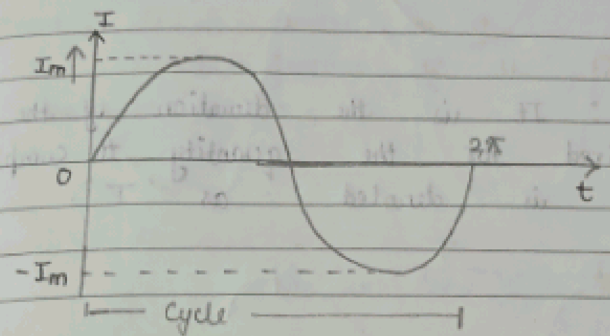
Date...................

Unit - 2 : AC circuit

Alternate current (AC) :-

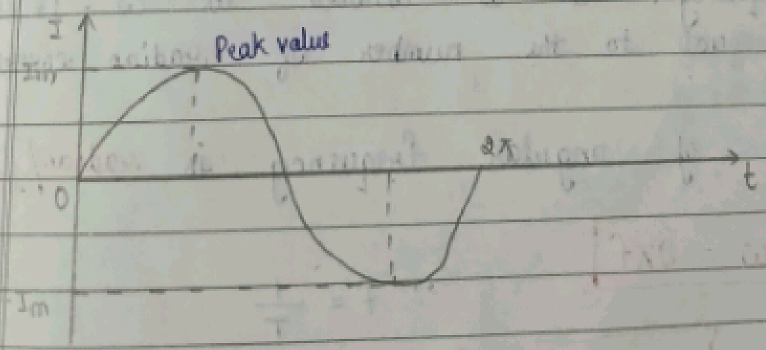
The AC means alternate current. A current reversed it direc^n periodically the mathematically representation as — $\boxed{I = I_0 \sin\omega t}$ or $\boxed{I = I_m \sin\omega t}$



Terminology :

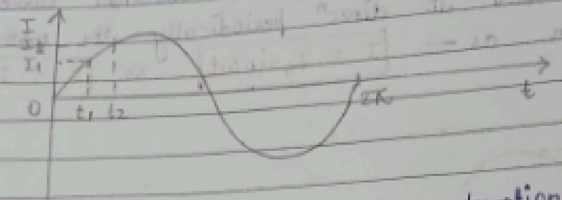
1. Cycle : The value of sin wave repeat after every 2π radians. One complete set of positive and negative values of the function is c/a a cycle.

2. Peak value : Peak value is the maximum value denoted as I_m, +Ve and -Ve of the quantity.

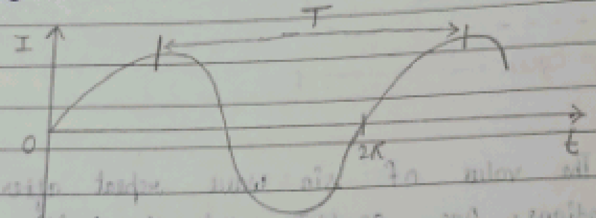


Date.........

3. Instantaneous value : It is the value of quantity any instant w. r. to time is known as instantaneous value.



4. Time period : It is the duration of the time required for the quantity to complete one cycle. It is denoted as 'T'.



5. Frequency : Frequency is the number of cycle occur in one second.

$$\boxed{f = \frac{1}{T}}$$

6. Angular frequency : It is denoted as 'ω'. It is equal to the number of radian covered in 1 sec.

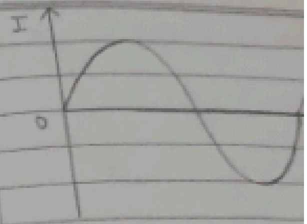
→ The unit of angular frequency is radian/sec.

$$\boxed{\omega = 2\pi f}$$

$$\because f = \frac{1}{T}$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

Spiral



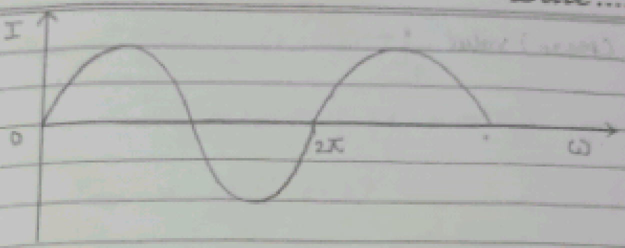
7. Phase : It is den... It is the fr... known as phase.

Values of AC quan...

1. Peak value
2. Average value
3. RMS (root m...

1. Peak value :- ... alternate q... value. This ... The peak value ... maximum...

Date.................



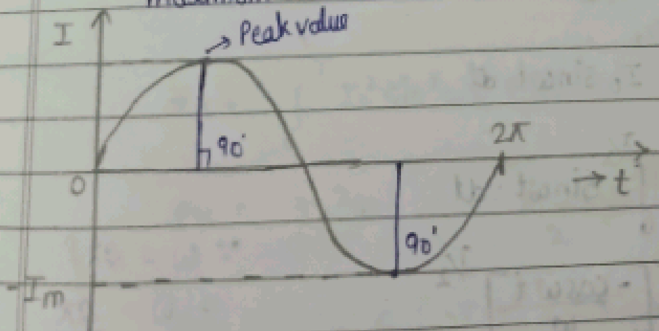
7. Phase : It is denoted as 'ϕ'.
It is the fraction of the time period or cycle known as phase.

$$\boxed{\phi = 2\pi}$$

Values of AC quantities —
There are three types of values in A.C circuit -

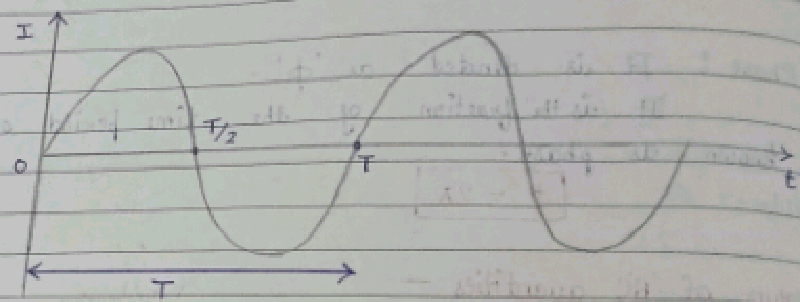
1. Peak value
2. Average value
3. RMS (root mean square) Effective value

1. Peak value :- The maximum value attained by the alternate quantity during one cycle is c/a as peak value. This is also c/a maximum value of AC quantities. The peak value in sinusoidal alternating quantity obtains its maximum value at 90°.



The peak value of alternating quantities voltage and current denoted by V_m and I_m

2. Average (Mean) value :-



$$I = I_0 \sin\omega t$$

$$I_{av.} = \frac{\text{Area under half cycle}}{\text{length of half cycle}}$$

$$I_{av.} = \frac{\int_0^{T/2} I\, dt}{T/2}$$

$$I_{av.} = \frac{\int_0^{T/2} I_0 \sin\omega t \cdot dt}{T/2}$$

$$I_{av.} = \frac{2}{T}\int_0^{T/2} I_0 \sin\omega t \cdot dt$$

$$I_{av.} = \frac{2I_0}{T}\int_0^{T/2} \sin\omega t \cdot dt$$

$$I_{av.} = \frac{2I_0}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2}$$

$\because \omega = 2\pi f$

$\Rightarrow \omega = \frac{2\pi}{T}$

R → Heat discipitation

L → Energy stored in magnetic form

C → Energy stored in electric form.

Date..................

$$I_{av} = \frac{-2I_0}{T}\left[\frac{1}{\omega}(-\cos\omega t)\right]_0^{T/2}$$

$$I_{av} = \frac{-2I_0}{2\pi}\left[\cos\frac{2\pi}{\cancel{T}}\times\frac{\cancel{T}}{2} - \cos 0\right]$$

$$I_{av} = \frac{-I_0}{\pi}[-1-1]$$

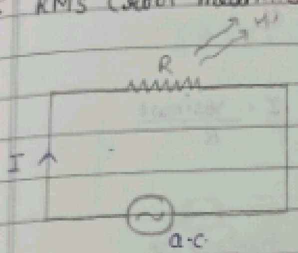
$$\boxed{I_{av} = \frac{2I_0}{\pi} = 0.637\, I_0}$$

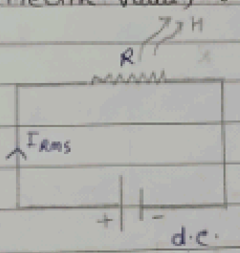
$$\boxed{I_{av} = 63.7\% \text{ of } I_0}$$

Imp

3. RMS (root mean square or effective value) :-

exam





$\boxed{I = I_0 \sin\omega t}$

$$I = I_0\sin\omega t$$

$$H = I^2RT \qquad \Rightarrow H = I_{rms}^2RT \text{ --- (1) for d.c. ckt}$$

$$H = I_0^2\sin^2\omega t \times R\times T \text{ --- (2)}$$

$$\int dH = \int_0^T I^2R\,dt$$

$$H = \int_0^T I_0^2\sin^2\omega t\, R\,dt$$

$$H = I_0^2R\int_0^T\left(\frac{1-\cos 2\omega t}{2}\right)dt$$

$$H = \frac{I_0^2R}{2}\left[T - \frac{1}{2\omega}\sin 2\omega t\right]_0^T$$

Spiral

$H = \frac{I_0^2 R}{2}\left[T - \frac{1}{2\omega}(\sin 4\pi - \sin 0)\right]$ Date..........

$$I_{Rms} = \frac{I_0^2 R}{2}[T] \quad \text{— (2)}$$

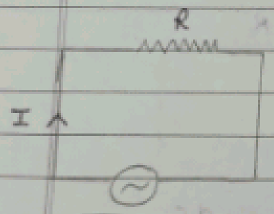
by eqⁿ (1) & (2) —

$$\frac{I_0^2 R}{2} T = I_{rms}^2 \times R \times T$$

$$\frac{I_0^2}{2} = I_{rms}^2$$

$$\boxed{I_{rms} = \frac{I_0}{\sqrt{2}}} \qquad \boxed{I_{rms} = 0.707\, I_0}$$

#. AC circuit with resistance only



$\boxed{V = V_0 \sin\omega t}$

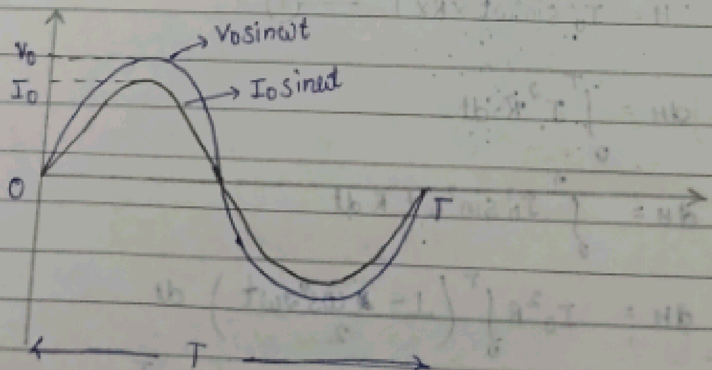
$$V = IR$$

$$I = \frac{V}{R} \Rightarrow I = \frac{V_0 \sin\omega t}{R}$$

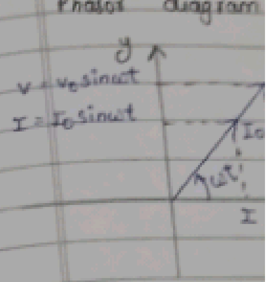
$$\boxed{I_0 = \frac{V_0}{R}}$$

$$\boxed{I = I_0 \sin\omega t}$$

*. Wave from presentation :-



Phasor diagram



#. A-C circuit

$dI =$

$\int dI =$

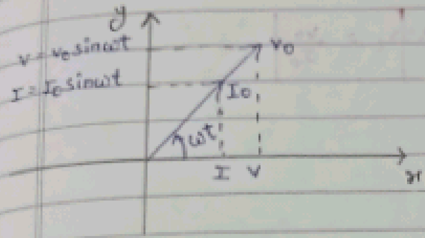
$I = V$

$I =$

Date 6 2022

Nikita

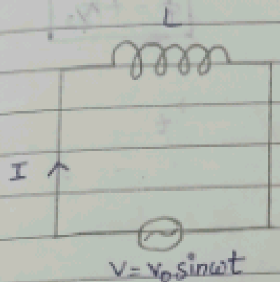
Phasor diagram —



$\boxed{\phi = 0^\circ}$

$\boxed{I \ \& \ V}$

A-C circuit with inductor only :—



$$e = -L\frac{dI}{dt}$$

$$\boxed{V = -e}$$

$$V = -\left[-L\frac{dI}{dt}\right]$$

$$\boxed{V = L\frac{dI}{dt}}$$

$$dI = \frac{V}{L}dt \Rightarrow dI = \frac{V_0 \sin\omega t}{L}dt$$

$$\int dI = \int \frac{V_0}{L}\sin\omega t \cdot dt \Rightarrow I = \frac{V_0}{L}\int \sin\omega t \cdot dt$$

$$I = \frac{V_0}{L}\left[\frac{-\cos\omega t}{\omega}\right] \Rightarrow I = -\frac{V_0}{\omega L}\cos\omega t$$

$$I = \frac{-V_0}{\omega L}\left(\sin(\pi/2 - \omega t)\right) \Rightarrow I = \frac{V_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{I = \frac{V_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)}$$

Spiral

Date..........

$\frac{V_0}{\omega L} = I_0$

$\omega L = \frac{V_0}{I_0}$

$\boxed{\omega_L = X_L}$

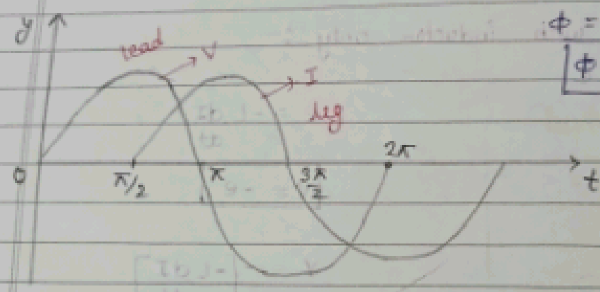
$\because \boxed{X_L = \frac{V_0}{I_0}}$

Wave form diagram –

$\phi = \omega t - (\omega t - \frac{\pi}{2})$

$\phi = \omega t - \omega t + \pi/2$

$\boxed{\phi = +\pi/2}$



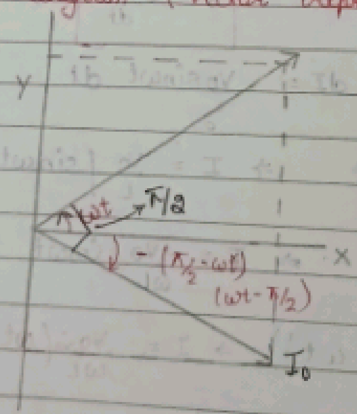
Phasor diagram (Vector representation of two quantities)

$V = V_0 \sin\omega t$

$I = I_0 \sin\omega t$

anticlockwise → (angle +ve)

clockwise → (angle −ve)



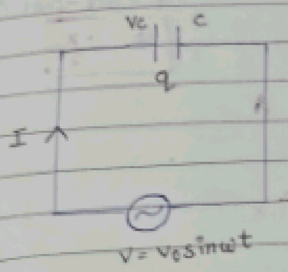
AC circuit

max. value of current

Wave form

Date..................

AC circuit with capacitor only :-



V_C = across the capacitor voltage

$$V_C = V$$

$$\frac{q}{C} = V_0 \sin\omega t$$

$$q = CV_0 \sin\omega t$$

$$I = \frac{dq}{dt}$$

$$I = \frac{d}{dt}(CV_0 \sin\omega t)$$

$$I = CV_0 \omega \cos\omega t$$

$$\boxed{I = CV_0 \omega \sin\left(\frac{\pi}{2} + \omega t\right)}$$

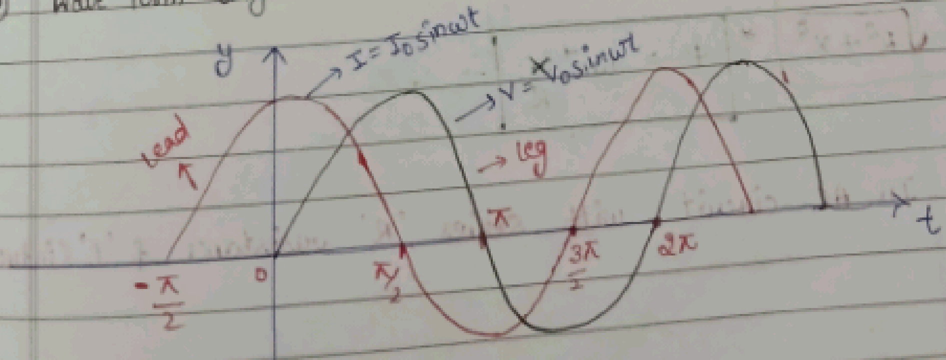
max value of current $\Leftarrow I_0 = CV_0\omega$

$$\frac{V_0}{I_0} = \frac{1}{C\omega} \quad \& \quad \boxed{X_C = \frac{1}{\omega C}}$$

X_C = capacitive reactance

$$\star \quad \boxed{I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)}$$

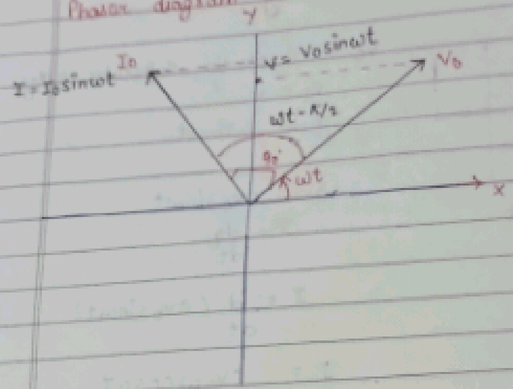
⊙ Wave form diagram



Date..............

$\boxed{Z = \frac{V}{I}}$

Phasor diagram —



Phase difference —

$\phi = \omega t - (\omega t + \pi/2)$

$\boxed{\phi = -\frac{\pi}{2}}$

Complex Impedance :-

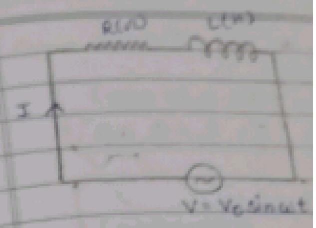
For an AC circuit the ratio of voltage phasor and the current phasor is known as complex quantity and c/a ~~known~~ as ~~clock~~ complex Impedance. It is represented by 'Z'. Its real part is c/a resistance and its imaginary parts is c/a reactance.

Complex Impedance (z) = Resistance + j (Reactance)

$$\boxed{Z = R + jX}$$

$$\boxed{Z = \sqrt{R^2 + X^2}} \quad \boxed{\phi = \tan^{-1}\frac{X}{R}},$$

In AC circuit, with series 'R' resistance & 'L' (Inductor):

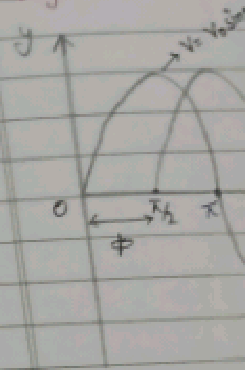


$X_L =$

$X_L =$

$\phi =$

wave form diagram —



Phasor diagram →

Spiral

...te........

...se difference —

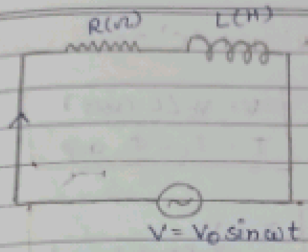
$= \omega t - (\omega t + \pi/2)$

$\boxed{\phi = -\frac{\pi}{2}}$

...ratio of ... is known as ... complex Impedance. ...l part is ... is c/a

'L' (inductor):-

Date........



$V = (V_0 \angle 0)$ Volt

$I = I_0 \angle -\phi$ $\boxed{Z = R}$

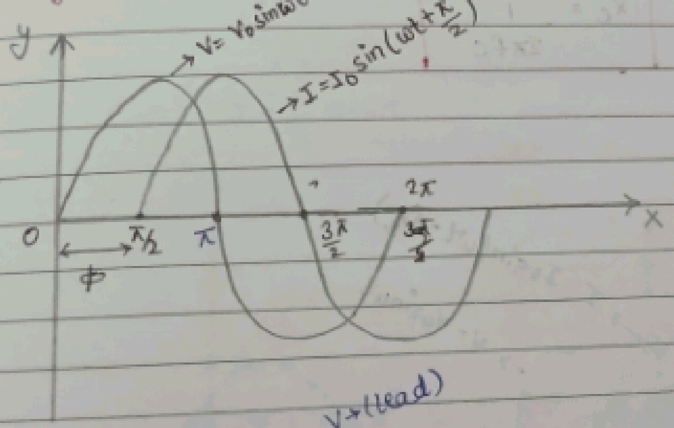
$\boxed{I = \frac{V}{Z}}$ → Z = Impedance

$$\boxed{\begin{aligned} Z &= R + jX_L \\ Z &= \sqrt{R^2 + X_L^2} \end{aligned}}$$

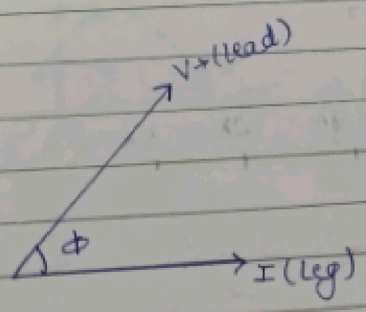
$$\boxed{\begin{aligned} X_L &= \omega L \\ X_L &= 2\pi f L \end{aligned}}$$

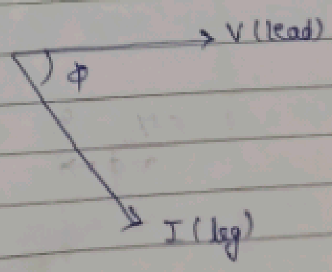
$$\boxed{\phi = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

wave form diagram —



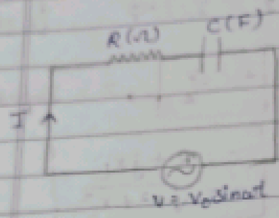
Phasor diagram →





Date.........

AC circuit with series 'R' and 'C' —



$V = V_0 \angle 0$ (volt)
$I = I_0 \angle \phi$ amp.

$I = \frac{V}{Z}$ (Impedance)

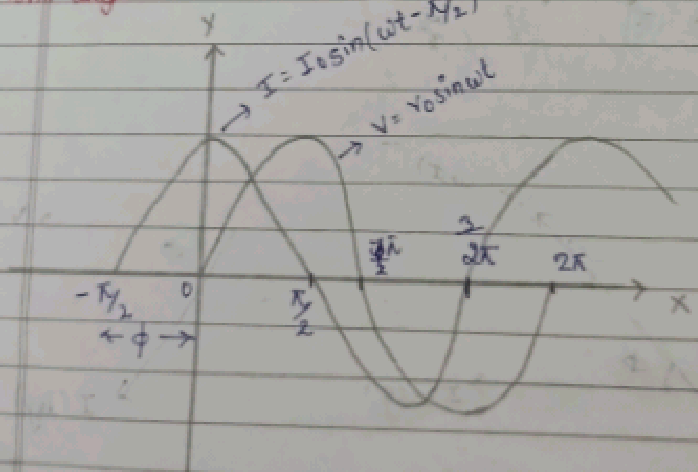
$\boxed{Z = R - jX_C}$

$\boxed{Z = \sqrt{R^2 + X_C^2}}$

$X_C = \frac{1}{\omega C}$

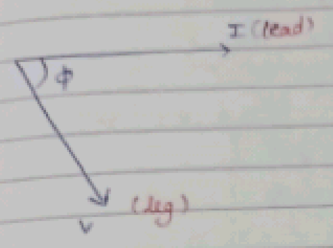
$X_C = \frac{1}{2\pi f C}$

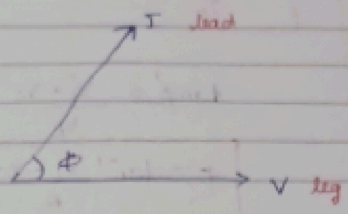
Wave form diagram —



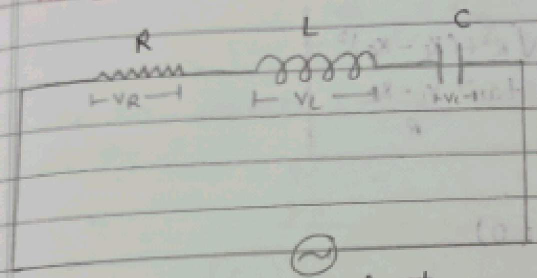
Date..................

⊙ Phasor diagram —





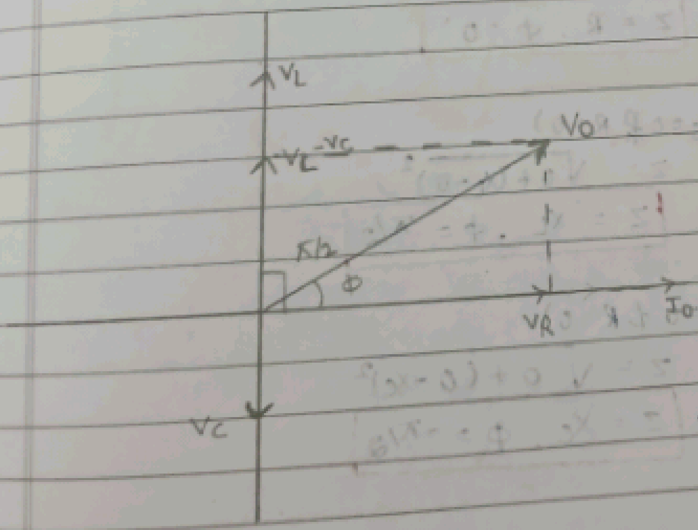
AC circuit with Series in RLC



$I = I_0 \sin \omega t$

Let $V_L > V_C$

$X_L > X_C$



$$V_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V_0 = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2}$$

$$V_0 = I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{V_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

Date........

$$\boxed{Z = \frac{V_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}}$$

$$\tan\phi = \frac{V_L - V_C}{V_R}$$

$$\boxed{\phi = \tan^{-1}\frac{X_L - X_C}{R}}$$

$$\boxed{V = V_0 \sin(\omega t + \phi)}$$

Summary :

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$\phi = \tan^{-1}\frac{X_L - X_C}{R}$$

1. Only R $(X_L = 0, X_C = 0)$

$$Z = \sqrt{R^2 + (0-0)^2}$$
$$Z = \sqrt{R^2}$$
$$\boxed{Z = R, \ \phi = 0^\circ}$$

2. Only L, $(X_C = 0 \ \& \ R = 0)$

$$Z = \sqrt{0 + (X_L - 0)^2}$$
$$\boxed{Z = X_L, \ \phi = \pi/2}$$

3. Only C, $(X_L = 0 \ \& \ R = 0)$

$$Z = \sqrt{0 + (0 - X_C)^2}$$
$$\boxed{Z = X_C, \ \phi = -\pi/2}$$

Date....................

4. R-L CKT $(X_C = 0)$

$$\boxed{\begin{aligned} Z &= \sqrt{R^2 + X_L^2} \\ \phi &= \tan^{-1}\left(\frac{X_L}{R}\right) \end{aligned}}$$

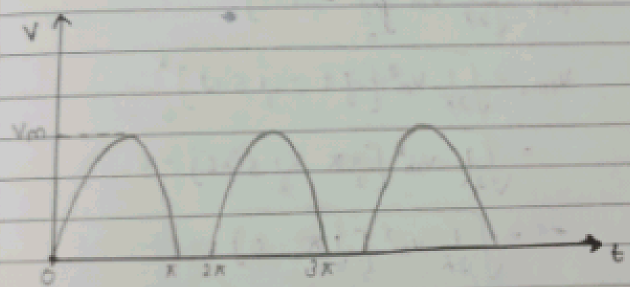
5. R-C CKT $(X_L = 0)$

$$\boxed{\begin{aligned} Z &= \sqrt{R^2 + X_C^2} \\ \phi &= -\tan^{-1}\left(\frac{X_C}{R}\right) \end{aligned}}$$

6. Only L-C CKT $(R = 0)$

$$\boxed{\begin{aligned} Z &= |X_L - X_C| \\ \phi &= \pm \pi/2 \end{aligned}}$$

Q. Find the average value, RMS value and from factor of the voltage shown in fig.



$$V_{av} = \frac{\text{area under 1 cycle}}{\text{time period}}$$

$$\int_0^{2\pi} v(t)\,dt = \frac{\int_0^{\pi} V_m \sin t\,dt}{2\pi}$$

$$= \frac{V_m \int_0^{\pi} \sin t \cdot dt}{2\pi}$$

$$= \frac{V_m[-\cos t]_0^{\pi}}{2\pi}$$

Date........

$$\frac{-V_m[\cos\pi - \cos 0]}{2\pi}$$

$$\frac{-V_m[-1-1]}{2\pi} = \frac{2V_m}{2\pi} = \frac{V_m}{\pi}$$

$$\boxed{V_{av.} = \frac{V_m}{\pi}}$$ Ans.

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{\pi} V_m^2 \sin^2 t \, dt}$$

$$V_{rms} = \sqrt{\frac{1}{T} V_m^2 \int_0^{\pi} \sin^2 t \cdot dt}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1-\cos t}{2}\right]_0^{\pi}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2} - \frac{1\cos t}{2}\right]_0^{\pi}}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2}t - \frac{1}{2}\sin t\right]_0^{\pi}}$$

$$= \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2}\pi - \frac{1}{2}\sin 0\right] - 0}$$

$$= \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2}\pi - 0\right]}$$

$$V_{rms} = \sqrt{\frac{1}{2\cancel{\pi}} V_m^2 \times \frac{1}{2}\cancel{\pi}}$$

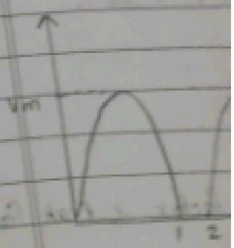
$$V_{rms} = \frac{V_m}{\sqrt{2\pi}} \times \sqrt{\frac{\pi}{2}}$$

$$\boxed{V_{rms} = \frac{V_m}{2}}$$ Ans.

From factor = $\frac{V_{r}}{V}$...

Peak factor = $\frac{V_m}{V_{r}}$...

Que-2 Find the ave... factor of vo...



V_{av}

V_{rms} =

V...

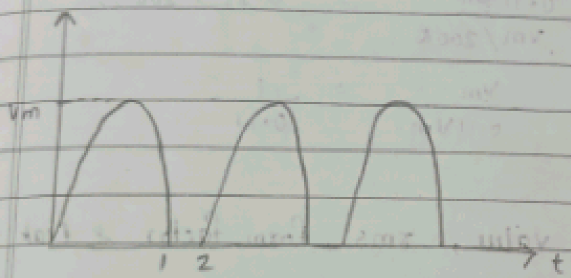
Date........

From factor $= \frac{V_{rms}}{V_{av}} = \frac{V_m/2}{V_m/\pi} = \frac{\pi}{2}$

Ans.

$$\boxed{\text{Peak factor} = \frac{V_m}{V_{rms}} = \frac{2V_{rms}}{V_{rms}} = 2}$$

Que. 2 Find the average value, rms value, form factor & Peak factor of voltage shown in fig.



$$= \int_0^t \frac{V_m \sin t \cdot dt}{2\pi}$$

$$V_m \int_0^t \frac{\sin t}{2\pi} \cdot dt$$

$$-\frac{V_m}{2\pi}[\cos t]_0^t$$

$$-\frac{V_m}{2\pi}[\cos 1 - \cos 0^\circ]$$

$$V_{av} = \frac{-V_m}{2\pi}[\cos 1 - 1] = \frac{-V_m}{2\pi}[0.99 - 1] = \frac{V_m}{200\pi}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^1 V_m^2 \sin^2 t \cdot dt} \Rightarrow V_{rms} = \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2}t - \frac{1}{4}\cos^2 t\right]_0^1}$$

$$V_{rms} = \sqrt{\frac{1}{2\pi} V_m^2 \left[\left(\frac{1}{2} - \frac{1}{4}\sin 2\right) - \left(0 - \frac{0}{4}\right)\right]}$$

$\cos 2 = 0.999$

$\sin 2 = 0.034$

Date.......

$$V_{rms} = \frac{V_m}{\sqrt{2\pi}} \sqrt{\frac{1}{2}\left(1 - \frac{1}{2}\sin 2\right)}$$

$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{\frac{1}{2}\left(1 - \frac{1}{2} \times 0.034\right)}$$

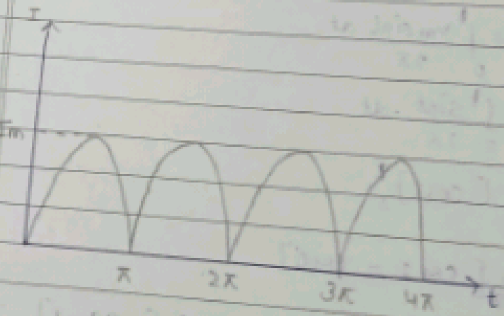
$$= \frac{V_m}{\sqrt{2\pi}} \sqrt{\frac{1}{2} \times 0.9830}$$

$$\boxed{V_{rms} = 0.11\, V_m}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{av}} = \frac{0.11\, V_m}{V_m / 200\pi} = 0.11 \times 200\pi$$

$$\text{Peak factor} = \frac{V_m}{V_{rms}} = \frac{V_m}{0.11\, V_m} = \frac{1}{0.11}$$

Que. Find the average value, rms, form factor & peak factor shown in fig.



$$I = I_0 \sin \omega t$$

$$I_{av} = \int_0^{\pi} I \, dt$$

$$I_{av} = \int_0^{\pi} \frac{I_0 \sin t \, dt}{\pi}$$

Date..................

$$I_{av} = \frac{-I_0}{\pi}\left[\cos t\right]_0^{\pi}$$

$$\frac{-I_0}{\pi}\left[\cos \pi - \cos 0\right]$$

$$\frac{-I_0}{\pi}(-1-1)$$

$$\boxed{I_{av} = \frac{2I_m}{\pi}}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^{\pi} I_m^2 \sin^2 t \, dt}$$

$$= \sqrt{\frac{1}{\pi} I_m^2 \int_0^{\pi} \left[\frac{1-\cos 2t}{2}\right]}$$

$$= \sqrt{\frac{1}{\pi} I_m^2 \left[\frac{1}{2}t - \frac{1}{4}\sin 2t\right]_0^{\pi}}$$

$$= \sqrt{\frac{1}{\pi} I_m^2 \left[\frac{1}{2}\pi - \frac{1}{4}\sin 2\times\pi\right] - 0}$$

$$= \sqrt{\frac{1}{\cancel{\pi}} I_m^2 \frac{1}{2}\times\cancel{\pi}}$$

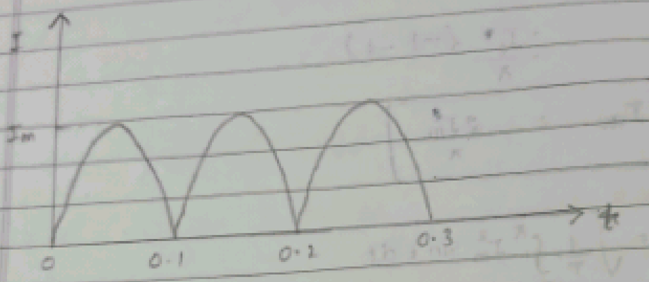
$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}}}$$

$$\text{form factor} = \frac{I_{rms}}{I_{av}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = \frac{\pi}{2\sqrt{2}}$$

$$\text{Peak factor} = \frac{I_m}{I_m}\times\sqrt{2} = \sqrt{2}$$

Date........

Que. Find the average value, rms value, form & Peak fac[tor] of the current shown in fig.



$$I_{av} = \int_0^{0.1} \frac{I_m \sin t \cdot dt}{0.1}$$

$$= \frac{-I_m}{0.1}\left[\cos t\right]_0^{0.1}$$

$$I_{av} = \frac{-I_m}{0.1}\left[\cos 0.1 - \cos 0\right]$$

$$I_{av} = \frac{-I_m}{0.1}\left[0.99 - 1\right] = -\frac{I_m}{0.1} \times \left[-0.01\right] = \boxed{\frac{I_m}{10}}$$

$\cos 2x = 1 - 2\sin^2 x$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^{0.1} I_m^2 \sin^2 t \cdot dt}$$

$$= \sqrt{\frac{1}{0.T} I_m^2 \int_0^{0.1} \frac{1}{2} - \frac{\cos 2t}{2} \cdot dt}$$

$$\frac{I_m}{\sqrt{0.1}}\sqrt{\left[\frac{1}{2}t - \frac{1}{4}\sin 2t\right]_0^{0.1}}$$

$$\frac{I_m}{\sqrt{0.1}}\sqrt{\left[\frac{1}{2}\times 0.1 - \frac{1}{4}\sin 2\times 0.1\right] - 0}$$

$I_{rms} = \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1}{20} - \frac{1}{4}s\ldots}$

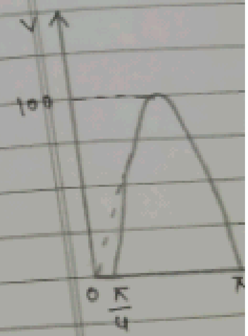
$I_{rms} = \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1}{20} - \ldots}$

$= \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1-0\ldots}{20}}$

$= \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{5}{200\ldots}}$

$I_{rms} = \ldots$

Que. Find the aver[age] ... shown in wav[e] ...



V_{av}

V_{av}

Peak Factor

Date.................

$$I_{rms} = \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1}{20} - \frac{1}{4}\sin 0.2}$$

$$I_{rms} = \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1}{20} - \frac{1}{4} \times 0.19} = \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1}{20} - \frac{0.19}{4}}$$

$$= \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1 - 0.19 \times 5}{20}} = \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1 - 0.95}{20}} = \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{0.05}{20}}$$

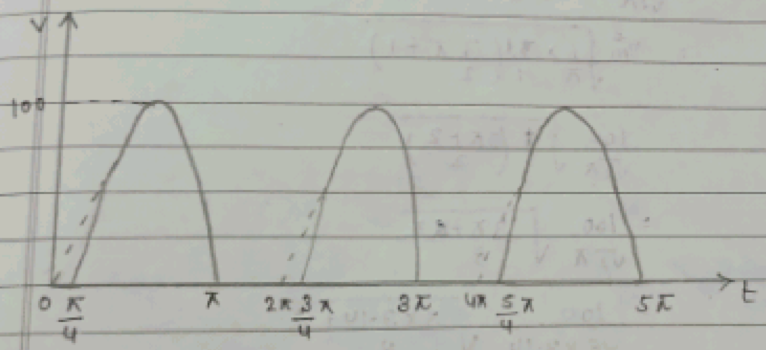
$$= \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{5}{2000}} = \frac{I_m}{\sqrt{0.1}}\sqrt{\frac{1}{400}} = \frac{I_m}{\sqrt{0.1} \times 20}$$

$$I_{rms} = \frac{I_m}{20 \times \sqrt{0.1}} = I_{rms} = \frac{I_m}{20 \times 0.31}$$

$$\boxed{I_{rms} = \frac{I_m}{6.2}}$$

Que. Find the average and effective value of the sinodial shown in wave form.
$\underbrace{\text{effective}}_{\text{rms}}$

Vav = 27.17 V



$$V_{av} = \int_{\pi/4}^{\pi} \frac{V_0 \sin t \, dt}{2\pi}$$

$$\frac{-V_0}{2\pi}\left[\cos t\right]_{\pi/4}^{\pi} = \frac{-V_0}{2\pi}\left[\cos\pi - \cos\frac{\pi}{4}\right]$$

$$V_{av} = \frac{-V_0}{2\pi}\left[-1 - \frac{\sqrt{3}}{2}\right] = \frac{-V_0}{2\pi}\left(-1 - \frac{1}{\sqrt{2}}\right)$$

$$= -\frac{50}{\pi}\left[\frac{-\sqrt{2}-1}{\sqrt{2}}\right] = \frac{50}{\sqrt{2}\pi}\left[1+\sqrt{2}\right]$$

Spiral

$$\boxed{V_{av} = 27.17\,V}$$

$$V_{rms} = \sqrt{\frac{1}{T} V_{rms}^2 \sin^2 t \cdot dt}$$

$$= \sqrt{\frac{1}{2\pi} V_m^2 \int_{\pi/4}^{\pi} \frac{1-\cos 2t}{2} \cdot dt}$$

$$= \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2}t - \frac{\sin 2t}{4}\right]_{\pi/4}^{\pi}}$$

$$= \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2}\pi - \frac{\sin 2\pi}{4}\right] - \left[\frac{1}{2} \times \frac{\pi}{4} - \frac{1}{4}\sin \pi/2\right]}$$

$$= \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2}\pi - 0\right] - \left[\frac{\pi}{8} - \frac{1}{4}\right]}$$

$$= \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{1}{2}\pi - \frac{\pi}{8} - 0 + \frac{1}{4}\right]}$$

$$= \sqrt{\frac{1}{2\pi} V_m^2 \left[\frac{3\pi}{8} + \frac{1}{4}\right]}$$

$$= V_m \sqrt{\frac{1}{\pi} \times \frac{1}{4}\left(\frac{3}{2}\pi + 1\right)}$$

$$= \frac{100}{\sqrt{2}\pi} \sqrt{\frac{1}{4}\left(\frac{3\pi + 2}{2}\right)}$$

$$= \frac{100}{\sqrt{2}\pi} \sqrt{\frac{3\pi + 2}{8}}$$

$$= \frac{100}{\sqrt{2} \times 3.14} \sqrt{\frac{3 \times 3.14 + 2}{8}}$$

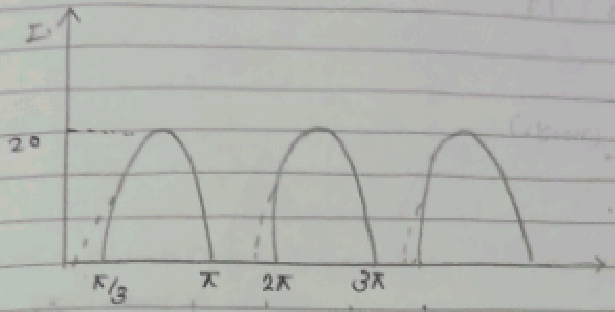
$$= \frac{100}{4.44} \sqrt{\frac{9.42 + 2}{8}}$$

$$= 22.52 \sqrt{\frac{11.42}{8}} = 22.52 \times \sqrt{1.4275}$$

$$= 22.52 \times 1.19 = 24.75$$

$$\boxed{V_{rms} = 24.75}$$

Que. Find the average and effective value of the sinusodal waveform shown in fig.



$$I = \int_{\pi/3}^{\pi} \frac{I_m \sin t}{2\pi} = \frac{I_m}{2\pi}\int_{\pi/3}^{\pi} \sin t \cdot dt = -\frac{I_m}{2\pi}\left[\cos t\right]_{\pi/3}^{\pi}$$

$$= -\frac{I_m}{2\pi}\left[\cos\pi - \cos\frac{\pi}{3}\right] = -\frac{I_m}{2\pi}\left[-1-\frac{1}{\sqrt{2}}\right]$$

$$= -\frac{I_m}{2\pi}\left[\frac{-\sqrt{2}-1}{\sqrt{2}}\right] = \frac{I_m}{2\pi}\left[\frac{1+\sqrt{2}}{\sqrt{2}}\right]$$

$$= \frac{I_m}{2\pi}\left[\frac{1+\sqrt{2}}{\sqrt{2}}\right] = \frac{I_m}{2\pi}\left[\frac{1}{\sqrt{2}}+1\right]$$

$$I_{av} = \frac{I_m}{2\pi}\left(\frac{1+\sqrt{2}}{\sqrt{2}}\right)$$

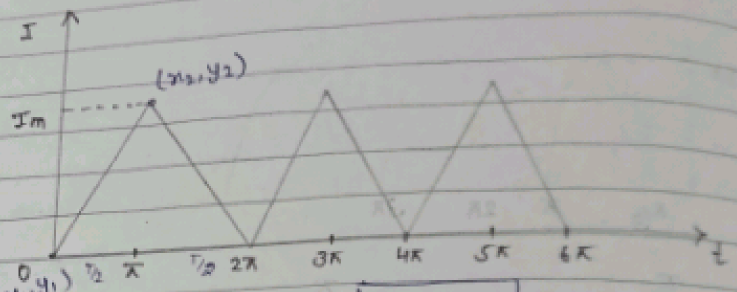
$$I_{rms} = \sqrt{\frac{1}{T}\int I_m^2 \sin^2 t \cdot dt} = \sqrt{\frac{1}{2\pi} I_m^2 \int_{\pi/3}^{\pi} \frac{1-\cos 2t}{2} \cdot dt}$$

$$= \sqrt{\frac{1}{2\pi} I_m^2 \left[\frac{1}{2}t - \frac{\sin 2t}{4}\right]_{\pi/3}^{\pi}}$$

$$= \sqrt{\frac{1}{2\pi} I_m^2 \left[\left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right) - \left(\frac{\pi}{6} - \sin\frac{\pi}{6}\right)\right]} = \sqrt{\frac{1}{2\pi} I_m^2 \left(\frac{\pi}{2}-1\right) - \left(\frac{\pi}{6}-\frac{1}{2}\right)}$$

$$\sqrt{\frac{1}{2\pi} I_m^2 \left(\frac{\pi}{2} - 1 - \frac{\pi}{6} + \frac{1}{2}\right)} = \sqrt{\frac{1}{2\pi} I_m^2 \left(\frac{\pi}{3} - \frac{1}{2}\right)} = I_{rms}$$

Q. Determine the average value, rms value and form factor for current wave form shown in fig.



$$m = \frac{y_2 - y_1}{x_2} \rightarrow \boxed{m = \frac{I_m}{\pi}} \rightarrow \text{(slope)}$$

$$I = \frac{I_m}{\pi} t$$

$$I_{av.} = \frac{\text{Area under 1 cycle}}{\text{half period}}$$

$$I_{av.} = \frac{\int_0^{\pi} \frac{I_m}{\pi} \cdot t \, dt}{\pi/2}$$

$$I_{av.} = \frac{I_m}{\pi^2} \left[\frac{t^2}{2}\right]_0^{\pi}$$

$$= \frac{I_m}{\pi^2} \left[\frac{\pi^2}{2}\right]$$

$$\boxed{I_{av.} = \frac{I_m}{2}}$$

$$I_{rms} = \sqrt{\frac{1}{T} \int \frac{I_m^2}{\pi^2} t^2 \, dt}$$

$$= \sqrt{\frac{1}{\pi^3} I_m^2 \left[\frac{t^3}{3}\right]_0^{\pi}}$$

$$= \sqrt{\frac{1}{3\pi^3} I_m^2 \left[\pi^3\right]}$$

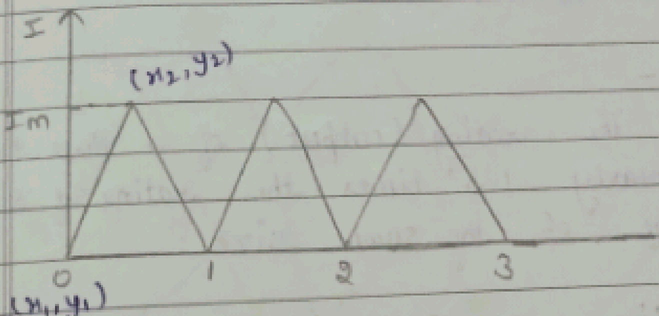
factor

$$\boxed{I_{rms} = \frac{1}{\cancel{6}}\frac{I_m}{\sqrt{3}}}$$

$$\text{form factor} = \frac{I_{rms}}{I_{av.}} = \frac{I_m/\sqrt{3}}{I_m/2} = \boxed{\frac{2}{\sqrt{3}}} \text{ Ans.}$$

$$\text{Peak factor} = \frac{I_m}{I_{rms}} = \frac{I_m}{\frac{I_m \times \sqrt{3}}{\sqrt{3}}} = \boxed{\sqrt{3}} \text{ Ans.}$$

Que. Find the average value, rms value and form factor of the current wave form –



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\boxed{m = \frac{I_m}{0.5}} \rightarrow \text{slope}$$

$$I = \frac{I_m}{0.5} \times t$$

$$I_{av.} = \frac{\int_0^{0.5} \frac{I_m}{0.5} \cdot t\,dt}{0.5} \Rightarrow I_{av.} = \frac{I_m}{0.5 \times 0.5}\left[\frac{t^2}{2}\right]_0^{0.5}$$

$$I_{av.} = \frac{I_m}{0.25} \times \left[\frac{0.25}{2} - 0\right] \Rightarrow \boxed{I_{av.} = \frac{I_m}{2}} \text{ Ans.}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^{0.5} \frac{I_m^2}{0.25} \cdot t^2\,dt} \Rightarrow I_{rms} = \sqrt{\frac{I_m^2}{0.25 \times 0.5}\left[\frac{t^3}{3}\right]_0^{0.5}}$$

$$I_{rms} = \sqrt{\frac{I_m^2}{0.125}\left[\frac{(0.5)^3}{3} - \frac{125}{3}\right]} \Rightarrow I_{rms} = \sqrt{\frac{I_m^2}{0.125}\left[\frac{0.125}{3} - 0\right]}$$

$$I_{rms} = \sqrt{\frac{I_m^2}{0.125} \times \frac{0.125}{3}} \Rightarrow \boxed{I_{rms} = \frac{I_m}{\sqrt{3}}} \text{ Ans.}$$

Date........

Form factor : $\frac{I_{rms}}{I_{av}}$: $\frac{I_m/\sqrt{3}}{I_m/2}$ = $\boxed{2/\sqrt{3}}$

Peak factor : $\frac{I_m}{I_{rms}}$ = $\frac{I_m}{I_m/\sqrt{3}}$ = $\boxed{\sqrt{3}}$

Three face AC system :

The advantage of using three ~~phase~~ face system over face system are as follows :-

(i) Constant Power : In single face the power deliv... as pulsating. The power is zero twice in each cycle. In three face power delivered as constant when load are balanced.

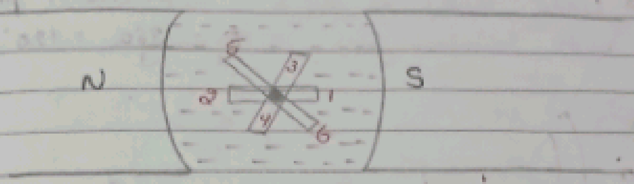
(ii) Higher rating :-
The rating (output) of a three face machine is nearly 1.5 times the rating of single p face machine of the same size.

(iii) Power transmition economical :-
Three face transmi... lines required much less conductor material whereas in single face much more require conductor materials.

(iv) Superiority of three face induction motor : Three fa... induction motor are self starting whereas single face induction motor have no starting torque.

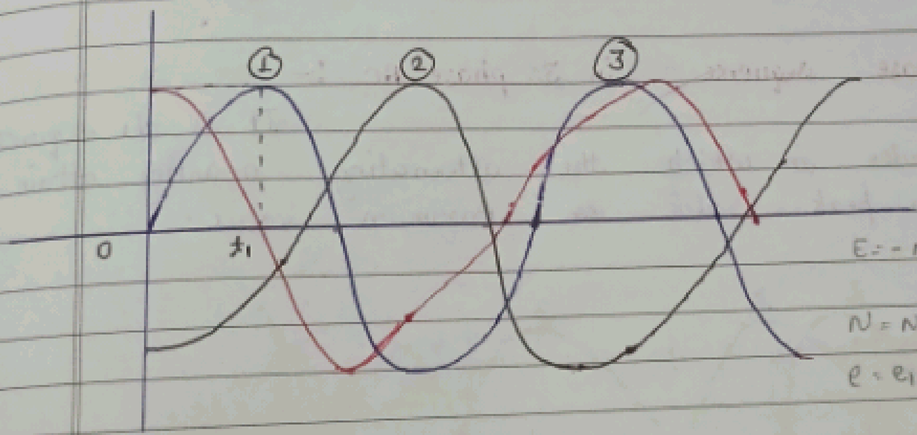
Date..................

Generation of three ~~face~~ phase system:-



↺ωt radian

Rotate these magnet anti clock wise (↺ ωt radian).



$$E = -N\frac{d\phi}{dt}$$

$$N = N_1 = N_2 = N_3$$

$$e = e_1 = e_2 = e_3$$

$$e_1 = E_m \sin\omega t \quad \text{--- (1)}$$

For conductor in magnet

$$e_2 = E_m \sin\left(\omega t - \frac{2\pi}{3}\right) \quad \text{--- (2)}$$

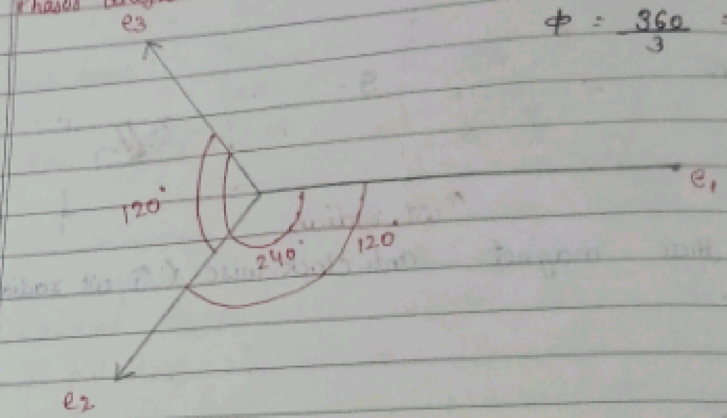
$$e_3 = E_m \sin\left(\omega t - \frac{4\pi}{3}\right) \quad \text{--- (3)}$$

Date........

$\phi = \frac{360 \text{ electrical degree}}{N \text{ (No of phase)}}$

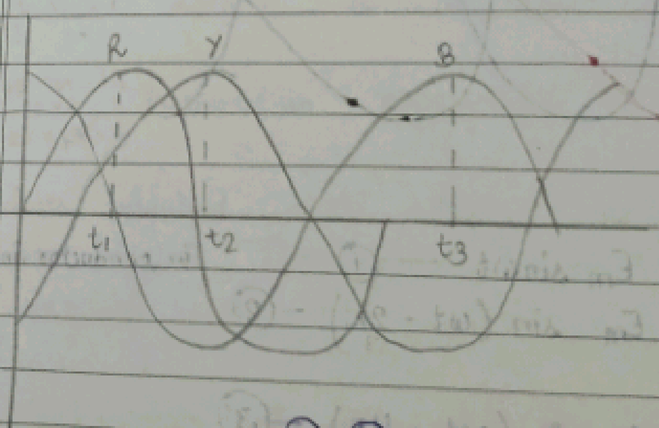
$\phi = \frac{360}{3} = 120^\circ$

Phasor diagram —



Phase sequence in 3 phase AC :-

It is the sequence order in which the alternating quantity attain the peak value or maximum value.



R Y B = +ive phase sequence

B Y R = -Ve phase sequence

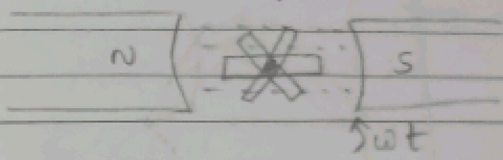
Date..................

Types of 3 phase A.C —

There are two types of three phase A.C —

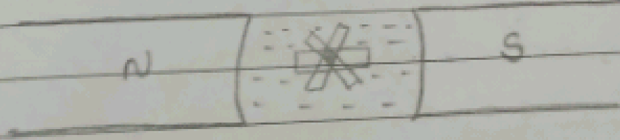
1. Symmetrical/balanced 2. Unsymmetrical three phase A.C.

1. $\boxed{e = -N\frac{d\phi}{dt}}$



$$\boxed{\begin{array}{l} = N = N_1 = N_2 = N_3 \\ = e = e_1 = e_2 = e_3 \\ \frac{d\phi}{dt} = \frac{d\phi_1}{dt} = \frac{d\phi_2}{dt} = \frac{d\phi_3}{dt} \end{array}}$$ balanced/ Symmetrical

2. $N_1 = 200, \quad N_2 = 300, \quad N_3 = 500$

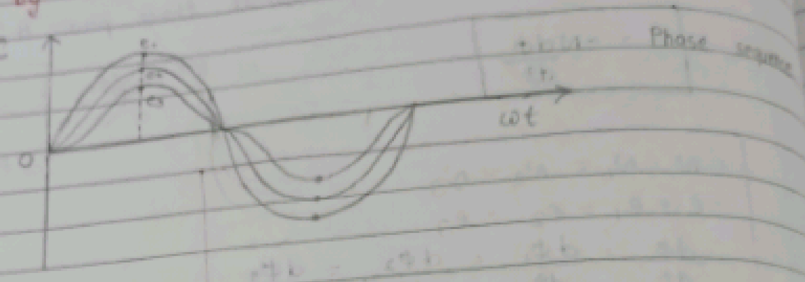


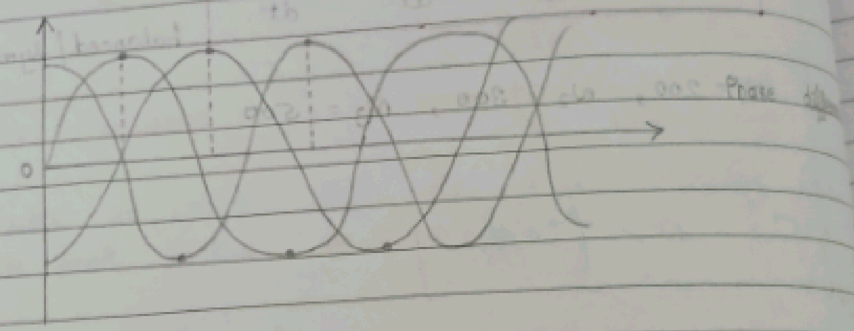
$\boxed{e = -N\frac{d\phi}{dt}}$

$$\boxed{\begin{array}{l} = N = N_1 \neq N_2 \neq N_3 \\ = e = e_1 \neq e_2 \neq e_3 \\ \frac{d\phi}{dt} = \frac{d\phi_1}{dt_1} \neq \frac{d\phi_2}{dt_2} \neq \frac{d\phi_3}{dt_3} \end{array}}$$ Unsymmetrical

Date..............

Identification of phase sequence and phase difference by wave form representation –





Connection of 3 phase A.C. →

→ There are 2 type of connection in 3 phase A.C –

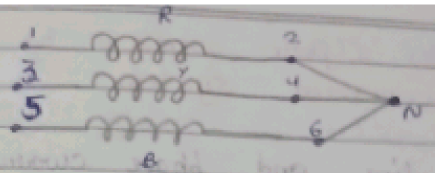
1. star connection
2. delta connection

1. Star connection :–

It is the type of connection where ~~which~~ one terminal from each binding is connected to the common point and remaining 3 terminals are connected to the CKT.

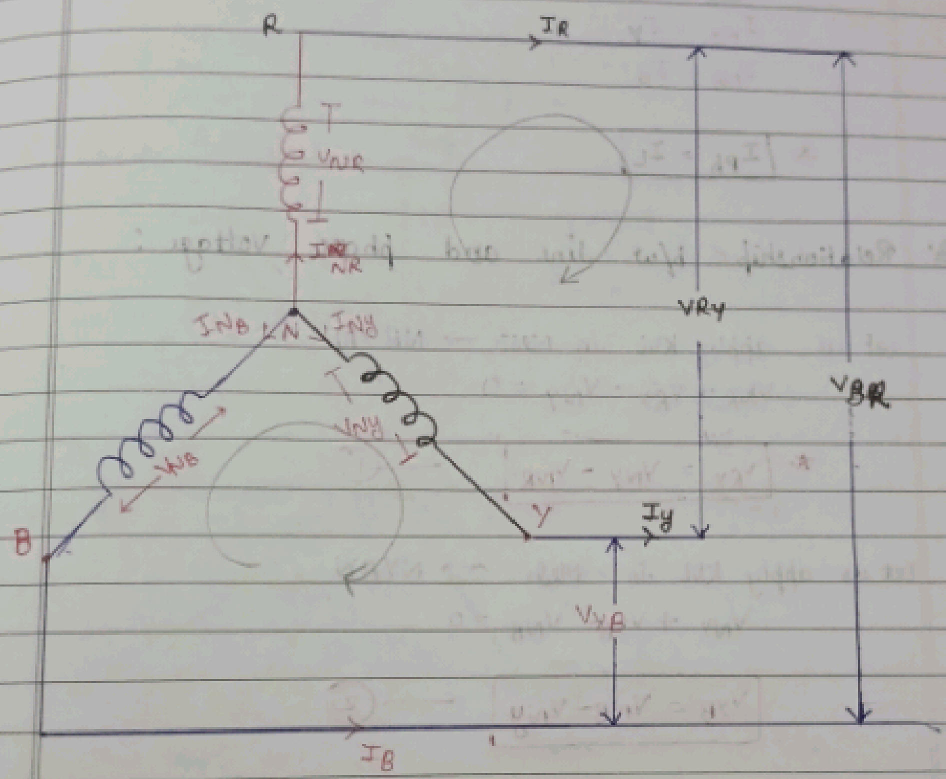
Spiral

Date..................



It is also known as wye, ⅄, Y connection.

⊙ Standard diagram of star connection R, Y, B :-



Line quantities →

1. Line voltage ⇒ $V_{RY} = V_{\cancel{R}YB} = V_{BR} = V_L$
2. Line current ⇒ $I_R = I_Y = I_B = I_L$

Phase quantities →

1. Phase voltage ⇒ $V_{NR} = V_{NY} = V_{NB} = V_{Phase}$
2. Phase current ⇒ $I_{NR} = I_{NY} = I_{NB} = I_{Phase}$

Spiral

#. Relationship b/w line & phase quantities for balanced 3 phase star connected system:

1. Relationship b/w line and phase current:

$$I_{NR} = I_R$$
$$I_{NY} = I_Y$$
$$I_{NB} = I_B$$

* $\boxed{I_{Ph} = I_L}$

2. Relationship b/w line and phase voltage:

let us apply KVL in Mesh → NRYN

$$V_{NR} + V_{RY} - V_{NY} = 0$$

* $\boxed{V_{RY} = V_{NY} - V_{NR}}$ —— (1)

Let us apply KVL in Mesh → NYBN

$$V_{NY} + V_{YB} - V_{NB} = 0$$

$\boxed{V_{YB} = V_{NB} - V_{NY}}$ —— (2)

Let us apply KVL in mesh → NRBN

$$V_{NR} - V_{BR} - V_{NB} = 0$$

$\boxed{V_{BR} = V_{NR} - V_{NB}}$ —— (3)

Date..................

from eq^n (1) $\longrightarrow \sqrt{a^2+b^2+2ab\cos\theta}$

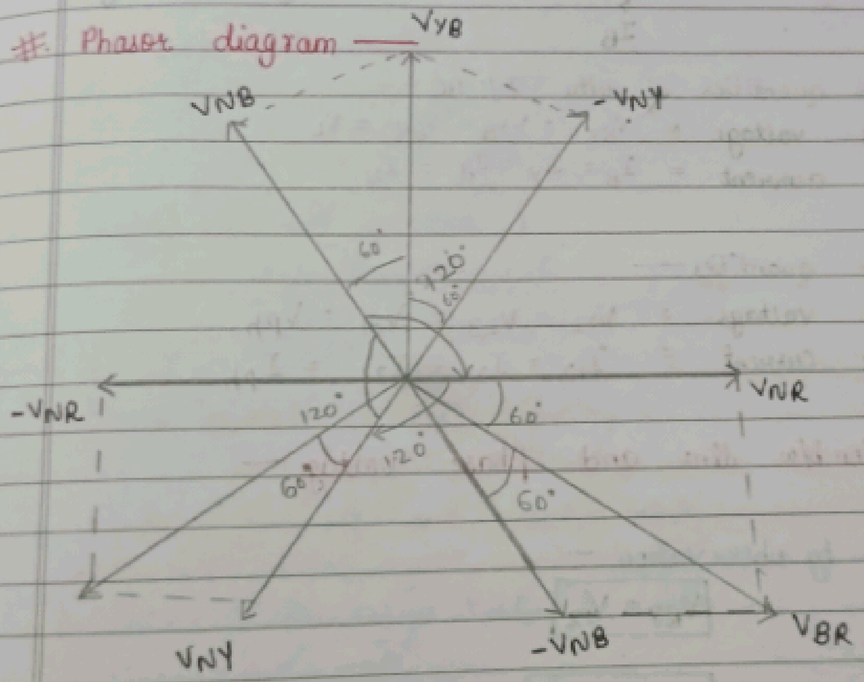
$$V_{RY} = \sqrt{V_{Ny}^2 + V_{NR}^2 + 2V_{NR}\cos\phi}$$

$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph}^2\cos 60^\circ}$$

$$= \sqrt{3V_{ph}^2}$$

$$* \boxed{V_{RY} = \sqrt{3}\, V_{ph}}$$

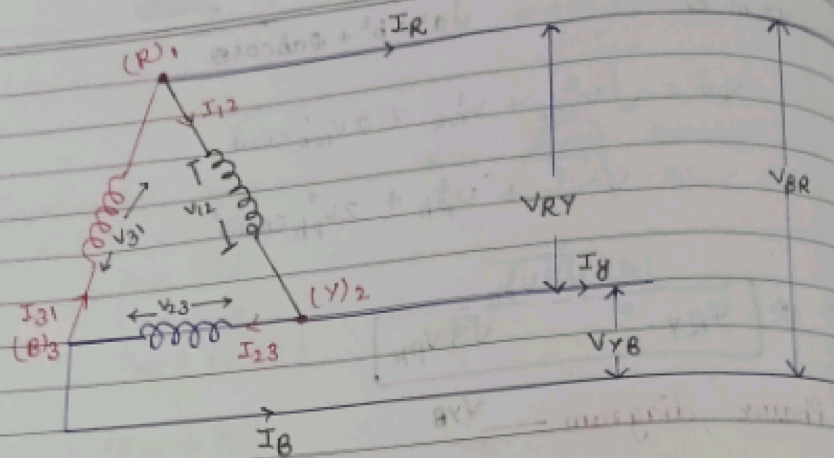
#. Phasor diagram —



2. Delta connection :-

It is the type of connection of three phase bindings where all the coils are connected back to back arrangement

Date..........



Line quantities of delta 3ϕ AC —

(a) line voltage = $V_{RY} = V_{YB} = V_{BR} = V_L$

(b) line current = $I_R = I_y = I_B = I_L$

Phase quantities —

(a) Phase voltage = $V_{12} = V_{23} = V_{31} = V_{ph}$

(b) Phase current = $I_{12} = I_{23} = I_{31} = I_{ph}$

Relation b/w line and phase voltage —

Just by observation —

$$\boxed{V_{RY} = V_{12}}$$

$$\boxed{V_{YB} = V_{23}}$$

$$\boxed{V_{BR} = V_{31}}$$

$$\boxed{V_L = V_{ph}}$$

Date..................

Relation b/w line and phase current in 3ϕ —

Line current = $I_R = I_Y = I_B = I_L$

Phase current = $I_{12} = I_{23} = I_{31} = I_{ph}$

Apply KCL at junction 1 —

$$I_{31} - I_{12} - I_R = 0$$

$$\boxed{I_R = I_{31} - I_{12}} \quad \text{— (1)}$$

Apply KCL at junction 2 —

$$I_{12} - I_{23} - I_y = 0$$

$$\boxed{I_y = I_{12} - I_{23}} \quad \text{— (2)}$$

Apply KCL at junction 3 —

$$I_{23} - I_{31} - I_B = 0$$

$$\boxed{I_B = I_{23} - I_{31}} \quad \text{— (3)}$$

Taking equation (1) for resulting —

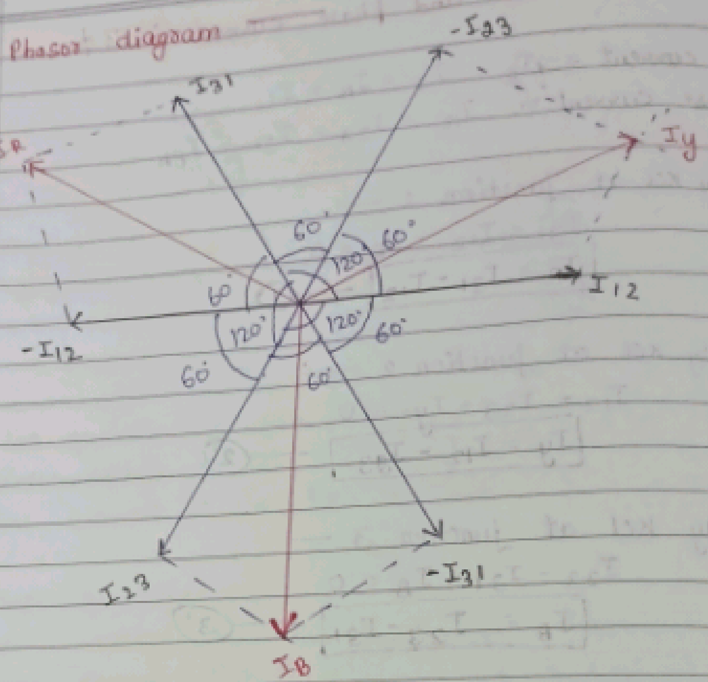
$$I_R = \sqrt{I_{31}^2 + I_{12}^2 + 2 I_{31} I_{12} \cos\phi}$$

$$I_R = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph} \cdot I_{ph} \cos 60^\circ}$$

$$I_R = \sqrt{I_{ph}^2 + I_{ph}^2 + I_{ph}^2}$$

★ $\boxed{I_R = \sqrt{3}\, I_{ph}}$ ★

Phasor diagram —



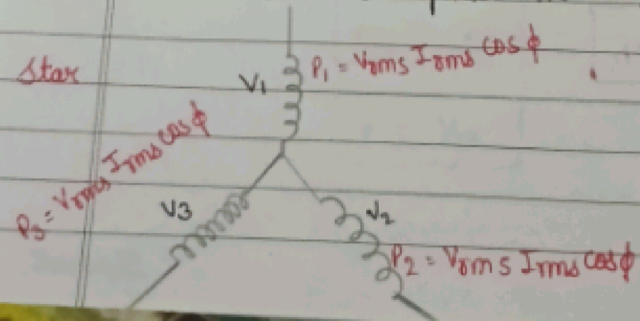
#. Power in three phase AC →

There are three types of power in single phase

1. Active Power (P) = $V_{rms} I_{rms} \cos\phi$ (watt)
2. Reactive Power (Q) = $V_{rms} I_{rms} \sin\phi$ (VAR) (volt ampere ...
3. Apparante Power (S) = $V_{rms} I_{rms}$ (VA) (volt ampere)

Power in three phase AC →

Star



The star connection -

$I_1 = I_2 = I_3 = I_{ph}$

$V_1 = V_2 = V_3 = V_{ph}$

$P_1 = P_2 = P_3 = P_T$

$\boxed{P_{star} = 3 V_{ph} I_{ph} \cos\phi}$

Delta

$P_3 = V_{rms} I_{r...}$

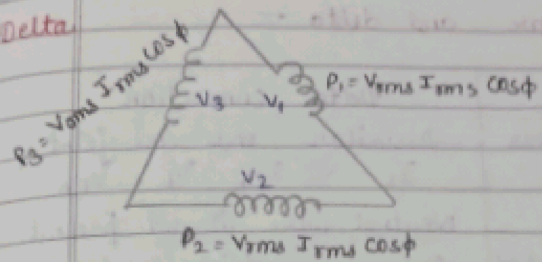
1. Activ...

2. Reacti...

3. App...

Date..................

Delta



Delta connection —

$V_1 = V_2 = V_3 = V_{ph}$

$I_1 = I_2 = I_3 = I_{ph}$

$P_1 = P_2 = P_3 = P_T$

$\boxed{P_{T_{delta}} = 3 V_{ph} I_{ph} \cos\phi}$

Star	Delta
$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$
$V_L = \sqrt{3} V_{ph}$	$V_L = V_{ph}$
$P_{ph} = 3 V_{ph} I_{ph} \cos\phi$	$P_{ph} = 3 V_{ph} I_{ph} \cos\phi$
$\boxed{P_L = \sqrt{3} V_L I_L \cos\phi}$	$\boxed{P_L = \sqrt{3} V_L I_L \cos\phi}$

1. Active power in term of phase

$\boxed{P_T = 3 V_{ph} I_{ph} \cos\phi}$

Active power in term of line

$\boxed{P_L = \sqrt{3} V_L I_L \cos\phi}$

2. Reactive power in term of phase

$\boxed{Q = 3 V_{ph} I_{ph} \cos\phi}$

Reactive power in term of line

$\boxed{Q = \sqrt{3} V_L I_L \cos\phi}$

3. Apprent power in term of phase —

$\boxed{S = 3 V_{ph} I_{ph}}$

Apprent power in term of line —

$\boxed{S = \sqrt{3} V_L I_L}$

Spiral

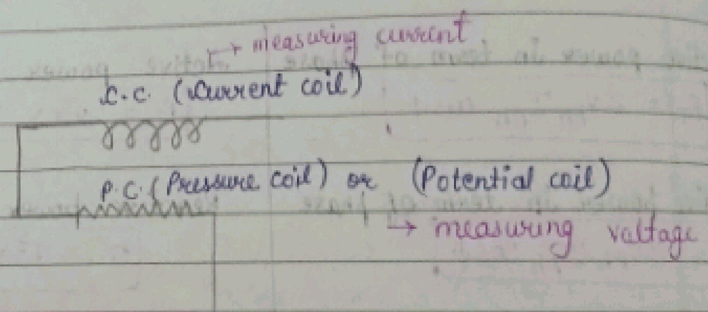
Comparision b/w star and delta :-

Star connection	Delta connection
→ Similar ends are joint together	Disimilar ends are joint
→ $I_L = I_{Ph}$, $V_L = \sqrt{3} V_{Ph}$	$I_L = \sqrt{3} I_{Ph}$ $V_L = V_{Ph}$
→ Neutral wire is present	Neutral wire is not present
→ Both domestic and industrial load can be handled.	Only industrial load can handled.
→ By earthing the neutral wire release and protective device can be provided by safetly.	Due to absence of the neutral wire it is not possible.

V.V.I.

Measurement of power in three phase AC system :-

Wattmeter :-



$$\boxed{P = VI\cos\phi}$$

Blondel's theorem :-

According to the blondel's theorem any 'N' phase balanced system the minimum no. wattmeter required for power measurement will be (N-1).

Whereas for 'N' phase unbalanced system the

Spiral

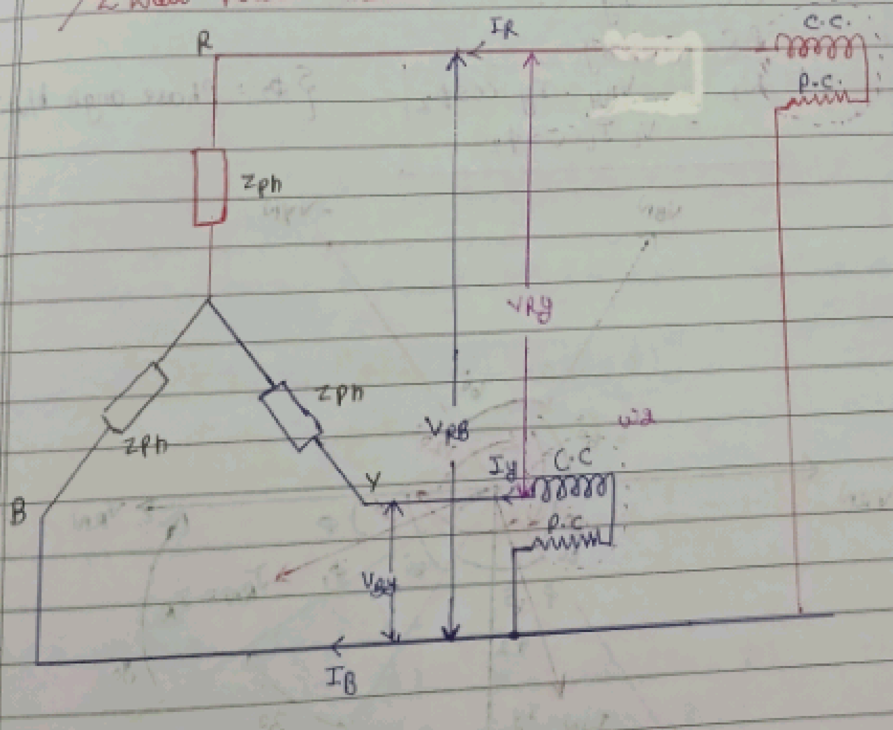
Date..................

minimum number of wattmeter required will be 'N'. Here,
N = number of phases

** Types of connected load in three phase (3ϕ) power measurement of balanced system :—

→ There are 4 types :
(i) Star connected lagging power factor load → Inductor
(ii) Star connected leading power factor load. → Capacitor
(iii) Delta connected lagging power factor load. → Inductor
(iv) Delta connected leading power factor load → Capacitor

1. Star connected lagging power factor load :—
/2 watt power method



Date..................

Since the system is balanced

$$\boxed{\begin{aligned} V_L &= V_{RY} = V_{YB} = V_{RB} \\ I_L &= I_R = I_y = I_B \end{aligned}}$$

Reading of wattmeter of W_1

$C.C. = I_R$

$P.C = V_{RB}$

$$\boxed{\begin{aligned} W_1 &= V_{RB}\, I_R \cos\phi_1 \\ &= V_L I_L \cos\phi_1 \end{aligned}}$$

$\{\phi_1 =$ Phase angle b/w V_{RB} & $I_R\}$

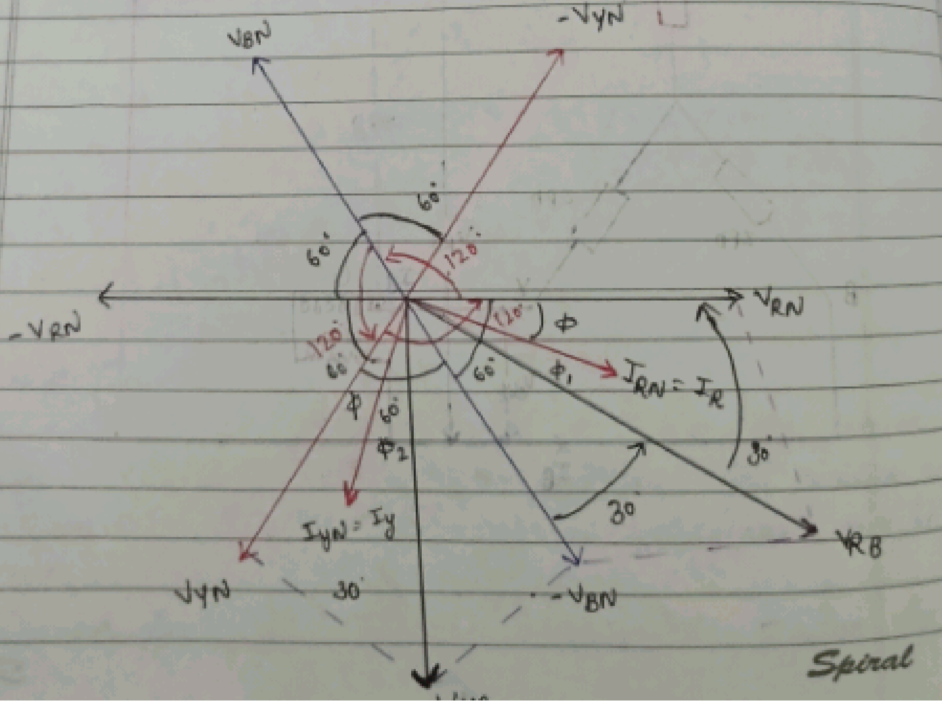
Reading of wattmeter of W_2

$C.C. = I_y$

$P.C = V_{By}$

$$\boxed{\begin{aligned} W_2 &= V_{By} \cdot I_y \cos\phi_2 \\ &= V_L I_L \cos\phi_2 \end{aligned}}$$

$\{\phi_2 =$ Phase angle b/w V_{By} & $I_y\}$



Spiral

Date..................

$\phi_1 = V_{RB} \,\&\, I_R$

$\phi_2 = V_{YB} \,\&\, I_Y$

for star connected balanced 3ϕ system –

$I_L = I_{Ph}$

$I_{RN} = I_R$

$\boxed{\phi_1 = 30^\circ - \phi}$

$\boxed{\phi_2 = 30^\circ + \phi}$

$W_1 = V_L I_L \cos(30^\circ - \phi)$ —— (1)

$W_2 = V_L I_L \cos(30^\circ + \phi)$ —— (2)

adding eq^n (1) and (2) ——

$\omega_1 + \omega_2 = V_L I_L \cos(30 - \phi) + V_L I_L \cos(30 + \phi)$

$\omega_1 + \omega_2 = V_L I_L [\cos(30 - \phi) + \cos(30 + \phi)]$

$\because \boxed{\cos C + \cos D = 2\cos\frac{C+D}{2} \cdot \cos\left(\frac{C-D}{2}\right)}$

$W_1 + W_2 = V_L I_L \left[2\cos\frac{(30 - \phi + 30 + \phi)}{2} \cdot \cos\frac{(30 - \phi - 30 - \phi)}{2}\right]$

$= V_L I_L [2\cos 30^\circ \cdot \cos(-\phi)]$

$= V_L I_L \left[2 \times \frac{\sqrt{3}}{2} \cdot \cos\phi\right]$

★ $\boxed{\omega_1 + \omega_2 = \sqrt{3} V_L I_L \cos\phi}$ —— (3)

subtracting eq^n (2) $-$ eq^n (1)

$\omega_2 - \omega_1 = V_L I_L \cos(30 + \phi) - V_L I_L \cos(30 - \phi)$

$= V_L I_L [\cos(30 + \phi) - \cos(30 - \phi)]$

Date..........

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\,\sin\frac{C-D}{2}$$

$$W_2 - W_1 = V_L I_L\left[-2\sin\frac{(30+\phi+30-\phi)}{2}\cdot\sin\frac{(30+\phi-30+\phi)}{2}\right]$$

$$= V_L I_L\left[-2\sin 30^\circ\cdot\sin\phi\right]$$

$$= V_L I_L\left[-2\times\frac{1}{2}\cdot\sin\phi\right]$$

$$= -V_L I_L \sin\phi$$

$$\boxed{W_1 - W_2 = V_L I_L \sin\phi} \quad (4)$$

Divide $eq^n\,(4)/(3)$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3}V_L I_L \cos\phi}$$

$$\boxed{\frac{W_1 - W_2}{W_1 + W_2} = \frac{1}{\sqrt{3}}\tan\phi}$$

Imp. ★ ⇒★ $$\boxed{\phi = \tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)}$$

★ Power factor :-

$$\boxed{\cos\phi = \cos\left[\tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]}$$