

Q₁ Given n array elements and a starting index and a ending index. Find sum in that range.

Ex1 : arr[] = [2 4 1 6 5]

l = 2

r = 4

⇒ loop from l-r, find sum

Tc: $O(n)$

Sc: $O(1)$

Pseudo Code !

```
int sum = 0
```

```
for (int i = l; i ≤ r; i++) {
```

```
    sum += arr[i];
```

```
}
```

```
return sum;
```

Q₂ Given n array elements and a starting index and a ending index. Find sum in that range. You are given Q queries.

arr[] = [⁰2 ¹4 ²1 ³6 ⁴5]

Q = 3

1) l = 0, r = 3 \Rightarrow 13

2) l = 1, r = 2 \Rightarrow 5

3) l = 2, r = 4 \Rightarrow 12

```
for (int j = 0 ; j < q ; j++) {  
    // l, r as input;
```

```
    int sum = 0
```

```
    for (int i = l ; i <= r ; i++) {
```

```
        sum += arr[i];
```

```
    }
```

```
    print (sum);
```

```
}
```

T_c : O(QN)

Q3 Given are Indian score after every over.

Overs	1	2	3	4	5	6	7	8	9	10
Runs SC	2	8	14	29	31	49	65	79	88	97

1) Runs scored in 10th over : $[10, 10]$
 $S[10] - S[9]$
 $97 - 88 = 9$

2) Runs scored in the last 5 overs. : $[6, 10]$
 $S[10] - S[5]$
 $97 - 31 = 66$

3) Runs scored in 7th over : $[7, 7]$
 $S[7] - S[6]$
 $65 - 49 = 16$

Cumulative sum from the start \Rightarrow Prefix Sum

Prefix Sum Array.

$Pf[i] \Rightarrow$ Sum of all elements from the start till i^{th} index.

$$\sum_{x=0}^i arr[x]$$

	0	1	2	3	4	5
$arr[]$	-9	10	2	1	8	5

	0	1	2	3	4	5
$Pf[]$	-9	1	3	4	12	17

1) $Sum(2,4) =$

$$Sum(0,4) = Sum(0,1) + Sum(2,4)$$

$$Sum(2,4) \Rightarrow Sum(0,4) - Sum(0,1)$$

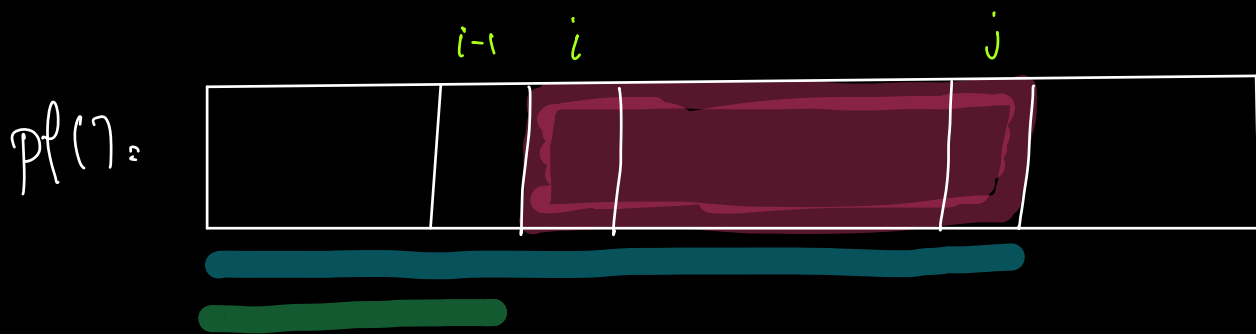
$$Sum(2,4) \Rightarrow Pf[4] - Pf[1]$$

$$12 - 1 \Rightarrow 11$$

$$2) \text{ Sum}(1, 3) = \text{pf}[3] - \text{pf}[0]$$

$$3) \text{ Sum}(2, 3) = \text{pf}[3] - \text{pf}[1]$$

$$\text{Sum}(i, j)$$



$$\text{Sum}(i, j) = \text{pf}[j] - \text{pf}[i-1]$$

$$(0, 3) = \text{pf}[3] \quad [\text{Edge Case}]$$

if ($i == 0$)

$$\text{Sum} = \text{pf}[j]$$

else

$$\text{Sum} = \text{pf}[j] - \text{pf}[i-1]$$

Revisit

Given n array elements and a starting index and an ending index. Find sum in that range. You are given Q queries.

// prepare prefix sum array.

$pf[0] = arr[0];$

for (int $i=1$; $i < n$; $i++$) {

$pf[i] = pf[i-1] + arr[i];$

}

for (int $i=0$; $i < q$; $i++$) {

// l, r are input.

int sum;

if ($l == 0$)

sum = $pf[r]$;

else

sum = $pf[r] - pf[l-1]$;

print sum;

n

q

nq

$n+q$

1

$O(n)$

$O(q)$

T.C: $O(n+q)$

S.C: $O(n)$

1) Can we think in terms of $n > q$ or $q > n$

$$T.C: O(n+q)$$

2) Can we modify the same array

→ depends if the original array is later needed

```
for (int i = 1; i < n; i++) {  
    arr[i] = arr[i-1] + arr[i];  
}
```

}

SC: $O(1)$ → if we modify original.

: $O(n)$ → if we don't

Equilibrium Index $\rightarrow n \geq 1$

Given an $arr[]$, find count of Equilibrium Index.

Equilibrium Index \Rightarrow Sum of all elements to left is equal to Sum of all elements to right

		0	1	2	3
$arr[]$	=	-3	2	4	-1
l	=	0	-3	-1	3
r	=	5	3	-1	0

Ans = 1

left \Rightarrow $pf[i-1]$

right $\rightarrow [i+1, n-1]$

$pf[n-i] - pf[i]$

$i = 0 \Rightarrow \text{Edge left} = \text{pf}[-1]$
 $\text{left} \Rightarrow 0$

$i = n-1 \Rightarrow \text{right} = 0$
 $\text{pf}[n-1] - \text{pf}[n-1] = 0$

$n = 1 \Rightarrow \text{pf}[0] - \text{pf}[0] = 0$

```
int cnt = 0; int left, right;  
for (int i = 0; i < n; i++) {
```

```
    if (i == 0)  
        left = 0
```

```
    else left = pf[i-1]
```

```
    right = pf[n-i] - pf[i]
```

```
    if (left == right)  
        cnt++;
```

```
}
```

```
return cnt;
```

Tc: $O(n)$

SC

$O(1)$

$O(n)$

If we
modify original
array.

Q4 Given N array elements & Q queries.

Each Q queries consists of i, j .

Return the number of even valued terms.
in $[i, j]$

Ex1 $arr[] = [\overset{0}{2} \overset{1}{3} \overset{2}{6} \overset{3}{8} \overset{4}{1} \overset{5}{10}]$

(i) $[0, 3] \Rightarrow 3$

(ii) $[4, 5] \Rightarrow 1$

	0	1	2	3	4	5
[2	3	6	8	1	10]

	1	0	1	1	0	1
--	---	---	---	---	---	---

	1	1	2	3	3	4
--	---	---	---	---	---	---

$[0, 3] \Rightarrow pf[3] \Rightarrow 3$

$[4, 5] \Rightarrow pf[5] - pf[3] \Rightarrow 1$

1) Update your array.

even element = 1 $O(n)$

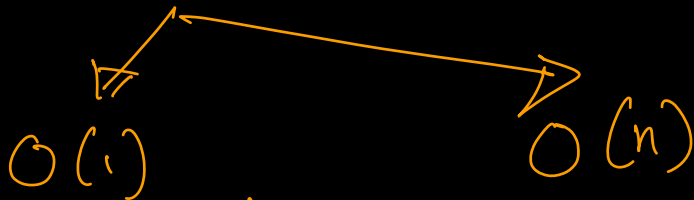
odd element $\Rightarrow 0$

2) Construct Pf on the $\Rightarrow O(n)$
updated array

3) Solve for each query $\Rightarrow O(q)$

$T_c: O(n+q)$

SC



if we modify
original array

leastod e \rightarrow prefix Sum.