. They looked like disks of Uranus and Neptune and hence were called "Planetary Nebulae".
The nebulae look green because of emission lines of oxygen while the planets look green due to methane in the atmosphere.

Figure 6: This is an image captured by Hubble of the nebula NGC 6818, also known as the Little Gem Nebula. It is located in the constellation of Sagittarius (The Archer), roughly 6,000 light-years away from us.

This ejection of mass in a planetary nebula is uneven, and they can have very complex shapes. NGC 6818 shows knotty filament-like structures and distinct layers of material, with a bright and enclosed central bubble surrounded by a larger, more diffuse cloud.

About 10,000 years later the residual core, called **Planetary Nebula Nucleus** (PNN) has cooled such that the ejecta have disperse into the general interstellar material and emits only a few ionising photons. The resulting mass remains as a white dwarf. The central temperatures reduce and the nuclear reactions also cease.

Stars of mass less than 6–10 $M_\odot$ thus end their lives as carbon/oxygen white dwarfs of masses 0.55 - 1.3 $M_\odot$, because their cores do not get hot enough for any reactions beyond helium fusion by the time they are dense enough to be degenerate.

2.7 Advanced Evolutionary Phases

(Initial masses greater than 6-10 $M_\odot$.)

The lifetime of massive stars is much lesser than smaller stars. They have more fuel to begin with, but they also use it more profligately leading to shorter lifetimes. This is not affected by their ability to fuse elements beyond hydrogen and helium.
The cores of massive stars which are fusing carbon and heavier elements are so hot that they produce a flux of neutrinos made in several different processes which do nothing to maintain stellar photon luminosity.

Dominant fuel	T_c	Duration	Important products
Carbon	5×10^8 K	10^3–10^4 yr	Ne, Na
Neon	8×10^8 K	10^2–10^3 yr	Mg, some O
Oxygen	1×10^9 K	< 1 yr	Si, some S, etc.
Silicon	3×10^9 K	days	^{56}Ni

Figure 7: Advanced Burning Phases of Massive Stars

Figure 7 shows the stages of heavy element burning. These stages may overlap. The group from Mn to Zn is called the "iron peak," because they are more abundant than those on either side, and Fe is both in the middle and most abundant.

Any fusion reactions beyond helium takes place in a very short duration. These reactions in the interior of the star go faster than the outer layers can respond creating an unstable environment.

The core of the iron peak is approaching its maximum mass that can be supported by pressure of degenerate electrons. The nuclei of atomic number 56 are tightly bound and cannot undergo any more fusion reactions and hence, the star must collapse.

2.8 Core Collapse

Core collapse can be triggered by the two processes given below.

First, photodisintegrations cool the gas, removing the support of thermal pressure. Photodisintegration is a nuclear process in which an atomic nucleus absorbs a high-energy gamma ray, enters an excited state, and immediately decays by emitting a subatomic particle. This process is endothermic for elements lighter than iron and exothermic for elements heavier than iron.

Second, the electrons are forced into higher energy states due to the increasing density until some of them have kinetic energies exceeding the neutron-proton mass difference. Protons are converted into neutrons by electron capture and with fewer electrons, degeneracy pressure drops.

The core collapses suddenly and catastrophically releasing huge amounts of energies. It takes only a few seconds for a core to change from a density of about 10^9 gm cm^{-3} to 10^{15} gm cm^{-3} which is the density of the nucleus of an atom.

The products of the core collapse are a neutron star and a supernova.

The explosion that occurs as a result of the core collapse is called a supernova. The remnants of this explosion is the extremely dense core called the neutron star.

Figure 8: A 3D illustration of a neutron star in the nebula.

3 Variable Stars

It has been shown that the light output of every star varies with time in an almost periodic fashion. The time period may be from dynamic time scale t_{dyn} to nuclear time scale t_{nuc}.

3.1 A Brief Overview

3.1.1 Eclipsing and Ellipsoidal Variables

If a pair of stars orbit each other, the observer may be located close enough to the orbit plane to have one pass in front of the other and block its light for a portion of each orbital period.
The eclipse tells us that the system is nearly edge on, eclipsing binaries are among the sorts particularly useful in measuring stellar masses.
Even in case there is no eclipse, the gravitational field of one star may distort the shape of its companion into an ellipsoid, so that you see a larger star area when the stars are side-on to you than when they are end on.

Figure 9: An illustration of a contact binary star

3.1.2 Spotted, Rotating Stars

Starspots are stellar phenomena, so-named by analogy with sunspots. Spots as small as sunspots have not been detected on other stars, as they would cause undetectably small fluctuations in brightness. The sun's brightness varies both at its rotation period and through the 11-year sunspot cycle. The variation is, however, only about 0.1% .

Larger fluctuations in brightness happen among younger, rapidly rotating stars and among close binary pairs, where the rotation period is locked to the orbit period. rapid rotation plus the convective atmosphere of these cool-surfaced stars permits the operation of a dynamo, producing a magnetic field, which, in turn, drives spot formation and other kinds of stellar activity.

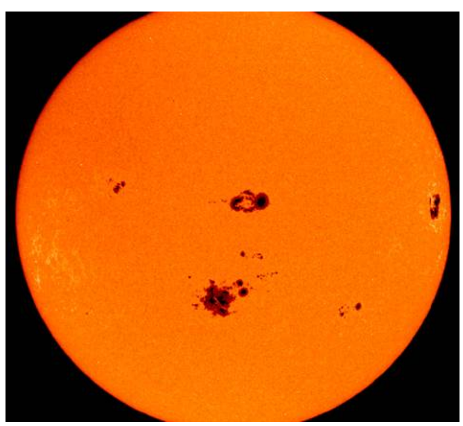

Figure 10: An image of Sun spots by NASA.

3.1.3 T Tauri Stars, FU Orionis Stars (FUORs), and Luminous Blue Variables

These are stars that are very young (pre-main sequence) or very massive and bright, or both. They are collecting material from a disk and blowing off material at their poles, and may be heavily spotted as well. Rapid rotation and magnetic fields are also part of the picture.

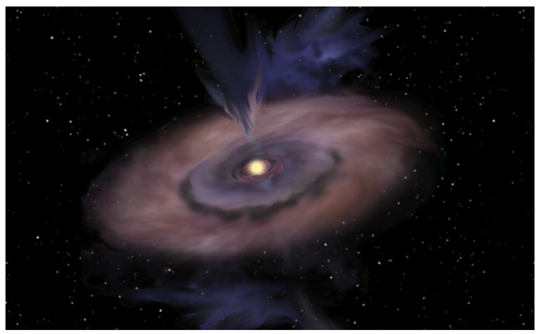

Figure 11

3.1.4 Last Helium Flash and Formation of Atmospheric Dust

These two physically different causes of variability appear together because one often happens after the other. A star that has already left the AGB phase can experience one last flash of its helium burning shell. This puffs up the envelope so that the star quickly comes to resemble a red giant again.

Highly evolved stars with carbon–rich atmospheres occasionally and unpredictably fade by many magnitudes in a few weeks and gradually recover over months. The missing light comes out as infrared, and the cause is sudden condensation of carbon dust in the cool stellar atmosphere, which then gets blown out again by radiation pressure.

3.2 Pulsational Variables

These are the most useful variable stars because the periods of time where they brighten and fade again are correlated with their absolute brightness and can be used to measure distances within Milky Way and nearby galaxies.

The period-luminosity relation can be used for measure distances to distant galaxies and so is used to measuring the speed of expansion of the universe, its age, and other things.

Stellar pulsations are caused by expansions and contractions in the outer layers as a star seeks to maintain equilibrium. These fluctuations in stellar radius cause corresponding changes in the luminosity of the star. Stellar pulsations can be purely radial or include material flopping around the latitudinal and longitudinal directions as well.

The variability of pulsational variables shows in their radial velocities as well as their light. In principle, one can integrate the velocity curves to get radius as a function of time and then, with a temperature from their colors or spectra, calculate the absolute luminosities. This is called the Baade-Wesselink method, and the results more or less agree with the results of parallax measurements and other ways of getting the brightnesses and distances of the stars concerned.

3.3 Explosive Variables

These are the stars that release a great deal of energy in a hurry. The cataclysmic variables are close star pairs with a white dwarf in orbit with a main sequence or red giant companion. The white dwarf accepts material from its companion.

One sort of variability arises when the rate of release of gravitational potential energy changes due to change in the rate of acceptance. When enough hydrogen–rich material has accumulated on the surface of the white dwarf, it fuses explosively.
Degenerate hydrogen need not even be very hot when it is as dense as a white dwarf causing spectacular supernovae.

3.3.1 Novae

A nova is an explosion from the surface of a white-dwarf star in a binary star system. A nova occurs when the white dwarf, which is the dense core of a once-normal star, "steals" gas from its nearby companion star. When enough gas builds up on the surface of the white dwarf it triggers an explosion. For a brief time, the system can shine up to a million times brighter than normal. As long as it continues to take gas from its companion star, the white dwarf can produce nova outbursts at regular intervals.

Figure 12: Artist's concept of a binary system similar to the one that originated the nova Sagittarii 2015 N.2. Credit: David A. Hardy y PPARC

Classical novae may yield upto $10^{44} - 10^{45}$ ergs upon eruption. The odd thing about novae is that pre and post novae stars appear to be identical. This suggests that the star has not suffered too much from the explosion and is waiting for enough mass to be accepted so that it can do its thing all over again.

3.3.2 Supernovae

Supernovae are the most spectacular variables of all. At maximum light, they are as bright as a whole, smallish galaxy. Recognizing them for what they are helped determine the size of milky way and existence of other galaxies. There is a sort of family resemblance among all supernovae—they get really bright in a matter of days and fade in months to years.

They blow out material of solar mass or more at large enough velocities that the remnants can be seen for thousands of years. A large galaxy experiences one to a few per century.

The supernovae are categorized into Type I and Type II based on their spectra. Type I spectra show no evidence of hydrogen while type II spectra have strong emission and absorption features due to hydrogen.In addition, Type II events always, or nearly always, occur in galaxies with recent, vigorous, star formation, while Type I events can also occur in elliptical galaxies and galactic bulges and halos.

The distinction between Type I and II supernova almost, but not exactly, corresponds to a very fundamental difference in what is going on in the two cases. Type II events (which are a commoner sort, though somewhat fainter and so harder to discover) are the products of the collapsing cores of massive stars. The basic energy source is the gravitational energy released in the collapse.

3.4 SN Remnants

The Supernova remnants we see age from 20 years to 10^4 years or more. Many of these are sources of x-rays , radio waves and visible light. Where there is a central pulsar, it continues to pour energy into the remnant until the gas is too dispersed to be seen. Pulsar-less remnants nevertheless remain bright for as long as the expanding ejecta are plowing into surrounding interstellar gas. They look brightest around the edges.

Two observable SN remnants are those of The Crab nebula and Cas A((meaning the brightest radio source in Casseiopia) which is the remnant of SN1685.

The Crab nebula is pulsar fed and recent x-ray images show energy from the pulsar being beamed out along the long axis of the prolate nebula, which is brightest at the center.
Our view of Cas A is partly blocked by dust, but the radio and x-ray images show that it is brightest around the edges, consistent with the absence of a detectable pulsar.

The gas in both of these SNRs includes lots of hydrogen, so presumably they would have been classified as Type II supernovae.

The SN remnants of Crab Nebula is shown in figure 6 and the Cas A remnants are shown below.

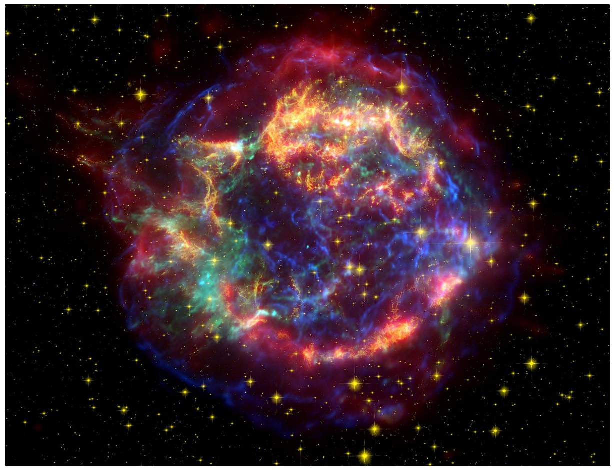

Figure 13: A false color image composed of data from three sources: Red is infrared data from the Spitzer Space Telescope, gold is visible data from the Hubble Space Telescope, and blue and green are data from the Chandra X-ray Observatory. The small, bright, baby-blue dot just off-center is the remnant of the star's core.

4 Star Formation

Star formation occurs across three distinct stages, which are completed after roughly 10 to 30 million years.

4.1 Compression of molecular clouds

A very large interstellar cloud provides the initial stage of star formation.The mass is dominantly in the form of cold atomic and molecular gas, with some dust.

Spiral arm shock waves and supersonic turbulence fragment and compress cold, giant molecular clouds (GMCs) into sheets and filaments, which contract gravitationally into dense cloud cores.

About 4 to 8 % of the Galaxy mass is in the form of an interstellar medium (ISM).The ISM is confined to the Galaxy disk by lines of magnetic flux and internal friction, and revolves at different speeds than the Galaxy spiral arm density waves.

As a result, spiral waves compress these gas clouds into dark, cold, giant molecular clouds (GMCs).

Clouds are normally supported against their own gravity and held within the galaxy disk by lines of magnetic flux but turbulence from the spiral arm shock waves and nearby supernova explosions can compress the filaments further into cloud cores with masses of $10^2 - 10^3\ M_\odot$.

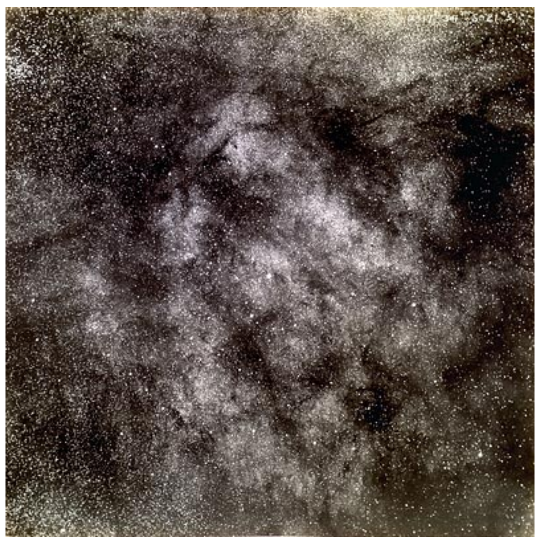

Figure 14: This shows a GMC from the early stages of milky way captured by E.E. Barnard

4.2 Formation of Protostellar Objects

At a critical density, cloud cores collapse under their own gravity to form protostellar objects (PSOs). These burn deuterium, gather mass and dissipate angular momentum inside a circumstellar accretion disk and a cloud "cocoon" of hot dust.

Each cloud core is a concentration of mass within a certain spatial radius. When this mass concentration exceeds a limit known as the Bonnor-

Ebert density, gravity overwhelms the thermal pressure and magnetic support within the cloud core and an isothermal free fall collapse occurs, forming one or more disklike, rotating protostellar objects (PSOs), each with a radius of about 1-5 AU and a mass of around 10–3 $M_{\odot}$.

The collapse removes thermal support underneath the surrounding gas, which falls toward the PSO. This "late" infalling gas forms a rotating accretion disk around the PSO with a radius up to approximately 100 AU.

When gravity contracts the PSO far enough to raise the central temperature above 2000 K, molecular hydrogen (H_2) is ionized and the protostar becomes opaque, creating a radiant surface or "photosphere". A second core collapse to around 0.3 AU occurs, deuterium fusion begins and raises core temperature to 10^6 K, and the protostar luminosity rises such that the radiation is almost in the infrared region.

Mass continues to accrete from the surrounding cloud into the circumstellar disk, and from the disk into the YSO.

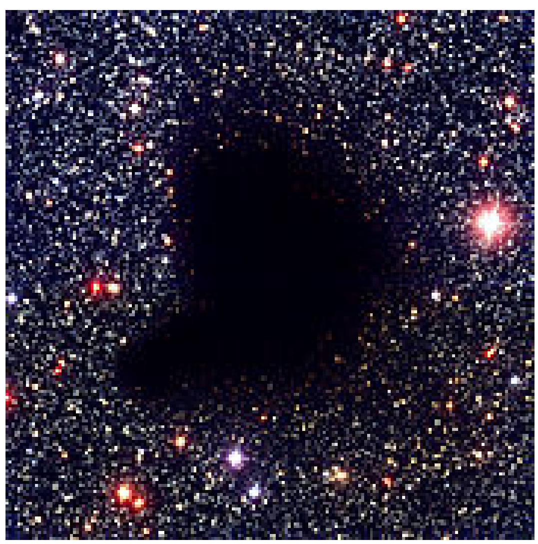

Figure 15: The cloud core visible as bok globules

4.3 Taking its Place in the Main Sequence

After roughly one million years of accretion, the PSO attains sufficient mass and density to initiate hydrogen fusion. Planets may form from the remnant disk, otherwise radiation from the new star evaporates the gas/dust "cocoon" and the star takes its place on the main sequence.

The protostar gains more than 90 % of its mass through irregular accretion events. At the same time, it dissipates roughly 99 % of the angular momentum it inherited.

The resulting enormous increase in surface temperature and UV radiation pushes back and photoevaporates the accretion disk and surrounding cloud "cocoon", ending mass accretion. At this point the object appears as a young stellar object (YSO) or T Tauri star and takes its place on the main sequence.

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