Decimal Number System

$$\begin{matrix} 3 & 2 & 1 & 0 \\ 2 & 4 & 1 & 5 \end{matrix} = 2000 + 400 + 10 + 5$$

$$2 \times 10^3 + 4 \times 10^2 + 1 \times 10^1 + 5 \times 10^0$$

BASE: 10 $\Rightarrow [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]$

$$\#\ \overset{3}{X_1}\overset{2}{X_2}\overset{1}{X_3}\overset{0}{X_4} \Rightarrow X_1 * 10^3 + X_2 * 10^2 + X_3 * 10^1 + X_4 \times 10^0$$

OCTAL NUMBER SYSTEM

BASE: 8 : [0 1 2 3 4 5 6 7]

$$(\overset{3}{1}\overset{2}{4}\overset{1}{5}\overset{0}{6})_8 \Rightarrow 1(8)^3 + 4(8)^2 + 5(8)^1 + 6(8)^0$$

$$512 + 256 + 40 + 6$$

$$\Rightarrow 768 + 46$$

$$\Rightarrow (814)_{10}$$

Quiz 1 : $(125)_8$ to Decimal

$$\overset{2}{(}\overset{1}{1}\overset{0}{2}5)_8 = 1(8)^2 + 2(8)^1 + 5(8)^0$$

$$64 + 16 + 5 = 85$$

Binary Number System

Base : 2 $\Rightarrow$ [0, 1]

Quiz 2 : $(1010)_2$ to Decimal.

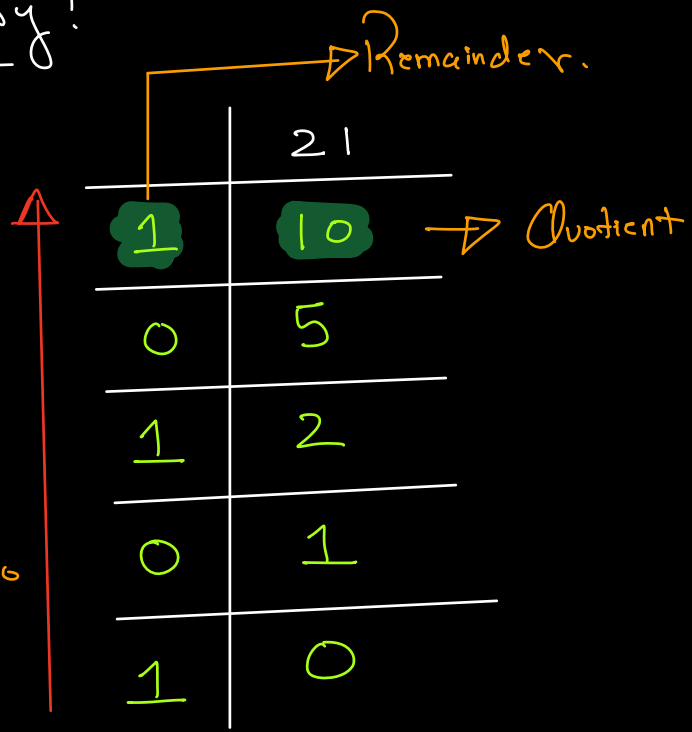
$$(1010)_2 = 1(2)^3 + 0(2)^2 + 1(2)^1 + 0(2)^0$$

$$8 + 0 + 2 + 0$$

$$\Rightarrow (10)_0$$

Decimal to Binary!

1) $(21)_{10}$



$\Rightarrow (10101)_2$

$\Downarrow$

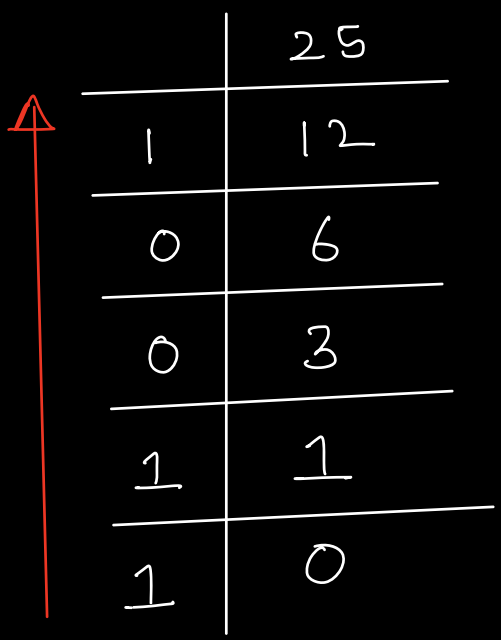
$1(2)^4 + 1(2)^2 + 1(2)^0$

$16 + 4 + 1 \Rightarrow 21$

Quiz 3

$(25)_{10}$

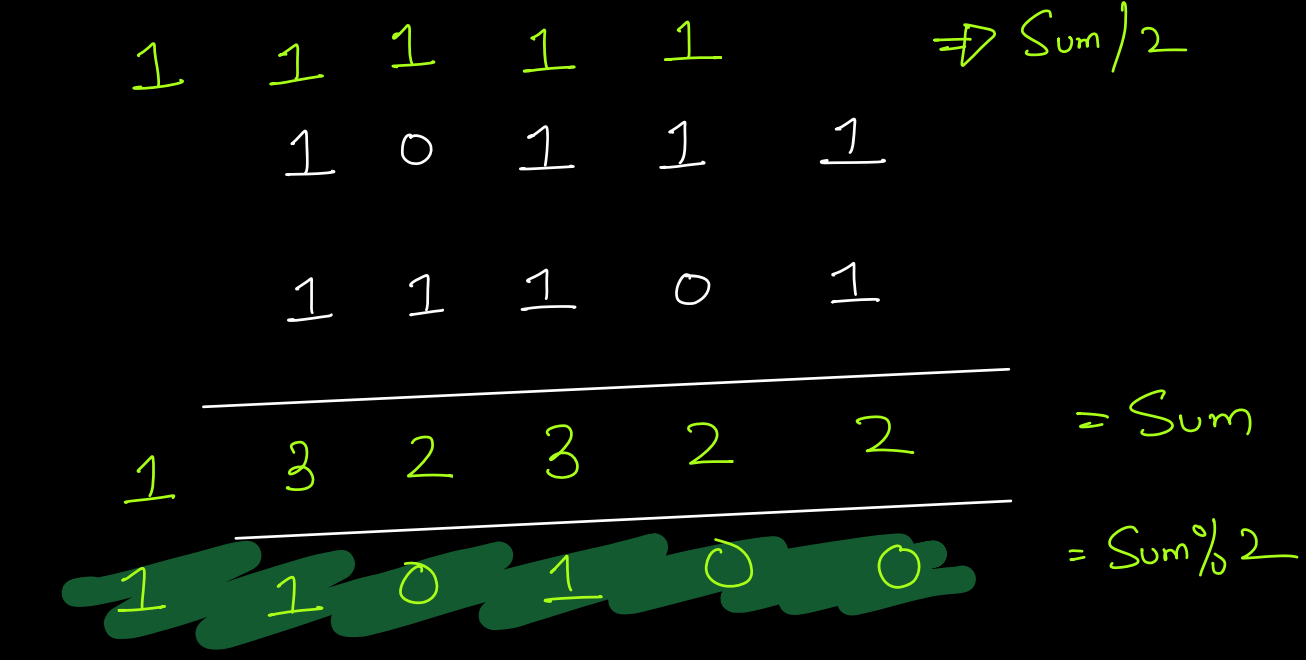
$(11001)_2$



Decimal Addition

0	1	1	1	⇒ Sum /10
	2	3	4	8
	1	7	6	5
	4	11	11	13 ⇒ Sum
	4	1	1	3 ⇒ Sum %10

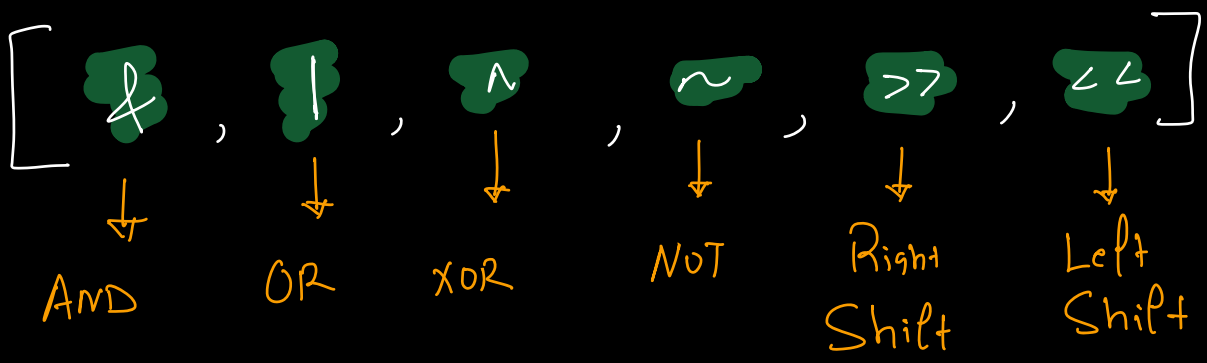
Binary Addition



Quiz 4

$$\begin{array}{r} 1\ 0\ 0\ 1\ 1 \\ 0\ 1\ 0\ 0\ 1 \\ \hline 1\ 1\ 1\ 0\ 0 \end{array}$$

Bitwise Operators



a	b	a&b	a|b	a^b	~a	~b
0	0	0	0	0	1	1
0	1	0	1	1	1	0
1	0	0	1	1	0	1
1	1	1	1	0	0	0

Bitwise Operations on Decimal Numbers.

1) 4 & 3

4 → 1 0 0

3 → 0 1 1

---

0 0 0

→ 0

2) 4 | 3

4 → 1 0 0

3 → 0 1 1

---

1 1 1

→ 7

3) 4 ^ 3

4 → 1 0 0

3 → 0 1 1

---

1 1 1

→ 7

Quiz 21 & 41

0 1 0 1 0 1

1 0 1 0 0 1

---

0 0 0 0 0 1

→ 1

Quiz 10 | 1

⇒ 1 0 1 0

1

1 0 1 1 ⇒ 11

Quiz 11 | 1

1 0 1 1

1

1 0 1 1 ⇒ 12

Q_1 : 14 | 1 ⇒ 15

Q_2 : 17 | 1 ⇒ 17

1) For a odd number, last bit is always 1.

2) For a even number, last bit is always 0.

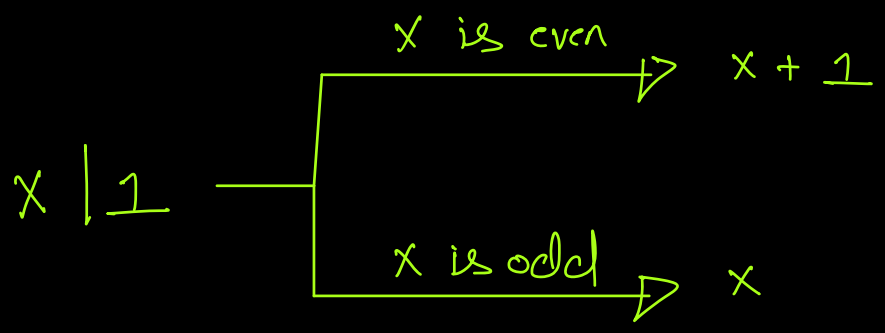
Odd | 1 = No change.

x_1	x_2	x_3	x_4	x_5	1
0	0	0	0	0	1
x_1	x_2	x_3	x_4	x_5	1

EVEN | 1 $\Rightarrow$ EVEN + 1

x_1	x_2	x_3	x_4	x_5	0
0	0	0	0	0	1
x_1	x_2	x_3	x_4	x_5	1

Imp: B | 0 $\Rightarrow$ B.



Quiz $x \,|\, 1$, x is even.

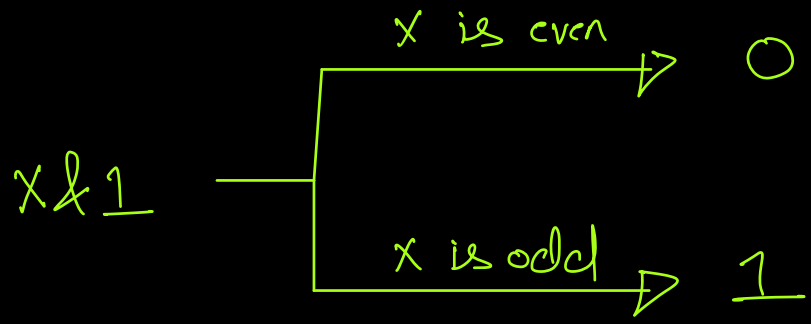
x_1	x_2	x_3	x_4	x_5	0
0	0	0	0	0	1
0	0	0	0	0	0

$\Rightarrow 0$

Quiz $x \,|\, 1$, x is odd.

x_1	x_2	x_3	x_4	x_5	1
0	0	0	0	0	1
0	0	0	0	0	1

$\Rightarrow 1$



Imp : $B \wedge 0 \Rightarrow B$

$0 \wedge 0 \Rightarrow 0$

$1 \wedge 0 \Rightarrow 1$

Quiz $x \wedge 1$, x is even.

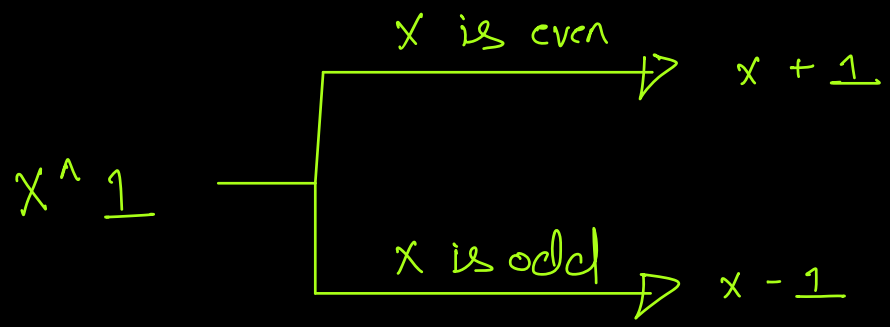
x_1	x_2	x_3	x_4	x_5	0
0	0	0	0	0	1
x_1	x_2	x_3	x_4	x_5	1

$\Rightarrow x + 1$

$x \wedge 1$, x is odd

x_1	x_2	x_3	x_4	x_5	1
0	0	0	0	0	1
x_1	x_2	x_3	x_4	x_5	0

$\Rightarrow x - 1$



Bitwise Properties

1) $A \mid A \Rightarrow A$

$$\begin{array}{r} 1010 \\ 1010 \\ \hline 1010 \end{array}$$

$1 \mid 1 \Rightarrow 1$

$0 \mid 0 \Rightarrow 0$

2) $A \& A \Rightarrow A$

$$\begin{array}{r} 1010 \\ 1010 \\ \hline 1010 \end{array}$$

$1 \& 1 = 1$

$0 \& 0 = 0$

3) $A \wedge A \Rightarrow 0$

$1 \wedge 1 = 0$

$0 \wedge 0 = 0$

$$\begin{array}{r} 1010 \\ 1010 \\ \hline 0000 \end{array}$$

4) $A \wedge 0 = A$

$$\begin{array}{r} 101010 \\ 000000 \\ \hline 101010 \end{array}$$

Commutative

1) $A \& B = B \& A$

2) $A | B = B | A$

3) $A \wedge B = B \wedge A$

Associativity

$$= \left\{ \begin{array}{l} a \times b \times c \\ = (a \times b) \times c \\ = (a \times c) \times b \\ = (b \times c) \times a \end{array} \right\}$$

$a \wedge b \wedge c$

$\Rightarrow (a \wedge b) \wedge c$

$\Rightarrow (a \wedge c) \wedge b$

$\Rightarrow (b \wedge c) \wedge a$

true for & and | also

Q $P \wedge Q \wedge R \wedge P \wedge Q \wedge R = ?$

$\Downarrow$

$A \wedge A = 0$

$(P \wedge P) \wedge (Q \wedge Q) \wedge (R \wedge R)$

$0 \wedge 0 \wedge 0 \Rightarrow 0$

$\Rightarrow$ 10:

Q Missing Number.

Given an array where every number appears for even number of times except one. Find that number.

Do this inplace: SC: O(1)

Eg : [1, 2, 1, 3, 1, 4, 3, 1, 4]

→ 1^1^1^1 → 0

→ 3^3 → 0

→ 4^4 → 0

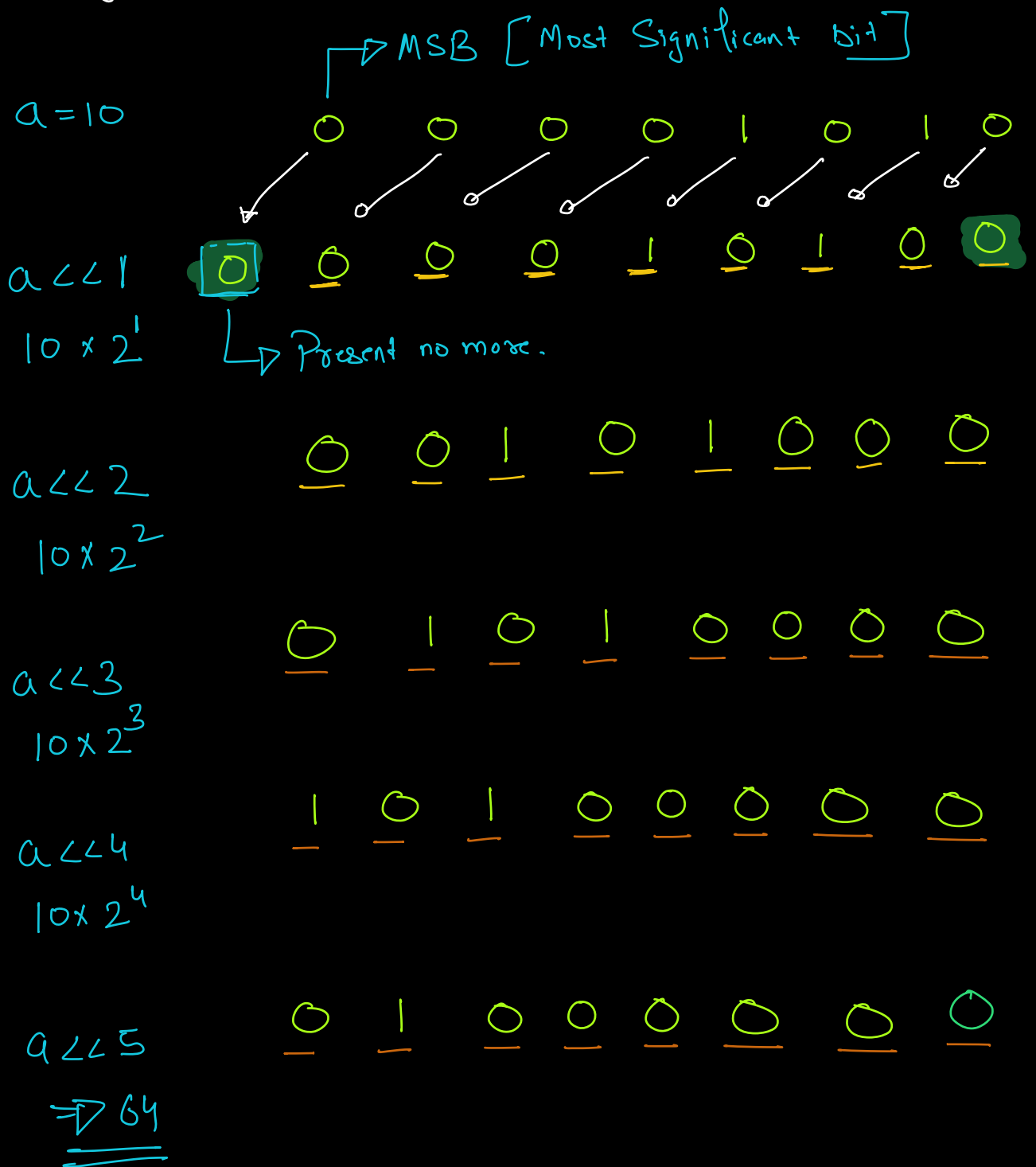
→ 2 → 2

⇒ ans = 0

```
for (int i = 0 ; i < n ; i++) {
    ans ^= arr[i];
}
return ans;
```

Left Shift.

byte a ⇒ 10 [8 bit number]



$$a << n \Rightarrow a \times 2^n$$

↳ If no overflow

$$1 << n \Rightarrow 1 \times 2^n$$

$$1 << n \Rightarrow 2^n \Rightarrow O(1)$$

Right Shift

byte a = 10

LSB ↑ { least Significant bit }

a = 160 : 1 0 1 0 0 0 0 0

$a >> 1 = \frac{a}{2^1}$: 0 1 0 1 0 0 0 0

$a >> 2 = \frac{a}{2^2}$: 0 0 1 0 1 0 0 0

$a >> 3 = \frac{a}{2^3}$: 0 0 0 1 0 1 0 0

$a >> 4 = \frac{a}{2^4}$: 0 0 0 0 1 0 1 0

$$a >> n = \frac{a}{2^n}$$

$$160 >> 4 = \frac{160}{2^4} = \frac{160}{16}$$

$$= 10$$

$$a >> 2n = \frac{a}{(2)^{2n}} = \frac{a}{4^n}$$

$160 >> 64$

$(2)^{2\times 3} = 2^6 = 64$ $(a)^{2n} = (2a)^n$

$\Rightarrow (4)^3 = 64$

$$\frac{160}{2^6} = \frac{160}{64} = 2$$