Ex - 27

(1) $\frac{dy}{dx} = y \tan x - 2\sin x$

$\frac{dy}{dx} - y \tan x = -2 \sin x$ — (1)

Which is lenias diff eq.

$P = -\tan x$, $Q = -2\sin x$

$I.f = e^{\int P dx} = e^{\int(-\tan x) dx} = e^{\log(\cos x)}$

$I.f = \cos x$

sol. of eq (1)

$y \times I.f = \int Q \times IF \, dx + c$

$y \times \cos x = \int -2\sin x \cos x \, dx + c$

$y \times \cos x = -2\int \sin x \cos x \, dx + c$

$y \times \cos x = -\int \sin(2x) \, dx + c$

$y \times \cos x = \frac{\cos 2x}{2} + c$

$y = \frac{\cos 2x + c}{2\cos x}$

$$y = \frac{\cos 2x}{2\cos x} + 2\sec x\, c$$

$$y = \frac{2\cos^2 x - 1}{2\cos x} + 2\sec x\, c$$

$$y = \cos x - \frac{1}{2}\sec x + 2\sec x\, c$$

$$y = \cos x + \frac{3}{2}\sec x\, c$$

$$\boxed{y = \cos x + K \sec x} \quad \left(\text{where } c = \frac{3}{2}\cdot c\right)$$

2. $\frac{dy}{dx} + y\tan x = \sec x$ — (1)

which is lenias diff. eq.

$I.f = e^{\int \tan x\, dx} = \sec x$

Sol. of eq (1)

$y \times I.f = \int Q \times If\, dx + c$

$y \times \sec x = \int \sec^2 x\, dx + c$

$\boxed{y \times \sec x = \tan x + c}$ Ans

3. $\frac{dy}{dx} + \frac{y}{x} = x^2$

which is lenias diff. eq.

$P = \frac{1}{x}$, $Q = x^2$

$I.f = e^{\int P dx} = e^{\log x} = x$

$y.x = \int x^2 . x\, dx + c$

$y.x = \frac{x^4}{4} + c$

$4y.x = x^4 + 4c$

ATQ x & $y = 1$ so $c = \frac{3}{4}x$

$\boxed{y = \frac{x^4}{4} + \frac{3}{4x}}$

4. $\frac{dr}{d\theta} = \alpha\theta^n - \frac{r}{\theta}$

$\frac{dr}{d\theta} + \frac{r}{\theta} = \alpha \cdot \theta^n$ — (1)

$P = \frac{1}{\theta}$, $Q = \alpha\theta^n$

which is lenias diff eq.

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$I.F = e^{\int \frac{1}{\theta} d\theta} = \theta$

Sol. of eq (1)

$r \times \theta = \int \alpha\, \theta^n \times \theta\, d\theta + c$

$r\theta = \alpha \int \theta^{n+1}\, d\theta + c$

$\boxed{r\theta = \alpha \frac{\theta^{n+2}}{n+2} + c}$ Ans

(5) $\frac{dy}{dx} + \frac{3x^2}{1+x^3} y = \frac{\sin^2 x}{1+x^3}$

Which is linear diff. eq.

$I.f = e^{\int \frac{3x^2}{1+x^3} dx}$

$I.F = e^{\log(1+x^3)}$ $\because \int \frac{F'(x)}{F(x)} = \log x$

$I.f = (1+x^3)$

Sol of eq (1)

$y \times (1+x^3) = \int \frac{\sin^2 x}{(1+x^3)} \times (1+x^3)\, dx$

$y \times (1+x^3) = \int \sin^2 x\, dx$

$y(1+x^3) = \frac{1}{2}\int (1-\cos 2x)\, dx$

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$\boxed{y(1+x^3) = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c}$

(6) $x(x-1)\frac{dy}{dx} - (x-2)y = x^3(2x-1)$

$\frac{dy}{dx} - \frac{(x-2)}{x(x-1)} y = \frac{x^3(2x-1)}{x(x-1)}$

$P = \frac{-(x-2)}{x(x-1)}$, $Q = \frac{x^3(2x-1)}{x(x-1)}$

$I.F = e^{\int \frac{-(x-2)}{x(x-1)} dx} = \frac{x-1}{x^2}$

$y\left(\frac{x-1}{x^2}\right) = \int \frac{x^3(2x-1)}{x(x-1)} \cdot \left(\frac{x-1}{x^2}\right) dx$

$\boxed{y = \frac{x^2(x^2-x+c)}{x-1}}$ Ans

(7) $(x^2+1)\frac{dy}{dx} + 2xy = 4x^2$

$\frac{dy}{dx} + \frac{2xy}{(x^2+1)} = \frac{4x^2}{x^2+1}$

$P = \frac{2x}{(x^2+1)}$, $Q = \frac{4x^2}{x^2+1}$

$I.f = e^{\int P dx} = x^2+1$

$y \cdot (x^2+1) = \int \frac{4x^2}{x^2} \ldots$

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$y.(x^2+1) = \int 4x^2\,dx + c$

$y.x^2+1 = \frac{4x^3}{3} + c$

$$\boxed{y = \frac{\frac{4x^3}{3} + c}{x^2+1}}$$

(8) $\cos^2 x \frac{dy}{dx} + y = \tan x$

$\frac{dy}{dx} + \frac{y}{\cos^2 x} = \tan x$

$P = \sec^2 x,\ Q = \tan x$

$I.F = e^{\int P dx} = e^{\int \sec^2 x\,dx}$

$I.F = e^{\tan x}$

$e^{\tan x}.y = (\tan x - 1).e^{(\tan x)} + c$

$$\boxed{y = \tan(x-1) + c.e^{(-\tan x)}}$$

(9) $x\cos x \frac{dy}{dx} + y(x\sin x + \cos x) = 1$

$\frac{dy}{dx} + y\left[\frac{x\sin x}{x\cos x} + \frac{\cos x}{x\cos x}\right] = \frac{1}{x\cos x}$

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$\frac{dy}{dx} + y\left[\tan x + \frac{1}{x}\right] = \frac{1}{x\cos x}$ —— (1)

which is linear diff. eq.

$I.f = e^{\int(\frac{1}{x} + \tan x)dx}$

$I.f = e^{\log x + \log \sec x} = e^{\log(x.\sec x)}$

$I.f = x\sec x$

Sol. of eq (1)

$y \times x\sec x = \int \frac{1}{x} \sec x \times x \sec x\,dx + c$

$= \int \sec^2 x\,dx + c$

$\boxed{y\,x\sec x = \tan x + c}$ Ans

(10) $x\log x \frac{dy}{dx} + y = 2\log x$

$\frac{dy}{dx} + \frac{y}{x\log x} = 2\log x$

$I.f = e^{\int P dx} = e^{\int \frac{1}{x\log x}} = \log x$

$y.\log x = \int 2\log x.\log x\,dx + c$

$y.\log x = \int 2(\log x)^2 dx + c$

$y \cdot \log x = 2 \times 2x \log x + 2x + c$

$y \cdot 4\log x = 4x \log x + 2x + c$

$$\boxed{y = \frac{\log x \, 4x + 2x + c}{4\log x}}$$

(11) $\sin y \frac{dy}{dx} = \cos y (1 - x\cos y)$

$\frac{dy}{dx} = \cot y (1 - x\cos y)$

$\frac{dy}{dx} = \cot y - x \times \cos y \times \cot y$

$\frac{dy}{dx} = \cot y - x \times \cot y \cos y$

$\frac{dy}{dx} - \cot y = -x \cot y \cos y$

$\tan y \sec y \frac{dy}{dx} - \sec y = -x$ — (1)

let $\sec y = v$

diff. w.r.t x

$\sec y \tan y \frac{dy}{dx} = \frac{dv}{dx}$

Put in eq (1)

$\frac{dv}{dx} - v = -x$

Which is Linear

$I.f = e^{\int dx} = e^{-x}$

$v \times e^{-x} = -\int x \times e^{-x} dx + c$

$v \times e^{-x} = -\left[x \times \frac{e^{-x}}{-1} - 1 \times e^{-x}\right] + ce^{x}$

$v = x + 1 + ce^{x}$

$\sec y = x + 1 + ce^{x}$

(12) $\sin 2x \frac{dy}{dx} = y + \tan x$

$\frac{dy}{dx} - \frac{y}{\sin 2x} = \frac{\tan x}{\sin 2x}$

$I.f = e^{\int \frac{1}{\sin 2x} dx} = e^{-\left(\frac{1}{2} \log \tan x\right)}$

$I.f = \frac{1}{\sqrt{\tan x}}$

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$$y \times \frac{1}{\sqrt{\tan x}} = \int \frac{\tan x}{\sin 2x} \cdot \frac{1}{\sqrt{\tan x}}\, dx + c$$

$$y \cdot \frac{1}{\sqrt{\tan x}} = \int \frac{\sqrt{\tan x}}{\sin 2x}\, dx + c$$

$$\boxed{y \cdot (\sqrt{\tan x})^{-1} = \sqrt{\tan x} + c}$$

(13) $(1+x^2)\frac{dy}{dx} + 2yx - 4x^2 = 0$

$$\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{4x^2}{1+x^2} \quad \text{—— (1)}$$

which is linear diff. eq.

$$I.f = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)}$$

$$I.f = 1+x^2$$

Sol. of eq (1) is

$$y \times (1+x^2) = 4\int \frac{x^2}{1+x^2}(1+x^2)\,dx + c$$

$$y \times (1+x^2) = 4 \times \int x^2\, dx + c$$

$$y(1+x^2) = \frac{4}{3}x^3 + c$$

Put $x = 0$, $y = 0$

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$$0(1+0) = \frac{4}{3} \times 0 + c$$

$$\boxed{c = 0}$$

$$\boxed{3y(1+x^2) = 4x^3}$$

(14) $(x+2y^3)\frac{dy}{dx} = y$

$$\frac{dy}{dx} = \frac{y}{x+2y^3}$$

$$\frac{dy}{dx} - \frac{y}{x+2y^3} = 0$$

$$I.f = e^{\int P\,dx} = e^{\int \frac{1}{x+2y^3} dx} = e^{\log(x+2y^3)}$$

$$I.f = x+2y^3$$

$$y \cdot x + 2y^3 = 0 + c$$

$$\boxed{y \cdot x = -2y^3 + c}$$

(15) $(y-x)\frac{dy}{dx} = a^2$

$$\frac{dx}{dy} = \frac{y-x}{a^2}$$

$$\frac{dx}{dy} + \frac{1}{a^2}x = \frac{y}{a^2} \quad \text{—— (1)}$$

$$I.F = e^{\int \frac{1}{a^2} dy} = e^{1/a^2 y}$$

sol. of eq (1)

$$x \times e^{1/a^2 y} = \frac{1}{a^2} \times \int y \times e^{1/a^2 y} dy + c$$

$$x \times e^{1/a^2 y} = \frac{1}{a^2} \left[y \times a^2 e^{1/a^2} - 1 \times a^4 e^{1/a^2} \right] + ce^{a^{-1/a^2 y}}$$

$$x = \frac{1}{a^2} \left[ya^2 - a^4 \right] + ce^{-1/a^2 y}$$

$$\boxed{x = y - a^2 + ce^{-1/a^2 y}}$$

(16) $2(x - 5y^3)\frac{dy}{dx} + y = 0$

$$\frac{dy}{dx} + \frac{y}{2(x-5y^3)} = 0$$

$$I.F = e^{\int P dx} = e^{\int \frac{1}{2(x-5y^3)} dx}$$

$$= e^{\log (x-5y^3)^{1/2}}$$

$$I.F = \sqrt{x - 5y^3}$$

$$y \cdot \sqrt{x - 5y^3} = \int 0 \, dx + c$$

$$y \cdot \sqrt{x - 5y^3} = c$$

(17) $\frac{dy}{dx} + \frac{1-2x}{x^2} y = 1$

$$I.F = e^{\int \frac{1-2x}{x^2} dx}$$

$$= e^{\int \left(\frac{1}{x^2} - \frac{2}{x}\right) dx}$$

$$= e^{\int \left(\frac{1}{x^2} - \frac{2}{x}\right) dx}$$

$$= e^{\left(\left(-\frac{1}{x}\right) - 2\log x\right)}$$

$$= e^{-1/x} \times e^{-2\log x}$$

$$= e^{-1/x} \times e^{\log 1/x^2}$$

$$= \frac{1}{x^2} e^{-1/x}$$

solution is

$$y \times \frac{1}{x^2} e^{-1/x} = \int \frac{1}{x^2} e^{-1/x} dx + c$$

let $-\frac{1}{x} = 4$

diff. w.r.t x

$\frac{1}{x^2}dx = dv$

$y \times \frac{1}{x^2} e^{-1/x} = \int e^{v} dv + c = e^{v} + c$

$y \times \frac{1}{x^2} e^{-1/x} = e^{-1/x} + c$

$\frac{y}{x^2} = 1 + ce^{1/x}$

$$\boxed{y = x^2(1 + ce^{1/x})}$$

(18) $\frac{dy}{dx} + \frac{x}{(1+x^2)} \cdot y = \frac{1}{2x(1+x^2)}$

$P = \frac{x}{1+x^2}$, $Q = \frac{1}{2x(1+x^2)}$

I.F. $= e^{\int P dx} = e^{\int \frac{x}{1+x^2} dx} = e^{\log(1+x^2)^{1/2}}$

I.F. $= (1+x^2)^{1/2}$

$y \times \sqrt{1+x^2} = \int \frac{\sqrt{1+x^2}}{2x(1+x^2)} \cdot dx + c$

$y \times \sqrt{1+x^2} = \int \frac{1}{2x\sqrt{1+x^2}} \cdot dx + c$

$y \times \sqrt{1+x^2} = \frac{1}{2}\int \frac{1}{\tan\theta \sqrt{1+\tan^2\theta}} \cdot \sec^2\theta \, d\theta + c$

$y \times \sqrt{1+x^2} = \frac{1}{2}\int \frac{1}{\tan\theta \sec\theta} \cdot \sec^2\theta \, d\theta + c$

$= \frac{1}{2}\int \frac{\sec\theta}{\tan\theta} d\theta + c$

$= \frac{1}{2}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}} d\theta + c$

$= \frac{1}{2}\int \operatorname{cosec}\theta \, d\theta + c$

$= \frac{1}{2} \log\left|\frac{\tan\theta}{2}\right| + c$

$$\boxed{y \times \sqrt{1+x^2} = \frac{1}{2}\log\left|\frac{\tan^{-1}x}{2}\right| + c}$$

(19) $(1 + y + x^2 y)dx + (x + x^3)dy = 0$

$(x + x^3)dy = -(1 + y + x^2 y)dx$

VVI

$$\frac{dy}{dx} = \frac{-[1+y(1+x^2)]}{x(1+x^2)}$$

$$= -\left[\frac{1}{x(1+x^2)} + \frac{y(1+x^2)}{x(1+x^2)}\right]$$

$$\frac{dy}{dx} + \frac{y}{x} = -\frac{1}{x(1+x^2)}$$

$$P = \frac{1}{x}, \quad Q = \frac{-1}{x(1+x^2)}$$

$$I.F = e^{\int P dx} = e^{\int \frac{1}{x} dx} = x$$

$$y \times x = \int \frac{-1}{x(1+x^2)} \times x \, dx + c$$

$$\boxed{xy = -\tan^{-1} x + C}$$

VVI

(20) $(1+y^2) + (x - e^{-\tan^{-1} y})\frac{dy}{dx} = 0$

$$(1+y^2) = \frac{-(x-e^{-\tan^{-1}y})}{1+y^2} \frac{dy}{dx}$$

$$\frac{dx}{dy} = \frac{-(x - e^{-\tan^{-1} y})}{1+y^2}$$

$$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{-\tan^{-1} y}}{1+y^2} \quad (1)$$

which is linear diff. eq.

$$I.F = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

Solution of eq (1)

$$x \times e^{\tan^{-1} y} = \tan^{-1} y + c$$

$\boxed{\text{Ex } 28}$

(1) $x\frac{dy}{dx} + y = xy^3$

$$\frac{dy}{dx} + \frac{y}{x} = y^3 \quad (1)$$

$$P = \frac{1}{x}, \quad Q = 1, \quad n = 3$$

$$v = y^{1-n} = y^{1-3} = y^{-2} = \frac{1}{y^2}$$

$$\frac{dv}{dx} = -2y^{-3}\frac{dy}{dx}$$

multiply by eq (1)

$$-2y^{-3} \times \frac{dy}{dx} - \frac{2y^{-3}}{x} \cdot y = -2y^{-3} \cdot y^3$$

$$-2y^{-3} \times \frac{dy}{dx} - \frac{2}{x} y^{-2} = -2$$

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$$\frac{dv}{dx} - \frac{2}{x}v = -2$$

$$I.F = e^{\int \frac{-2}{x} dx} = x^{-2}$$

$$v \cdot x^{-2} = \int x^{-2} \cdot -2$$

$$v \cdot x^{-2} = \frac{2x^{-1}}{1}$$

$$x^{-2} \cdot y^{-2} = 2x^{-1}$$

(2) $\frac{dy}{dx} + \frac{y}{x} = x^2 y^6$

$$\frac{1}{y^6}\frac{dy}{dx} + \frac{y}{x \cdot y^6} = x^2$$

$\frac{1}{y^5} = t$

$$-\frac{1}{5}\frac{dt}{dx} + \frac{1}{x} \cdot t = x^2$$

$\frac{-5}{y^6}\frac{dy}{dx} = \frac{dt}{dx}$

$$\frac{dt}{dx} - \frac{5}{x} t = -5x^2$$

$\frac{1}{y^6} = \frac{dy}{dx} = -\frac{1}{5}\frac{dt}{dx}$

$$\frac{dt}{dx} + Pt = Q$$

$$P = \frac{-5}{x}, \quad Q = -5x^2$$

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$$I.F = e^{\int P dx} = e^{\int \frac{-5}{x} dx} = x^{-5}$$

$$t \times x^{-5} = \int -5 x^{+2} \cdot x^{-5} dx + c$$

$$t \times x^{-5} = \int -5 x^{-3} dx + c$$

$$t \times x^{-5} = \frac{+5 x^{-2}}{+2} + c$$

$$\frac{x^{-5}}{y^5} = \frac{5x^{-2}}{2} + c$$

$$x^{-5} y^{-5} = \frac{5x^{-2}}{2} + c$$

(3) $(1-x^2)\frac{dy}{dx} + xy = xy^2$

$$\frac{dy}{dx} + \frac{x}{1-x^2} \times y = \frac{x}{1-x^2} \cdot y^2 \quad \text{--- (1)}$$

$$y^{-2}\frac{dy}{dx} - \frac{x}{x^2-1} \times y^{-1} = \frac{-x}{x^2-1} \quad \text{--- (2)}$$

let $y^{-1} = t$

diffe. both side wrt x

$$\to y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$$

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$$y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$$

Put in eq (2)

$$-\frac{dv}{dx} - \frac{x}{x^2-1} \times v = \frac{-x}{x^2-1}$$

$$\frac{dv}{dx} + \frac{x}{x^2-1} \cdot v = \frac{x}{x^2-1} \quad \text{--- (3)}$$

which is lenior

$$I.F. \; e^{\frac{1}{2}\int \frac{2x}{x^2-1}dx} = e^{\frac{1}{2}\log x(x^2-1)}$$

$$I.F = \sqrt{x^2-1}$$

solve

$$v \times \sqrt{x^2-1} = \int \frac{x}{(x^2-1)} \times \sqrt{x^2-1}\,dx + c$$

$$v \times \sqrt{x^2-1} = \int \frac{x}{\sqrt{x^2-1}}\,dx + c$$

let $x^2-1 = t^2$

$$2x\,dx = 2t\,dt$$

$$x\,dx = t\,dt$$

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$$v \times \sqrt{x^2-1} = \int \frac{t\,dt}{t} + c$$

$$v \times \sqrt{x^2-1} = \int dt + c$$

$$v \times \sqrt{x^2-1} = t + c$$

$$\frac{1}{y} \times \sqrt{x^2-1} = \sqrt{x^2-1} + c$$

$$\boxed{\frac{1}{y} = 1 + \frac{c}{\sqrt{x^2-1}}}$$

(4) $$\frac{dy}{dx} + \frac{y}{x} = y^2 \sin x$$

$$y^{-2}\frac{dy}{dx} + \frac{1}{x} \times \frac{1}{y} = \sin x \quad \text{--- (1)}$$

let $\frac{1}{y} = v$

diff. w.r.t x

$$y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$$

Put in (I)

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$$-\frac{dv}{dx} + \frac{1}{x} v = \sin x$$

$$\frac{dv}{dx} - \frac{1}{x} v = -\sin x$$

which is Leibnitz diff. eq.

$I.f = e^{\int -1/x \, dx} = e^{-\log x} = \frac{1}{x}$

Sol. is

$$v \times \frac{1}{x} = \int -\frac{1}{x} \sin x \, dx + c$$

$$\frac{1}{xy} = -\int \frac{1}{x} \sin x \, dx + c$$

(5) $$\frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2}$$

$P = -\frac{1}{x}$, $Q = \frac{1}{x^2}$, $n = 2$

$v = y^{1-n} = y^{1-2} = y^{-1}$

$$\frac{dv}{dx} = -y^{-2}\frac{dy}{dx}$$

$$-y^{-2}\frac{dy}{dx} + \frac{y^{-1}}{x} = -\frac{y^{-1}}{x^2}$$

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$\frac{dv}{dx}$

$$\frac{1}{y^2}\cdot\frac{dy}{dx} - \frac{y}{y^2 \cdot x} = \frac{1}{x^2}$$

$$\frac{dy}{dx}\cdot\frac{1}{y^2} - \frac{1}{y \cdot x} = \frac{1}{x^2}$$

$$-\frac{dv}{dx} - \frac{v}{x} = \frac{1}{x^2}$$

$$\frac{dv}{dx} + \frac{v}{x} = -\frac{1}{x}$$

$I.f = e^{\int P dx} = x$

$$v \cdot x = \int -\frac{1}{x} \cdot x \, dx + c$$

$$v \cdot x = -x + c$$

$$y^{-1} \cdot x = -x + c$$

(6) $$3\frac{dy}{dx} + \frac{2}{(x+1)} \cdot y = \frac{x^3}{y^2} \quad \text{--- (1)}$$

$n = -2$

$v = y^{1-n} = y^{1+2} = y^3$

$$\frac{dv}{dx} = 3y^2 \cdot \frac{dy}{dx}$$

multiply y^2 both side in eq ①

$$3y^2\frac{dy}{dx} + \frac{2y^3}{(x+1)} = x^3$$

$$\frac{dv}{dx} + \frac{2v}{(x+1)} = x^3$$

$$I.f. = e^{\int \frac{1}{(x+1)} dx} = 1+x$$

$$(1+x) \cdot v = \int \left(\frac{1+x}{x^3}\right) dx + c$$

$$(1+x) \cdot v = \int \left(\frac{1}{x^3} + \frac{1}{x^2}\right) dx + C$$

$$(1+x) \cdot v = \frac{x^{-2}}{-2} + \frac{x^{-1}}{-1} + C$$

$$(1+x) \cdot v = \frac{x^{-2}}{-2} - x^{-1} + C$$

$$y^3(1+x) = \frac{x^{-2}}{-2} - x^{-1} + C$$

⑦ $$\frac{dy}{dx} + \frac{1}{x}\tan y = \frac{1}{x^2}\frac{\sin^2 y}{\cos y}$$

devide both side by $\frac{\cos y}{\sin^2 y}$

$$\frac{\cos y}{\sin^2 y} \cdot \frac{dy}{dx} + \frac{1}{x}\tan y \cdot \frac{\cos y}{\sin^2 y} = \frac{1}{x^2}$$

$$\cot y \operatorname{cosec} y \frac{dy}{dx} + \frac{1}{x}\tan y \cdot \cot y \times \frac{1}{\sin y} = \frac{1}{x^2}$$

$$\cot y \operatorname{cosec} y \frac{dy}{dx} + \frac{1}{x}\tan y \cdot \cot y \times \frac{1}{\sin y} = \frac{1}{x^2}$$

$$\cot y \operatorname{cosec} y \frac{dy}{dx} + \frac{1}{x}\tan y \cdot \cot y \frac{1}{\sin y} = \frac{1}{x^2}$$

$$\cot y \operatorname{cosec} y \frac{dy}{dx} + \frac{1}{x}\operatorname{cosec} y = \frac{1}{x^2} \quad ①$$

let $\operatorname{cosec} y = v$

$$\frac{-dv}{dx} = \cot y \operatorname{cosec} y \frac{dy}{dx}$$

from eq ①

$$\frac{-dv}{dx} + \frac{1}{x} \cdot v = \frac{1}{x^2}$$

$$\frac{dv}{dx} - \frac{1}{x}v = -\frac{1}{x^2}$$

which is lenial diff. eq^n

$$I.F. = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$$

Sol is

$$v \times \frac{1}{x} = -\int \frac{1}{x^2} \times \frac{1}{x}\, dx + c$$

$$\frac{v}{x} = -\int \frac{1}{x^3}\, dx + c$$

$$\frac{v}{x} = \frac{1}{2x^2} + c$$

$$v = \frac{1}{2x} + cx$$

$$\boxed{\operatorname{cosec} y = \frac{1}{2x} + cx}$$

(8) $\frac{dy}{dx} = \frac{1}{xy(x^2y^2+1)}$

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9. $\cos x \frac{dy}{dx} = y(\sin x - y)$

$\cos x \frac{dy}{dx} - y \sin x = -y$

$\frac{dy}{dx} - y \frac{\sin x}{\cos x} = -y$

$\frac{dy}{dx} - y \tan x = y$

I.F. $e^{\int P dx} = e^{\int \tan x dx} = \sec x$

$\sec x \cdot y = \int y \cdot \sec x \, dx + c$

10. $y(2xy + e^x) dx - e^x dy = 0$

$y(2xy + e^x) dx = e^x dy$

$= (2xy^2 + e^x \cdot y) dx = e^x dy$

$= 2xy^2 dx = -ye^x dx + e^x dy$

$2x \, dx = \frac{-ye^x dx + e^x dy}{y^2}$

$2x \, dx = ye^x dx - e^x dy$

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$2x \, dx = -\frac{y \, d(e^x) - e^x dy}{y^2}$

$\int 2x \, dx = -\int d\left(\frac{e^x}{y}\right)$

$\frac{2x^2}{2} = -\frac{e^x}{y} + c$

$\boxed{x^2 + \frac{e^x}{y} = c}$ Ans

Ex. 29

1. $(1 + 4x + 2y^2) dx + (1 + 4xy + 2x^2) dy = 0$ — (i)

Here $m = 1 + 4xy + 2y^2$ — (ii)
$n = 1 + 4xy + 2x^2$ — (iii)

Partialy diff. eq (ii) & (iii)

$\frac{dm}{dy} = 4x + 4y \quad \Big| \quad \frac{dn}{dy} = 4x + 4y$

$\boxed{\frac{dm}{dy} = \frac{dn}{dx}} \Rightarrow$ Exact

$\int_{y \text{ constant}} 1 + 4xy + 2y^2 dx + \int_{\text{w.r. of } x} 1 \, dy$

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$= x + 2x^2y + 2xy^2 + y = c$

Q.2 $(x^4 - 2y^2x + y^4)\,dx - (2x^2y - y^3 ...)$

$m = x^4 - 2xy^2 + y^4$ — ①

$N = -2x^2y + 4xy^3 - \sin y$ — ②

diff. w.r.t y (partialy)

$\frac{dm}{dy} = 0 - 4xy + 4y^3$ — ③

eq ② diff w.r.t x (partialy)

$\frac{dN}{dx} = -4xy + 4y^3 - 0$ — ④

from eq ③ & ④

$$\boxed{\frac{dm}{dy} = \frac{dN}{dx}}$$

sol. of eq — ⓐ is

$\frac{dm}{dy} = \int (x^4 - 2xy^2 + y^4)\,dx + \int_{\text{w.r.t. of } x} \sin y\,dy$

$= \frac{x^5}{5} - \frac{2x^2}{2}y^2 + y^4x + \cos y = c$

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$x^5 - 5y^2 + 5xy^4 + 5\cos y = c$

② $x(x^2+3y^2)\,dx + y(y^2+3x^2)\,dy = 0$

$y(y^2+3x^2)\,dy = -x(x^2+3y^2)\,dx$

$\frac{dy}{dx} = \frac{-x(x^2+3y^2)}{y(y^2+3x^2)}$

$\frac{dm}{dy} = 3x^2+3y^2$, $\frac{dN}{dx} = 3x^2+3y^2$

$\frac{dm}{dy} = \frac{dN}{dx} \Rightarrow$ exact

$\int_{y\ \text{constant}} (x^3 + 3xy^2)\,dx + \int_{\text{w.r.t. of } x} y^3\,dy = c$

$= \frac{x^4}{4} + \frac{3x^2}{2}y^2 + \frac{y^4}{4} = c$

④ $(2ax + by)\,y\,dx + (ax + 2by)\,x\,dy = 0$

$m = 2axy + by^2$, $n = ax^2 + 2bxy$

$\frac{dm}{dy} = 2ax + 2by$, $\frac{dn}{dx} = 2ax + 2by$

$\boxed{\frac{dm}{dy} = \frac{dn}{dx}} \Rightarrow$ exact

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$\int_{y\ \text{constant}} 2axy + by^2 dx + \int_{w.r.t.\ of\ x} y^3 dy = c$

$= ax^2y + by^2x + \frac{y^4}{4} + c$

(5) $(x^2 - ay)dx - (ax - y^2)dy = c$

$\frac{dm}{dy} = -a \quad , \quad \frac{dn}{dx} = -a$

$\int x^2 - ay\, dx + \int -(ax - y)^2 dy = c$

$\frac{x^3}{3} + \frac{y^3}{3} = axy + c$

(6) $\cos x(\cos x - \sin\alpha \sin y)dx + \cos y(\cos y$
$\sin\alpha \sin x)dy = 0$

$m = \cos^2 x - \sin\alpha \sin y\, dx$

$\frac{dm}{dy} = 0 - \sin\alpha \frac{\cos y}{\cos x}$

$\frac{dn}{dx} = -\sin\alpha \cos y \cos x$

$\boxed{\frac{dm}{dy} = \frac{dn}{dx}} \Rightarrow$ Exact

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$= \int_{y\ \text{constant}} \cos^2 x - \sin\alpha \sin y \cos x\, dx + \int_{w.r.t.\ of\ x} \cos^2 y - \sin\alpha \cos y\, dy$

$= \frac{\sin 2x}{2} - \sin\alpha \sin x - \sin x + \frac{\sin^2 y}{2}$

$= \frac{\sin 2x}{2} + \sin\alpha \sin x \sin x + \frac{\sin^2 y}{2} + c$

(7) $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

$m = (e^y + 1)\cos x \qquad n = e^y \sin x$

Partial diff w.r.t y — Partial diff. w.r.t x

$\frac{dm}{dy} = e^y \cos x \qquad \frac{dn}{dx} = e^y \sin x$

$\boxed{\frac{dm}{dy} = \frac{dn}{dx}} \Rightarrow$ Exact

$\int_{y\ \text{constant}} (e^y + 1)(\cos x)dx + \int_{w.r.t.\ of\ x} e^y dy = c$

$= (e^y + x) - \sin x + e^y = c$

(8) $(x^2 + y^2)dx + 2xy\, dy = c$

$m = x^2 + y^2 \qquad n = 2xy$

$\frac{dm}{dy} = 2x \qquad \frac{dn}{dy} = 2x$

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$\boxed{\frac{dm}{dy} = \frac{dn}{dx}}$ ⇒ Exact

$= \int_{y\ constant} x^2 + y\,dx + \int_{w.r.t.\ of\ x} 1\,dy = c$

$= \frac{x^3}{3} + y^2 + y = c$

(9) $(1+e^{x/y})\,dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$ (1)

where $m = (1+e^{x/y})$ — (2)

$n = e^{x/y}\left(1-\frac{x}{y}\right)$ — (3)

diff. eq (2) w.r.t y (Partially)

$\frac{dm}{dy} = 0 + e^{x/y} \times \left(\frac{-x}{y^2}\right)$ — (4)

diff. eq (3) w.r.t x (Partially)

$\frac{dn}{dx} = e^{x/y}\left[0-\frac{1}{y}\right] + \left[1-\frac{x}{y}\right] e^{x/y}\cdot\frac{1}{y}$

$\frac{dN}{dx} = \frac{-x}{y^2} e^{x/y}$ — (5)

from eq (4) & (5)

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$\boxed{\frac{dm}{dy} = \frac{dN}{dx}}$ ⇒ Exact

solution of (1) is

⇒ $\int_{y\ constant} (1+e^{x/y})\,dx + \int_{w.r.t.\ of\ x} 0\,dy = c$

⇒ $x + \frac{e^{x/y}}{1/y} + c_1 = c$

⇒ $x + ye^{x/y} = c - c_1 = K$

(10) $[y[1+(1/x)] + \cos y]\,dx + [x + \log x - x \sin y]\,dy = 0$

$m = y\left(1+\frac{1}{x}\right) + \cos y$ | $n = x + \log x - x \sin y$

$\frac{dm}{dy} = 1 + \frac{1}{x} - \sin y$ | $\frac{dn}{dx} = 1 + \frac{1}{x} - \sin y \cdot 1$

$\frac{dn}{dx} = 1 + \frac{1}{x} - \sin y$

$\boxed{\frac{dm}{dy} = \frac{dn}{dx}}$ ⇒ Exact

$\int_{y\ constant} y[1+(1/x)] + \cos y\,dx + \int_{w.r.t.\ of\ x} 0\,dy = c$

$= y(x + \log x) + \cos y \cdot x = c$

Ex 33

(1) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0$

$D^2y - 4Dy + y = 0$

$(D^2 - 4D + 1) = 0$

Put $D = m$

$m^2 - 4m + 1 = 0$

$C.F = e^{2x}[C_1 \cosh\sqrt{3}x + C_2 \sinh\sqrt{3}x]$

Sol. is

$y = CF + PI$

$y = e^{2x}[C_1\cosh\sqrt{3}x + C_2\sinh\sqrt{3}x]$

(2) $\frac{d^2y}{dx^2} + y = 0$, given $y = 2$ for $x = 0$ &
$y = -2$ for $x = \pi/2$

$A \cdot E = m^2 + 1 = 0 \Rightarrow m^2 - 1$

$m = \pm\sqrt{-1}$

$m = 0 \pm \sqrt{i}$

$CF = e^{0x}[C_1\cos x + C_2\sin x]$

$Cf = [C_1\cos x + C_2\sin x]$

Put $x = 0, y = 2$

$C_1\cos 0 + C_2\sin 0 = 2$

$\boxed{C_1 = 2}$

Put $x = \frac{\pi}{2}, y = -2$

$-2 = C\cos\frac{\pi}{2} + C_2\sin\frac{\pi}{2}$

$C_2 = -2$

$\boxed{y = 2\cos x - 2\sin x}$

(3) $\frac{d^2y}{dx^2} + 2P\frac{dy}{dx} + (P^2+q^2)y = 0$

$D^2y + 2PDy + (P^2+q^2)y = 0$

$A.E = m^2 + 2Pm + P^2 + q^2$

$CF = e^{\alpha x}[C_1\cosh\sqrt{\beta}x + C_2\sinh\sqrt{\beta}x]$

$e^{-px}[c_1\cos qx + c_2\sin qx]$

(4) $\frac{d^2y}{dx^2} - \frac{7dy}{dx} + 12y = 0$

$D^2y - 7Dy + 12y = 0$

A.E $m^2 - 7m + 12 = 0$

$m_1 = 3, \quad m_2 = 4$

$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$CF = c_1 e^{3x} + c_2 e^{4x}$

(5) $\frac{d^2x}{dt^2} - \frac{3dx}{dt} + 2x = 0$ given $t=0, x=0$ & $\frac{dx}{dt} = 0$

$\frac{dx}{dt} = 0$

so $x = 0$

(6) $(D^3 - D^2 - D + 1)y = 0$

A.E $m^3 - m^2 - m + 1 = 0$

$m = 1$

$(m^3 - m^2) - (m+1)$

$m^2(m-1) - 1(m-1)$

$(m-1)(m^2-1)$

$(m-1)(m-1)(m+1)$

$(m-1)^2(m+1)$

$m = 1, \quad m = -1$

$\boxed{CF = c_1 e^{-x} + c_2 e^{x}}$

(7) $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

$D^3y - 6D^2y + 11Dy - 6y = 0$

$m^3 - 6m^2 + 11m - 6 = 0$

$m = 1$

$(m^3 - 6m^2) + (11m - 6)$

$\cancel{m^2(m-6)+}$

$m = 1, 2, 3$

$CF = c_1 e^{x} + c_2 e^{2x} + c_3 e^{3x}$

(8) $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$

A.E $m^4 + 8m^2 + 16 = 0$

~~$m^2(m^2+8) + 16$~~

let $y = m^2$

$y^2 + 8y + 16$

$y = -4$

$y = \pm 2i$

C.f $= e^0 [c_1 \cos i2 + c_2 \sin 2x]$

(9) $(D^4 - 2D^2 + 1)y = 0$

A.E $m^4 - 2m^2 + 1 = 0$

let $y = m^2$

$y^2 - 2y + 1 = 0$

$y = 1$

C.f $= (C_1 + C_2 x)e^{m_1 x} + C_3 e^{m_3 x} + C_4 e^{m_4}$

Ex. 34

C.f $= (c_1 + c_2 x)e^{x} + (c_3 + c_4 x)e^{-x}$

Ex 34

(1) $\frac{d^2y}{dx^2} - 4y = e^x + \sin 2x$

A.E $= m^2 - 4 = 0$

$m = \pm 2$

Cf $= c_1 e^{2x} + c_2 e^{-2x}$

$PI = \frac{1}{D^2-4} e^x + \sin 2x$

$P.I = \frac{1}{D^2-4} e^x + \frac{1}{D^2-4} \sin 2x$

$PI = \frac{1}{1-4} e^x + \frac{1}{-4-4} \sin 2x$

$P.I = \frac{1}{-3} e^x + \frac{1}{-8} \sin 2x$

$Y = Cf + PI$

$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{3} e^x - \frac{1}{8} \sin 2x$

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(2) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = a \sin 2x$

A.E $= m^2 - 4m + 1 = 0$

$= \frac{+4 \pm \sqrt{16-4}}{2}$

$= \frac{4 \pm \sqrt{12}}{2}$

$= \frac{4 \pm 2\sqrt{3}}{2}$

$= 2 \pm \sqrt{3}$

Cf $= e^{x}[c_1 \cosh \sqrt{3}x + c_2 \sinh \sqrt{3}x]$

Cf $= c_1 e^2 \cosh(\sqrt{3}x + c_2)$

$PI = \frac{1}{D^2 - 4D + 1} \cdot a \sin 2x$

$P.I = a \cdot \frac{1}{D^2 - 4D + 1} \sin 2x$

$PI = a \cdot \frac{1}{-4 - 4D + 1} \sin 2x$

$PI = a \cdot \frac{1}{+4D + 3} \sin 2x$

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$PI = a \cdot \frac{(4D-3)\sin 2x}{(4D+3)(4D-3)}$

$PI = a \cdot \frac{(4D-3)\sin 2x}{((4D)^2 - (3)^2)}$

$PI = a \cdot \frac{(4D-3)\sin 2x}{4 \times -16 + 9}$

$PI = a \cdot \frac{4D \sin 2x - 3 \sin 2x}{+73}$

$PI = \frac{a \cdot 8\cos 2x - 3\sin 2x}{+73}$

$Y = Cf + PI$

$Y = e^{x}[c_1 \cosh \sqrt{3}x + c_2 \sinh$

$Y = c_1 e^2 \cosh(\sqrt{3}x + c_2)$

(3) $(D^2 - 3D + 2)y = \sin 3x$

A.E $= m^2 - 3m + 2 = 0$

$= \frac{3 \pm \sqrt{9-8}}{2}$

$= \frac{3 \pm 1}{2} = 2, 1$

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$Cf = C_1 e^{x} + C_2 e^{2x}$

$PI = \frac{1}{D^2 - 3D + 2} \sin 3x$

$PI = \frac{1}{-9 - 3D + 2} \sin 3x$

$PI = \frac{1}{3D + 7} \sin 3x$

$PI = \frac{(3D-7)}{(3D-7)(3D+7)} \sin 3x$

$PI = \frac{(3D-7) \sin 3x}{((3D)^2 - (7)^2)}$

$PI = \frac{(3D-7) \sin 3x}{(9 \times -9) - 49}$

$PI = \frac{(3D-7) \sin 3x}{-81 - 49}$

$PI = \frac{3D \sin 3x - 7 \sin 3x}{-130}$

$PI = \frac{3 \times 3 \cos 3x - 7 \sin 3x}{-130}$

$y = Cf + PI =$

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$y = C_1 e^{x} + C_2 e^{2x} + \frac{-9\cos 3x - 7\sin 3x}{+130}$

(4) $(D^3 + 1) y = \cos 2x$

$A.E = \cancel{m^3 + 1 = 0}$
$\cancel{m(m^2+1) = 0}$
$\cancel{m = 0, 1, -1}$

$m^3 + 1 = 0$
$m^3 + m^2 - m^2 + m - m = 0$
$m^2(m+1) - m(m+1) + 1(m+1) = 0$
$(m+1)(m^2 - m + 1) = 0$

$m = -1, \; m = \frac{1 \pm i\sqrt{3}}{2},$

$Cf = C_1 e^{-x} + e^{\frac{x}{2}} \left(C_2 \cos \frac{\sqrt{3}}{2} x + \sin \frac{\sqrt{3}}{2} x \right)$

$PI = \frac{1}{(D^3 + 1)} \cos 2x$

$PI = \frac{1}{D^2 (D) + 1} \cos 2x$

$PI = \frac{1}{-4D + 1} \cos 2x$

$PI = \frac{1}{4D - 1} \cos 2x$

$$P.I = \frac{(4D+1)}{(4D-1)(4D+1)} \cos 2x$$

$$P.I = \frac{(4D+1)\cos 2x}{((4D)^2-(1)^2)}$$

$$P.I = \frac{4D\cos 2x + \cos 2x}{(16 \times -4) - 1}$$

$$P.I = \frac{-8\sin 2x + \cos 2x}{4 \times (-16) - 1}$$

$$P.I = \frac{-8\sin 2x + \cos 2x}{-64-1}$$

$$P.I = \frac{+8\sin 2x + \cos 2x}{+65}$$

$$y = CF + PI$$

$$y = C_1 e^{-x} + e^{x/2}\left(C_2 \cos\frac{\sqrt{3}}{2}x + \sin\frac{\sqrt{3}}{2}x\right)$$

$$+ \frac{8\sin 2x + \cos 2x}{65}$$

(5) $(D^2 - 4D + 4)y = \cos 2x$

A.E: $m^2 - 4m + 4$

$m = 2 \pm \sqrt{3}$

$$CF = e^{2x}\left[C_1 \cosh\sqrt{3}x + C_2 \sinh\sqrt{3}x\right]$$

$$PI = \frac{1}{D^2 - 4D + 4} \cdot \cos 2x$$

$$= \frac{1}{-4 - 4D + 4} \cos 2x$$

$$= \frac{1}{-4D} \cos 2x$$

$$= \frac{D}{-4D^2} \cos 2x$$

$$= -\frac{D\cos 2x}{16}$$

$$= \frac{-2\cos 2x}{16}$$

$$= \frac{-\cos 2x}{8}$$

$$y = CF + PI$$

$$y = e^{2x}\left[C_1 \cosh\sqrt{3}x + C_2 \sinh\sqrt{3}x\right] - \frac{\cos 2x}{8}$$

(6) $(D^2 - 4D + 4)y = e^{-4x} + 5\cos 3x$

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$A.E = m^2 - 4m + 4 = 0$

$\frac{4 \pm \sqrt{16-16}}{2}$

$\frac{4}{2} = 2$

$CF = (C_1 + C_2 x) e^{2x}$

$P.I = \frac{1}{D^2-4D+4} (e^{-4x} + 5\cos 3x)$

$P.I = \frac{1}{D^2-4D+4} e^{-4x} + \frac{1}{D^2-4D+4} 5\cos 3x$

$P.I = \frac{1}{16+16+4} e^{-4x} + \frac{1}{-9-4D+4} 5\cos 3x$

$P.I = \frac{1}{36} e^{-4x} + \frac{1}{4D+5} 5\cos 3x$

$P.I = \frac{1}{36} e^{-4x} + \frac{(4D-5)}{((4D)^2-(5)^2)} 5\cos 3x$

$P.I = \frac{1}{36} e^{-4x} + \frac{4D-5}{(16\times -9)-25} 5\cos 3x$

$P.I = \frac{1}{36} e^{-4x} + \frac{4D\, 5\cos 3x - 25\cos 3x}{-169}$

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$P.I = \frac{1}{36} e^{-4x} + \frac{60\sin 3x - 25\cos 3x}{+169}$

$y = CF + PI$

$y = (C_1 + C_2 x) e^{2x} + \frac{1}{36} e^{-4x} + \frac{60\sin 3x - 25\cos 3x}{169}$

(7) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

$A.E.\ m^2 + 1 = 0$
$m = \pm 1$

$CF = C_1 e^{x} + C_2 e^{-x}$

$P.I = \frac{1}{D^2+1} \operatorname{cosec} x = \frac{1}{(D-i)(D+i)} \operatorname{cosec} x$

$= \left[\frac{A}{D-i} + \frac{B}{D+i}\right] \operatorname{cosec} x$

$= \frac{1}{(D+i)(D-i)} = \frac{A}{(D-i)} + \frac{B}{(D+i)}$

$= 1 = A(D+i) + B(D-i)$

Put $D = i$

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Put $D = i$

$1 = A(i+i) + 0$

$A = \frac{1}{2i}$

Put $D = -i$

$1 = A(-i+i) + B(\ldots)$

$B = -\frac{1}{2i}$

$$P.I = \frac{1}{2i}\left[\frac{1}{D-i}\operatorname{cosec} x - \frac{1}{D+i}\operatorname{cosec} x\right]$$

$$= \frac{1}{2i}\left[e^{ix}\int e^{-ix}\operatorname{cosec} x\,dx - e^{-ix}\int e^{ix}\operatorname{cosec} x\,dx\right]$$

$e^{ix} = \cos x + i\sin x$ — (1)

$e^{-ix} = \cos x - i\sin x$ — (2)

$\frac{e^{ix} - e^{-ix}}{2i} = \cos x$

$\frac{e^{ix} - e^{-ix}}{2i} = \sin x$

$$= \frac{1}{2i}\left[e^{ix}\operatorname{cosec} x\,dx - e^{-ix}\int e^{ix}\operatorname{cosec} x\,dx\right]$$

$$= \frac{1}{2i}\left[e^{ix}\int((\cos x - i\sin x)\operatorname{cosec} x - e^{-ix}\int (\cos x + i\sin x)\operatorname{cosec} x\,dx\right]$$

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$$= \frac{1}{2i}\left[e^{ix}\int(\cot x - i)\,dx - e^{-ix}\int(\cot x + i)\,dx\right]$$

$$= \frac{1}{2i}\left[e^{ix}(\log\sin x - ix) - e^{-ix}[\log\sin x + ix]\right]$$

$$= \frac{1}{2i}\left[(e^{ix} - e^{-ix})\log\sin x - ix\,(e^{ix} + e^{-ix})\right]$$

$$= \frac{e^{ix} - e^{-ix}}{2i}\log\sin x - \frac{ix}{i}\,\frac{e^{ix} + e^{-ix}}{2}$$

$P.I = \sin x\,\log\sin x - x\cos x$

$y = CF + PI$

$$y = c_1 e^{x} + c_2 e^{-x} + \frac{e^{ix} - e^{-ix}}{2i}\log\sin x - x\cos x$$

(8) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$

$D^2 - 3D + 2 = e^{5x}$

A.E $= m^2 - 3D + 2 = 0$

$m^2 - 2m - m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$(m-1)(m-2) = 0$

$m = 1,\ m = 2$

$CF = (C_1 + C_2 x)e^{2x}$

$CF = (C_1 e^{x} + C_2 e^{2x})$

$PI = \frac{1}{D^2 - 3D + 2} \cdot e^{5x}$

$PI = \frac{1}{25 - 15 + 2} e^{5x}$

$PI = \frac{1}{12} e^{5x}$

$y = CF + PI$

$y = C_1 e^{x} + C_2 e^{2x} + \frac{1}{2} e^{5x}$

(9) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = e^{2x}$

$A.E = m^2 - 7m + 6 = 0$

$m = 1, 6$

$CF = C_1 e^{x} + C_2 e^{6x}$

$PI = \frac{1}{D^2 - 7D + 6} \cdot e^{2x}$

$PI = \frac{e^{2x}}{4 - 14 + 6}$

$PI = \frac{e^{2x}}{-4}$

$y = CF + PI$

$y = C_1 e^{x} + C_2 e^{6x} + \frac{e^{2x}}{-4}$

(10) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = \sin 3x$

$A.E = m^2 - 2m + 5 = 0$

$\frac{+2 \pm \sqrt{4 - 20}}{2}$

$\frac{+2 \pm \sqrt{-16}}{2}$

$\frac{+2 \pm i4}{2} = 1 \pm i2$

$CF = e^{x}(C_1 \cos 2x + C_2 \sin 2x)$

$PI = \frac{1}{D^2 - 2D + 5} \cdot \sin 3x$

$\frac{1}{-9 - 2D + 5} \sin 3x$

$\frac{1}{2D + 4} \sin 3x$

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$$= \frac{(2D-4)}{(2D^2)-(4)^2} \cdot \sin 3x$$

$$= \frac{2D\sin 3x - 4\sin 3x}{4x-9-16}$$

$$= \frac{6\cos 3x - 4\sin 3x}{-36-16}$$

$$= \frac{6\cos 3x - 4\sin 3x}{-52}$$

$= y = Cf + PI$

$$= y = e^{x}(c_1\cos 2x + c_2\sin 2x) + \frac{6\cos 3x - 4\sin 3x}{-52}$$

(11) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 9y = 40\sin 5x$

A.E = $m^2 - 8m + 9 = 0$

$$= \frac{8 \pm \sqrt{64-36}}{2}$$

$$\frac{8 \pm \sqrt{28}}{2}$$

$$\frac{4 \pm 2\sqrt{7}}{2} = 2 \pm \sqrt{7}$$

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$Cf = e^{2x}(C_1\cosh\sqrt{7}x + C_2\sinh\sqrt{7}x)$

$$PI = \frac{1}{D^2-8D+9}(40\sin 5x)$$

$$PI = \frac{1}{-25-8D+9}(40\sin 5x)$$

$$PI = \frac{1}{8D+16}(40\sin 5x)$$

$$PI = \frac{1}{8(D+2)}(40\sin 5x)$$

$$PI = \frac{(D-2)}{8((D)^2-(2)^2)} 40\sin 5x$$

$$PI = \frac{D40\sin 5x - 80\sin 5x}{8(-25)-4}$$

$$PI = \frac{120\cos 5x - 80\sin 5x}{8(-29)}$$

$$PI = \frac{120\cos 5x - 80\sin 5x}{232}$$

$y = Cf + PI$

$$y = e^{2x}(C_1\cosh\sqrt{7}x + C_2\sinh\sqrt{7}x) + \frac{120\cos 5x - 80\sin 5x}{232}$$

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12. $\frac{d^2y}{dx^2} + 9y = \cos x + \sin 2x$

A.E. $m^2 + 9 = 0$

$m = \pm 9$

$CF = C_1 e^{9x} + C_2 e^{-9x}$

$PI = \frac{1}{D^2+9} \cdot \cos x + \sin x$

$= \frac{1}{D^2+9} \cos x + \frac{1}{D^2+9} \sin 2x$

$= \frac{1}{8} \cos x + \frac{1}{5} \sin 2x$

~~$PI = \frac{5\cos x + 8\sin 2x}{40}$~~

~~$y = CF + PI$~~

~~$y = C_1 e^{9x} + C_2 e^{-9x} + \frac{5\cos x + 8\sin 2x}{40}$~~

$PI = \frac{\cos 2x + \sin x}{5}$

$y = CF + PI = C_1 e^{9x} + C_2 e^{-9x} + \frac{\cos 2x + \sin x}{5}$

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(13) $\frac{d^2y}{dx^2} + 4y = \tan 2x$

~~$AE = m^2 + 4 = 0$~~

~~$m = \pm 4$~~

~~$CF = C_1 e^{4x} + C_2 e^{-4x}$~~

PI: $AE = m^2 + 4 = 0$

$m = 0 \pm 2i$

$CF = C_1 \cos 2x + C_2 \sin 2x$

$PI = \frac{1}{D^2+4} \tan 2x = \frac{1}{(D-2i)(D+2i)} \tan 2x$

$= \left(\frac{A}{(D-2i)} - \frac{B}{(D+2i)}\right) \tan 2x$

$= \left[\frac{1}{D-2i} - \frac{1}{D+2i}\right]$

$= \frac{1}{4}\left[\frac{1}{D-2i} - \frac{1}{D+2i}\right] \tan 2x$

$= \frac{1}{4i}\left[\frac{1}{D-2i} \tan 2x - \frac{1}{D+2i} \tan 2x\right]$

$= \frac{1}{4i}\left[e^{2ix} \int e^{-2ix} \tan 2x - e^{-2ix} \int e^{2ix} \tan 2x \, dx\right]$

$\Rightarrow \frac{1}{4i}\left[e^{2ix}\int(\cos 2x - i\sin 2x)\tan 2x\,dx\right.$

$e^{-2ix}\int(\cos 2x + i\sin 2x)\tan$

$= \frac{1}{4i}\left[e^{2ix}\int\left(\sin 2x - \frac{i\sin^2 2x}{\cos 2x}\right)dx - e^{-2ix}\right.$

$\left.\int\left(\sin 2x + \frac{i\sin^2 2x}{\cos 2x}\right)dx\right]$

$= \frac{1}{4i}\left[e^{2ix}\int\left(\sin 2x - \frac{i(1-\cos^2 2x)}{\cos 2x}\right)dx - e^{-2ix}\right.$

$\int\left(\sin 2x + \frac{i(1-\cos^2 2x)}{\cos 2x}\right)$

$= \frac{1}{4i}\left[e^{2i}\left\{\frac{-\cos 2x}{2} - i\left(\frac{1}{2}\log(\sec 2x + \tan 2x) - \frac{\sin 2x}{2}\right)\right\}\right]$

$= \frac{1}{4i}\left[\frac{-\cos 2x}{2}\left\{e^{2ix} - e^{-2ix}\right\} - \frac{1}{2}\log\{\sec 2x + \tan 2x\}\, i\left\{e^{2ix} + e^{-2ix}\right\} + \frac{\sin 2x}{2}\left\{e^{2ix} + e^{-2ix}\right\}\right.$

$= \frac{1}{4}\left(-\cos 2x \times \sin 2x - \log\{\sec 2x + \tan 2x\}\cos 2x + \sin 2x \times \cos 2x\right)$

$y = CF + PI$

$= c_1\cos 2x + c_2\sin 2x + \frac{1}{4}\left(-\cos 2x \times \sin 2x - \log\{\sec 2x + \tan 2x\}\cos x + \sin 2x \times \cos 2x\right)$

(15) Q $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^{-x}$

A.E $m^2 + m + 1 = 0$

$\frac{-1 \pm \sqrt{1-4}}{2}$

$\frac{-1 \pm \sqrt{3}i}{2}$

$CF = e^{-x/2}\left(c_1\cos\frac{\sqrt{3}}{2}x + c_2\sin\sqrt{\frac{3}{2}}x\right]$

$PI = \frac{1}{D^2 + D + 1}e^{-x}$

$= \frac{1}{1}e^{-x}$

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$y = CF + PI$

$y = e^{-x/2}\left[C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}}{2}x\right] + e^{x}$

(16) $\frac{d^2y}{dx^2} + 2p\frac{dy}{dx} + (p^2+q^2)y = e^{\alpha x}$

A.E $m^2 + 2pm + (p^2+q^2) = 0$

$m = \frac{-2p \pm 2q\,i}{2}$

$CF = (C_1 e^{-x/2} \cos(qx) + C_2 e^{-x/2} \cos qx)$

$PI = \frac{1}{m^2+2pm+(p^2+q^2)} e^{\alpha x}$

$PI = \frac{e^{\alpha x}}{(p+\alpha)^2+q^2}$

$y = CF + PI$

$y = C_1 e^{-x/2} \cos qx + C_2 e^{-x/2} \cos qx + \frac{e^{\alpha x}}{(p+\alpha)^2+q^2}$

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(17) $(D^3+1)y = (e^x+1)^2$

$(D^3+1)y = e^{2x} + 2e^x + 1$

AE: $m^3+1=0$

$m=-1$

$m^3 + m^2 - m^2 - m + m + 1 = 0$

$m^2(m+1) - m(m+1) + 1(m+1) = 0$

$(m+1)(m^2-m+1) = 0$

$m=-1$

$m^2 - m + 1 = 0$

$m = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$CF = C_1 e^{-x} + e^{\frac{1}{2}x}\left[C_2 \cos \frac{\sqrt{3}}{2}x + C_3 \sin \frac{\sqrt{3}x}{2}\right]$

$PI = \frac{1}{D^3+1}\left[e^{2x} + 2e^x + 1\right]$

$= \frac{1}{D^3+1} e^{2x} + \frac{1}{D^3+1} 2e^x + \frac{1}{D^3+1} e^{0x}$

$= \frac{1}{(2)^3+1} e^{2x} + \frac{1}{(1)^3+1} 2e^x + \frac{1}{0+1} \times e^{0x}$

$= \frac{1}{9} e^{2x} + \frac{2}{2} e^x + 1$

Ex 35

$\frac{1}{9} e^{2x} + e^{x} + 1$

Sol. is

$y = CF + PI$

$y = c_1 e^{-x} + e^{\frac{1}{2}x}\left[c_2 \cos\frac{\sqrt{3}}{2}x + c_3 \sin\frac{\sqrt{3}}{2}x\right.$
$\left. + \frac{1}{9}e^{2x} + e^{x} + 1\right]$

Ex 35

(1) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^{x}$

AE: $m^2 - 2m + 4 = e^{x}$

$= \frac{2 \pm \sqrt{4-16}}{2}$

$= \frac{2 \pm \sqrt{3} \times 2i}{2}$

$= 1 \pm \sqrt{3}\, i$

$Cf = e^{x}(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x)$

$PI = \frac{1}{D^2 - 2D + 4} e^{x}$

$PI = \frac{1}{1-2+1} e^{x}$

RI ~ Y

$PI = \frac{xe^{x}}{2D-2}$

$PI = \frac{xe^{x}}{2-2}$

$PI = \frac{x^2 e^{x}}{2}$

$= \frac{Y}{(D-2)^2}$

$= Y$

$y = cf + PI$

$y = e^{x}(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x)$

(2) (a) $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2 + x$

AE = $m^3 + 2m^2 + m = 0$

$m = 0,\ m = -1,\ -1$

$Cf = c_1 e^{0x} + c_2 e^{-1x} + c_3 e^{-1x}$

$Cf = c_1 + c_2 e^{-x} + c_3 e^{-x}$

$PI = \frac{1}{D^3 + 2D^2 + D} e^{2x} + \frac{1 \cdot x^2}{D^3 + 2D^2 + D} + \frac{x}{D^3 + 2D^2 + D}$

$\frac{1 \cdot e^{2x}}{8+8+2} + [D^3 + 2D + D]^{-1} x^2 + [D^3 + 2D^2 + D]^{-1} x$

$$= \frac{e^{2x}}{18} + \frac{1}{D(D+1)^2}x^2 + \frac{1}{D(D+1)^2}x$$

$$= \frac{e^{2x}}{18} + D^{-1}(D+1)^{-2}x^2 + D^{-1}(D+1)^{-2}x$$

$$= \frac{e^{2x}}{18} + D^{-1}\cdot(1-2D+3D^2-4D^3 \ldots)x^2 + D^{-1}(1-2D+3D^2-4D^3\ldots)x$$

$$= \cancel{\frac{e^{2x}}{18} + (D-2D^2+3D^3\ldots)x^2 + D-2D^2+3}$$

$$= \frac{e^{2x}}{18} + (D^{-1} - 2 + 3D - 4D^2)x^2 + (D^{-1} - 2 + 3D - 4D^2)x$$

$$= \frac{e^{2x}}{18} + (D^{-1}x^2 - 2x^2 + 3Dx^2 - 4D^2x^2) + (D^{-1}x - 2x + 3Dx - 4D^2x)$$

$$= \frac{e^{2x}}{18} + \frac{x^3}{3} - 2x^2 + 6x - 8 + \frac{x^2}{2} - 2x + 3 - 4$$

$$= \frac{e^{2x}}{18} + \frac{x^3}{3} - 2x^2 + 6x + \frac{x^2}{2} - 2x - 9$$

$$= \frac{e^{2x}}{18} + \frac{x^3}{3} - 2x^2 + \frac{x^2}{2} + 4x$$

$y = CF + PI$

$$y = c_1 + c_2e^{-x} + c_3e^{-x} + \frac{e^{2x}}{18} + \frac{x^3}{3} - 2x^2 + \frac{x^2}{2} + 4x$$

(b) $$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = xe^x + e^x$$

A.E. $m^3 - 3m^2 + 3m - 1 = 0$

$m(m^2 - 3m + 3) - m$

$m = 1$

CF $= c_1e^x + c_2e^x x + c_3 e^x x^2$

$$PI = \frac{1}{D^3-3D^2+3D-1}\cdot xe^x + \frac{1}{D^3-3D^2+3D-1}\cdot e^x$$

$$= e^x\frac{1}{(D-1)^3}x + \frac{1}{(D-1)^3}e^x$$

$$= e^x\frac{1}{(D+1-1)^3}x + \frac{1}{(D-1)^3}e^x$$

$$= e^x\cdot\frac{x}{D^3} + \frac{1}{(D-1)^3}e^x$$

$$= e^x\cdot\frac{x^4}{24} + \frac{xe^x}{3D^2-6D+3}$$

$$= \frac{e^{x} \cdot x^4}{24} + \frac{x^2 e^{x}}{6D - 6}$$

$$= \frac{e^{x} \cdot x^4}{24} + \frac{x^3 e^{x}}{6}$$

$y = cf + PI$

$$= c_1 e^{x} + c_2 e^{x} x + c_3 e^{x} x^2 + e^{x} \cdot \frac{x^4}{4}$$

(3) $$\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2 = 0$$

A.E = $m^3 - 4m^2 + 5m - 2 = 0$

$m = 1, m = 2$

$m^2 - 3m + 2$

$y = (c_1 + c_2 x) e^{x} + c_3 e^{2x}$

(4) $$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$$

A.E = $m^3 - 4m^2 - 6m - 1 = x^2$

$m = -1, \quad m = \frac{5 \pm \sqrt{29}}{2}$

$$PI = \frac{1}{D^3 - 4D^2 - 6D} x^2$$

$$CF = e^{\frac{5x}{2}}\left(c_1 \cosh \frac{\sqrt{29}}{2} x + c_2 \sinh \frac{\sqrt{29}}{2} x\right)$$

A.E = $m^3 - m^2 - 6m = 0$

$m(m^2 - m - 6) = 0$

$m = 0, \; m = 3, \; m = -2$

$CF = c_1 + c_2 e^{-2x} + c_3 e^{3x}$

$$PI = \frac{1}{D^3 - D^2 - 6D} \cdot 1 + x^2$$

$$PI = \frac{1}{D^3 - D^2 - 6D} \cdot e^{0x} + \frac{1}{(D^3 - D^2 - 6D)} \cdot x^2$$

$$PI = \frac{x^3 e^{0x}}{1} + \frac{1}{-6D\left[\frac{1 - D^3 - D^2}{6D}\right]} x^2$$

$$PI = \frac{x^3}{1} + \frac{1}{6D}\left[\frac{1 - D^2 - D}{6}\right]^{-1} x^2$$

$$PI = -\frac{1}{6} \times \frac{1}{D}(1) - \frac{1}{6D}\left[\frac{1 - D^2 - D}{6}\right]^{-1} x^2$$

$$PI = -\frac{1}{6} x - \frac{1}{6D}\left[1 + \frac{D^2 - D}{6} + \frac{(D^2 - D)^2}{36}\right] x^2$$

$PI = -\frac{1}{6}x - \frac{1}{6D}\left[1 + \frac{D^2 - D}{6} + \frac{1}{36}(D^2 \ldots\right.$

$PI = -\frac{1}{6}x - \frac{1}{6D}\left[x^2 + \frac{1}{6}(2-2x) + \frac{1}{36}(2)\right.$

$PI = -\frac{1}{6}x - \frac{1}{6D}\left[x^2 + \frac{1}{6}(2-2x) + \frac{1}{36}(2)\right.$

$PI = -\frac{1}{6}x - \frac{1}{6D}\left(x^2 + \frac{1}{3} - \frac{1}{3}x + \frac{1}{18}\right)$

$PI = -\frac{1}{6}x - \frac{1}{6}\left[\frac{x^3}{3} + \frac{1}{3}x - \frac{1}{6}x^2 + \frac{1}{18}x\right]$

$PI = -\frac{1}{6}x - \frac{1}{6}\left[\frac{x^3}{3} - \frac{1}{6}x^2 + \frac{7x}{18}\right]$

$PI = -\frac{1}{6}x - \frac{1}{18}x^3 + \frac{1}{36}x^2 - \frac{7}{108}x$

$PI = -\frac{1}{18}x^3 + \frac{1}{36}x^2 - \frac{25}{108}x$

$y = CF + PI$

$y = c_1 + c_2 e^{3x} + c_3 e^{-2x} - \frac{1}{18}x^3 + \frac{1}{36}x^2 - \frac{25}{108}x$

(5) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2$

A.E. $m^2 - 4m + 4 = 0$

$\frac{+4 \pm \sqrt{16-16}}{2}$

$\frac{4 \pm 0}{2}$

$m = 2, 2$

$CF = (c_1 + c_2 x)e^{2x}$

$PI = \frac{1}{D^2 - 4D + 4} x^2$

$= \frac{1}{(D-2)^2} x^2$

$\cancel{= \frac{1}{4\left(\frac{D^2 - 4D + 4}{4}\right)} x^2}$

$\cancel{= \frac{16}{4}(D - 1)^{-2} x^2}$

$\cancel{= 4}$

$\cancel{= \frac{1}{(D-a)^2}} = e^{ax} D^{-2} e^{-ax}$

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$= e^{2x} D^{-2} (e^{-2x} x^2)$

$\therefore (D-2)^{-2} = \frac{1}{4}\left(1 - \frac{D}{2}\right)^{-2}$

$= \frac{1}{4}\left(1 - \frac{D}{2}\right)^{-2} \cdot x^2$

$= \frac{1}{4}\left(1 + \frac{2D}{2} + \frac{3D^2}{2} + \frac{4D^3}{2} \dots\right) x^2$

$= \frac{1}{4}\left(x^2 + Dx^2 + \frac{3D^2x^2}{2}\right)$

$= \frac{1}{4}\left(x^2 + 2x + \frac{3}{2} \times 2\right)$

$= \frac{x^2}{4} + \frac{2x}{4} + \frac{3}{4}$

$= \frac{x^2}{4} + \frac{1x}{2} + \frac{3}{4}$

$y = CF + PI$

$y = (c_1 + c_2 x) e^{2x} + \frac{x^2}{4} + \frac{x}{2} + \frac{3}{4}$

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(5) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2$

A.E $m^2 - 4m + 4 = x^2$

$\frac{-4 \pm \sqrt{16-16}}{2}$

$\frac{-4 \pm 0}{2} = -2, -2$

$CF = (c_1 + c_2 x) e^{-2x}$

$PI = \frac{1}{D^2 - 4D + 4} \cdot x^2$

$PI = \frac{1}{(D-2)^{+2}} x^2$

$PI = \cancel{(D-2)^{-2} \cdot x^2}$

$PI = \frac{1}{4}\left(1 - \frac{D}{2}\right)^{-2} \cdot x^2$

$PI = \frac{1}{4}\left(1 + \frac{2D}{2} + \frac{3D^2}{2}\right) x^2$

$PI = \frac{1}{4} x^2 + 2x + \frac{3}{2} \times 2$

$PI = \frac{1}{4}\left(x^2 + 2x + 3\right.$

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$y = CF + PI$

$y = (C_1 + C_2 x) e^{-2x} + \frac{1}{4}x^2 + 2x + 3$

(6) $(D^2 + D - 2) y = e^x$

A.E: $m^2 + m - 2 = 0$

$\frac{-1 \pm \sqrt{1+8}}{2}$

$\frac{-1 \pm 3}{2}$

$m = 1, -2$

$CF = C_1 e^x + C_2 e^{-2x}$

$PI = \frac{1}{D^2 + D - 2} \cdot e^x$

$PI = \frac{x e^x}{2D + 1}$

$PI = \frac{x e^x}{3}$

$y = CF + PI = C_1 e^x + C_2 e^{-2x} + \frac{x}{3} e^x$

(9) $\frac{d^3 y}{dx^3} + y = 3e^{-x} + 5e^{2x}$

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A.E $m^3 + 1 = 0$

$m = -1, \; m = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$CF = C_1 e^{-x} (e^{x/2} (C_2 \cos \frac{\sqrt{3}}{2} x + C_3 \sin \frac{\sqrt{3}}{2} x))$

$PI = \frac{1}{D^3 + 1} \cdot 3e^{-x} + 5e^{2x}$

$\frac{1}{D^3 + 1} 3e^{x} + \frac{1}{D^3 + 1} 5e^{2x}$

$\frac{x\, 3e^{-x}}{2D^2 + 1} + \frac{1}{9} 5e^{2x}$

$\frac{3x e^{-x}}{3} + \frac{1}{9} 5e^{2x}$

$y = CF + PI$

$y = C_1 e^{-x} (e^{x/2} (C_2 \cos \frac{\sqrt{3}}{2} x + C_3 \sin \frac{\sqrt{3}}{2} x)) +$

$\frac{3x e^{-x}}{3} + \frac{1}{9} 5e^{2x}$

(11) $(D^2 - 1) y = \cosh x + a^x$

$$\cosh x = \frac{e^{x}+e^{-x}}{2} \qquad e^{ix} = \cos x + i\sin x$$

$$\sinh x = \frac{e^{x}-e^{-x}}{2} \qquad e^{-ix} = \cos x - i\sin x$$

$$AE = m^2 - 1 = 0$$
$$m = \pm 1$$

$$CF = c_1 e^{x} + c_2 e^{-x}$$

$$PI = \frac{1}{(D^2-1)} \cosh x \cos x + \frac{1}{(D^2-1)} a^{x}$$

$$= \frac{1}{(D^2-1)} \left(\frac{e^{x}+e^{-x}}{2}\right) \cdot \cos x + \frac{1}{(D^2-1)} e^{x\log a}$$

$$= \frac{1}{(D^2-1)2} e^{x} \cdot \cos x + \frac{e^{-x}\cos x}{2(D^2-1)} + \frac{1}{D^2-1} e^{x\log a}$$

$$= \frac{1}{2} e^{x} \times \frac{1}{(D+1)^2-1} + \frac{1}{2} e^{-x} \frac{1}{(D-1)^2-1} \cos x +$$

$$\frac{1}{2} e^{x\log a}$$

$$= \frac{1}{2} e^{x} \times \frac{1}{D^2+2D+1-1} \cos x + \frac{1}{2} e^{-x} \frac{1}{D^2-2D+1-1} \cos x + \frac{e^{x\log a}}{(\log a)^2-1}$$

$$= \frac{1}{2} e^{x} \times \frac{1}{-1+2D} \cos x + \frac{1}{2} e^{-x} \times \frac{1}{-1-2D} \cos x + \frac{1}{(\log a)^2-1}$$

$$= \frac{1}{2} e^{x} \times \frac{2D+1}{(2D-1)(2D+1)} \cos x - \frac{1}{2} e^{-x} \times \frac{2D-1}{(2D+1)(2D-1)} \cos x + \frac{1}{(\log a)^2-1}$$

$$= \frac{1}{2} e^{x} \times \frac{(2D+1)}{4D^2-1} \cos x - \frac{1}{2} e^{-x} (2D-1) \cos x + \frac{1}{(\log a)^2-1} a^{x}$$

$$= \frac{-1}{10} e^{x} (2D+1) \cos x + \frac{1}{10} e^{-x} (2D-1) \cos x + \frac{1}{(\log a)^2-1} a^{x}$$

$$= \frac{-1}{10} e^{x} (-2\sin x + \cos x) + \frac{1}{10} e^{-x} (-2\sin x - \cos x) + \frac{1 \cdot a^{x}}{(\log a)^2-1}$$

$$= \frac{1}{5} e^{x} \sin x - \frac{1}{10} e^{x} \cos x - \frac{1}{5} e^{-x} \sin x - \frac{1}{10} e^{-x} \cos x + \frac{1}{(\log a)^2-1}$$

$= \frac{1 \times 2}{5}\left[\frac{e^{x}-e^{-x}}{2}\right]\sin x - \frac{1}{5}\left[\frac{e^{x}+e^{-x}}{2}\right]$

$+ \frac{1}{(\log a)^2 - 1} \cdot a^x$

$= \frac{2}{5}\sin hx \sin x - \frac{1}{5}\sin x \cos x + \frac{1}{(\log a)^2 \ldots}$

(12) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{3x}$

AE: $m^2 - 2m + 1 = 0$

$\frac{+2 \pm \sqrt{4-4}}{2}$

$\frac{2 \pm 0}{2} = 1, 1$

CF: $c_1 e^x + c_2 e^x$

PI $= \frac{1}{(D^2 - 2D + 1)} \cdot x^2 \cdot e^{3x}$

$= \cancel{\frac{1}{2D\left(\frac{D}{2}\right)}} = \frac{1}{D(D-1)} x^2 \cdot e^{3x}$

$= \cancel{\frac{1}{D}(D-1)^{-1} x^2 \cdot e^{3x}}$

$e^{3x} \cancel{\frac{1}{(D+3)(D+2)} \cdot x^2}$

$= e^{3x} \cdot \frac{1}{(D+3)^2 - 2(D+3) + 1} \cdot x^2$

$= e^{3x} \cdot \frac{1}{D^2 + 6D + 9 - 2D - 6 + 1} x^2$

$= e^{3x} \cdot \frac{1}{D^2 + 4D + 4} x^2$

$= e^{3x} \cdot \frac{1}{4\left[1 + \frac{D^2 + 4D}{4}\right]} x^2$

$= \frac{e^{3x}}{4}\left[1 + \frac{D^2 + 4D}{4}\right] x^2$

$= \frac{e^{3x}}{4}\left[1 - \frac{D^2 + 4D}{4} + \left(\frac{D^2 + 4D}{4}\right)^2\right] x^2$

$= \frac{e^{3x}}{4}\left[x^2 - \frac{1}{4}(2 + 8x) + 2\right]$

$= \frac{e^{3x}}{4}\left[x^2 - \frac{1}{2} - 2x + 2\right]$

$= \frac{e^{3x}}{4}\left[x^2 - 2x + \frac{3}{2}\right]$

$y = cf + pi$

$y = (c_1 + c_2 x)e^{x} + \frac{e^{3x}}{4}\left(x^2 - 2x + \frac{3}{2}\right)$

(13) $\frac{d^2y}{dx^2} + \alpha^2 y = \sin\alpha x$

A.E : $m^2 + \alpha^2 = 0$

$\frac{-0 \pm \sqrt{(0)^2 - 4\alpha^2}}{2}$

$\frac{0 \pm 2i\sqrt{\alpha^2}}{2}$

$0 \pm \alpha i$

CF : $e^{0x}(c_1 \cos\alpha x + c_2 \sin\alpha x)$

PI => $\frac{1}{D^2 + \alpha^2} \cdot \sin\alpha x$

$PI = \frac{-x}{2a} \cos\alpha x$

$y = cf + PI$

$y = c_1 \cos\alpha x + c_2 \sin\alpha x - \frac{x}{2a} \cos\alpha x$

(14) $\frac{d^2y}{dx^2} - 4y = x + \sin 2x + \cos^2 x$

AE : $m^2 - 4 = 0$

$m = \pm 2$

Cf : $c_1 e^{2x} + c_2 e^{-2x}$

$PI = \frac{1}{D^2 - 4} \cdot x + \sin 2x + \cos^2 x$

$= \frac{1}{D^2-4} x + \frac{1}{D^2-4}\sin 2x + \frac{1}{D^2-4}\cos^2 x$

$= \frac{x}{4\left(1 - \frac{D^2}{4}\right)} + \frac{\sin 2x}{D^2 - 4} + \frac{1 \cos^2 x}{D^2 - 4}$

$= \frac{1}{4}\left[1 - \frac{D^2}{4}\right]^{-1} x + \frac{\sin 2x}{-8} + \frac{\cos x}{-5} + \frac{\cos x}{-5}$

$= \frac{1}{4}\left[1 + \frac{D^2}{4} + \left(\frac{D^2}{4}\right)^2\right] x + \frac{\sin 2x}{-8} + \frac{\cos x}{-5} + \frac{\cos x}{-5}$

$= \frac{1}{4}\left[x + \frac{D^2 x}{4}\right] + \frac{\sin 2x}{-8} + \frac{\cos x}{-5} + \frac{\cos x}{-5}$

$= \frac{1}{4}\left[x + \frac{1}{4}\right] - \frac{\sin 2x}{8} - \frac{\cos x}{5} - \frac{\cos x}{5}$

$$= \frac{x}{4} + \frac{1}{8} - \frac{\sin 2x}{8} + \frac{\cos x}{5} + \frac{\cos x}{5}$$

$$+ \frac{1 - \cos 2x}{D^2 - 4}$$

$$= \frac{2x}{8} + \frac{1}{8} - \frac{\sin 2x}{8} + \frac{1}{-4} + \frac{x}{2a} \cos 2x$$

$$= \frac{x}{4} + \frac{1}{8} - \frac{\sin 2x}{8} - \frac{1}{4} + \frac{x}{4} \cos 2x$$

$y = Cf + PI$

$$y = c_1 e^{2x} + c_2 e^{-2x} + \frac{x}{4} + \frac{1}{8} - \frac{\sin 2x}{8} - \frac{1}{4}$$
$$+ \frac{x}{4} \cos 2x$$

(15) $(D^2 - 2D + 5)\, y = e^{2x} \sin x$

$A.E = m^2 - 2m + 5$

$$m = \frac{+2 \pm \sqrt{4 - 20}}{2}$$

$$m = \frac{2 \pm i4}{2}$$

$m = 1 \pm 2i$

$CF = e^{x}(c_1 \cos 2x + c_2 \sin 2x)$

$$PI = \frac{1}{D^2 - 2D + 5} \cdot e^{2x} \cdot \sin x$$

$$= \frac{1}{4 - 4 + 5} \cdot e^{2x} \cdot \frac{1}{-1 - 2D + 5} \sin x$$

$$= \frac{1}{5} e^{2x} \cdot \frac{\sin x}{2D - 4}$$

$$= \frac{1}{5} e^{2x} \cdot \frac{(2D + 4) \sin x}{(2D + 4)(2D - 4)}$$

$$= \frac{1}{5} e^{2x} \cdot \frac{2D \sin x + 4 \sin x}{4D^2 - 16}$$

$$= \frac{1}{5} e^{2x} \cdot \frac{2\cos x + 4 \sin x}{-20}$$

$$= \frac{e^{2x}}{-100} \cdot 2\cos x + 4 \sin x$$

$y = cf + PI$

$$y = e^{x}(c_1 \cos 2x + c_2 \sin 2x) - \frac{e^{2x}}{100} \cdot 2\cos x - 4\sin x$$

17. $(D^2+4D+4)y = 2\sinh 2x$

A.E $= m^2+4m+4$

$\frac{-4 \pm \sqrt{16-16}}{2}$

$= \frac{-4 \pm 0}{2}$

$m = -2, -2$

$Cf = (C_1 + C_2 x)e^{-2x}$

$PI = \frac{1}{D^2+4D+4} 2\sinh 2x$

$PI = \frac{1}{-4+4D+4} 2\sinh 2x$

$PI = \frac{1}{4D} 2\sinh 2x$

$PI = \frac{D}{4D^2} 2\sinh 2x$

$PI = \frac{4\cosh 2x}{-8}$

$PI = \frac{2}{D^2+4D+4} \cdot \frac{e^{2x}}{2} - \frac{e^{-2x}}{2}$

$PI = \frac{1}{D^2+4D+4} e^{2x} - e^{-2x}$

$PI = \frac{e^{2x}}{4+8+4} - \frac{xe^{-2x}}{2D+4}$

$PI = \frac{e^{2x}}{16} - \frac{x^2 e^{-2x}}{2}$

$y = Cf + PI$

$y = (C_1+C_2x)e^{-2x} + \frac{e^{2x}}{16} - \frac{x^2e^{-2x}}{2}$

(18) $\frac{D^3y}{dx^2} - y = (e^x+1)^2$

A.E $= m^3 - 1 = 0$

$(m-1)(m^2+m+1)$

$m = 1, \; m = \frac{-1 \pm i\sqrt{3}}{2}$

$Cf = C_1 e^x + e^{-x/2}\left(C_1 \cos \frac{\sqrt{3}}{2}x + C_2 \sin \frac{\sqrt{3}x}{2}\right)$

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$PI = \frac{1}{D^3 - 1} \cdot e^{2x} + 1 + 2e^{x}$

$\Rightarrow \frac{1}{8-1} \cdot e^{2x} + \frac{1}{-1} + \frac{2xe^{x} \cdot x}{3D^2}$

$\Rightarrow \frac{1}{7} e^{2x} - 1 + x e^{x}$

$PI = \frac{1}{7} e^{2x} + xe^{x} - 1$

$y = Cf + PI$

$y = c_1 e^{x} \left(e^{-x/2} \left[c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} \right. \right.$

(19) $(D^2 - 4D + 3)y = e^{x} \cos 2x + \cos 3x$

$A.E = m^2 - 4m + 3$

$\frac{4 \pm \sqrt{16 - 12}}{2}$

$\frac{4 \pm \sqrt{4}}{2}$

$2 \pm \frac{\sqrt{4}}{2}$

2 ± 1

$m = 3 \ , \ m = 1$

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$Cf = c_1 e^{x} + c_2 e^{3x}$

$PI = \frac{e^{x} \cdot 1 \cdot \cos 2x}{D^2 - 4D + 3} + \frac{\cos 3x}{D^2 - 4D + 3}$

$= \frac{e^{x} \cdot 1 \cdot \cos 2x}{(D+1)^2 - 4(D+1) + 3} + \frac{\cos 3x}{-4D - 6}$

$= \frac{e^{x} \cdot 1 \cdot \cos 2x}{D^2 - 2D} + \frac{(4D - 6) \cos 3x}{(4D - 6)(4D + 6)}$

$= \frac{e^{x} \cdot 1 \cdot \cos 2x}{(2D + 4)} + \frac{4D \cos 3x - 6 \cos 3x}{16 D^2 - 36}$

$= \frac{e^{x} \, 2D \cos 2x - \cos 2x}{4 D^2 - 16} + \frac{12 \cdot - \sin 3x - 6 \cos 3x}{144 - 36}$

$= \frac{-e^{x} 4 \sin 2x - \cos 2x}{-16 - 16} + \frac{-12 \sin 3x - 6 \cos 3x}{108}$

$PI = \frac{-e^{x} \sin 2x - \cos 2x}{8} - \frac{1}{9} \sin 3x - 6 \cos 3x$

$y = Cf + PI$

$y = c_1 e^{x} + c_2 e^{3x} \; \frac{-e^{x} \sin 2x - \cos x}{8} - \frac{1}{9} \sin 3x - 6 \cos 3x$

20: $(D^2+2D+1)y = e^x + x^2 - \sin x$

A.E $= m^2+2D+1$

$$\frac{-2 \pm \sqrt{4-4}}{2}$$

$$= \frac{-2 \pm 0}{2}$$

$m = -1, -1$

C.F. $= (C_1 + C_2 x) e^{-x}$

P.I. $= \frac{1}{D^2+2D+1} \cdot e^x + \frac{1}{D^2+2D+1} x^2 - \frac{\sin x}{D^2+2\cdot+1}$

$\frac{1}{4} e^x + \frac{x^2}{D(1+2+D)} - \frac{\cos x}{-1+2+1}$

$\frac{1}{4} e^x + \frac{x^2}{2D\left(\frac{D}{2}+1\right)} - \frac{\cos x}{2}$

$\frac{1}{4} e^x + \left[1+\frac{D}{2}\right]^{-1} x^2 - \frac{\cos x}{2}$
$\qquad 2D$

$\frac{1}{4} e^x + \frac{1}{2D}\left[1-\frac{D}{2}+\left(\frac{D}{2}\right)^2+\left(\frac{D}{2}\right)^3\right] x^2 - \frac{\sin x}{2}$

$\frac{1}{4} e^x + \frac{1}{[2D]}\left[x^2 - \frac{D x^2}{2} + \frac{D^2 x^2}{4} + \frac{D^3 x^2}{8}\right] - \frac{\cos x}{2}$

$\frac{1}{4} e^x \frac{1}{2D}\left[x^2 - \frac{2x}{2} + \frac{2 \cdot 1}{4 \cdot 2}\right] + \frac{\cos x}{2}$

$\frac{1}{4} e^x + \frac{1}{2D}\left[x^2 - x + \frac{1}{2}\right] + \frac{\cos x}{2}$

$\frac{1}{4} e^x + \left[\frac{x^2}{2D} - \frac{x}{2D} + \frac{1}{4D}\right] + \frac{\cos x}{2}$

$\frac{1}{4} e^x + \frac{2x^2 - 2x + 1}{4D} + \frac{\cos x}{2}$

$\frac{1}{4} e^x + \frac{2x^3}{3} - \frac{2x^2}{2} + x + \frac{\cos x}{2}$

$\frac{1}{4} e^x + \frac{2x^3}{3} - x^2 + x + \frac{\cos x}{2}$

$= \frac{1}{4} e^x + \frac{x^2}{D^2+2D+1} - \frac{\sin x}{D^2+2+1}$

$= \frac{1}{4} e^x + \frac{x^2}{(D+1)^2} - \frac{\sin x}{D^2+2+1}$

$= \frac{1}{4} e^x + [D+1]^{-2} x^2 - \frac{\cos x}{-1+2+1}$

$= \frac{1}{4} e^x + [1-2D+3D^2] x^2 - \frac{\cos x}{2}$

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$$= \frac{1}{4}e^{x} + x^2 - 2Dx^2 + 3D^2x^2 + \frac{\cos x}{2}$$

$$= \frac{1}{4}e^{x} + x^2 - 4x + 6 + \frac{\cos x}{2}$$

$$y = CF + PI$$

$$y = (c_1 + c_2 x)e^{-x} + \frac{1}{4}e^{x} + x^2 - 4x + 6 + \frac{\cos x}{2}$$

(Q) $(D^2 - 3D + 2)y = \sin 3x + x^2 + x + e^{4x}$

A.E: $m^2 - 3m + 2 = 0$

$$= \frac{3 \pm \sqrt{9-8}}{2}$$

$$= \frac{3 \pm 1}{2}$$

$$m = 2, 1$$

$$CF = c_1 e^{x} + c_2 e^{2x}$$

$$PI = \frac{1}{D^2 - 3D + 2} \cdot \sin 3x + x^2 + x + e^{4x}$$

$$= \frac{1 \cos 3x}{-9 - 3D + 2} + \frac{x^2}{2\left(\frac{D^2}{2} - \frac{3D}{2} + 1\right)} + \frac{x}{2\left(\frac{D^2}{D} - \frac{3D}{2} + 1\right)} + \frac{e^{4x}}{6}$$

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$$= \frac{1 \cos 3x}{-9 - 3D + 2} + \frac{1}{2}\left[x^2\left[1 + \frac{3D}{2} - \frac{D^2}{2}\right]^{-1}\right] +$$

$$\frac{1}{2}\left[x\left[1 + \frac{3D}{2} - \frac{D^2}{2}\right]^{-1}\right] + \frac{e^{4x}}{6}$$

$$= \frac{\cos 3x}{-7 - 3D} + \frac{1}{2}\left[x^2 + \frac{3Dx^2}{2} + \frac{D^2x^2}{2} + \left(\frac{3D}{2}\right)^2 + \left(\frac{D^2}{2}\right)^2\right] + \frac{1}{2}\left[x - \frac{3D}{2} + \frac{D^2}{2} + \left(\frac{3D}{2}\right)^2 - \frac{D^2}{2}\right] + \frac{e^{4x}}{6}$$

=

Ex 2.30

(1) $(x^3e^x - my^2)dx + mxy\,dy = 0$

So $(x^3e^x - my^2)dx + mxy\,dy = 0$ — (1)

where $M = x^3e^x - my^2$ | $N = mxy$

$\frac{dM}{dy} = 0 - 2my$ | $\frac{dN}{dx} = my$

$\frac{dM}{dy} \neq \frac{dN}{dx}$ Not exact

$\frac{1}{N}\left(\frac{dM}{dy} - \frac{dN}{dx}\right) = \frac{1}{N}\left(-2my - my\right)$

$= -\frac{1}{N}(3my$

$= \frac{-3my}{mxy} = -\frac{3}{x}$

$I.f = e^{-3\int 1/x} = e^{-3\log x} = \frac{1}{x^3}$

$I.f \times (1) = \left(\frac{x^3e^x - my^2}{x^3}\right)dx + \frac{mxy}{x^3}dy = 0$

$\left(e^x - \frac{my^2}{x^3}\right)dx + \frac{my}{x^2}dy = 0$

$M' = e^x - \frac{my^2}{x^3}$, $N' = \frac{my}{x^2}$

$\frac{dM'}{dy} = 0 - \frac{2my}{x^3}$, $\frac{dN'}{dx} = \frac{-2my}{x^3}$

$\frac{dM'}{dy} = \frac{dN'}{dx}$ = Exact

Solution is

$\int_{y\ \text{constant}}\left(e^x - \frac{my^2}{x^3}\right)dx + \int_{\text{w.r.t } x} 0\,dy = C$

$\boxed{e^x + \frac{my^2}{2x^2}} = C$ Ans

(2) $y^2 = (xy - x^2)(dy/dx)$

$y^2 = (xy - x^2)\,dy = 0$ — (I)

where $M = y^2$ | $N = -xy + x^2$

diff partially

$\frac{dM}{dy} = 2y$ $\qquad$ $\frac{dN}{dx} = y + 2x$

$\boxed{\frac{dM}{dy} \neq \frac{dN}{dx}}$

which is homogenus

$$I.F = \frac{1}{Mx+Ny} = \frac{1}{y^2x - x^2y^2 + x^2y^2} = \frac{1}{x^2y}$$

$$(1) \times IF = \frac{y^2}{x^2y}\,dx - \left(\frac{xy}{x^2y} - \frac{x^2}{x^2y}\right)dy = 0$$

$$= \frac{y}{x^2}\,dx - \left(\frac{1}{x} - \frac{1}{y}\right)dy = 0$$

where $m' = \frac{y}{x^2}$, $N' = -\frac{1}{x} + \frac{1}{y}$

diff w.r.t partialy

$$\frac{dm'}{dy} = \frac{1}{x^2}, \quad \frac{dN'}{dx} = \frac{1}{x^2}$$

$$\boxed{\frac{dm'}{dy} = \frac{dN'}{dx}}$$ so exact

solution is $\int_{y\text{-constant}} \frac{y}{x^2}\,dx + \int_{\text{w.r.t of } x} \frac{1}{y}\,dy = c$

$$= -\frac{y}{x} + \log y = c$$

$$= y - x\log y + cx = 0$$

(3) $(3x+2y^2)\,y\,dx + 2x(2x+3y^2)\,dy = 0$

sol) $(3xy+2y^3)dx + (4x^2+6xy^2)\,dy = 0$ — (1)

where $m = 3xy - 2y^3$ | $N = 4x^2+6y^2x$

$\frac{dm}{dy} = 3x+6y^2$ | $\frac{dN}{dx} = 8x+6y^2$

$$\boxed{\frac{dm}{dy} \neq \frac{dN}{dx}}$$

I.F = $x^h y^k$

$IF = (3x^{h+1}y^{k+1} + 2x^h y^{k+3})dx + (4x^{h+2}y^k + 6x^{h+1}y^{k+2})\,dy = 0$

$m' = 3x^{h+1}y^{k+1} + 2x^h y^{k+3}$

$$\frac{dm'}{dy} = 3(k+1)x^{h+1}y^k + 2(k+3)x^h y^{k+2}$$

$N' = 4x^{h+2}y^k + 6x^{h+1}y^{k+2}$

$$\frac{dN'}{dx} = 4(h+2)x^{h+1}y^k + 6(h+1)x^h y^{k+2}$$

compare both side coffiecent of x^{h+1} & y^k & $x^h y^{k+2}$

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$3(K+1) = 4(h+2)$ $\quad 2(K+3) = 6(h+1)$

$3K+3 = 4h+8$ $\quad 2K+6 = 6h+6$

$3K - 4h - 5 = 0$ — (A) $\quad 2K - 6h = 0$

Solve eq A & B

$3K - 4h - 5 = 0$

$2K - 6h = 0$

$6K - 8h - 10 = 0$

$6K - 18h = 0$

$10h - 10 = 0$

$h = \frac{10}{10} = 1$

Put h value in B eq :-

$2K - 6(1) = 0$

$2K - 6 = 0$

$\boxed{K = 3}$

Put h & K value in eq (2)

$= 3(K+1)x^{h+1}y^{K} + 2(K+3)x^{h}y^{K+2} = 4(h+2)x^{h+1}y^{K} + 6(h+1)x^{h}y^{K+2}$

$= 3(3+1)x^{1+1}y^{3} + 2(3+3)x^{1}y^{3+2} = 4(1+2)x^{1+1}y^{3} + 6(1+1)x^{1}y^{3+2}$

$= 12x^2y^3 + 12xy^5 = 12x^2y^3 + 12xy^5$

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$\boxed{\frac{dm'}{dy} = \frac{dN'}{dx}}$ ⇒ Exact

Solution is

$\int 3x^2y^4 + 2xy^6 dx + \int 0 \, dy = c$

y constant $\quad$ w.r.t. of x

$3\frac{x^3y^4}{3} + \frac{2x^2y^6}{2} = c$

$x^3y^4 + x^2y^6 = c$

$\boxed{x^2y^4(x+y^2) = c}$

(4) $y(axy + e^x)dx - e^x dy = 0$

Sol. $(axy^2 + xe^x)dx - e^x dy = 0$ — (1)

where $m = axy^2 + ye^x$, $N = -e^x$

diff. wrt y partially

$\frac{dm}{dy} = 2xy + e^x$, $\frac{dN}{dx} = -e^x$

$\boxed{\frac{dm}{dy} \neq \frac{dN}{dx}}$ ⇒ Not Exact

$\frac{1}{m}\left(\frac{dN}{dx} - \frac{dM}{dy}\right)$

$\frac{1}{m}\left(-e^{x} - 2xy - e^{x}\right)$

$\frac{1}{m}\left(2e^{x} - 2xy\right)$

$\frac{2(e^{x} + xy)}{(e^{x} + axy)}$

$= \frac{-2}{y}$

$I.f = e^{\int \frac{-2}{y} dy}$

$= e^{-2\log y}$

$= e^{\log y^{-2}}$

$\boxed{I.f = \frac{1}{y^2}}$

(I) $\times$ I.f

$\left(\frac{axy^2}{y^2} + \frac{y e^{x}}{y^2}\right) dy - \frac{e^{x}}{y^2} dy = 0$

where $m' = ax + \frac{e^{x}}{y}$, $n' = -\frac{e^{x}}{y^2}$

diff partially

$dm' = a - \frac{e^{x}}{y^2}$, $\frac{dN'}{dx} = -\frac{e^{x}}{y^2}$

solution is

$\int_{y\ \text{constant}} ax + \frac{e^{x}}{y} dx + \int_{\text{w.r.t. } x} 0 dy = c$

$\frac{ax^2}{2} + \frac{e^{x}}{y} = c$

$\frac{ax^2}{2} + \frac{e^{x}}{y} - c = 0$

$\boxed{ax^2y + 2e^{x} - cy = 0}$

(5) $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$

where $m = 3x^2y^4 + 2xy$, $N = 2x^3y^3 - x^2$

diff partially

$\frac{dm}{dy} = 12x^2y^3 + 2x$, $\frac{dN}{dx} = 6x^2y^3 - 2x$

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$\boxed{dm \neq dn}$

$\frac{1}{m}\left(\frac{dN}{dx} - \frac{dm}{dy}\right)$

$\frac{1}{m}\left(6x^2y^3 - 12x^2y^3 - 2x\right)$

$\frac{1}{m}\left(-6x^2y^3 - 4x\right)$

$-2\left(\frac{3x^2y^3 + 2x}{3x^2y^3 + 2x\,y}\right)$

$= -\frac{2}{y}$

$If = e^{-2\int\frac{1}{y}dy} = e^{-2\log y}$

$= e^{\log y^{-2}} = \frac{1}{y^2}$

$I \times If =$

$\left(\frac{3x^2y^4}{y^2} + \frac{2xy}{y^2}\right)dx + \left(\frac{2x^3y^3}{y^2} - \frac{x^2}{y^2}\right)dy =$

$\left(3x^2y^2 + \frac{2x}{y}\right)dx + \left(2x^3y - \frac{x^2}{y^2}\right)dy =$

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where $m' = 3x^2y^2 + \frac{2x}{y}$, $N' = 2x^3y - \frac{x^2}{y^2}$

diff. partially

$\frac{dm'}{dy} = 6x^2y - \frac{2x}{y^2}$, $\frac{dN'}{dx} = 6x^2y - \frac{2x}{y^2}$

$\boxed{\frac{dm'}{dy} = \frac{dN'}{dx}}$ ⇒ Exact

solution is

$\int\limits_{y\ constant} 3x^2y^2 + \frac{2x}{y}\,dx + \int\limits_{w.r.t.\ of\ x} 0\,dy = c$

$\frac{3}{3}x^3y^2 + \frac{x^2}{y} = c$

$\boxed{x^3y^3 + \frac{x^2}{y} = c}$

Q. 6 $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$

as $(xy^2 + 2x^2y^3)dx + (x^2y - x^3y^2)dy = 0$ — (1)

where $m = xy^2 + 2x^2y^3$, $N = x^2y - x^3y^2$

diffr. partially

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$\frac{dm}{dy} = 2xy + 6x^2y^2$, $\frac{dn}{dx} = 2xy\ 3x^2$

$\boxed{\frac{dm}{dy} \neq \frac{dn}{dx}}$ Not Exact

$I.f = \frac{1}{mx - ny}$

$= \frac{1}{(xy^2 + 2x^2y^3)x - (x^2y - x^3y^2)y}$

$= \frac{1}{x^2y^2 + 2x^3y^3 - x^2y^2 + x^3y^3}$

$I.f = \frac{1}{3x^3y^3}$

(I) × I.F

$= \left(\frac{xy^2}{3x^3y^3} + \frac{2x^2y^3}{3x^3y^3}\right)dx + \left(\frac{x^2y}{3x^3y^3} - \frac{x^3y^2}{3x^3y^3}\right)dy$

$= \left(\frac{1}{3x^2y} + \frac{2}{3x}\right)dx + \left(\frac{1}{3xy^2} - \frac{1}{3y}\right)dy = 0$

where $m' = \frac{1}{3x^2y} + \frac{2}{3x}$ | $n' = \frac{1}{3xy^2} - \frac{1}{3y}$

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$\frac{dm'}{dy} = \frac{-1}{3x^2y^2} + 0$, $\frac{dN'}{dx} = \frac{-1}{3x^2y^2} - 0$

$\boxed{\frac{dm'}{dy} = \frac{dN'}{dx}}$ ⇒ Exact

Solution is

$\int \frac{1}{3x^2y} + \frac{2}{3x}\,dx + \left(-\frac{1}{3}\right)\cdot\int \frac{1}{y}\,dy = c$

y constant w.r.t. of x

$-\frac{1}{3}\frac{1}{xy} + \frac{2}{3}\log x + \left(-\frac{1}{3}\log y\right) = C$

$\frac{2}{3}\log x + \left(-\frac{1}{3}\log y\right) = \frac{1}{3}\frac{1}{xy} + c$

$\frac{1}{3}\left(\log x^2 + \log y^{-1} = \frac{1}{3}\frac{1}{xy} + C\right)$

$\boxed{\log\frac{x^2}{y} = \frac{1}{xy} + C}$

Q.7 $(2y\,dx + 3x\,dy) + 2xy(3y\,dx + 4x\,dy) = 0$

Sol. $2y\,dx + 3x\,dy + 6xy^2\,dx + 3x^2y\,dx = 0$

$(2y + 6y^2x)\,dx + (3x + 8x^2y)\,dy = 0$

where $M = 2y + 6xy^2$; $N = 3x + 3x^2y$

diff. partially

$\frac{dM}{dy} = 2 + 12xy$, $\frac{dN}{dx} = 3 + 16xy$

$\boxed{\frac{dM}{dy} \neq \frac{dN}{dx}}$ Not Exact

I.F $= x^h y^k$

$= (2x^h y^{k+1} + 6x^h y^{k+2})\,dx + (3x^{h+1} y^k + 8x^{h+2} y^{k+1})\,dy = 0$

where

$M' = 2x^h y^{k+1} + 6x^{h+1} y^{k+2}$;

$N' = 3x^{h+1} y^k + 8x^{h+2} y^{k+1}$

$\frac{dM'}{dy} = 2(k+1)x^h y^k + 6(k+2)x^{h+1} y^{k+1}$

$\frac{dN'}{dx} = 3(h+1)x^h y^k + 8(h+2)x^{h+1} y^{k+1}$

$\frac{dM'}{dy} = \frac{dN'}{dx}$

compare in both side coefficient of $x^h y^k$ & $x^{h+1} y^{k+1}$

$2(k+1) = 3(h+1)$
$2k + 2 = 3h + 3$
$3h - 2k + 1 = 0$ — (A)

$6(k+2) = 8(h+2)$
$6k + 12 = 8h + 16$
$8h - 6k + 4 = 0$
$4h - 3k + 2 = 0$ — (B)

$3h - 2k + 1 = 0$ — (A)
$4h - 3k + 2 = 0$ — (B)

(A) × 4 & B × 3

$12h - 8k + 4 = 0$
$12h - 9k + 6 = 0$

$k - 2 = 0$
$k = 2$

Put the value of k in eq (B)

$4h - 3 \times (2) + 2 = 0$
$h = 1$

$6xy^2 + 24x^2y^3 = 6xy^2 + 24x^2y^3$

$\frac{dm'}{dy} = \frac{dN}{dx} \Rightarrow$ Exact

Solution is

$\int_{y\ \text{constant}} 2xy^3 + 6x^2y^4\, dx + \int_{\text{w.r.t. of } x} 0\, dy = 0$

$\frac{2x^2y^3}{2} + \frac{6x^3y^4}{3} = 0$

$\boxed{x^2y^3 + 2x^3y^4 = C}$

(8) $2xy^2\, dx = e^x (dy - y\, dx)$

$2xy^2\, dx = e^x dy - e^x y\, dx$

$(2xy^2 + e^x y)\, dx - e^x dy = 0$ — (1)

$m = 2xy^2 + e^x y$, $N = -e^x$

$\frac{dm}{dy} = 4xy + e^x$, $\frac{dN}{dx} = -e^x$

$\boxed{\frac{dm}{dy} \neq \frac{dn}{dx}}$ Not Exact

$\frac{1}{m}\left(\frac{dN}{dx} - \frac{dm}{dy}\right) = \frac{1}{m}(-4xy - e^x - e^x)$

$= \frac{1}{m}(-4xy - 2e^x)$

$= \frac{-2(2xy + e^x)}{2(xy + e^x)y} = \frac{-2}{y}$

$I.f = e^{\int \frac{-2}{y} dy} = e^{-2\log y}$

$= e^{\log \frac{1}{y^2}} = \frac{1}{y^2}$

(1) $\times I.f$: $\left(\frac{2xy^2}{yx} + \frac{e^x y}{yx}\right) dx - \frac{e^x}{y^2} dy = c$

$2x + \frac{e^x}{y^2}\, dx - \frac{e^x}{y^2}\, dy = c$

$m' = 2x + \frac{e^x}{y}$, $N' = \frac{-e^x}{y^2}$

diff. partially

$\frac{dm'}{dy} = \frac{e^x}{y^2}$, $\frac{dn'}{dx} = \frac{-e^x}{y^2}$

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$$\frac{dm'}{dy} = \frac{dn'}{dx} \Rightarrow \text{Exact}$$

solution is

$$\underset{y\ \text{constant}}{\int} 2x + \frac{e^x}{y}\, dx + \underset{\text{w.r.t. of } x}{\int} 0\, dy = c$$

$$x^2 + \frac{e^x}{y} + c_1 = c$$

$$\boxed{x^2 + \frac{e^x}{y} = K}$$

(9) $(x^4y^4 + x^2y^2 + xy)y\,dx + (x^4y^4 - x^2y^2 + xy)x\,dy = 0$

Sol.^{n} $(x^4y^5 + x^2y^3 + xy^2)dx + (x^5y^4 - x^3y^2 + x^2y)dy = 0$

where $m = x^4y^5 + x^2y^3 + xy^2$

diff. partially

$$\frac{dm}{dy} = 5x^4y^4 + 3x^2y^2 + 2xy$$

$$n = x^5y^4 - 3x^3y^2 + x^2y$$

$$\frac{dn}{dy} = 5x^4y^4 - 3x^2y^2 + 2xy$$

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$$\frac{dm}{dy} \neq \frac{dn}{dx} \Rightarrow \text{Not Exact}$$

$$\text{I.F.} = \frac{1}{mx + ny}$$

$$= \frac{1}{x^5y^5 + x^3y^3 + x^2y^2 - x^5y^5 + x^3y^3 - x^2y^2}$$

$$= \frac{1}{2x^3y^3}$$

(1) $\times$ I.F

$$= \left(\frac{x^4y^5}{2x^3y^3} + \frac{x^2y^3}{2x^3y^3} + \frac{xy^2}{2x^3y^3}\right)dx + \left(\frac{x^5y^4}{2x^3y^3} - \frac{x^3y^2}{2x^3y^3} + \frac{x^2y}{2x^3y^3}\right)dy = 0$$

$$= \left(\frac{xy^2}{2} + \frac{1}{2x} + \frac{1}{2x^2y}\right)dy + \left(\frac{x^2y}{2} - \frac{1}{2y} + \frac{1}{2xy^2}\right)dy = 0$$

$$= \frac{1}{2}\left(xy^2 + \frac{1}{x} + \frac{1}{x^2y}\right)dx + \frac{1}{2}\left(x^2y - \frac{1}{y} + \frac{1}{xy^2}\right)dy = 0$$

where $M' = \frac{1}{2}\left(xy^2 + \frac{1}{x} + \frac{1}{x^2 y}\right)$,

$N' = \frac{1}{2}\left(x^2 y - \frac{1}{y} + \frac{1}{xy^2}\right)$

diff. partially

$$\frac{dM'}{dy} = \frac{1}{2}\left(2xy + 0 - \frac{1}{x^2y^2}\right),$$

$$\frac{dN'}{dx} = \frac{1}{2}\left(2xy - 0 + \frac{1}{x^2y^2}\right)$$

$$\boxed{\frac{dN'}{dx} = \frac{dM'}{dy}} \Rightarrow \text{Exact}$$

solution is

$$\frac{1}{2}\int_{y\text{-constant}} xy^2 + \frac{1}{x} + \frac{1}{x^2 y}\, dx + \left(-\frac{1}{2}\right)\int \frac{1}{y}\, dy = c$$

$$\frac{1}{2}\left[\frac{x^2y^2}{2} + \log x + -\left(\frac{1}{xy}\right) + \left(-\frac{1}{2}\right)\right.$$

$$\left.[\log y\right] = c$$

$$= \frac{1}{2}x^2y^2 + \log\frac{x}{y} - \frac{1}{xy} = c$$

(10) $(y^2 + 2x^2 y)\,dx + (2x^3 - xy)\,dy = 0$ — (1)

where $M = y^2 + 2x^2 y$, $N = 2x^3 - xy$

$$\frac{dM}{dy} = 2y + 2x^2, \quad \frac{dN}{dx} = 6x^2 - y$$

$$\boxed{\frac{dM}{dy} \neq \frac{dN}{dx}}$$

If $= x^h y^k$

I.F × (1)

$(x^h y^{k+2} + 2x^{k+2} y^{k+1})\,dx + (2x^{h+3} y^k - x^{h+1} y^{k+1})\,dy = 0$ (2)

$M' = (x^h y^{k+2} + 2x^{k+2} y^{k+1})$, $N' = 2x^{h+3} y^k - x^{h+1} y^{k+1}$

$$\frac{dM'}{dy} = (k+2)x^h y^{k+1} + 2(k+1)x^{k+2} y^k$$

$$\frac{dN'}{dx} = 2(h+3)x^{h+2} y^k - (h+1)x^h y^{k+1}$$

$(k+2)x^h y^{k+1} + 2(k+1)x^{h+2} y^{k+1} = 2(h+3)x^{h+2} y^k - (h+1)x^h y^{k+1}$ — (3)

compare in both side coefficients of $x^h y^{k+1}$ & $x^{h+2} y^k$

$(k+2) = -(h+1)$ | $2(k+1) = 2(h+3)$

$(k+2) = -h-1$ | $2k+2 = 2h+6$

$k+h+3 = 0$ — (A) | $2k-2h-4=0$ — (B)

from eq (A) & (B)

$k+h+3=0$ — (A)
$2k-2h-4=0$ — (B)

$$k = -\frac{1}{2}$$

$$h = -\frac{5}{2}$$

Put the value of h & k in eq (B)

$\left(-\frac{1}{2}+2\right)x^{-5/2}y^{-1/2+1} + 2(-1/2+1)x^{-5/2+1}y^{-1/2+1} = 2\left(-\frac{5}{2}+3\right)x^{-5/2+2+1/2}y - (-5/2+1)x^{-5/2}y^{-1/2}$

$$\boxed{\frac{dm'}{dy} = \frac{dn'}{dx}}$$ Exact

$\int_{y\ \text{constant}} x^{-5/2}y^{1/2} + x^{-1/2}y^{1/2}dx + \int 0\,dy = c$ w.r.t. of x

$= \frac{x^{-5/2+1}}{-\frac{5}{2}+1}y^{3/2} + \frac{2x^{-1/2+1}}{-1/2+1}y^{1/2} = c$

$$\boxed{= -\frac{2}{3}x^{-3/2}y^{-3/2} + 4x^{1/2}y^{1/2} = c}$$

(11) $(x^2+y^2+2x)dx + 2y\,dy = 0$

$(x^2+y^2+2x)dx + 2y\,dy = 0$ — (1)

where $m = x^2+y^2+2x$, $n = 2y$

diff. Partially

$\frac{dm}{dy} = 0+2y+0$, $\frac{dn}{dx} = 0$

$\boxed{\frac{dm}{dy} \neq \frac{dn}{dx}}$ ⇒ Not Exact

$\frac{1}{N}\left(\frac{dm}{dy} - \frac{dn}{dx}\right)$

$\frac{1}{N}(2y-0)$

$\frac{\partial y}{\partial y} = 1$

I.f $= e^{\int 1 dx}$

$= e^{x}$

(1) × I.f

$e^{x}(x^2+y^2+3x)dx + e^{x}y\,dy = 0$

$m' = e^{x}(x^2+y^2+2x)$, $N' = e^{x}2y$

diff. partially

$\frac{dm'}{dy} = e^{x}(0+2y+0)$, $\frac{dn'}{dx} = 2ye^{x}$

$\boxed{\frac{dm'}{dy} = \frac{dn'}{dx}} \Rightarrow$ Exact

Solution is

$\int e^{x} + e^{x}y^2 + 2xe^{x}\,dx + \int 0\,dy = 0$

y constant   W.r.t. of x

$x^2e^{x} - 2xe^{x} + 2e^{x} + y^2e^{x} + 2xe^{x} - 2e^{x} = c$

$\boxed{x^2e^{x} + y^2e^{x} = c}$

(12) $x^2y\,dx - (x^3+y^3)\,dy = 0$

Sol. $x^2y\,dx - x^3 - y^3\,dy = 0$ — (1)

where
$m = x^2y$, $n = -x^3 - y^3$

$\frac{dm}{dy} = x^2$, $\frac{dN}{dx} = -3x^2$

$\boxed{\frac{dm}{dy} \neq \frac{dN}{dx}} \Rightarrow$ Not Exact

which is homogeneous

I.f. $= \frac{1}{mx+Ny} = \frac{1}{x^3y - x^3y - y^4}$

I.f. $= -\frac{1}{y^4}$

(1) × I.f $= \frac{-x^2}{y^4}y\,dx + \frac{x^3}{y^4} + \frac{y^3}{y^4}\,dy = 0$

$= \frac{-x^2}{y^3}dx + \frac{x^3}{y^4} + \frac{1}{y}dy = 0$

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where $m' = \frac{-x^2}{y^3}$, $n' = \frac{x^3}{y^4} + \frac{1}{y}$

diff. partially

$\frac{dm'}{dy} = \frac{3x^2}{y^4}$, $\frac{dn'}{dx} = \frac{3x^2}{y^4}$

$$\boxed{\frac{dm'}{dy} = \frac{dn'}{dx}}$$

solution is

$$\int_{y\ \text{constant}} \frac{-x^2}{y^3}\,dx + \int_{\text{w.r.t. } x} \frac{1}{y}\,dx = c$$

$$-\frac{x^3}{3y^3} + \log y = c$$

(13) $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

$(xy^3 + y)dx + (2x^2y^2 + 2x + 2y^4)dy = 0$ —(1)

where $m = xy^2 + y$; $n = 2xy^2 + 2x + 2y^4$

$\frac{dm}{dy} = 3xy^2 + 1$, $\frac{dn}{dx} = 4xy^2 + 2$

$\frac{dm}{dy} \neq \frac{dn}{dx} \Rightarrow$ Not Exact

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$\frac{1}{m}(4xy^2 + 2 - 3xy^2 - 1) = \frac{1}{m}(xy^2 + 1)$

$= \frac{xy^2 + 1}{2(xy^2 + 1)} = \frac{1}{y}$

I.f. $= e^{\int \frac{1}{y} dy} = e^{\log y} = y$

I.f × (1) $(xy^3 + y)y\,dx + (2x^2y^2 + 2x + 2y^4)y\,dy = 0$

$xy^4 + y^2 dx + 2x^2y^3 + 2xy + 2y^5 dy = 0$

$m' = xy^4 + y^2$, $n' = 2x^2y^3 + 2xy + 2y^5$

$\frac{dm'}{dy} = 4xy^3 + 2y$, $\frac{dN'}{dx} = 4xy^3 + 2y + 0$

$\frac{dm'}{dy} = \frac{dN'}{dx} \Rightarrow$ Exact

Sol. is

$$\int_{y\ \text{constant}} xy^4 + y^2\,dx + \int_{\text{w.r.t. of } x} 2y^5\,dy = c$$

$$\frac{x^2y^4}{2} + xy^2 + \frac{2y^6}{6} = c$$

$$\frac{1}{2}x^2y^4 + xy^2 + \frac{1}{3}y^6 = c$$

$$\boxed{3x^2y^4 + 6xy^2 + 2y^6 = c}$$

(14) $(y-2x^3)dx - x(1-xy)dy = 0$

$(y-2x^3)dx - (x-x^2y)dy = 0$ — (1)

where $M = y-2x^3$, $N = x + x^2y$

diff. partially

$\frac{dM}{dy} = 1-0$, $\frac{dN}{dx} = -1 + 2xy$

$\boxed{\frac{dM}{dy} \neq \frac{dN}{dx}}$ ⇒ Not Exact

$\frac{1}{N}\left(\frac{dM}{dy} - \frac{dN}{dx}\right)$

$\frac{1}{N}(1+1-2xy)$

$\frac{1}{N}(2-2xy)$

$\frac{2(1-xy)}{-x(1-xy)}$

$= \frac{2}{-x}$

$I.F. = e^{\int -\frac{2}{x}dx} = e^{\log x^{-2}} = \frac{1}{x^2}$

(1) × I.F.

$\left(\frac{y}{x^2} - \frac{2x^3}{x^2}\right)dx - \left(\frac{x}{x^2} - \frac{x^2y}{x^2}\right)dy = 0$

$\left(\frac{y}{x^2} - 2x\right)dx - \left(\frac{1}{x} - y\right)dy = 0$

$M' = \frac{y}{x^2} - 2x$, $N' = -\frac{1}{x} + y$

$\frac{dM'}{dy} = \frac{1}{x^2} - 0$, $\frac{dN'}{dx} = \frac{1}{x^2} + 0$

$\boxed{\frac{dM'}{dy} = \frac{dN'}{dx}}$ ⇒ Exact

Solution is

$\int \frac{y}{x^2} - 2x\,dx + \int y\,dc = c$

y constant w.r.t. of x

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$$-\frac{y}{x} - \frac{2x^2}{2} + \frac{y^2}{2} = c$$

$$-\frac{y}{x} - x^2 + \frac{y^2}{2} = c$$

$$\boxed{xy^2 = cx + 2y + 2x^3}$$

(15) $(xy\sin xy + \cos xy)y\,dx + (xy\sin xy - \cos xy)x\,dy = 0$

Sol. $(xy^2\sin xy + y\cos xy)dx + (x^2y\sin xy - x\cos xy)dy = 0$

where $m = xy^2\sin xy + y\cos xy$

$n = x^2y\sin xy - x\cos xy$

diff. partially

$$\frac{dm}{dy} = 2yx\sin xy + x^2y^2\cos xy + \cos y - xy\sin y$$

$$\frac{dn}{dx} = 2xy\sin xy + x^2y^2\cos xy - \cos y + xy\sin xy$$

$$\boxed{\frac{dm}{dy} \neq \frac{dN}{dx}} \Rightarrow \text{Not exact}$$

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$$I.F = \frac{1}{mx - ny} = \frac{1}{xy^2\sin xy + xy\cos xy - x^2y^2\sin xy + xy\cos xy}$$

$$I.f = \frac{1}{2xy\cos xy}$$

(1) $\times$ I.f =

$$\left(\frac{xy^2\sin xy + y\cos xy}{2xy\cos xy}\right)dx + \left(\frac{x^2y\sin xy - x\cos xy}{2xy\cos xy}\right)dy = 0$$

$$= \frac{1}{2}\left(y\tan xy + \frac{1}{x}\right)dx + \frac{1}{2}\left(x\tan xy - \frac{1}{y}\right)dy = 0 \quad (2)$$

$$= m' = \frac{1}{2}\left(y\tan x + \frac{1}{x}\right), \quad n' = \frac{1}{2}\left(x\tan xy - \frac{1}{y}\right)$$

= diff partially

$$\frac{dm'}{dy} = \frac{1}{2}\left[yx\sec^2 x + 1 \times \tan xy\right),$$

$$\frac{dn'}{dx} = \frac{1}{2}\left[xy\sec^2 xy + 1 \times \tan xy\right)$$

$$\boxed{\frac{dm'}{dy} = \frac{dn'}{dx}} \Rightarrow \text{Exact}$$

Solution of (2) eq

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$$= \int \frac{1}{2}\left(y \tan xy + \frac{1}{x}\right) dx + \frac{1}{2}\int -\frac{1}{y}\,dy = c$$

y-constant w.r.t x

$$= \frac{1}{2}\left(y \times \frac{\log \sec xy}{y} \times \log x\right) - \frac{1}{2}\log y = \log c$$

$$= \log \sec xy + \log x - \log y = \log k$$

$$\log \frac{x \sec xy}{y} = \log k$$

$$\boxed{x \sec xy = yk}$$