Unit 3 of Differential Equation

* Linear Differential equation of second order with variable coefficient *

① Homogenous cauchy's linear D.E.

$(x^n D^n + a_1 x^{n-1} D^{n-1} + a_2 x^{n-2} D^{n-2} + \dots + a_n)y = Q(x)$

step-① $D = \frac{d}{dx}$

step ② put $x^4 D^4 = D_1(D_1-1)(D_1-2)(D_1-3)$

$x^3 D^3 = D_1(D_1-1)(D_1-2)$

$x^2 D^2 = D_1(D_1-1)$

$xD = D_1$

$x^5 D^5 = D_1(D_1-1)(D_1-2)(D_1-3)(D_1-4)$

step ③

$D_1 = \frac{d}{dz} \rightarrow x = e^z, \log x = z$

step-④ $f(D)y = Q(x)$

ex-

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = 0 \qquad \text{put } D = \frac{d}{dx}$$

$x^2 D^2 y - xDy - 3y = 0$

$(x^2 D^2 - xD - 3)y = 0$ Cauchy's Homogenous L.D.E.

put $x = e^z$ or $z = \log x$

$x^2 D^2 = D_1(D_1-1)$, $xD = D_1$, $D_1 = \frac{d}{dz}$

$(D_1(D_1-1) - D_1 - 3)y = 0$

$(D_1^2 - D_1 - D_1 - 3)y = 0$

$(D_1^2 - 2D_1 - 3)y = 0$

This is L.D.E. with constant coefficient

A.E. is $D_1^2 - 2D_1 - 3 = 0$

$(D_1 - 3)(D_1 + 1)$

$D_1 = 3 \quad D_1 = -1$ The roots are real and ~~imaginary~~ different

$\boxed{\therefore D_1 = -1, 3}$

$\boxed{C.F. = c_1 e^{-z} + c_2 e^{3z}}$ $\therefore PI = 0$

general solution is

$y = C.F. + P.I = c_1 e^{-z} + c_2 e^{3z}$

$\boxed{\therefore e^z = x \quad y = c_1 x^{-1} + c_2 x^3}$

<u>Example</u> - 2 $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$

$\therefore \frac{d}{dx} = D \quad x^2 D^2 y - 2x Dy - 4y = x^4$

$(x^2 D^2 - 2xD - 4)y = x^4$

This is Cauchy's Homogeneous L.D.E.

put $\Rightarrow x^2 D^2 = D_1(D_1 - 1) \quad xD = D_1, x = e^z$ या $\log x = z$

$(D_1(D_1 - 1) - 2(D_1) - 4)y = e^{4z}$

$(D_1^2 - D_1 - 2D_1 - 4)y = e^{4z}$

$(D_1^2 - 3D_1 - 4)y = e^{4z}$

$(D_1 - 4)(D_1 + 1)$ roots $\therefore$ A.E $\Rightarrow D_1^2 - 3D_1 - 4 = 0$

$\boxed{D_1 = 4, -1}$

$\boxed{CF = c_1 e^{-z} + c_2 e^{4}}$ $P.I = \frac{1}{f(D)} \times Q(z)$

$$P.I. = \frac{1}{(D_1^2 - 3D_1 - 4)} \times e^{4z} = \frac{z}{2D_1 - 3} \times e^{4z}$$

$a = 4 \quad D_1 = a$

$$= \frac{x}{2(4) - 3} \times e^{4z} = \frac{z}{8-3} \times e^{4z}$$

$$P.I. = \frac{x}{5} \times e^{4z}$$

general solution is $= C.F. + P.I.$

$$= c_1 e^{-z} + c_2 e^{4z} + \frac{z}{5} e^{4z}$$

$\therefore x = e^z, \quad z = \log x$

$$\boxed{y = c_1 x^{-1} + c_2 x^4 + \frac{\log x}{5} x^4}$$

<u>Example</u>: $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 4y = \sin(\log x^2)$

$\frac{d}{dx} = D$

$$x^2 D^2 y + x D y + 4y = \sin(\log x^2)$$

$$(x^2 D^2 + x D + 4)y = \sin(\log x^2) = \sin(2\log x)$$

put $x^2 D^2 = D_1(D_1 - 1), \quad xD = D_1 \quad x = e^z \quad z = \log x$

$$(D_1(D_1 - 1) + D_1 + 4)y = \cancel{\sin(2\log x)} = \sin 2z$$

$$(D_1^2 - D_1 + D_1 + 4)y = \cancel{\sin(2\log x)} = \sin 2z$$

$$(D_1^2 + 4)y = \cancel{\sin(2\log x)} = \sin 2z$$

$\therefore A.E. = D_1^2 + 4 = 0 \qquad$ <u>roots</u> $= D_1^2 = -4$

$D_1^2 = i^2 4$

roots are imaginary $\qquad \boxed{D_1 = \pm i2}$

$$C.F. = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$$

$$\boxed{C.F. = (c_1 \cos \beta e^z + c_2 \sin \beta e^z)} \text{ या } (c_1 \cos 2z + c_2 \sin 2z)$$

$$P.I. = \frac{1}{D_1^2 + 4} \times \sin(2z)$$

$$P.I = \frac{z}{2D_1+0}\sin 2z = \frac{z}{2(2)}\sin 2z$$

$$P.I = \frac{z}{4}\sin 2z$$

General solution is $y = C.F + P.I$

$$\boxed{y = c_1\cos 2z + c_2\sin 2z + \frac{z}{2}\sin 2z}$$

$x = e^z \quad z = \log x$

$y = c_1$

$$\Rightarrow P.I. = \frac{z}{2D_1}\sin 2z$$

$$= \frac{z}{2}\int \sin 2z\,dz =$$

$$= \frac{z}{2}\frac{(-\cos 2z)}{2} = \frac{-z\cos 2z}{4}$$

$\Rightarrow$ General solution is $y = C.F + P.I.$

$$y = c_1\cos 2z + c_2\sin 2z - \frac{z}{4}\cos 2z$$

$\therefore e^z = x$ or $z = \log x$

$$y = c_1\cos 2\log x + c_2\sin 2\log x - \frac{\log x}{4}\cos 2\log x$$

$$\boxed{y = \log x\left(c_1\cos 2x + c_2\sin 2x - \frac{1}{4}\cos 2x\right)}$$

or

$$\boxed{y = c_1\cos\log x^2 + c_2\sin\log x^2 - \frac{\log x}{4}\cos 2\log x^2}$$

This the required solution.

Example-④ solve the D.E.

$$\boxed{x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x\log x}$$ → Very Important

put $x\frac{d}{dx} = D$ $\quad x^2D^2y + 4xDy + 2y = x\log x$

$(x^2D^2 + 4xD + 2)y = x\log x$

put $x^2D^2 = D_1(D_1-1)$ $\quad xD = D_1$ $\quad e^z = x$ $\quad \log x = z$

$\therefore (D_1(D_1-1) + 4(D_1) + 2)y = e^z z$ or, ze^z

$(D_1^2 - D_1 + 4D_1 + 2)y = ze^z$

$(D_1^2 + 3D_1 + 2)y = ze^z$

A.E $= \boxed{D_1^2 + 3D_1 + 2 = 0}$ roots are (D_1-2) (D_1-1)

$\boxed{D_1 = -2}$ $\boxed{D_1 = -1}$ $D_1 = -2, -1$

roots are real and different

$\boxed{C.F. = c_1e^{m_1x} + c_2e^{m_2x}}$ ⇒ $c_1e^{m_1z} + c_2e^{m_2z}$

$C.f = c_1e^{-2z} + c_2e^{-1x}$

$P.I. = \frac{1}{f(D)} \times Qx = \frac{1}{(D_1^2+3D_1+2)} ze^z$

$= \frac{z}{2D_1 - 3} \times ze^z$ ⇒ $\frac{z}{2(1)-3} \times ze^z$

$= \frac{z}{-1} \times ze^z$ ⇒ $-z \times ze^z$

$\therefore \frac{1}{f(D)} \times e^{ax}V$ ⇒ $e^{ax}\frac{1}{f(D+a)} \times V$

$= e^z \times \frac{1}{(D_1+1)^2 + 3(D_1+1) + 2} \times z$

$= e^z \times \frac{1}{(D_1^2 + 1 + 2\times D_1 + 3D_1 + 3 + 2)} \times z$

$$= e^z \frac{1}{(D_1^2 + 2D_1 + 3D_1 + 6)} \times z$$

$$= e^z \frac{1}{(D_1^2 + 5D_1 + 6)} \times z$$ → How find the case of Higher order

$\therefore$ make binomial expansion

$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 \ldots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots$

$$\frac{e^z}{6} \times \frac{1}{\left(\frac{D_1^2}{6} + \frac{5D_1}{6} + 1\right)} \times z$$

$$\frac{e^z}{6} \times \frac{1}{\left[1 + \left(\frac{D_1^2}{6} + \frac{5D_1}{6}\right)\right]} \times z$$

$$= \frac{e^z}{6} \times \left[1 + \left(\frac{D_1^2}{6} + \frac{5D_1}{6}\right)\right]^{-1} \times z$$

$(1+x)^{-1}$

$$= \frac{e^z}{6} \times \left[1 - \frac{5D_1}{6} - \frac{D_1^2}{6} + \ldots\right] \times z$$

$$= \frac{e^z}{6} \times \left[z - \frac{5}{6} Dz - \frac{D_z^2}{6} + \ldots\right]$$

$$= \frac{e^z}{6} \times \left[z - \frac{5}{6} \times 1 - 0\right] \Rightarrow \frac{e^z}{6} \times \left[z - \frac{5}{6}\right]$$

P.I. $= \frac{e^z}{36} \times (6z - 5)$

General solution is

$y = C.F. + P.I.$

$y = c_1 e^{-2z} + c_2 e^{-z} + \frac{e^z}{36}(6z-5)$

put $x = e^z$ $\log x = z$

$y = c_1 x^{-2} + c_2 x^{-1} + \frac{x}{36}(6\log x - 5)$

$\boxed{y = \frac{c_1}{x^2} + \frac{c_2}{x} + \frac{x}{36}(6\log x - 5)}$ final Answer

Example - 5

$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log x$

put $D = \frac{d}{dx}$ $\therefore x^2 D^2 y - 2xDy - 4y = x^2 + 2\log x$

$(x^2D^2 - 2xD - 4)y = x^2 + 2\log x$

$\Rightarrow$ put $x^2D^2 = D_1(D_1 - 1)$ $xD = D_1$

$x = e^z$, $\log x = z$

$(D_1(D_1-1) - 2D_1 - 4)y = e^{2z} + 2z$

$(D_1^2 - D_1 - 2D_1 - 4)y = e^{2z} + 2z$

$(D_1^2 - 3D_1 - 4)y = e^{2z} + 2z$

A.E is $D_1^2 - 3D_1 - 4 = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1 - 4 \times -4}}{2}$

$= \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$

$(D_1 - 4)(D+1) = 0$ roots are real and different

$(D_1 = 4 \quad D_1 = -1)$

$C.F. = c_1 e^{m_1 x} + c_2 e^{m_2 x}$ or $c_1 e^{m_1 z} + c_2 e^{m_2 z}$

$$\boxed{C.F. = c_1 e^{4z} + c_2 e^{-z}}$$

$$P.I = \frac{1}{f(D)} \times \theta(z) = \frac{1}{(D_1^2 - 3D_1 - 4)} \times e^{2z} + 2z$$

$$= \frac{1}{D_1^2 - 3D_1 - 4} e^{2z} + \frac{1}{D_1^2 - 3D_1 - 4} 2z$$

$$= \frac{1}{(2)^2 - 3(2) - 4} e^{2z} + \frac{2}{(D_1^2 - 3D_1 - 4)} \times z$$

$$= -\frac{1}{6} e^{2z} + \frac{2}{-4\left(\frac{D_1^2}{-4} - \frac{3D_1}{-4} - \frac{4}{-4}\right)} \times z$$

$$= -\frac{1}{6} e^{2z} - \frac{1}{2\left(\left(+\frac{3D_1}{4} - \frac{D_1^2}{4}\right) + 1\right)} \times z$$

$$= -\frac{1}{6} e^{2z} - \frac{1}{2} \times \frac{1}{\left[1 + \left(\frac{3D_1}{4} - \frac{D_1^2}{4}\right)\right]} \times z$$

$$= -\frac{1}{6} e^{2z} - \frac{1}{2}\left[1 + \left(\frac{3D_1}{4} - \frac{D_1^2}{4}\right)\right]^{-1} \times z$$

$$= -\frac{1}{6} e^{2z} - \frac{1}{2}\left[1 - \frac{3D_1}{4} + \frac{D_1^2}{4}\right] \times z$$

$$= -\frac{1}{6} e^{2z} - \frac{1}{2}\left[z - \frac{3}{4} \times 1 + 0\right]$$

$$\boxed{PI = -\frac{1}{6} e^{2z} - \frac{z}{2} - \frac{3}{8}}$$

General solution -

$$Y = C.F. + P.I.$$

$$Y = c_1 e^{4z} + c_2 e^{-z} - \frac{1}{6} e^{2z} - \frac{z}{2} - \frac{3}{8}$$

$x = e^z \quad \log x = z$

$y = c_1 x^4 + c_2 x^{-1} - \frac{1}{6} x^2 - \frac{\log x}{2} - \frac{3}{8}$

$y = c_1 x^4 + \frac{c_2}{x} - \frac{x^2}{6} - \frac{\log x}{2} - \frac{3}{8}$ final Answer

example - (6) $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x\log x$

put $\frac{d}{dx} = D$ $\quad x^2 D^2 y - 3x Dy + 5y = x \log x$

$\rightarrow (x^2 D^2 - 3xD + 5)y = x\log x$

$x^2 D^2 = D_1(D_1 - 1) \quad xD = D_1 \quad x = e^z, \quad \log x = z$

$\rightarrow (D_1(D_1-1) - 3D_1 + 5)y = ze^z$

$(D_1^2 - D_1 - 3D_1 + 5)y = ze^z$

$(D_1^2 - 4D_1 + 5)y = ze^z$

A.E. $= D_1^2 - 4D_1 + 5 = 0$

roots are imaginary.

$2 \pm i$

C.f. $= e^{\alpha z}(c_1 \cos\beta z + c_2 \sin\beta z)$

C.f. $= e^{2z}(c_1\cos z + c_2 \sin z)$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{+4 \pm \sqrt{(-4)^2 - 4\times 1 \times 5}}{2\times 1}$

$= \frac{4 \pm \sqrt{16-20}}{2} = \frac{4 \pm \sqrt{-4}}{2}$

$= \frac{4 \pm \sqrt{i^2 4}}{2} = \frac{4 \pm 2i}{2}$

$= \frac{\cancel{2}(2 \pm i)}{\cancel{2}} = 2 \pm i$ roots

P.I. $= \frac{1}{f(D)} \times Q(z)$

P.I. $= \frac{1}{D_1^2 - 4D_1 + 5} \times ze^z$

P.I. $= e^z \frac{1}{D_1^2 - 4D_1 + 5} \times z$

$$P.I. = e^z \frac{1}{(D_1+1)^2 - 4(D_1+1) + 5} \times z$$

$$P.I. = e^z \frac{}{D_1^2 + 1^2 + 2D_1 - 4D_1 - 4 + 5} \times z$$

$$P.I. = e^z \frac{}{D_1^2 + 2 - 2D_1} \times z$$

$$P.I. = e^z \frac{}{2\left(\frac{D_1^2}{2} + \frac{2}{2} - \frac{2D_1}{2}\right)} \times z = e^z \frac{}{2\left(1 + \frac{D_1^2}{2} - D_1\right)} \times z$$

$$P.I. = e^z \cdot \frac{1}{2}\left[1 + \left(\frac{D_1^2}{2} - D_1\right)\right]^{-1} \times z$$

$$P.I. = e^z \times \frac{1}{2}\left[1 - \frac{D_1^2}{2} + D_1\right] \times z$$

$$P.I = e^z \times \frac{1}{2}\left[z - \frac{1}{2} \times 0 + 1\right] \Rightarrow \frac{e^z}{2}\left[z + 1\right].$$

$$P.I. = \frac{e^z \times z}{2} + \frac{e^z}{2} \text{ or } \frac{e^z \times z + e^z}{2}$$

General solution is $y = C.F. + P.I.$

$$y = e^{2z}(c_1 \cos z + c_2 \sin z) + \frac{e^z}{2}(z+1)$$

Put $z = \log x \qquad x = e^z$

$$y = x^2 (c_1 \cos \log x + c_2 \sin \log x) + \frac{x}{2}(\log x + 1)$$

This is the required solution. Final Answer

Example :- (1) $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin(\log x)$

put $\frac{d}{dx} = D$

$x^2 D^2 y - 3x D y + 5y = x^2 \sin(\log x)$

$\therefore \quad (x^2 D^2 - 3xD + 5) y = x^2 \sin(\log x)$

$\therefore x^2 D^2 = D_1(D_1 - 1) \cdot \quad xD = D_1 \quad x = e^z, \quad \log x = z$

$(D_1(D_1 - 1) - 3D_1 + 5) y = e^{2z} \sin z \qquad D_1 = \frac{d}{dz}$

$(D_1^2 - D_1 - 3D_1 + 5) y = \cancel{x^2 \sin(\log x)} \cdot e^{2z} \sin(z)$

$(D_1^2 - 4D_1 + 5) y = \cancel{x^2 \sin(\log x)} \quad e^{2z} \sin z$

A.E. $= D_1^2 - 4D_1 + 5 = 0$ root are imaginary $2 \pm i$

C.F. $= e^{2z} (c_1 \cos z + c_2 \sin z)$

P.I. $= \frac{1}{D_1^2 - 4D_1 + 5} \times \cancel{x^2 \sin(\log x)} \; e^{2z} \sin z$

P.I. $= e^{2z} \frac{1}{D_1^2 - 4D_1 + 5} \sin z$

$= e^{2z} \frac{1}{(D_1+2)^2 - 4(D_1+2) + 5} \sin z$

$= e^{2z} \frac{1}{D_1^2 + 4 + 2D_1 \times 2 - 4D_1 - 8 + 5} \sin z$

$= e^{2z} \frac{1}{D_1^2 + 4 + 4\cancel{D_1} - 4\cancel{D_1} - 3} \sin z$

$= e^{2z} \frac{1}{D_1^2 + 1} \sin z$

$= \cancel{e^{2z}} \frac{}{[1 + \cancel{(D_1^2)}]} \cancel{\sin z}$

$$= e^{2z}\left[1+(D_1^2)\right]^{-1}\sin z$$

$$= e^{2z}\left[1-(D_1^2)+(D_1^2)^2\ldots\right]\sin z$$

$$= e^{2z}\left[\sin z - \right.$$

$$\Rightarrow P.I. = e^{2z}\frac{1}{D_1^2+1}\times\sin z$$

$$= e^{2z}\frac{z}{2D_1}\times\sin z$$

$$\Rightarrow \frac{ze^{2z}}{2}\times\frac{1}{D_1}\times\sin z \Rightarrow \frac{ze^{2z}}{2}\int\sin z$$

$$\Rightarrow \frac{ze^{2z}}{2}(-\cos z) \Rightarrow -\frac{ze^{2z}\cos z}{2}$$

$$P.I \Rightarrow -\frac{z}{2}\left(e^{2z}\cos z\right)$$

General solution is $y = C.F. + P.I.$

$$y = e^{2z}(c_1\cos z + c_2\sin z) - \frac{z}{2}\left(e^{2z}\cos z\right)$$

put $x = e^z \quad z = \log x$

$$\boxed{y = x^2\left(c_1\cos\log x + c_2\sin\log x\right) - \frac{\log x}{2}\left(x^2\cos\log x\right)}$$

Final Answer this is the required solution.

Important

Example : (8) $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$

$\boxed{\text{put } D = \frac{d}{dx}}$ $x^2 D^2 y + 4x Dy + 2y = e^x$

$\Rightarrow (x^2 D^2 + 4xD + 2) y = e^x$

put $= x^2 D^2 = D_1(D_1 - 1)$ $xD = D_1$ $x = e^z$ $z = \log x$

$D_1 = \frac{d}{dz}$

$\Rightarrow [D_1(D_1-1) + 4D_1 + 2] y = e^{e^z}$

$(D_1^2 - D_1 + 4D_1 + 2) y = e^{e^z}$

$(D_1^2 + 3D_1 + 2) y = e^{e^z}$ A.E $= D_1^2 + 3D_1 + 2 = 0$

roots are :- $(D_1+1)(D_1+2)$

$D_1 = -1, -2$ root are real and different

$C.F. = C_1 e^{-z} + C_2 e^{-2z}$

* $P.I. = \frac{1}{D_1^2 + 3D_1 + 2} \times e^{e^z}$

$P.I. = \left[\frac{1}{D_1+1} - \frac{1}{D_1+2} \right] e^{e^z}$ → Partia fraction method use किया

$P.I. = \frac{1}{D_1+1} \times e^{e^z} - \frac{1}{D_1+2} \times e^{e^z}$

$\therefore D_1 - (-1)$ $D_1 - (-2)$

formula $\frac{1}{D-m} Q(x) = e^{mx} \int Q(x) e^{-mx} dz$

$P.I. = e^{-z} \int e^{e^z} e^z dz - e^{-2z} \int e^{e^z} e^{2z} dz$

* put $e^z = t$, $e^z dz = dt$ $\rightarrow$ $\underbrace{e^z}_{t} . \underbrace{e^z dz}_{dt}$

$$P.I. = e^{-z}\int e^t dt - e^{-2z}\int e^t\, t\, dt$$

$$= e^{-z}e^t - e^{-2z}\left\{te^t - \int 1.e^t dt\right\}$$

$$= e^{-z}e^t - e^{-2z}\left\{te^t - e^t\right\}$$

$$= \left\{e^{-z} - e^{-2z}(t-1)\right\}e^t$$

$$P.I. = \left\{e^{-z} - e^{-2z}(e^z-1)\right\} e^{e^z}$$

$$= \left\{\cancel{e^{-z}} - \cancel{e^{-z}} + e^{-2z}\right\} e^{e^z}$$

$$P.I = \left\{e^{-2z}\right\} e^{e^z}$$

$$\boxed{P.I = e^{-2z}.e^{e^z}}$$

general solution is

$$y = C.F. + P.I$$

$$y = c_1 e^{-z} + c_2 e^{-2z} + e^{-2z}.e^{e^z}$$

put $x = e^z$ $\quad \log x = z$

$$y = c_1 x^{-1} + c_2 x^{-2} + x^{-2}.e^x$$

$$\boxed{y = \frac{c_1}{x} + \frac{c_2}{x^2} + \frac{1}{x^2}\times e^x}$$

this is the required solution

Exercise = 36

Q(1) $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 12\log x$ put $D = \frac{d}{dx}$

$x^2D^2y + xDy = 12\log x$

$(x^2D^2 + xD)y = 12\log x$

put $x^2D^2 = D_1(D_1-1)$ $xD = D_1$ $e^z = x$ $z = \log x$

$D_1 = \frac{d}{dz}$

$(D_1(D_1-1) + D_1)y = 12z$

$(D_1^2 - D_1 + D_1)y = 12z$

$(D_1^2)y = 12z$

A.E. $= D_1^2 = 0$ The root are real and equal

$(D_1)\times(D_1) = 0$ $\quad$ C.f. $= (c_1 + c_2 z)e^{D_1 z}$

$D_1 = 0$ $D_1 = 0$

C.f. $= (c_1 + c_2 z)e^{0z}$

$\boxed{\text{C.F.} = (c_1 + c_2 z)}$

P.I. $\frac{1}{D_1^2} \times 12z \Rightarrow 12\frac{1}{D_1}\int z\,dz \Rightarrow \frac{12}{2}\frac{1}{D_1}z^2$

$\Rightarrow \frac{12}{2}\int z^2\,dz \Rightarrow \frac{12}{2} \times \frac{z^3}{3} \Rightarrow \frac{12}{6}z^3$

$\boxed{\text{P.I.} \Rightarrow 2z^3}$ General solution

$y = \text{C.f.} + \text{P.I.}$

$y = (c_1 + c_2 z) + 2z^3$

put $x = e^z$ $z = \log x$

$y = c_1 + c_2 \log x + 2(\log x)^3$

The required solution

Que ② $x^2 \dfrac{d^2y}{dx^2} - 2y = x^2 + \dfrac{1}{x}$

Put $xD = \dfrac{d}{dx}$ $x^2 D^2 y - 2y = x^2 + \dfrac{1}{x}$

$\therefore$ $(x^2 D^2 - 2)y = x^2 + \dfrac{1}{x}$

$(D_1^2 - \cancel{D_1} - 2)y =$

Put $x^2 D^2 = D_1(D_1 - 1)$ $x = e^z$ $z = \log x$

$(D_1(D_1 - 1) - 2)y = e^{2z} + e^{-z}$

$(D_1^2 - D_1 - 2)y = e^{2z} + e^{-z}$

A.E $D_1^2 - D_1 - 2 = 0$ $(D_1 - 2)(D_1 + 1)$

$(m_1 - 2)(m_1 + 1)$ $m = +2, -1$

root are real & different.

$C.F = c_1 e^{+2z} + c_1 e^{-z}$

$P.I = \dfrac{1}{D_1^2 - D_1 - 2} \times e^{2z} + \dfrac{1}{D_1^2 - D_1 - 2} e^{-z}$

Q ③ $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x \log x$

put $D = \frac{d}{dx}$

$x^2 D^2 y - xDy + 2y = x \log x$

$(x^2 D^2 - xD + 2)y = x \log x$

put $x^2 D^2 = D_1(D_1 - 1)$ $\quad xD = D_1$ $\quad x = e^z$ $\quad \log x = z$

$(D_1(D_1 - 1) - D_1 + 2)y = ze^z$

$(D_1^2 - D_1 - D_1 + 2)y = ze^z$

$(D_1^2 - 2D_1 + 2)y = ze^z$ $\quad$ A.E $D_1^2 - 2D_1 + 2 = 0$

roots $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+2 \pm \sqrt{(-2)^2 - 4 \times 1 \times 2}}{2 \times 1}$

$= \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm \sqrt{+4i^2}}{2} = \frac{2 \pm 2i}{2} = \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$

root are imaginary $\boxed{m = 1 \pm i}$

$$\boxed{C.F. = e^{1z}(c_1 \cos z + c_2 \sin z)}$$

$$P.I. = \frac{1}{D_1^2 - 2D_1 + 2} \times z e^z$$

$$= e^z \frac{1}{(D_1+1)^2 - 2(D_1+1) + 2} \times z$$

$$= e^z \frac{1}{D_1^2 + 1 + \cancel{2D_1} - \cancel{2D_1} + \cancel{2} - \cancel{2}} \times z$$

$$= e^z \frac{1}{D_1^2 + 1} \times z$$

$$= e^z \left[1 + (D_1^2)\right]^{-1} \times z$$

$$= e^z \left[1 - D_1^2\right] \times z$$

$$P.I = e^z [z - 0] \Rightarrow e^z \times z$$

$$\boxed{P.I = e^z \times z}$$

General solution is $y = C.F. + P.I$

$$y = e^z(c_1 \cos z + c_2 \sin z) + e^z \times z$$

Put $x = e^z \quad z = \log x$

$$\boxed{y = x\,(c_1 \cos(\log x) + c_2 \sin(\log x) + x \times \log x)}$$

Que- (4)

$$x^3 \frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} + xy = \sin(\log x)$$

$\therefore$ Put $D = \frac{d}{dx}$

$$x^3 D^2 y + 3x^2 Dy + xy = \sin(\log x)$$

$x^2D^2y + 3xDy + y = \frac{\sin}{x}(\log x)$

$(x^2D^2+3xD+1)y = \frac{\sin}{x}(\log x)$

$x^2D^2 = D_1(D_1-1) \quad xD = D_1 \qquad e^z = x \quad \log x = z$

$(D_1(D_1-1)+3D_1+1)\,y = \frac{\sin}{e^z}\, z$

$(D_1^2 - D_1 + 3D_1+1)y = e^{-z} \times \sin z$

$(D_1^2+2D_1+1)y = e^{-z} x \sin z$

$A.E = D_1^2+2D_1+1 = 0 \qquad$ roots :=

$m^2+2m+1=0 \quad$ ~~$(m+1)(m+1)$~~

$(m+1)(m+1) \quad , \; m = -1, -1$

root are equal & real

$C.F. = (c_1+c_2z)e^{mz} \Rightarrow (c_1+c_2z)e^{-1z}$

$C.F = (c_1+c_2z)e^{-z}$

$P.I. = \frac{1}{D_1^2+2D+1} \times e^{-z} \times \sin z$

$P.I = e^{-z} \cancel{\frac{1}{D_1^2+2D+1}} \sin z - \left[\frac{2D_1+2}{(D_1^2+2D+1)^2}\right]$

$P.I. = e^{-z} \frac{1}{D_1^2+2D+1} \times \sin z$

$P.I. = e^{-z} \frac{1}{(D_1-1)^2+2(D_1-1)+1} \sin z$

$P.I = e^{-z} \frac{1}{D_1^2+\cancel{1}-\cancel{2D_1}+\cancel{2D_1}-\cancel{2}+\cancel{1}} \sin z$

$P.I. = e^{-z} \frac{1}{D_1^2} \sin z$

$P.I. = e^{-z} \frac{1}{D} \times \int \sin z = e^{-z} \int -\cos z = -e^{-z} \sin z$

$P.I = -e^{-z}\sin z$ General solution is $y = CF + PI$

$y = (c_1 + c_2 z)e^{-z} - e^{-z}\sin z$

$x = e^{z} \quad z = \log x$

$y = (c_1 + c_2 \log x)x^{-1} - x^{-1}\sin \log x$

$y = [c_1 + c_2(\log x)]\frac{1}{x} - \frac{1}{x}\sin(\log x)$

Que-5 $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^{-1}$

put $D = \frac{d}{dx}$ $\quad x^2D^2y - 2xDy + 2y = x^{-1}$

$(x^2D^2 - 2xD + 2)y = x^{-1}$

Put $x^2D^2 = D_1(D_1-1) \quad xD = D_1 \quad e^{z} = x \quad \log x = z$

$(D_1(D_1-1) - 2D_1 + 2)y = e^{-z}$

$(D_1^2 - D_1 - 2D_1 + 2)y = e^{-z}$

$(D_1^2 - 3D_1 + 2)y = e^{-z} \quad A.E = D_1^2 - 3D_1 + 2 = 0$

roots are requal & different $\quad m^2 - 3m + 2 = 0$

$m = +2, +1 \quad (m-2)(m-1) = 0$

$\boxed{C.F. = c_1e^{2z} + c_2e^{z}}$ $\quad P.I. = \frac{1}{D_1^2 - 3D_1 + 2} \times e^{-z}$

$P.I = \frac{1}{(-1)^2 - 3(-1) + 2} = \frac{1}{1+3+2} = \frac{1}{6}e^{-z}$

$\boxed{P.I = \frac{1}{6}e^{-z}}$ General solution is $y = cf + P.I$

$y = c_1e^{2z} + c_2e^{z} + \frac{1}{6}e^{-z}$

$x = e^{z} \quad \log x = z$

$y = c_1 x^2 + c_2 x + \frac{1}{6}x^{-1}$

Que-(6) $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$

Put $D=\frac{d}{dx}$ $\quad x^2D^2y+4xDy+2y=e^x$

$(x^2D^2+4xD+2)y=e^x$

Put $x^2D^2=D_1(D_1-1)$ $\quad xD=D_1$ $\quad z=\log x$ $\quad e^z=x$

Que-(6) Done in previous example-(08)

Que-(7) $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+y=\frac{\log x\sin(\log x)+1}{x}$

Put $D=\frac{d}{dx}$ $\Rightarrow x^2D^2y-3xDy+y=\frac{\log x\sin(\log x)+1}{x}$

$(x^2D^2-3xD+1)y=\frac{\log x\sin(\log x)+1}{x}$

put $x^2D^2=D_1(D_1-1)$ $\quad xD=D_1$ $\quad e^z=x$ $\quad z=\log x$

$(D_1(D_1-1)-3D_1+1)y=\frac{z\sin(z)+1}{e^z}$

$(D_1^2-D_1-3D_1+1)y=\frac{z\sin(z)}{e^z}+\frac{1}{e^z}$

$(D_1^2-4D_1+1)y=ze^{-z}\sin(z)+e^{-z}$

A.E. $=m^2-4m+1=0$

root are $2\pm\sqrt{3}$

~~imagi~~ complex

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{+4\pm\sqrt{(-4)^2-4\times1\times1}}{2a}$

$=\frac{4\pm\sqrt{16-4}}{2}$

$=\frac{4\pm\sqrt{12}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$

$\boxed{C.F=e^{2z}(c_1\cosh\sqrt{3}z+c_2\sinh\sqrt{3}z)}$

$$P.I. = \frac{1}{D_1^2 - 4D_1 + 1} \times e^{-2}z \sin z + e^{-2}$$

$$P.I. = e^{-2} \frac{1}{D_1^2 - 4D_1 + 1} \times z \sin z + \frac{e^{-2}}{D_1^2 - 4D_1 + 1}$$

$$P.I. = e^{-2} \frac{}{(D_1 - 1)^2 - 4(D_1 - 1) + 1} \times z \sin z + \frac{e^{-2}}{(-1)^2 - 4(-1) + 1}$$

$$P.I. = e^{-2} \frac{\cancel{2}}{D_1^2 + 1 - 2D_1 - 4D_1 + 4 + 1} z \times \sin z + \frac{e^{-2}}{1 + 4 + 1}$$

$$P.I = e^{-2} \frac{\cancel{2}}{D_1^2 + \cancel{6} - 6D_1} z \times \sin z + \frac{1}{6} e^{-2}$$

$$P.I = e^{-2} \frac{z \cancel{2}}{D_1^2 + \cancel{6} - 6D_1} \sin z - \frac{(2D_1 - 6)}{(D_1^2 - 6D_1 + 6)^2} \sin z + \frac{1}{6} e^{-2}$$

$$P.I. = e^{-2} \left[\frac{z}{-1 + 6 - 6D_1} \sin z - \frac{(2D_1 - 6)}{(-1 - 6D_1 + 6)^2} \sin z \right] + \frac{1}{6} e^{-2}$$

$$P.I. = e^{-2} \left[z \frac{}{5 - 6D_1} \sin z - \frac{2D_1 - 6}{(5 - 6D_1)^2} \sin z \right] + \frac{1}{6} e^{-2}$$

$$P.I. = e^{-2} \left[z \times \frac{5 + 6D_1}{5^2 - (6D_1)^2} \sin z - \frac{2D_1 - 6}{25 + 36D_1^2 - 60D_1} \sin z \right] + \frac{1}{6} e^{-2}$$

$$P.I. = e^{-2} \left[z \times \frac{5 + 6D_1}{25 - 36D_1^2} \sin z - \frac{2D_1 - 6}{25 - 36D_1^2 - 60D} \sin z \right]$$

Que. (8) $(x^2D^2 - xD + 4)y = \cos(\log x) + x\sin(\log x)$

Put $x^2D^2 = D_1(D_1 - 1)$ $\quad xD = D_1$ $\quad x = e^z$ $\quad \log x = z$

$(D_1(D_1-1) - D_1 + 4)y = \cos(z) + e^z \sin(z)$

$\therefore (D_1^2 - D_1 - D_1 + 4)y = \cos z + e^z \sin z$

$\therefore$ A.E $= (D_1^2 - 2D_1 + 4) = 0$

$m^2 - 2m + 4 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+2 \pm \sqrt{(-2)^2 - 4 \times 4}}{2}$$

$$= \frac{+2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{2 \pm \sqrt{-12}}{2}$$

$$\frac{2 \pm \sqrt{-12}}{2}$$

$$= \frac{2 \pm \sqrt{12i^2}}{2} = \frac{2 \pm 2i\sqrt{3}}{2}$$

$$= \frac{\cancel{2}(1 \pm i\sqrt{3})}{\cancel{2}}$$

$= 1 \pm i\sqrt{3}$ roots

roots are imaginary $1 \pm i\sqrt{3}$

$\boxed{C.F. = e^{1z}(c_1 \cos\sqrt{3}z + c_2 \sin\sqrt{3}z)}$

$$P.I. = \frac{1}{D_1^2 - 2D_1 + 4} \times \cos z + e^z \frac{\sin z}{D_1^2 - 2D_1 + 4}$$

$$P.I = \frac{1}{-1 - 2D_1 + 4} \cos z + e^z \frac{1}{(D_1+1)^2 - 2(D_1+1) + 4} \sin z$$

$$P.I. = \frac{1}{3 - 2D_1} \cos z + e^z \frac{1}{D_1^2 + 1 + \cancel{2D_1} - \cancel{2D_1} + 2 + 4} \sin z$$

$$P.I. = \frac{3 + 2D_1}{(3 - 2D_1) \times (3 + 2D_1)} \cos z + e^z \frac{1}{D_1^2 + 7} \sin z$$

$$P.I. = \frac{3 + 2D_1}{3^2 - (2D_1)^2} \cos z + e^z \frac{1}{1 + 7} \sin z$$

$$PI = \frac{3 + 2D_1}{9 - 4D_1^2} \cos z + e^z \frac{1}{8} \sin z$$

$$P.I. = 3\cos z + 2 D_1 \cos z + \frac{e^z}{8}$$

Que. 9 is done in example 7 starting

Que-10 $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$ Done

Que-11

$$(x+a)^2\frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$

$\Rightarrow$ $x+a$

Que. 12 $\cancel{(1+x)^2 \frac{d^2y}{dx^2} - 4(x+1)\frac{dy}{dx}} +$

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x)$$

(13) $(1+2x)^2 \frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$

Que-(14) $(5+2x)^2 \dfrac{d^2y}{dx^2} - 6(5+2x)\dfrac{dy}{dx} + 8y = 0$

Ex-37

Exact 2nd order Differential Equations:-

$$P_0 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = Q \quad - (I)$$

$Q, P_0, P_1, P_2 = f(x)$

condition for Exact :-

$$\boxed{P_2 - P_1' + P_0'' = 0} \Rightarrow \text{Exact}$$

solution is of (I) or the first integral is

$$P_0 \frac{dy}{dx} + (P_1 - P_0')y = \int Q\,dx + c$$

solve it we get Answer

Ex-37

(2) $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 2x$

$P_0 = 1-x^2 \quad P_1 =$

(3) $\frac{d^2y}{dx^2} + 2e^x \frac{dy}{dx} + 2e^x y = x^2$

$P_0 = 1$	$P_1 = 2e^x$	$P_2 = 2e^x$
$P_0' = 0$	$P_1' = 2e^x$	
$P_0'' = 0$		

Consider $P_2 - P_1' + P_0'' = 0$

$$\cancel{2e^x} - \cancel{2e^x} + 0 = 0$$

$$0 = 0 \quad \text{exact}$$

Solution is of (I)

$$P_0 \frac{dy}{dx} + (P_1 - P_0')y = \int Q\,dx + c$$

$$1\frac{dy}{dx}+(2e^x-0)y=\int x^2dx+c_1$$

$$\Rightarrow \frac{dy}{dx}+2e^x y=\int x^2dx+c_1$$

$$\Rightarrow \frac{dy}{dx}+2e^x y=\frac{x^3}{3}+c_1$$ which is linear :-

$P=2e^x$

$$\Rightarrow I.F.=e^{\int 2e^x dx}=\cancel{e^{\log 2e^x}}=e^{2e^x}$$

$\Rightarrow$ solution of linear equation —

$$y\cdot If=\int Q.If\,dx+c$$

$$\Rightarrow y\cdot e^{2e^x}=\int\left[\frac{x^3}{3}+c_1\right]e^{2e^x}dx+c_2$$

$$\Rightarrow \boxed{ye^{2e^x}=\frac{1}{3}\int x^3e^{2e^x}dx+c_1\int e^{2e^x}dx+c_2}$$ Answer

$$y=\frac{1}{3}\int x^3\frac{\cancel{e^{2e^x}}}{\cancel{e^{2e^x}}}dx+\int\frac{c_1\cancel{e^{2e^x}}}{\cancel{e^{2e^x}}}dx+c_2$$

$$y=\frac{1}{3}\int x^3dx+\int c_1dx+c_2$$

$$\boxed{y=\frac{1}{12}x^4+c_1x+c_2}$$

Ans

Not exact for 2nd order

$$P_0\frac{dy}{dx}+P_1\frac{dy}{dx}+P_2y-I$$

$P_2-P_1'+P_0''\neq 0$ Not exact

$I.f.=x^m$

$I\times IP$ $$\boxed{P_0\frac{dy}{dx}+P_1\frac{dy}{dx}+P_2y=q}$$

Consider

$P_2-P_1'+P_0''=0$ exact

solution —

$$P_0\frac{dy}{dx}+(P_1-P_0')y=\int q\,dx+c_1$$

solve it we get Answer

Ex-37

Q.(4) $(2x^2+3x)\frac{d^2y}{dx^2}+(6x+3)\frac{dy}{dx}+2y=(x+1)e^x$

$P_0 = 2x^2+3x$ $\quad P_1 = 6x+3$ $\quad P_2 = 2$ $\quad Q = (x+1)e^x$

$P_0' = 4x+3$ $\quad P_1' = 6$

$P_0'' = 4$

$P_2 - P_1' + P_0'' = 0$

$2-6+4=0$

$\boxed{0 = 0}$ exact 2nd order D.E

Formula

$P_0\frac{dy}{dx}+(P_1-P_0')y = \int Q\,dx + c_1$ Integration ?

$(2x^2+3x)\frac{dy}{dx}+(6x+3-4x-3)y = \int (x+1)e^x dx + c_1$

$$\underbrace{x(2x+3)}_{\text{common}}\frac{dy}{dx} + 2xy = (x+1)e^x - e^x + c_1$$

$$\frac{dy}{dx} + \frac{2}{2x+3}y = \frac{x \cdot e^x + c_1}{x(2x+3)}$$

$$I.f. = e^{\int \frac{2}{2x+3}dx}$$

$$= e^{\log(2x+3)}$$

$$= 2x+3$$

$$y \cdot I.f. = \int I.f.\ Q'\,dx + c_2$$

$$y(2x+3) = \int (2x\cancel{+3}) \frac{(xe^x + c_1)}{x(2x\cancel{+3})} dx + c_2$$

$$y(2x+3) = e^x + c_1 \log x + c_2 \text{ Ans}$$

$(x^2 - x)\frac{d^2y}{...} \quad ... (2x+1)\frac{dy}{...} + 2y = 0$

Imp Que (10) Ex 3.7 Important

$$\sin^2 x \frac{d^2y}{dx^2} = 2y \quad \text{or} \quad \frac{d^2y}{dx^2} = 2y \operatorname{cosec}^2 x .$$

$$\frac{d^2y}{dx^2} - 2y \operatorname{cosec}^2 x = 0$$

I.f. = $\cot x$ multiply by $\cot x$

~~apply formula~~ :-

$$\cot x \frac{d^2y}{dx^2} - 2y \cot x \operatorname{cosec}^2 x = 0$$

$\Rightarrow$

$P_0 = \cot x$, $P_1 = 0$, $P_2 = -2 \cot x \operatorname{cosec}^2 x$

$P_0' = \operatorname{cosec}^2 x$ $P_1' = 0$

$P_0'' = 2 \cot x \operatorname{cosec}^2 x$

#

$\Rightarrow$ $P_2 - P_1' + P_0'' = -2 \cancel{\cot x \operatorname{cosec}^2 x} - 0 + 2 \cot x \cancel{\operatorname{cosec}^2 x}$

$\quad = 0$ Exact

$$\Rightarrow P_0 \frac{dy}{dx} + (P_1 - P_0')y = \int Q\, dx + c_1$$

$$\cot x \frac{dy}{dx} + (0 + \operatorname{cosec}^2 x) y = 0 + c_1$$

$$\cot x \frac{dy}{dx} + \operatorname{cosec}^2 x\, y = c_1$$

$$\frac{dy}{dx} + \frac{\operatorname{cosec}^2 x}{\cot x} y = \frac{c_1}{\cot x}$$

$$\text{I.f.} = e^{-\int \frac{-\operatorname{cosec}^2 x}{\cot x} dx} = e^{-\log \cot x} = \frac{1}{\cot x} = \tan x$$

$$y \cdot \text{IF} = \int \text{IF}\, Q\, dx + c_2$$

$$y \cdot \tan x = \int \tan x \frac{c_1}{\cot x} dx + c_2$$

$$y \tan x = c_1 \int \tan^2 x \, dx + c_2$$

$$y \tan x = c_1 \int (\sec^2 x - 1) dx + c_2$$

$$\boxed{y \tan x = c_1 (\tan x - x) + c_2}$$

Que. (9) $\frac{d^2y}{dx^2} + 2\tan x \frac{dy}{dx} + 3y = \tan^2 x \cdot \sec x$

$\rightarrow \frac{1}{\cos x}$ multiply

$\Rightarrow \cos x \frac{d^2y}{dx^2} + 2\tan x \cos x \frac{dy}{dx} + 3\cos x \, y = \tan^2 x$

$P_0 = \cos x$, $P_1 = 2\sin x$, $P_2 = 3\cos x$, $\theta = \tan^2 x$

$P_0' = -\sin x$ $\quad P_1' = 2\cos x$

$P_0'' = -\cos x$

consider - $P_2 - P_1' + P_0'' = 3\cos x - 2\cos x - \cos x$

$P_2 - P_1' + P_0'' = 3\cos x - 3\cos x = 0$

$\rightarrow$ exact

Solution (first integral is)

$P_0 \frac{dy}{dx} + (P_1 - P_0')y = \int \theta \, dx + c_1$

$\cos x \frac{dy}{dx} + [2\sin x + \sin x] y = \int \tan^2 x \, dx + c_1$

$\cos x \frac{dy}{dx} + 3\sin x \, y = \int (\sec^2 x - 1) dx + c_1$

$\cos x \frac{dy}{dx} + 3\sin y = \tan x - x + c_1$

$$\frac{dy}{dx} + 3\tan x\, y = \tan x \sec x - x \sec x + c_1 \sec x$$

which is linear

$$I.F. = e^{3\int \tan x\, dx} = e^{3\log \sec x} = e^{\log \sec^3 x} = \sec^3 x$$

Solution –

$$y \times \sec^3 x = \int (\tan x \sec x - x \sec x + c_1 \sec x) \sec^3 x\, dx + c_2$$

$$y \times \sec^3 x = \int \tan x \sec^4 x\, dx - \int x \sec^4 x\, dx + c_1 \int \sec^4 x\, dx + c_2$$

$$= \underbrace{\int \tan x \sec x \sec^3 x\, dx}_{I_1} - \underbrace{\int x \sec^2 x \times \sec^2 x\, dx}_{I_2}$$

$$+ c_1 \underbrace{\int \sec^2 x \sec^2 x\, dx}_{I_3} + c_2$$

$I_1 \rightarrow$

$$I_1 = \int \tan x \sec x \sec^3 x\, dx \qquad \text{Let } \sec x = t$$

$$\sec x \tan x\, dx = dt$$

$$\boxed{I_1 = \int t^3\, dt = \frac{t^4}{4} = \frac{\sec^4 x}{4}}$$

$I_2 \Rightarrow \int x \sec^2 x\, (\tan^2 x + 1)\, dx$

$$I_2 \Rightarrow \int \underset{I}{x}\, \underset{II}{\sec^2 x \tan^2 x}\, dx + \int \overset{I}{x}\, \overset{II}{\sec^2 x}\, dx$$

$$\boxed{I_2 = \frac{x \tan^3 x}{3} + x \tan x - 1 \log \sec x}$$

$I_3 \Rightarrow c_1 \int \sec^2 x \times (1 + \tan^2 x)\, dx$ पता है इसे करना

$$= c_1 \int \sec^2 x\, dx + c_1 \int \sec^2 x \tan^2 x \frac{\sin}{dx}$$

$$\boxed{I_3 = c_1 \tan x + \frac{c_1 \tan^3 x}{3}}$$

$$y = \frac{\sec^4 x}{4} - \frac{x \tan^3 x}{3} + x\tan x - \log \sec x$$

$$+ c_1 \tan x + \frac{c_1}{3} \tan^3 x + c_2$$

Ex-37
Que-(8)

$$2x^2 \frac{d^2y}{dx^2} + 15x\frac{dy}{dx} - 7y = 3x^2$$

$P_0 = 2x^2 \quad P_1 = 15x \quad P_2 = -7$

let I.f. $= x^m$

$$x^m . 2x^2 . \frac{d^2y}{dx^2} + 15x\, x^m \frac{dy}{dx} - 7yx^m = 3x^{m+2}$$

$$2x^{m+2} \frac{d^2y}{dx^2} + 15x^{m+1}\frac{dy}{dx} - 7yx^m = 3x^{m+2}$$

$P_0 = 2x^{m+2} \quad P_1 = 15x^{m+1} \quad \& \; P_2 = -7x^m$

It is homogenrous

$x = e^z \quad z = \log x$

$$2D'(D'-1)y + 15D'y - 7y = 3e^{2z}$$

$$y(2D'^2 - 2D' + 15D' - 7) = 3e^{2z}$$

$$y(2D_1^2 + 13D_1 - 7) = 3e^{2z}$$

$$2m^2 + 13m - 7 = 0$$

$$2m^2 + 14m - m - 7 = 0$$

$$2m(m+7) - (m+7) = 0$$

$$\boxed{m = -7, 1/2}$$

roots

$$y = \frac{c_1}{x^7} + c_2\sqrt{x} + \frac{x^2}{9}$$

this a required answer.

Que. (7) $x^4\frac{d^2y}{dx^2}+x^2(x-1)\frac{dy}{dx}+xy=x^3-4$ —(I)

$\Rightarrow P_0=x^4$ $\quad P_1=x^2(x-1)$ $\quad P_2=x$

$P_0'=4x^3$ $\quad P_1'=3x^2-2x$

$P_0''=12x^2$

$x-3x^2-2x+12x^2\neq 0$

I.F. $=x^m$ → multiply in —(I)

$\Rightarrow (x^{4+m})\frac{d^2y}{dx^2}+(x^{3+m}-x^{2+m})\frac{dy}{dx}+x^{1+m}y=x^3-4$

$\Rightarrow P_0=x^{4+m}$

$P_0'=4+m\,x^{3+m}$

$P_0''=(4+m)(3+m)x^{2+m}$

$P_1=x^{3+m}-x^{2+m}$

$P_1'=3+m\,x^{2+m}-2+m\,x^{1+m}$

$P_2=x^{1+m}$

$P_2-P_1'+P_0''$

$\Rightarrow x^{1+m}-(3+m)x^{2+m}-(2+m)x^{1+m}+(4+m)(3+m)x^{2+m}$

$\Rightarrow x^{1+m}(1+2+m)+x^{2+m}(-(3+m)+(4+m)(3+m))=0$

$x^{1+m}(1+2+m)+x^{2+m}(-3-m+12+4m+3m+m^2)=0$

$x^{1+m}(1+2+m)+x^{2+m}(m^2+6m+9)=0$

$m^2+6m+9=0$

$$m^2+3m+3m+9=0$$
$$m(m+3)+3(m+3)=0$$
$$m=3 \qquad I.F.=x^{-3}$$

$$x^{4-3}\frac{d^2y}{dx}+(x^{3+m}-x^{2+m})\frac{dy}{dx}+x^{1+m}y=x^3-4$$

$$x'\frac{d^2y}{dx^2}+(1-x^{-1})\frac{dy}{dx}+x^{-2}y=x^3-4$$

$$x'x\frac{d^2y}{dx^2}+\left(1-\frac{1}{x}\right)\frac{dy}{dx}+\frac{1}{x^2}y=x^3-4$$

$$q_0=x \qquad q_1'=+\frac{1}{x^2} \qquad q_2=\frac{1}{x^2}$$
$$q_0'=1$$

$$q_2-q_1'+q_0''=0$$
$$1-\frac{1}{x^2}+\frac{1}{x^2}=0$$

$$x\frac{d^2y}{dx}+\left(1-\frac{1}{x}-1\right)\frac{dy}{dx}=\int x^3-4\,dx+c$$

$$x\frac{dy}{dx}-\frac{1}{x}y=\frac{x^4}{4}-4x+c$$

$$\frac{dy}{dx}-\frac{1}{x^2}y=\frac{x^3}{4}-4+\frac{c_1}{x}$$

$$e^{-\int\frac{1}{x^2}}=e^{+1/x}$$

⇒

Que-(6) $(x^2-x)\dfrac{d^2y}{dx^2} + 2(2x-1)\dfrac{dy}{dx} + 2y = 0$

$2-(4)+2=0$

$x^2-x\dfrac{dy}{dx} + (4x+2-2x+1)y = \int 0\cdot dx + c$

$x^2-x\dfrac{dy}{dx} + (2x+3)y = c$

$\dfrac{dy}{dx} + \dfrac{2x+3}{x^2-x}y = \dfrac{c}{x^2-x}$

I.f. $= e^{\int \frac{2x}{x^2-x} + \frac{3}{x^2-x} dx}$

$= e^{2\log(x-1)} \cdot e^{3\log(1-1/x)}$

$\Rightarrow (x-1)^2 \cdot \left(1-\dfrac{1}{x}\right)^3$

$\Rightarrow (x-1)^5 \dfrac{1}{x^3}$

$$y\frac{(x-1)^5}{x^3} = \int \frac{c}{x^2-x} \cdot \frac{(x-1)^5}{x^3}\,dx + K$$

$$y\frac{(x-1)^5}{x^3} = c\int \frac{1}{x(x-1)} \frac{(x-1)^5}{x^3}\,dx + K$$

$$y\frac{(x-1)^5}{x^3} = c\int \frac{(x-1)^4}{x^4}\,dx + K$$

$$y\frac{(x-1)^5}{x^3} = c\int \frac{x^4-4x^3+6x^2-4x+1}{x^4}\,dx + K$$

$$y\frac{(x-1)^5}{x^3} = c\left[x - 4\log(x) + \frac{6}{x} + \frac{2}{x^2} + \frac{1}{3x^3}\right] + K$$

$$y(x-1)^5 = c(x^4 - 4x^3\log x - 6x^2 + 2x - 1/3) + Kx^3$$

Que ⑨ $\frac{d^2y}{dx^2} + 2\tan x\frac{dy}{dx} + 3y = \tan^2 x \sec x$

$$\cos\frac{d^2y}{dx^2} + 2\tan x\cos x\frac{dy}{dx} + 3\cos x\, y = \tan^2 x$$

$P_0 = \cos x$ | $P_1 = 2\sin x$ | $P_2 = 3\cos x$

$P_0' = -\sin x$ | $P_0'' = -\cos x$ | $P_1' = 2\cos x$ | $Q = \tan^2 x$

$P_2 - P_1' + P_0''$

$\therefore$ $3\cos x - 2\cos x - \cos x = 0$

$$\boxed{P_2 - P_1' + P_0'' = 0}$$

$$P_0\frac{dy}{dx} + (P_1 - P_0')y = \int Q\cdot dx + c_1$$

$$\cos x\frac{dy}{dx} + (2\sin x + \sin x)y = \int \tan^2 x\,dx + c_1$$

Part of C.F. Known

Ex - 38

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad -(I)$$

complete solution is $y = u \times v$ $\quad -(2)$

$$\frac{d^2v}{dx^2} + \left\{\frac{2}{u}\frac{du}{dx} + P\right\}\frac{dv}{dx} = \frac{R}{u}$$

part of C.f Known

(1) $1 + P + Q = 0$; $u = e^{+x}$

(2) $1 - P + Q = 0$; $u = e^{-x}$

(3) $P + Qx = 0$; $u = x$

(4) $1 + \frac{P}{a} + \frac{Q}{a^2} = 0$; $u = e^{ax}$

(5) $2 + 2Px + Qx^2 = 0$; $u = x^2$

(6) $m(m-1) + mxP + Qx^2 = 0$; $u = x^m$

Let $\frac{dv}{dx} = z$ $\quad \frac{d^2v}{dx^2} = \frac{dz}{dx}$

$$\frac{dz}{dx} + \left[\frac{2}{u}\frac{du}{dx} + P\right]z = \frac{R}{u}$$

which linear equation -

$$I.F. = e^{\int\left(\frac{2}{u}\frac{du}{dx} + P\right)dx}$$

$$= e^{\int \frac{2}{u}du} \times e^{\int P dx}$$

$$= e^{2\log u} \times e^{\int P dx}$$

$$= u^2 e^{\int P dx}$$

Solution — $z \times u^2 e^{\int P dx} = \int \frac{R}{u} \times u^2 e^{\int P dx} dx + c_1$

$$z\, u^2 \times e^{\int P dx} = \int R u e^{\int P dx} dx + c_1$$

$$\frac{dv}{dx} \times u^2 \times e^{\int P dx} = \int R u e^{\int P dx} dx + c_1$$

Solve it we get $v = ?$

put the v, du & v in (2) we get $y = ?$

Ex-38

Que-(1) $$\underbrace{\frac{d^2y}{dx^2} - x^2}_{P} \frac{dy}{dx} + \underbrace{xy}_{Q} = \underbrace{x}_{R} \quad —(1)$$

where $P = -x^2$, $Q = x$, $R = x$

$$P + Qx = -x^2 + x \cdot x \Rightarrow -\cancel{x^2} + \cancel{x^2} = 0$$

$P + Qx = 0$, $u = x$, $\boxed{0 = 0}$

$$\boxed{P + Qx = 0 \quad u = x, \quad \frac{du}{dx} = 1}$$

complete solution -

$$y = u \times v = x \times v \quad —(2)$$

we know that

$$\frac{d^2v}{dx^2} + \left[\frac{2}{u}\frac{du}{dx} + P\right]\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left[\frac{2}{x} \times 1 - x^2\right]\frac{dv}{dx} = \frac{\cancel{x}}{\cancel{x}}$$

$$\frac{d^2v}{dx^2} + \left[\frac{2}{x} - x^2\right]\frac{dv}{dx} = 1 \quad —(3)$$

Let $\frac{dv}{dx} = z \Rightarrow \frac{d^2v}{dx^2} = \frac{dz}{dx}$

put in - (3)

$$\frac{dz}{dx} + \left[\frac{2}{x} - x^2\right] \times z = 1$$

which is linear diff equation

$$I.f. = e^{\int (\frac{2}{x} - x^2)\, dx}$$

$$I.f = e^{2\log x - \frac{x^3}{3}}$$

$$I.f = e^{\log x^2} \times e^{-\frac{x^3}{3}} \Rightarrow x^2 \times e^{-\frac{x^3}{3}}$$

$$I.f. = x^2 \times e^{-x^3/3}$$

solution is

$$\Rightarrow z \times x^2 e^{-\frac{x^3}{3}} = \int 1 \times x^2 \times e^{-\frac{x^3}{3}}\, dx + c_1 \qquad \therefore t = \frac{x^3}{3}$$

Didn't understand

$$\Rightarrow z \cdot x^2 e^{-\frac{x^3}{3}} = \int e^{-t}\, dt + c_1 = -e^{-t} + c_1$$

$$= -e^{-x^3/3} + c_1$$

$$\Rightarrow z = -\frac{1}{x^2} + c_1 \times \frac{1}{x^2 \cdot e^{-\frac{x^3}{3}}}$$

$$\Rightarrow \frac{dv}{dx} = \frac{-1}{x^2} + \frac{c_1 \cdot e^{\frac{x^3}{3}}}{x^2}$$

V.A.S and integrate

$$\Rightarrow \int dv = \int \left[-\frac{1}{x^2} + \frac{c_1\, e^{\frac{x^3}{3}}}{x^2}\right] dx + c_2$$

$$v = \frac{1}{x} + c_1 \int \frac{e^{\frac{x^3}{3}}}{x^2}\,dx + c_2$$

Complete solution is – $y = u \times v$

$$y = x\left[\frac{1}{x} + c_1 \int \frac{e^{x^3/3}}{x^2}\,dx + c_2\right]$$

$$y = 1 + c_1 x \int \frac{e^{x^3/3}}{x^2}\,dx + c_2 x$$

final Answer

⇒ Que-(4) $\frac{d^2y}{dx^2} + (1-\cot x)\frac{dy}{dx} - y\cot x - \sin^2 x$

$P = 1 - \cot x \qquad Q = -\cot x \qquad R = \sin^2 x$

$1 - P + Q \Rightarrow \cancel{1} - \cancel{1} + \cancel{\cot x} - \cancel{\cot x} = 0$

$0 = 0$

$u = e^{-x} \quad \frac{du}{dx} = -e^{-x}$

Comple solution $\Rightarrow y = u \times v$ – (2)

we know that -

$$\frac{d^2v}{dx^2} + \left[\frac{2}{u}\frac{du}{dx} + P\right]\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left[\frac{2}{e^{-x}} \times (-e^{-x}) + 1 - \cot x\right]\frac{dv}{dx} = \frac{\sin^2 x}{e^{-x}}$$

$$\frac{d^2v}{dx} + \left[-2+1-\cot x\right]\frac{dv}{dx} = e^x \sin^2 x$$

$$\Rightarrow \frac{d^2v}{dx^2} - \left[1+\cot x\right]\frac{dv}{dx} = e^x \sin^2 x \quad - (3)$$

$$\Rightarrow \frac{dv}{dx} = z \qquad \Rightarrow \frac{d^2v}{dx^2} = \frac{dz}{dx}$$

put in (3)

$$\frac{dz}{dx} - \left[1+\cot x\right] z = e^x \sin^2 x \quad - (3)$$

<u>which is linear</u> -

$$I.f = e^{\int(-1+\cot x)\,dx} = \frac{e^{-x} + \log \operatorname{cosec}}{} \; ?$$

$$= e^{-x} \operatorname{cosec} x$$

<u>solution of 3 eq</u>

$$z \times e^{-x} \cdot \operatorname{cosec} x = \int \cancel{e^{-x}} \operatorname{cosec} x \; \cancel{e^{x}} \sin^2 x \, dx + c_1$$

$$= \int \operatorname{cosec} x \times \sin x \times \frac{1}{\operatorname{cosec} x} dx + c_1$$

$$= \int \sin x \, dx + c$$

$$z \times e^{-x} \operatorname{cosec} x = -\cos x + c_1$$

$$z = -e^x \cos x \sin x + c_1 e^x \sin x$$

$$\frac{dv}{dx} = -e^x \cos x \sin x + c_1 e^x \sin x$$

V.A.S and integrate

$\int dv = \int -e^x \sin x \cos x \, dx + \int c_1 e^x \sin x \, dx$

$\therefore$ using $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a \cos bx + b \sin bx\right] + c$?

$\Rightarrow \int dv = \frac{1}{-2}\int 2e^x \sin x \cos x \, dx + c_1 \int e^x \sin x \, dx + c_2$

$\Rightarrow v = -\frac{1}{2}\int e^x \sin 2x \, dx + c_1 \int e^x \sin x \, dx + c_2$

$v = -\frac{1}{2}\left[\frac{e^x}{1+4}\left\{\sin 2x - 2\cos 2x\right\}\right] + c_1\left[\frac{e^x}{1+1}\right.$

$\left.\left\{1 \cdot \sin x - 1 \cos x\right\}\right] + c_2$

$v = -\frac{e^x}{10}\left\{\sin 2x - 2\cos 2x\right\} + \frac{c_1}{2} e^x \left\{\sin x - \cos x\right\} + c_2$

$y = \underline{\underline{u \times v}} \Rightarrow e^{-x}\left[-\frac{e^x}{10}\left\{\sin 2x - 2\cos 2x\right\}\right.$

$\left. + \frac{c_1}{2} e^x (\sin x - \cos x) + c_2\right]$

$$\boxed{y = -\frac{1}{10}(\sin 2x - 2\cos 2x) + \frac{c_1}{2}\left[\sin x - \cos x\right] + c_2 e^{-x}}$$

Final Answer

Extra ?

Qu (8) Ex-5B

$$\sin^2 x \frac{d^2y}{dx^2} = 2y$$, given that $y = \cot x$ is a solution.

$$\frac{d^2y}{dx^2} - 2y \operatorname{cosec}^2 x = 0$$

$Q = \phi - 2\operatorname{cosec}^2 x \qquad P = 0$

CS: $u = \cot x \qquad \frac{du}{dx} = -\operatorname{cosec}^2 x$

we know that $$\frac{d^2v}{dx^2} + \left[\frac{2}{u}\frac{du}{dx} + P\right]\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} - \frac{2}{\cos x} \times \sin x \times \operatorname{cosec}^2 x \frac{dv}{dx} = 0$$

$$\frac{d^2v}{dx^2} - 2\sec x \times \operatorname{cosec} x \frac{dv}{dx} = 0$$

$$\frac{1}{2}\int \frac{dz}{z} = 2\int \operatorname{cosec} 2x\, dx + c_1$$

$$\frac{1}{2}\log z = 2\log \tan x + \log c_1$$

$$\log z = 4\log \tan x + \log c_1$$

$$\log z = \log(\tan x)^4 + \log c_1$$

$$\log z = \log \tan^4 x \times c_1$$

$$z = \tan^4 x + c_1$$

$$\frac{dv}{dx} = \tan^2 x \times \tan^2 x$$

V.A.S and Integrate

$$\int dv = c_1 \int \tan^2 x \times \tan^2 x\, dx + c_2$$

$$\int dv = c_1 \int (\sec^2 x - 1)\tan^2 x \, dx + c_2$$

$$v = c_1 \int (\sec^2 x \times \tan^2 x - \tan^2 x) \, dx + c_2$$

$$v = c_1 \left[\int \sec^2 x \times \tan^2 x \, dx - \int (\sec^2 x - 1) \, dx \right] + c_2$$

let $\tan x = t$

$$v = c_1 \left[\int t^2 dt - \tan x + x + c_2 \right]$$

$$v = c_1 \left[\left(\frac{t^3}{3} - \tan x + x \right) \right] + c_2$$

$$v = c_1 \left[\frac{\tan^3 x}{3} - \tan x + x \right] + c_2$$

C.S. $y = v \cdot \cot x$

$$y = \cot x \left[\frac{c_1}{3} \tan^3 x - \tan x + x + c_2 \right]$$

$$y = \left[\frac{c_1}{3} \tan^2 x - 1 + x \cot x + c_2 \cot x \right]$$

Ex- 38

Que. $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$

given that $y = e^{a \sin^{-1} x}$

$u = e^{a \sin^{-1} x}$

$$\Rightarrow \frac{du}{dx} = e^{a\sin^{-1}x} \times \left[\frac{a}{\sqrt{1-x^2}} \right] = \frac{a}{\sqrt{1-x^2}} e^{a\sin^{-1}x}$$

$\Rightarrow P = \frac{-x}{1-x^2}$, $Q = \frac{-a^2}{1-x^2}$, $R = 0$

C.S. $\Rightarrow y = u \times v = e^{a \sin^{-1} x}$

We know that

$$\frac{d^2v}{dx^2}+\left[\frac{2}{u}\frac{du}{dx}+P\right]\frac{dv}{dx}=\frac{R}{u}$$

$$\frac{d^2v}{dx^2}+\left[\frac{2}{\cancel{e^{a\sin^{-1}x}}}\times\frac{a}{\sqrt{1-x^2}}\times\cancel{e^{a\sin^{-1}x}}-\frac{x}{1-x^2}\right]\frac{dv}{dx}=0$$

$$\Rightarrow \frac{d^2v}{dx^2}+\left[\frac{2a}{\sqrt{1-x^2}}-\frac{x}{1-x^2}\right]\frac{dv}{dx}=0$$

Let $\frac{dv}{dx}=z \Rightarrow \frac{d^2v}{dx^2}=\frac{dz}{dx}$

$$\frac{dz}{dx}+\left[\frac{2a}{\sqrt{1-x^2}}-\frac{x}{1-x^2}\right]z=0$$

$$\frac{dz}{dx}=\left[\frac{x}{1-x^2}-\frac{2a}{\sqrt{1-x^2}}\right]z$$

V.A.S and Integrate.

इसका $\sin^{-1}x$ होगा

$$\int\frac{dz}{z}=\frac{1}{2}\int\frac{2x}{1-x^2}dx-2a\int\frac{1}{\sqrt{1-x^2}}dx+c_1$$

$$\log z=-\frac{1}{2}\log(1-x^2)-2a\sin^{-1}x+c_1$$

$$\log z=\log\frac{1}{\sqrt{1-x^2}}-2a\sin^{-1}x+c_1$$

$$\log z-\log\frac{1}{\sqrt{1-x^2}}=-2a\sin^{-1}x+c_1$$

$$\log\frac{z\sqrt{1-x^2}}{c_1}=-2a\sin^{-1}x+\log c_1$$

$$\frac{z\sqrt{1-x^2}}{c_1}=e^{-2a\sin^{-1}x}$$

$$z=\frac{c_1 e^{-2a\sin^{-1}x}}{\sqrt{1-x^2}}$$

$$\frac{dv}{dx} = \frac{c_1 e^{-2a\sin^{-1}x}}{\sqrt{1-x^2}} \quad \text{v.A.S and Inte.}$$

$$\int v = c_1 \int \frac{e^{-2a\sin^{-1}x}}{\sqrt{1-x^2}} dx + c_2$$

Let $\sin^{-1}x = t$

$$\frac{1}{\sqrt{1-x}} dx = dt \Rightarrow v = c_1 \int e^{-2at} dt + c_2$$

$$\Rightarrow v = c_1 \frac{e^{-2at}}{-2a} + c_2$$

Complete solution-

$$y = e^{a\sin^{-1}x} \times \left[c_1 \frac{e^{-2at}}{-2a} + c_2\right]$$

$$y = e^{a\sin^{-1}x} \times \left[c_1 \frac{e^{-2a\sin^{-1}x}}{-2a} + c_2\right]$$

$$y = \frac{c_1}{-2a} e^{-a\sin^{-1}x} + c_2 e^{a\sin^{-1}x}$$

or

$$y = ke^{-a\sin^{-1}x} + c_2 e^{a\sin^{-1}x}$$

Que (10) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 9y = 0$

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{9y}{x^2} = 0$$

$$x^2 D^2 y + xDy - 9y = 0$$

Put $\Rightarrow$ $x^2D^2 = D_1(D_1-1) \quad xD = D_1$

$x = e^z \quad z = \log x$

$D_1(D_1-1)y + D_1 y - 9y = 0$

$D_1^2 - D_1 + D_1 - 9 = 0$

$D_1^2 - D_1 + D_1 - 9 = 0 \Rightarrow D_1^2 - 9 = 0$

$m^2 - 9 = 0$

$m^2 = 9$

$\boxed{m = \pm 3}$ roots are real and different

$C.F. = c_1 e^{3z} + c_2 e^{-3z}$

$P.I. = \frac{1}{D_1^2 - 9} \times 0 = 0$

Complete solution or General solution -

$y = C.F. + P.I.$

$y = c_1 e^{3z} + c_2 e^{-3z}$

Put $x = e^z$, $z = \log x$

$\boxed{y = c_1 x^3 + c_2 x^{-3}}$

Que - 2 $(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x(1-x^2)^{3/2}$

$\frac{d^2y}{dx^2} + \frac{x}{1-x^2}\frac{dy}{dx} - \frac{1}{1-x^2}y = x\frac{(1-x^2)^{3/2}}{1-x^2}$

$\frac{d^2y}{dx^2} + \frac{x}{1-x^2}\frac{dy}{dx} - \frac{1}{1-x^2}y = x(1-x^2)^{1/2}$

$P + Qx = 0$

$\frac{x}{1-x^2} - \frac{x}{1-x^2} = 0$ $\boxed{\because P + Qx = 0 \quad \therefore u = x}$

$y = u \times v$

$\frac{d^2v}{dx^2} + \left[\frac{2}{u}\frac{du}{dx} + P\right]\frac{dv}{dx} = \frac{R}{u}$

$$\frac{d^2v}{dx^2}+\left[\frac{2}{x}+\frac{x}{1-x^2}\right]\frac{dv}{dx}=\frac{x(1-x^2)^{1/2}}{x}$$

let $z=\frac{dv}{dx}$ $\qquad \frac{d^2v}{dx^2}=\frac{dz}{dx}$

$$\frac{dz}{dx}+\left[\frac{2}{x}+\frac{x}{1-x^2}\right]z=(1-x^2)^{1/2}$$

$$I.f=e^{\int 2/x-\frac{x}{1-x^2}} \Rightarrow x^2\times e^{2/2\int\frac{x}{1-x^2}}$$

$$I.f.=x^2\times e^{1/2\log(1-x^2)} \Rightarrow x^2\times\frac{1}{\sqrt{1-x^2}}$$

$\Rightarrow \cancel{\frac{x^2}{\sqrt{1-x^2}}} \Rightarrow$

$$\Rightarrow z.\frac{x^2}{\sqrt{1-x^2}}=\int\cancel{(1-x^2)^{1/2}}.\frac{x^2}{\cancel{\sqrt{1-x^2}}}+c$$

$$\Rightarrow z.\frac{x^2}{\sqrt{1-x^2}}=\int x^2+c$$

$$z.\frac{x^2}{\sqrt{1-x^2}}=\frac{x^3}{3}+c$$

$$\frac{dv}{dx}=\frac{\cancel{x^3}.\sqrt{1-x^2}}{3\,\cancel{x^2}}+\frac{c}{x^2}\sqrt{1-x^2}$$

$$\int dv=\int\frac{x\sqrt{1-x^2}}{3}+c\int\frac{\sqrt{1-x^2}}{x^2}+k$$

$$v=\int\frac{x(x-x^2)^{1/2}}{3}+c\int\frac{\sqrt{1-x^2}}{x^2}+k$$

let $=1-x^2=v$
$-2x=dv$

let $x=\sin\theta$
$dx=\cancel{\sin\theta}\cos\theta$

$v = \frac{-1}{9} x \sqrt{1-x^2}$

$V = -\frac{1}{9} x (1-x^2)^{3/2} + c(-\cot\theta - \theta) + K$

$v = \frac{-1}{9}(1-x^2)^{3/2} + c(\sin^{-1}(x) + \sqrt{1-x^2})K$

$y = u.v$

$y = x(-1/9(1-x^2)^{3/2} + c(\sin^{-1}(x) + \sqrt{1-x^2})K +$

Answer

Que (8) $x\frac{d^2y}{dx^2} - (2x-1)\frac{dy}{dx} + (x-1)y = 0$

$\frac{d^2y}{dx^2} - \left(\frac{2x-1}{x}\right)\frac{dy}{dx} + \left(\frac{x-1}{x}\right)y = 0$

$\frac{d^2y}{dx^2} - \left(2-\frac{1}{x}\right)\frac{dy}{dx} + \left(1-\frac{1}{x}\right)y = 0$

$1+P+Q=0$ $\qquad$ $1+P+Q=0$

$y\cancel{\pm}/2 + \cancel{\frac{y}{x}} + y - 1/2 = 0$ $\qquad$ $v = e^x$

$\frac{d^2v}{dx^2} + \left[\frac{2}{e^x}\frac{du}{dx} + P\right]\frac{dv}{dx} = \frac{R}{u}$

$\frac{d^2v}{dx^2} + \left[\frac{2}{e^x}\cdot e^x + \left(-2+\frac{1}{x}\right)\right]\frac{dv}{dx} = 0$

$\frac{d^2v}{dx^2} + \left[2-2+\frac{1}{x}\right]\frac{dv}{dx}$

$\frac{dz}{dx} + \frac{1}{x}z = 0$ $\quad$ I.f. $= e^{\int 1/x} = x$

$z.x = \int 0 + c_1\, dx$

$z = c_1$

$z \cdot x = c_1$

$$\frac{dv}{dx} = \frac{c_1}{x} \Rightarrow \int dv = \int c_1/x + c_2$$

$$v = c_1 \log x + c_2$$

$$y = u \cdot v$$

$$y = e^x(c_1 \log x + c_2)$$

Que. (5) $\frac{xd^2y}{dx} - (2x+1)\frac{dy}{dx} + (x+1)\,y = (x^2+x-1)e^{2x}$

$$\frac{d^2y}{dx^2} - \left(\frac{2x+1}{x}\right)\frac{dy}{dx} + \left(\frac{x+1}{x}\right)y = \frac{(x^2+x-1)e^{2x}}{x}$$

$1 + P + Q$

$1 - \frac{2x-1}{x} + \frac{x+1}{x} = 0$, $1+P+Q=0$
$u = e^x$

$x - 2x - 1 + x + 1 = 0$

$$\frac{d^2v}{dx^2} + \left[\frac{2}{e^x}\frac{dv}{dx} + P\right]\frac{dv}{dx} = \frac{R}{u}$$

$\cancel{\frac{d^2v}{dx^2} + [x - (2x+1}$

$\cancel{\frac{d^2v}{dx^2} + \left[\frac{2}{e^x} x e^x + \left[\frac{2x+1}{x}\right]\right]\frac{dv}{dx} = \frac{(x^2+x-1)e^{2x}}{x \cdot e^x}}$

$\cancel{\frac{d^2v}{dx^2} - \frac{1}{x}\frac{dv}{dx} = \left(x + 1 - \frac{1}{x}\right)e^x}$

$\Rightarrow \frac{d^2v}{dx}$

Now continue –

$$\frac{d^2v}{dx^2} + \left[\frac{2}{\cancel{e^x}} x \cancel{e^x} + \left[\frac{2x+1}{x}\right]\right]\frac{dv}{dx} = \frac{(x^2+x-1)}{e^x \cdot x} e^{2x}$$

$$\frac{d^2v}{dx} + \left[2 - \left(\frac{2x+1}{x}\right)\right]\frac{dy}{dx} = \left(\frac{x^2+x-1}{x}\right)e^x$$

$$\frac{d^2v}{dx^2} - \frac{1}{x}\frac{dv}{dx} = \left(x+1-\frac{1}{x}\right)e^x$$

$$\frac{dz}{dx} - \frac{1}{x}z = \left(x+1-\frac{1}{x}\right)e^x$$

$$I.f. = e^{\int -1/x} = 1/x$$

$$z \times \frac{1}{x} = \int \left(1+\frac{1}{x}-\frac{1}{x^2}\right)e^x dx$$

$$\therefore \quad = \int (f(x) + f'(x))e^x = dx \Rightarrow f(x) = e^x + c$$

$$\frac{dv}{dx} \times \frac{1}{x} = \left|1+\frac{1}{x}\right| e^x + c$$

$$\frac{dv}{dx} = (x+1)e^x + c_1 x$$

$$\int dv = \int (x+1)e^x + c_1 x$$

$$v = (x+1)e^x - \int e^x\, dx + \frac{c_1 x^2}{2}$$

$$v = (x+1)e^x - e^x = xe^x + c_1\frac{x^2}{2}$$

$$y = e^x\left(xe^x + c_1\frac{x^2}{2}\right)$$

$\boxed{\text{Que - (6), (7) are left of ex-(38)}}$

Imp.

Que. (6) $x\frac{d^2y}{dx^2}(x\cos x - 2\sin x) + (x^2+2)\frac{dy}{dx}\sin x - 2y(x\sin x + \cos x) = 0$

$$\frac{d^2y}{dx^2} + \underbrace{\frac{(x^2+2)\sin x}{x(x\cos x - 2\sin x)}}_{P}\frac{dy}{dx} - \underbrace{\frac{2(x\sin x + \cos x)}{x(x\cos x - 2\sin x)}}_{Q}y = \underbrace{0}_{R}$$

Consider $2+2Px+Qx^2 = 2 + \frac{2.x(x^2+2)\sin x}{x(x\cos x - 2\sin x)} - \frac{2(x\sin x+\cos x)x}{x(x\cos x - 2\sin x)}$

$$= \frac{2x^2\cancel{\cos x} - 4x\cancel{\sin x} + 2x^3\cancel{\sin x} - 2x^3\cancel{\sin x} - 2x^2\cancel{\cos x}}{x(x\cos x - 2\sin x)}$$

$\boxed{Q = 0}$

Or

$\boxed{u = x^2}$ $\therefore 2+2Px+Qx^2 = 0$

$y = u \times v = x^2 v$

we know that

$$\frac{d^2v}{dx^2} + \left[\frac{2}{u}\frac{du}{dx} + P\right]\frac{dv}{dx} = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + \left[\frac{2}{x^2} \times 2x + \left[\frac{(x^2+2)\sin x}{x(x\cos x - 2\sin x)}\right]\right]\frac{dv}{dx} = 0$$

Let $\frac{dv}{dx} = z \Rightarrow \frac{d^2v}{dx^2} = \frac{dz}{dx}$

$\cancel{\frac{dz}{dx}}$ ~~f~~ $\cancel{[\frac{4}{x}}$ +

$$\frac{dz}{dx} + \left[\frac{4}{x} + \frac{(x^2+2)\sin x}{x(x\cos x - 2\sin x)}\right]z = 0$$

$$\frac{dz}{dx} = \left[-\frac{4}{x} - \frac{(x^2+2)\sin x}{x(x\cos x - 2\sin x)}\right]z$$

V.A.S and Inte

$$\int \frac{dz}{z} + \int\left[\frac{4}{x} + \frac{(x^2+2)\sin x}{x(x\cos x - 2\sin x)}\right]dx = \log c_1$$

$$\log z + 4\log x - \log(x(x\cos x - 2\sin x)) = \log c_1$$

$$\log z + \log x^4 - \log[x(x\cos x - 2\sin x)] = \log c_1$$

$$\log z = \log\frac{1}{x^4} + \log[x(x\cos x - 2\sin x)] + \log c_1$$

$$\log z = \log c_1\left[\frac{x(x\cos x - 2\sin x)}{x^4}\right]$$

$$z = c_1\left[\frac{x(x\cos x - 2\sin x)}{x^4}\right]$$

$$\frac{dv}{dx} = c_1\left[\frac{x^2\cos x - 2x\sin x}{x^4}\right]$$

v.A.s and integrate

$$\int dv = c_1\int \frac{(x^2\cos x - 2x\sin x)\,dx}{x^4} + c_2$$

$$v = c_1\int \frac{d}{dx}\left(\frac{\sin x}{x^2}\right)dx + c_2$$

$$v = c_1 \times \frac{\sin x}{x^2} + c_2$$

$$y = u \times v$$

$$y = x^2\left[c_1\frac{\sin x}{x^2} + c_2\right]$$

$$\boxed{y = c_1\sin x + c_2 x^2}$$

Ex- 39

Removal of the first Derivative or change of Dependent variable, Reduction to Normal form.

Article – $$\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R \quad — \text{(I)}$$

C.S. $y = u \times v$ — (2)

$$\frac{d^2v}{dx^2}+I.v=\frac{R}{u} \quad — (3)$$

where $I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$

Note – (1) I = constant or zero ⇒ linear D.E. of 2nd order

(2) $I = \frac{\text{constant}}{x^2}$ ⇒ Homogeneous 2nd order L.D.E.

Ex. = 39

~~(φ) I = constant or zero ⇒~~

Q.(4) $$\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} + 3y = 2\sec x \quad — (1)$$

अगर P की value trigonometry है और Q की value trigonometry नहीं, तो part of c.f. known apply नहीं होगा

C.S. $y = u.v$ — (2)

we known that $$\frac{d^2v}{dx^2}+I.v=\frac{R}{u} \quad — (3)$$

$I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$

~~$I = 3 + \sec^2 x - \tan^2 x$~~

~~$I = 3+$~~ $I = 3 - \frac{1}{2}(-2\sec^2 x) - \frac{1}{4}\times 4\tan^2 x$

$I = 3 + \sec^2 x - \tan^2 x \Rightarrow 3 + 1 = 4$

linear D.E. 2nd order —

$P = 2\tan x \Rightarrow \frac{dP}{dx} = -2\sec^2 x$

$Q = 3$

$R = 2\sec x$

$u = e^{-1/2 \int P dx} = e^{-1/2 \int -2\tan x \, dx} = e^{\int \tan x}$

$\Rightarrow e^{\log \sec x} = \sec x$

Or $u = \sec x$

put in (3)

$$\frac{d^2v}{dx^2} + 4v = \frac{2\sec x}{\sec x} = 2$$

A.E. $= m^2 + 4 = 0$

$\Rightarrow m = \pm 2i$

C.F. $= c_1 \cos 2x + c_2 \sin x$

$$P.I. = \frac{1}{D^2+4} \times 2 = \frac{1 \times 2e^{0x}}{D^2+4} = \frac{2 \times e^{0x}}{4} = 1/2$$

P.I. $= 1/2$

solution — $v = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{2}$

C.S. $\Rightarrow y = u \times v$

$$y = \sec x \left[c_1 \cos x + c_2 \sin x + 1/2 \right]$$

(6) $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2-3)y = e^{x^2}$ — (1)

$P = -4x \qquad Q = 4x^2 - 3 \qquad R = e^{x^2}$

$\frac{dP}{dx} = -4$

C.S. $\Rightarrow y = u \times v$ — (2)

We know that $\frac{d^2v}{dx^2} + Iv = \frac{R}{u}$ — (3)

where $I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$

$I = 4x^2 - 3 + \frac{1}{\cancel{2}}(-\cancel{4}^{2}) - \frac{1}{\cancel{4}}(\cancel{16}^{4}x^2)$

$I = 4x^2 - 3 + 2 - 4x^2$

$I = -1$

$u = e^{-1/2\int P dx} = e^{-1/2\int -4x dx} = e^{\int 2x dx} = e^{x^2}$

$\therefore u = e^{x^2}$

put in ③

$\frac{d^2v}{dx^2} - 1v = $ $\frac{\cancel{e^{x^2}}}{\cancel{e^{x^2}}} = 1$

A.E. $= m^2 - 1 = 0 \Rightarrow m = \pm 1$

C.F. $= c_1 e^{x} + c_2 e^{-x}$

P.I. $= \frac{1}{D^2-1} \times (e^0 x) = \frac{1}{0^2-1} \times 1 = -1$

General solution

$v = $ ~~B~~ C.I. + P.I.

$v = c_1 e^{x} + c_2 e^{-x} - 1$

Comple solution -

$y = u \times v$

$\boxed{y = e^{x^2}[c_1 e^{x} + c_2 e^{-x} - 1]}$

① $\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 5y = 0$ - ①

Cs. $y = u \times v$ - ②

we know that

$\frac{dv}{dx^2} + I \cdot V = R/u$ - ③

$I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$

$P = -2\tan x \Rightarrow \frac{dP}{dx} = +2\sec^2 x$

$Q = 5$

$R = 0$

$\Rightarrow I = 5 - \frac{1}{2}\times 2\sec^2 x - \frac{1}{\cancel{4}}\times(-\cancel{4}\tan^2 x)$

$I = 5 - \sec^2 x + \tan^2 x$

$I = -(5 + \sec^2 x - \tan^2 x)$

$\boxed{I = -4}$

$\Rightarrow u = e^{-1/2\int P\,dx} \Rightarrow u = e^{-1/2\int -2\tan x}$

$u = e^{-1/\cancel{2}\times -\cancel{2}\int \tan x} \Rightarrow u = e^{\int \tan x} \Rightarrow e^{\log \sec x} = \sec x$

$u = \sec x$

put in - (3)

$\frac{d^2v}{dx} + I\cdot v = R/u$

$\frac{d^2v}{dx^2} + (-4)\times v = \frac{0}{\sec} \to 0$

$\frac{d^2v}{dx^2} - 4v = 0$

$D^2v - 4v = 0$

$(D^2-4)v = 0$

$\Rightarrow m^2 - 4 = 0$

$m = \sqrt{4}$

$m = \pm 2$

$\Rightarrow CF = c_1 e^{-2x} + c_2 e^{2x}$

$PI = \frac{1}{(D^2-4)} \times 0$

$\boxed{PI := 0}$

solution -

$v = CF + PI$

$\boxed{v = c_1 e^{-3x} + c_2 e^{2x}}$

complete solution

$y = u\cdot v$

$\boxed{y = \sec x\,(c_1 e^{-2x} + c_2 e^{2x})}$

Q(2)

(2) $\frac{d^2y}{dx^2} - 2bx\frac{dy}{dx} + b^2x^2y = 0$ — (1)

C.S. $\Rightarrow y = u.v$ — (2)

We know that

$\frac{d^2v}{dx^2} + I.v = \frac{R}{u}$ — (3)

$P = -2bx$ | $Q = b^2x^2$

$\frac{dp}{dx} = -2b$ | $R = 0$

$I = Q - \frac{1}{2}\frac{dp}{dx} - \frac{1}{4}p^2$

$I = b^2x^2 - \frac{1}{2}(-2b) - \frac{1}{4}(-2bx)^2$

$I = b^2x^2 - \cancel{\frac{1}{2}} \times -\cancel{2}b - 1\cancel{\frac{1}{4}} \times \cancel{4}b^2x^2$

$I = \cancel{b^2x^2} + b - \cancel{b^2x^2}$

$\boxed{I = b}$ $\quad u = e^{-1/2\int p\,dx} = e^{-1/2\int -2bx\,dx} = e^{-\frac{1}{2}\times -2\int bx\,dx}$

$u = e^{\int bx} \Rightarrow e^{bx^2/2}$

put in (3) $\frac{d^2v}{dx^2} + I.v = \frac{R}{u}$

$\frac{d^2v}{dx^2} + b = \frac{0}{e^{bx/2}} \to 0$

$D^2v + bv = 0$ | $m = \sqrt{i^2 b}$

$(D^2+b)v = 0$ | $\boxed{m = \pm i b}$

$m^2 + b = 0$ | C.F. $= e^{0x}(c_1\cos\sqrt{b}x + c_2\sin\sqrt{b}x)$

$m = \sqrt{-b}$ | P.I. $= \frac{1}{D^2+b} \times 0$ $\quad \boxed{P.I = 0}$

Solution — $v = C.F + P.I$

$$\boxed{v = c_1 \cos\sqrt{6}x + c_2 \sin\sqrt{6}x}$$

C.S. $\Rightarrow$ $y = u.v$

$$\boxed{y = e^{bx/2}\left[c_1 \cos\sqrt{6}x + c_2 \sin\sqrt{6}x\right]}$$

Que③ $\dfrac{d^2y}{dx^2} + x^{-1/3}\dfrac{dy}{dx} + \left(\tfrac{1}{4}x^{-2/3} - \tfrac{1}{6}x^{-4/3} - 6x^{-2}\right)y = 0$ — ①

C.S. $= y = u.v$ — ②

$\dfrac{d^2v}{dx^2} + I.v = R/u$ — ③

$P = x^{-1/3}$

$\dfrac{dP}{dx} = -\tfrac{1}{3}x^{-4/3}$

$Q = \tfrac{1}{4}x^{-2/3} - \tfrac{1}{6}x^{-4/3} - 6x^{-2}$

$R = 0$

$I = Q - \tfrac{1}{2}\dfrac{dP}{dx} - \dfrac{1}{4}P^2$

$I = \dfrac{x^{-2/3}}{4} - \dfrac{x^{-4/3}}{6} - 6x^{-2} - \tfrac{1}{2}\times\left(-\tfrac{1}{3}\times x^{-4/3}\right) - \dfrac{1}{4}\left(x^{-1/3}\right)^2$

$I = \cancel{\dfrac{x^{-2/3}}{4}} - \cancel{\dfrac{x^{-4/3}}{6}} - 6x^{-2} + \cancel{\dfrac{x^{-4/3}}{6}} - \cancel{\tfrac{1}{4}x^{-2/3}}$

$$\boxed{I = -6x^{-2}}$$

$$\boxed{I = -6/x^2}$$

$u = e^{-1/2\int P\,dx} = e^{-1/2\,\frac{x^{-1/3+1}}{-1/3+1}}$

$= e^{-1/2\times\frac{x^{2/3}}{2/3}}$

$$\boxed{u = e^{-3/4\,x^{2/3}}}$$

put in ③

$$\frac{d^2V}{dx^2} + I.V = R/\mu \Rightarrow \frac{d^2V}{dx^2} + (-6x^{-2})V = R/\mu$$

$$\frac{d^2V}{dx^2} - 6x^{-2}V = R/\mu \Rightarrow \frac{d^2V}{dx^2} - \frac{6}{x^2}V = R/\mu$$

$$D^2V - \frac{6V}{x^2} = R/\mu \Rightarrow \left(D^2 - 6/x^2\right)V = \left.\begin{matrix}\frac{R}{\mu} \to 0 \\ \cdot \to 0\end{matrix}\right\} = 0$$

$$\therefore \left(D^2 - \frac{6}{x^2}\right)V = \cancel{0}\ \frac{R}{\mu}$$

multiply by x^2D^2 both side -

$$\left(x^2D^2 - \frac{x^2\,6}{x^2}\right)V = \frac{R}{\mu} \times x^2 \to 0$$

$$(x^2D^2 - 6)V = 0$$

$\therefore x^2D^2 = D_1(D_1 - 1)$

$z = \log x$

$x = e^z$

replaced

Done

$$(D_1(D_1 - 1) - 6)V = 0$$
$$(D_1^2 - D_1 - 6)V = 0 \Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \frac{+1 \pm \sqrt{1 - 4 \times -6}}{2}$$

$$\Rightarrow \frac{1 \pm \sqrt{1 + 24}}{2} \Rightarrow \frac{1 \pm \sqrt{25}}{2}$$

$$\Rightarrow \frac{1 \pm 5}{2}$$

$$\Rightarrow \frac{1+5}{2} \;;\; \frac{1-5}{2}$$

$$\Rightarrow \frac{6}{2} = 3 \;;\; -4/2 = 2$$

$\boxed{\text{roots} = 3, -2}$

$\boxed{m = 3, -2}$

$$\boxed{C.F. = c_1 e^{3z} + c_2 e^{-2z}}$$

$P.I = \frac{1}{D_1^2 - D_1 - 6} \times 0 \Rightarrow PI = 0$

Solution $= v = Cf + P.I$

$v = c_1 e^{3z} + c_2 e^{-2z}$ $\quad \boxed{z = \log x \quad x = e^z}$

$\boxed{v = c_1 x^3 + c_2 x^{-2}}$

C.S: $y = u \cdot v$

$$\boxed{y = e^{3/4} x^{-2/3}\left[c_1 x^3 + \frac{c_2}{x^2}\right]}$$ Ans

Que (5) $\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} + 3y = 2\sec x$ — (1)

C.S $\Rightarrow y = u.v.$ (2) we know that $\frac{d^2v}{dx^2} + I.V. = \frac{R}{u}$

$P = -2\tan x$ | $Q = 3$ | $R = 2\sec x$

$\frac{dP}{dx} = +2\sec^2 x$

$I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2 \Rightarrow I = 3 - \frac{\cancel{2}\sec^2 x}{\cancel{2}} - \frac{1}{4}(-2\tan x)^2$

$I = 3 - \sec^2 x - \frac{4}{4}\tan^2 x$

$I = 3 - \sec^2 x - \tan^2 x \Rightarrow 3 - 1 = 2$

$\boxed{I = 2}$ $\quad u = e^{-1/2\int P dx} = e^{-1/2\int -2\tan x}$

$u = e^{-2/2 \int \tan x} \Rightarrow u = e^{\log \sec x} \Rightarrow \sec x$

$\boxed{u = \sec x}$ put in -(3) $\frac{d^2v}{dx^2} + I.V. = R/u$

$\frac{d^2v}{dx^2} + 2v = 2\frac{\cancel{\sec x}}{\cancel{\sec x}} \Rightarrow \frac{d^2v}{dx^2} + 2v = 2$

$D^2V + 2V = 2$

$(D^2+2)V = 2$

$m^2+2=0$

$m^2 = -2$

$m = \sqrt{-2}$

$m = \sqrt{2i^2}$

$m = \pm i\sqrt{2}$

$\boxed{m = \pm i\sqrt{2}}$

$C.F. = (c_1 \cos\sqrt{2} + c_2 \sin\sqrt{2})$

$P.I. = \dfrac{1}{D^2+2} \times 2$

$P.I. = \dfrac{2}{0+2} \times e^{0x}$

$\boxed{P.I. = 2/3}$

Solution: $V = C.F + P.I$

$\boxed{V = c_1 \cos\sqrt{2} + c_2 \sin\sqrt{2} + 2/3}$

complete solution $\Rightarrow y = u \cdot v$

$y = \sec x\,[c_1 \cos\sqrt{2} + c_2 \sin\sqrt{2} + 2/3]$

Que-10 $\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} + (x^2+5)y = xe^{-x^2/2}$ —(1)

$\Rightarrow y = u \cdot v$ —(2) $\dfrac{d^2V}{dx^2} + I.V. = R/u$ —(3)

$\Rightarrow P = 2x$ | $Q = x^2+5$

$\dfrac{dP}{dx} = 2$ | $R = xe^{-x^2/2}$

$I = Q - \frac{1}{2}\,dP/dx - \frac{1}{4}P^2 \Rightarrow I = x^2+5-\frac{1}{2}\times\cancel{2} - \frac{1}{4}(2x)^2$

$I = x^2+5-\frac{1}{2}\times 2 - \frac{1}{4}\times 4x^2 \Rightarrow I = \cancel{x^2}+5-1-\cancel{x^2}$

$I = 5-1$ $\boxed{I = 4}$

$u = e^{-1/2\int P dx}$

$u = e^{-1/2\int 2x\,dx}$

$u = e^{-2/2\int x\,dx}$

$u = e^{-x^2/2}$

$\boxed{u = e^{-x^2/2}}$

put in —(3) u की value.

$\dfrac{d^2V}{dx^2} + I.V. = R/u$

$\dfrac{d^2V}{dx^2} + 4V = \dfrac{x\cancel{e^{-x^2/2}}}{\cancel{e^{-x^2/2}}}$

$\dfrac{d^2V}{dx^2} + 4V = x$

$\Rightarrow D^2 v + 4v = x$

$(D^2+4)v = x$

$D^2 = -4$

$D = \sqrt{4i^2}$

$\boxed{D = \pm i2}$

~~C.f. = $c_1 \cos \sqrt{2}$~~

$\boxed{C.f. = c_1 \cos 2x + c_2 \sin 2x}$

$$P.I. = \frac{1}{D^2+4} \times x$$

$$P.I. = \frac{1}{4\left(1+\frac{D^2}{4}\right)} x \cdot x$$

$$\Rightarrow P.I. = \frac{1}{4}\left(1+\frac{D^2}{4}\right)^{-1} x\, x \longrightarrow \text{expansion } (1+x)^{-1}$$

$$P.I. = \frac{1}{4}\left[1-\frac{D^2}{4}+\left(\frac{D^2}{4}\right)^2\right] x$$

$$P.I. = \frac{1}{4}\left[x - \frac{D^2}{4} x 1 + \frac{D^2}{4} x 0\right]$$

$$P.I. = \frac{1}{4}\left[\frac{x}{4} - \cancel{\frac{D^2}{4} x 0}\right]$$

$$\boxed{P.I. = \frac{x}{4}}$$

$v = C.f + P.I$

~~$y = c_1 \cos \sqrt{2} + c_2 \sin \sqrt{2} + x/5$~~

~~$y = u \times v$~~

~~$y = e^{-x^2}$~~

$$\boxed{v = c_1 \cos 2x + c_2 \sin 2x + x/4}$$

$y = u \cdot v$

$$\boxed{y = e^{-x^2/2} \times \left[c_1 \cos 2x + c_2 \sin 2x + x/4\right]}$$ Answer

Que - 9, 8, 7

Que (7) $\left(\frac{d^2y}{dx^2} + y\right) \cot x + 2\left(\frac{dy}{dx} + y \tan x\right) = \sec x$

$$\frac{d^2y}{dx^2}\cot x + 2\frac{dx}{dy} + (y\cot x + 2y\tan x) = \sec x$$

$$\frac{d^2y}{dx^2} + 2\tan x\frac{dy}{dx} + (1+2\tan^2 x)y = \sec x \times \frac{1}{\cot x}$$

$$\frac{d}{dx} = D \qquad = \sec x \cdot \tan x$$

$$D^2y + 2\tan x\, Dy + (1+2\tan^2 x)y = \sec x \cdot \tan x$$

$$y = u \cdot v$$

$$\frac{d^2v}{dx^2} + I \cdot V = \frac{R}{u}$$

$\therefore P = 2\tan x$ | $Q = 1+2\tan^2 x$

$\frac{dP}{dx} = -2\sec^2 x$ | $R = \frac{\sin x}{\cos^2 x}$

$$I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{1}{4}P^2$$

$$I = 1+2\tan^2 x + \frac{1}{2}(-\cancel{2}\sec^2 x) - \frac{1}{\cancel{4}}(\cancel{4}\tan^2 x)$$

$$I = 1 + 2\tan^2 x - \sec^2 x - \tan^2 x$$

$$I = 1+\tan^2 x - \sec^2 x$$

$$= 1 - (\sec^2 x - \tan^2 x)$$

$$I = 1 - 1 = 0 \qquad \boxed{I = 0}$$

$$u = e^{-1/2\int P\,dx} = e^{-1/\cancel{2}\int \cancel{2}\tan x\,dx} = e^{-\log \sec x}$$

$$u = \sec x^{-1} = \frac{1}{\sec x} = \cos x$$

$$\boxed{u = \cos x}$$

put in eq - (3)

$$\frac{d^2v}{dx^2} + IV = \frac{R}{u}$$

$$\frac{d^2v}{dx^2} + 0.v = \cancel{8}\sec x^2 \tan x \Rightarrow \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \times \sec x$$

$$\frac{d^2v}{dx^2} = \sec^2 x \tan x$$

Integrate

$$\frac{dv}{dx} = \int \sec^2 x \tan x \, dx + c_1$$

let $\tan x = t$

$\sec^2 x \, dx = dt$

$$\frac{dv}{dx} = \int t \, dt + c_1 \Rightarrow \frac{dv}{dx} = \frac{(t)^2}{2} + c_1$$

$$\Rightarrow \frac{dv}{dx} = \frac{1}{2}\tan^2 x + c_2$$

V.A.S and integrate

$$\int dv = \frac{1}{2}\int \tan^2 x \, dx + c_1 \, dx + c_2$$

$$v = \frac{1}{2}\int (\sec^2 x - 1)\, dx + c_1 x + c_2$$

$$v = \frac{1}{2}\int (\tan x - x) + c_1 x + c_2$$

$$y = u \times v$$

$$y = \cos x \left[\frac{1}{2}\tan x - x + c_1 x + c_2\right]$$

$$\boxed{y = \frac{1}{2}\sin x - 4\cos x + c_1 x \cos x + c_2 \cos x}$$

Que. 9 and 8 are left in Ex - 39 Ans

* Change of Independent Variable * Ex-40

Ex-40

100% Important

$\Rightarrow$ $$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad \text{— (I)}$$

changed

$\Rightarrow$ $$\boxed{\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1y = R_1} \quad \text{— (2)}$$

Where

# $P_1 = \dfrac{\dfrac{d^2z}{dx^2} + P\dfrac{dz}{dx}}{\left(\dfrac{dz}{dx}\right)^2}$	$Q_1 = \dfrac{Q}{\left(\dfrac{dz}{dx}\right)^2}$	$R_1 = \dfrac{R}{\left(\dfrac{dz}{dx}\right)^2}$

Note – ① Q_1 = Constant (with sign)

② P_1 = constant or zero

Linear differention equaction 2nd order

= CF + PI वाली बनेगी

③ $P_1 = \dfrac{\text{Constant}}{z^2}$ → Homogeneous Differential equation of 2nd second.

V.V. Important

Que (6) $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2 y = x^4$

Sol^n- $y'' - \frac{1}{x} y' + 4x^2 y = x^4$ — (1)

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

$P = -\frac{1}{x}$ $\quad Q = 4x^2$ $\quad R = x^4$

$Q_1 = 4$ $\quad Q_1 = \frac{Q(x)}{\left(\frac{dz}{dx}\right)^2}$ $\quad 4 = \frac{4x^2}{\left(\frac{dz}{dx}\right)^2}$

$\left(\frac{dz}{dx}\right)^2 = \frac{\cancel{4}x^2}{\cancel{4}}$ $\Rightarrow \boxed{\left(\frac{dz}{dx}\right)^2 = x^2}$ $\Rightarrow \frac{dz}{dx} = \sqrt{x^2}$

$\boxed{\frac{dz}{dx} = x}$ $\quad \int dz = x\,dx$

$\boxed{z = \frac{x^2}{2}}$ $\Rightarrow \boxed{x^2 = 2z}$ $\quad \boxed{\frac{d^2z}{dx^2} = 1}$

$$P_1 = \frac{\frac{d^2z}{dx} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} = \frac{1 + -\frac{1}{\cancel{x}} \cancel{x}}{x^2} = \frac{1-1}{x^2} \Rightarrow 0$$

$\boxed{P_1 = 0}$ $\quad R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{x^4}{x^2} = x^2$

$\boxed{R_1 = x^2}$

$\boxed{R_1 = 2z}$

$$\Rightarrow \frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1 y = R_1$$

$\Rightarrow \frac{d^2y}{dz^2} + 0\frac{dy}{dz} + 4y = 2z$

$\Rightarrow \frac{d^2y}{dz^2} + 4y = 2z$

$(D_1^2 + 4)y = 2z$

A.E. $\Rightarrow m^2 + 4 = 0$

$m^2 = -4$

$m = \sqrt{i^2 4}$

$m = \pm 2i$

C.F. $= C_1 \cos 2z + C_2 \sin 2z$

P.I. $= \frac{1}{D_1^2 + 4} \cdot 2z$

P.I. $= \frac{1}{4} \frac{1}{\left(1 + \frac{D_1^2}{4}\right)} \cdot 2z$

P.I. $= \frac{1}{4}\left[1 - \frac{D_1^2}{4} - \frac{D_1^4}{16}\right] \cdot 2z$

P.I $= \frac{1}{4}[2z - 0 - 0]$

P.I $= \frac{1}{4}(2z)$

P.I. $= \frac{\cancel{2}z}{\cancel{4}}$

$\boxed{\text{P.I.} = \frac{z}{2}}$

$y = $ C.f + P.I

$y = C_1 \cos 2z + C_2 \sin 2z + z/2$

replaced $z = x^2/2$

$\Rightarrow y = C_1 \cos \cancel{2}\frac{x^2}{\cancel{2}} + C_2 \sin \cancel{2}\frac{x^2}{\cancel{2}} + \frac{x^2}{4}$

$\boxed{y = C_1 \cos x^2 + C_2 \sin x^2 + x^2/4}$

Que (8) $\frac{d^2y}{dx^2} - \frac{dy}{dx}\cot x - y \sin^2 x = \cos x - \cos^3 x$

Sol^n

$\Rightarrow \frac{d^2y}{dx^2} - \cot \frac{dy}{dx} - \sin^2 x y = \cos x \cdot \sin^2 x$ —(1)

$P = -\cot x$ $Q = -\sin^2 x$

$R = \cos x \cdot \sin^2 x$

$\frac{d^2y}{dz^2} + P_1 \frac{dy}{dz} + Qy = R_1$

$P_1 = \frac{\frac{d^2z}{dx} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$

$Q_1 = \frac{Q(x)}{\left(\frac{dz}{dx}\right)^2}$

$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2}$

$\boxed{Q_1 = -1}$ $- \sin^2 x$

& -1 constant coefficient

$$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2}$$

$$\Rightarrow -1 = \frac{-\sin^2 x}{\left(\frac{dz}{dx}\right)^2}$$

$$\Rightarrow \boxed{\left(\frac{dz}{dx}\right)^2 = \sin^2 x}$$

$$\boxed{\frac{dz}{dx} = \sin x}$$

$$\boxed{\frac{d^2z}{dx^2} = \cos x}$$

$$\int dz = \int \sin x \, dx$$

$$\boxed{z = -\cos x}$$

$$P_1 = \frac{\cos x - \cot x \sin x}{-\sin^2 x}$$

$$= \frac{\cos x - \frac{\cos x \cdot \cancel{\sin x}}{\cancel{\sin x}}}{-\sin^2 x}$$

$$\boxed{P_1 = 0}$$

$$R_1 = \frac{\cos x \cdot \cancel{\sin^2 x}}{\cancel{\sin^2 x}}$$

$$\boxed{R_1 = \cos x}$$

$$z = -\cos x$$

$$-z = \cos x$$

$$\boxed{R_1 = -z}$$

$$\frac{d^2y}{dz^2} + P_1 \frac{dy}{dz} + Q_1 y = R_1$$

$$\frac{d^2y}{dz^2} - y = -z$$

$$[D_1^2 - 1]y = -z$$

$$A.E. = D_1^2 - 1 = 0$$

$$= D_1 = \pm 1$$

$$\boxed{C.F. = c_1 e^{-z} + c_2 e^{z}}$$

$$P.I. = \frac{1}{D_1^2 - 1}(-z) \Rightarrow \frac{-z}{-(1 - D_1^2)}$$

$$P.I. = (1 - D_1^2)^{-1} z \rightarrow \text{expansion}$$

$$P.I. = [1 + D_1^2 + D_1^4]\, z$$

$$P.I. = (z + 0)$$

$$\boxed{P.I. = z}$$

$$y = C.F. + P.I.$$

$$\boxed{y = c_1 e^{-z} + c_2 e^{z} + z}$$

replaced $z = -\cos x$

$$y = c_1 e^{\cos x} + c_2 e^{-\cos x} - \cos x$$

Que (7) $x\frac{d^2y}{dx^2}+(4x^2-1)\frac{dy}{dx}+4x^3y=2x^3$ | $\frac{d^2y}{dz^2}+P_1\frac{dy}{dz}+Q_1y=R_1$

Soln

$$\frac{d^2y}{dx}+(4x-1/x)\frac{dy}{dx}+4x^2y=2x^2$$

$P=4x-1/x$, $Q=4x^2$, $R=2x^2$

Let $Q_1=4$

$$Q_1=\frac{Q}{\left(\frac{dz}{dx}\right)^2} \Rightarrow 4=\frac{\cancel{4}x^2}{\left(\frac{dz}{dx}\right)^2}$$

$$\boxed{\left(\frac{dz}{dx}\right)^2=x^2} \Rightarrow \boxed{\frac{dz}{dx}=x}$$

$$\boxed{\frac{d^2z}{dx^2}=1}$$

$$\int dz=\int x\,dx \Rightarrow \boxed{z=\frac{x^2}{2}}$$

$$P_1=\frac{\frac{d^2z}{dx^2}+P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2} \Rightarrow \frac{1+(4x-1/x)\cdot x}{x^2}$$

$$P_1=\frac{1+4x^2-x/x}{x^2} \Rightarrow \frac{4\cancel{x^2}}{\cancel{x^2}}$$

$$\boxed{P_1=4}$$

$$R_1=\frac{R}{\left(\frac{dz}{dx}\right)^2} \Rightarrow \frac{2\cancel{x^2}}{\cancel{x^2}}$$

$$\boxed{R_1=2}$$

$$\frac{d^2y}{dz^2}+4\frac{dy}{dz}+4y=2$$

$$D^2y+4Dy+4y=2$$

$$(D^2+4D+4)y=2$$

A.E. = ~~m^2+4D+4~~

$$m^2+4m+4=0$$

$$(m+2)(m+2)$$

$$\boxed{m=-2,-2}$$

roots are equal and real.

$$\boxed{C.F.=(C_1+C_2z)e^{-2z}}$$

$$P.I.=\frac{1}{(D_1+2)^2}\cdot 2$$

$$=2\frac{1}{(D_1+2)}\cdot e^{0z}$$

$$P.I.=2\frac{1}{(0+2)^2}\cdot 1=1/2$$

$$\boxed{P.I=1/2}$$

$y=$ C.F. + PI

$$\Rightarrow (C_1+C_2z)e^{-2z}+1/2$$

$$\boxed{y=(C_1+C_2x^2/2)e^{-x^2}+1/2}$$

Q(3) $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3(\sin x^2)$

$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} - 4x^2y = 8x^2(\sin x^2)$

$P = -1/x,\ Q = -4x^2,\ R = 8x^2(\sin x^2)$

$Q_1 = -4$

$Q_1 = \frac{Q(x)}{\left(\frac{dz}{dx}\right)^2} \Rightarrow -4 = \frac{-4x^2}{\left(\frac{dz}{dx}\right)^2}$

$\Rightarrow \boxed{\left(\frac{dz}{dx}\right)^2 = x^2}$

$\boxed{\frac{dz}{dx} = x}$

$\boxed{\frac{d^2z}{dx^2} = 1}$

$\int dz = \int x\,dx$

$\boxed{z = \frac{x^2}{2}}$

$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$

$P_1 = \frac{1 + (-1/x)x}{x^2} \Rightarrow 1-1 = 0$

$P_1 = 0$

$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} \Rightarrow \frac{8\cancel{x^2}(\sin x^2)}{\cancel{x^2}}$

$R_1 = 8\sin x^2$

$\boxed{z = \frac{x^2}{2}} \Rightarrow \boxed{2z = x^2}$

$\boxed{R_1 = 8\sin 2z}$

$\frac{d^2y}{dz^2} + P_1\frac{dy}{dz} + Q_1y = R_1$

$\Rightarrow \frac{d^2y}{dz^2} + 0 + (-4)y = 8\sin 2z$

$\Rightarrow D^2 - 4 \quad D = \pm 4$

$\Rightarrow \boxed{D = \pm 2}$

$C.F. = c_1e^{2z} + c_2e^{-2z}$

$P.I. = \frac{1}{D^2-4} \times 8\sin 2z$

$P.I. = 8\frac{1}{D^2-4}\sin 2z$

$P.I. = 8 \times \frac{1}{-2^2-4} \times \sin 2z$

$P.I. = \frac{\cancel{8}}{-\cancel{8}} \times \sin 2z$

$\boxed{P.I. = -\sin 2z}$

$y = C.F. + P.I.$

$y = c_1e^{2z} + c_2e^{-2z} - \sin 2z$

$y = c_1e^{2x^2/2} + c_2e^{-2x^2/2} - \sin x^2$

$\boxed{y = c_1e^{x^2} + c_2e^{-x^2} - \sin x^2}$

Que (3) $\frac{d^2y}{dx^2}+(3\sin x-\cot x)\frac{dy}{dx}+2y\sin^2 x = e^{-\cos x}\cdot\sin^2 x$

Sol^n

$P = 3\sin x-\cot x \quad Q = 2y\sin^2 x$

$R = e^{-\cos x}\cdot\sin^2 x$

$\boxed{Q_1 = 2}$

$Q_1 = \frac{Q}{\left(\frac{dz}{dx}\right)^2} \Rightarrow 2 = \frac{2\sin^2 x}{\left(\frac{dz}{dx}\right)^2}$

$\boxed{\left(\frac{dz}{dx}\right)^2 = \sin^2 x}$

$\boxed{\frac{dz}{dx} = \sin x} \rightarrow \boxed{\frac{d^2z}{dx^2} = \cos x}$

$\int dz = \int \sin x\,dx$

$\boxed{z = -\cos x}$

$P_1 = \frac{\frac{d^2z}{dx^2}+P\frac{dz}{dx}}{\left(\frac{dz}{dx}\right)^2}$

$P_1 = \frac{\cos x+(3\sin x-\cot x)\cdot\sin x}{\sin^2 x}$

$P_1 = \frac{\cos x+3\sin^2 x-\cot x\sin x}{\sin^2 x}$

$P_1 = \cancel{\cos x}+3\sin^2 x-\frac{\cancel{\cos x}}{\sin x}\cdot\cancel{\sin x}$

$P_1 = \frac{3\cancel{\sin^2 x}}{\cancel{\sin^2 x}}$ $\boxed{P_1 = 3}$

$R_1 = \frac{R}{\left(\frac{dz}{dx}\right)^2} = \frac{e^{-\cos x}\cdot\cancel{\sin^2 x}}{\cancel{\sin^2 x}}$

$\boxed{R_1 = e^{-\cos x}}$

$\boxed{R_1 = e^{z}}$

$\boxed{z = -\cos x}$

$\frac{d^2y}{dz^2}+P_1\frac{dy}{dx}+Q_1 y = R_1$

$\Rightarrow \frac{d^2y}{dz^2}+3\frac{dy}{dz}+2y = e^z$

$(D_1^2+3D_1+2)\,y = e^z$ — (3)

$A.E. = D_1^2+3D_1+2 = 0$

$(D_1+1)(D_1+2) = 0$

$\boxed{D_1 = -1, -2}$

$C.F. = c_1e^{-z}+c_2e^{-2z}$

$P.I. = \frac{1}{D_1^2+3D_1+2}\times e^z$

$P.I. = \frac{1}{1+3(1)+2}\times e^z$

$P.I. = \frac{1}{6}\times e^z$

$\boxed{P.I. = \frac{e^z}{6}}$

$y = c_1e^{-z}+c_2e^{-2z}+e^z/6$

replaced

$\boxed{z = -\cos x}$

$$\boxed{y = c_1 e^{\cos x} + c_2 e^{2\cos x} + \frac{e^{-\cos x}}{6}}$$

Que-(1), (2), (4), (5) are left.

Ex-4.1

* **Method of Variation of Parameters** *

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \quad -①$$

C.S ⇒ $y = Au + Bv$ — ②

where A and B are parameter —

C.F. ⇒ $c_1 e^x + c_2 e^{-x}$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$A \quad u \quad B \quad v$

$$u\frac{dA}{dx} + v\frac{dB}{dx} = 0 \quad -③$$

$$\frac{du}{dx}\cdot\frac{dA}{dx} + \frac{dv}{dx}\cdot\frac{dB}{dx} = R \quad -④$$

Solve eq. ③ or ④ we find the value of A and B put in ② -

we get C.S - $\boxed{y = Au + Bv}$ Ans

Note —

$u =$ $\qquad v =$

$v =$ $\qquad u =$

$$v = u\int \frac{1}{u^2} e^{-\int P dx} \cdot dx$$

Que ① (Ex. 4.1) $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$ — ①

Sol^n — A.E. ⇒ $m^2+1 = 0$ $\qquad m = \pm i$

C.F. $= c_1 \cos x + c_2 \sin x$

C.S ⇒ $y = A\cos x + B\sin x$ — ②

$u = \cos x$, $v = \sin x$

$\frac{du}{dx} = -\sin x \quad \frac{dv}{dx} = \cos x$

$u\frac{dA}{dx} + v\frac{dB}{dx} = 0 \quad — (3)$

$\frac{du}{dx} \cdot \frac{dA}{dx} + \frac{dv}{dx} \cdot \frac{dB}{dx} = R \quad — (4)$

$\cos \frac{dA}{dx} + \sin x \frac{dB}{dx} = 0 \quad — (5)$

$-\sin \frac{dA}{dx} + \cos x \frac{dB}{dx} = \operatorname{cosec} x \quad — (6)$ → ? where this come from

$(5) \times \sin x + (6) \times \cos x$

$\cancel{\sin x \cos x \frac{dA}{dx}} + \sin^2 x \frac{dB}{dx} = 0$

$\cancel{- \sin x \cos x \frac{dA}{dx}} + \cos^2 x \frac{dB}{dx} = \operatorname{cosec} x + \cos x$

$\Rightarrow \sin^2 x \frac{dB}{dx} + \cos^2 x \frac{dB}{dx} = \cot x$

$\Rightarrow (\sin^2 x + \cos^2 x) \frac{dB}{dx} = \cot x$

$\frac{dB}{dx} = \cot x$

V.A.S and int.

$\int dB = \int \cot x \, dx$

$\boxed{B = \log \sin x + c_2}$

put in 5 $\frac{dB}{dx} = \cot x$

$\cos x \frac{dA}{dx} + \sin x \times \cot x = 0$

→ $\cancel{\cos x} \frac{dA}{dx} = -\cancel{\cos x}$

$\frac{dA}{dx} = -1$

V.A.S and Inte.

$\int dA = -\int dx + c_1$

$\boxed{A = -x + c_1}$

from (2)

C.S. $\Rightarrow y = (-x + c_1)\cos x + (\log \sin x + c_2)\sin x$

$y = c_1 \cos x + c_2 \sin x - x\cos x + \sin x \log \sin x$

Que ⑥ $\frac{d^2y}{dx^2} + (1-\cot x)\frac{dy}{dx} - y\cot x = \sin^2 x$

consider

$P = 1-\cot x \;,\; Q = -\cot x \;,\; R = \sin^2 x$

$\hookrightarrow$ ~~1+P+cotx~~.

$1-P+Q \Rightarrow 1-1+\cot x - \cot x = 0$

$1-P+Q = 0 \;;\; u = e^{-x} \Rightarrow \frac{du}{dx} = -e^{-x}$

$v = u\int \frac{1}{u^2} e^{-\int P dx} dx = e^{-x}\int \frac{1}{e^{-2x}} \times e^{-\int(1-\cot x)dx} \cdot dx$

$v = e^{-x}\int \frac{1}{e^{-2x}} \times e^{-\int dx} \times e^{+\int \cot x \, dx} dx$

$\Rightarrow e^{-x}\int \frac{1}{e^{-2x}} \times e^{-x} \times e^{\log \sin x} dx$

$v = e^{-x}\int \frac{1}{e^{-x}} \sin x \, dx \Rightarrow e^{-x}\int e^{x} \sin x \, dx$

$= e^{-x}\left[\frac{e^x}{1+1}\{1\sin x - 1\cos x\}\right]$

$v = \frac{1}{2}[\sin x - \cos x]$

$\frac{dv}{dx} = (\cos x + \sin x]$

C.S. $\Rightarrow y = Ae^{-x} + B \times \frac{1}{2}[\sin x \cos x]$

$y = Ae^{-x} + B(\sin x - \cos x] - ②$

we know that

$\frac{u\,dA}{dx} + \frac{v\,dB}{dx} = 0 \quad -③$

$$\frac{dy}{dx}\frac{dA}{dx}+\frac{dv}{dx}\frac{dB}{dx}=R \quad -(4)$$

$$e^{-x}\cancel{\frac{dA}{dx}}+(\sin-\cos x)\frac{dB}{dx}=0 \quad -(5)$$

$$-e^{-x}\cancel{\frac{dA}{dB}}+(\cos x+\sin x)\frac{dB}{dx}=\sin^2 x \quad -(6)$$

eq. (5)+(6) $\Rightarrow$ $\quad 2\,\cancel{\sin x}\,dB = \sin^{\cancel{2}} x$

$$\boxed{\frac{dB}{dx}=\frac{1}{2}\sin x}$$

V.A.S and Integrate $\quad \int dB=\frac{1}{2}\int \sin x\,dx$

$$\boxed{B=-\frac{1}{2}\sin x+c_1}$$

How ?

put the value of $\frac{dB}{dx}=\frac{1}{2}\sin x$ in (5)

$$e^{-x}\frac{dA}{dx}+\frac{1}{2}(\sin x-\cos x)\sin x=0$$

$$e^{-x}\frac{dA}{dx}=-\frac{1}{2}\sin^2 x+\frac{1}{2}\sin x\cos x$$

$$\frac{dA}{dx}=-\frac{1}{2}e^{x}\sin^2 x+\frac{1}{2}e^{x}\sin x\cos x$$

VAS and Inte.

$$\int dA=-\frac{1}{2}\int e^{x}\sin^2 x\,dx+\frac{1}{4}\int e^{x}\sin 2x\,dx$$

$$A=-\frac{1}{2}\int e^{x}\left(1-\frac{\cos 2x}{2}\right)dx+\frac{1}{4}\int e^{x}\sin 2x\,dx+c_2$$

$$A=-\frac{1}{4}\int e^{x}dx$$

$$+\frac{1}{4}\int e^{x}\cos 2x\,dx$$

$$+\frac{1}{4}\int e^{x}\sin 2x\,dx+c_2$$

$$= -\tfrac{1}{4}e^{x} + \tfrac{1}{4}\int \frac{e^{x}}{1+4}\left(\cos x + 2\sin^{2} x\right)$$

$$\hookrightarrow + \frac{1}{4}\left[\frac{e^{x}}{1+4}\left(\sin 2x - 2\cos 2x\right)\right]$$

$$A = -\tfrac{1}{4}e^{x} + \frac{1}{20}e^{x}\left(\cos 2x + 2\sin 2x\right) + \frac{1}{20}e^{x}\left[\sin 2x - 2\cos 2x\right) + c_2$$

$$y = Ae^{-x} + B(\sin x - \cos x)$$

$$y = -\tfrac{1}{4} + \tfrac{1}{20}\left(\cos 2x + 2\sin 2x\right) + \frac{1}{20}\left(\sin^{2} x - 2\cos 2x + c_2 e^{-x}\right)$$

$$+ (\sin x - \cos x)\left(-\tfrac{1}{2}\cos x + c_1\right)$$

Que (4) $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$

$x = e^z \qquad z = \log x$

$dz = \frac{1}{x}dx \qquad x^2D^2 = D_1(D_1-1) \qquad xD = D_1$

Put in (1)

$(D(D_1-1)) + D - 1)y = e^{2z} \cdot e^{e^z}$

$(D^2-1)y = e^{2z} \cdot e^{e^z}$

A.E. $\Rightarrow m^2 - 1 = 0 \qquad m = \pm 1$

$\boxed{C.F. = c_1 \overset{u}{e^z} + c_2 e^{-z}} \rightarrow v$

$y = A(x)\, e^z + B(x)\, e^{-z}$ or $y = PI + CF$

$\boxed{u = y_1 = e^z = x}$ (A) $\qquad \boxed{v = y_2 = e^{-z} = 1/x}$ (B)

$A(x) = \int \frac{-v(x)R}{uv' - u'v}\, dz$

$A(x) = \int \frac{e^{-z} \cdot e^{2z} \cdot e^{e^z}}{-e^z e^{-z} - e^z \cdot e^{-z}}\, dz$

$A(x) = +\frac{1}{2}\int e^z \cdot e^{e^z}\, dz \Rightarrow +\frac{1}{2}\int \cancel{x} \cdot e^x \cdot \frac{1}{\cancel{x}}\, dx$

$\boxed{A(x) = +\frac{1}{2} e^x + c_3}$

$B(x) = \int \frac{u(z)R}{uv' - u'v}\, dz + c_4$

$B(x) = -\frac{1}{2}\int e^z \cdot e^{2z} \cdot e^{e^z}\, dz$

$= -\frac{1}{2}\int e^{3z} \cdot e^{e^z}\, dz$

$= -\frac{1}{2}\int x^3 \cdot e^x \cdot \frac{1}{x}\, dx$

$= -\frac{1}{2}\int \underset{(1)}{x^2} \cdot \underset{(2)}{e^x}\, dx$ ILATE By part

$$= -\tfrac{1}{2}\left[x^2e^x - 2\left\{xe^x - e^x\right\}\right]$$

$$Bx = -\tfrac{1}{2}x^2e^x + xe^x - e^x$$

formula use

$$\int uv\,dx = u\int v\,dx - \int\left[\frac{d}{dx}(u)\int v\,dx\right]dx$$

$$\boxed{P.I = Ax\,u(x) + B(x)\,vx}$$

or

$$P.I. = uy_1 + vy_2$$

$$P.I. = x\cdot\tfrac{1}{2}e^x + \tfrac{1}{x}\left[-\tfrac{1}{2}x^2e^x + xe^x - e^x\right]$$

$$= \cancel{\tfrac{1}{2}xe^x} - \cancel{\tfrac{1}{2}xe^x} + e^x - \frac{e^x}{x}$$

$$\boxed{P.I. = e^x(1 - 1/x)}$$

$$y = C.F. + P.I$$

$$\boxed{y = c_1x + c_2\tfrac{1}{x} + e^x(1 - 1/x)}$$

Important

Que-③ $\left(\dfrac{d^2y}{dx^2} - y\right) = \dfrac{2}{1+e^x}$

$A.E. = m^2 - 1 = 0 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1$

$$\boxed{C.F. = c_1e^x + c_2e^{-x}}$$

$$\boxed{P.I. = A(x)\,u(x) + B(x)\,vx} \text{ or } uy_1 + vy_2$$

$$\boxed{u = y_1 = e^x} \qquad \boxed{v = y_2 = e^{-x}}$$

$A(x) = ?$ $\quad B(x) = ?$

$$\boxed{R = \frac{2}{1+e^x}}$$

$$A(x) = -\int\frac{v(x)\times R}{uv' - u'v}\,dx$$

$$A(x) = -\int \frac{e^{-x} \cdot 2}{-2(1+e^{x})} dx \quad uv' - u'v = (e^{x} \cdot (-e^{-x}) - e^{x} e^{-x})$$

$$= -\frac{e^x}{e^x} - \frac{e^x}{e^x}$$

$$\boxed{uv' - u'v = -1 - 1 = 2}$$

$$A(x) = \frac{-\cancel{2}}{-\cancel{2}} \int \frac{1}{e^x(1+e^x)} dx \Rightarrow \int \frac{1}{e^x(1+e^x)} dx$$

Partial fraction

$$A(x) \int \frac{1}{e^x(1+e^x)} = \int \left(\frac{A}{e^x} + \frac{B}{1+e^x} \right) dx \quad — (1)$$

$$\frac{1}{e^x(1+e^x)} = \frac{A(1+e^x) + Be^x}{e^x(1+e^x)}$$

$$1 = A(1+e^x) + Be^x$$

$$1 = A + Ae^x + Be^x$$

$$1 = A + e^x(A+B)$$

$$\boxed{A = 1} \quad A + B = 0 \Rightarrow 1 + B = 0$$

$$\boxed{B = -1}$$

put A and B in eq - (1)

$$A(x) = \int \left(\frac{1}{e^x} + \frac{(-1)}{1+e^x} \right) dx \Rightarrow \int \left(\frac{1+e^x}{e^x(1+e^x)} \right.$$

$$A(x) = \int e^{-x} dx - \int \frac{e^{-x}}{e^{-x}+1} dx$$

$$A(x) = -e^{-x} + \int \frac{dt}{t} \qquad \text{let } e^{-x} + 1 = t$$

$$-e^{-x} dx = dt$$

$$A(x) = -e^{-x} + \log t \qquad e^{-x} dx = -dt$$

$$A(x) = -e^{x} + \log(1+e^{-x})$$

$$\boxed{A(x) = -e^{-x} + \log(1+e^{-x})}$$

$$B(x) = \int \frac{u(x) \times R}{uv' - u'v} dx$$

$$B(x) = \int \frac{e^x + \cancel{2}}{-\cancel{2}\,(1+e^x)}\,dx$$

$$= -\int \frac{e^x}{1+e^x}\,dx \quad \text{let } 1+e^x = t$$
$$e^x\,dx = dt$$

$$B(x) = -\int \frac{dt}{t} = -\log t = -\log(1+e^x)$$

$$\boxed{B(x) = -\log(1+e^x)}$$

$$P.I. = A(x)\,u(x) + B(x)\,V(x)$$

$$P.I. = \left[-e^{-x} + \log(1+e^{-x})\right]e^x + e^{-x}\left[-\log(1+e^x)\right]$$

$$\boxed{P.I. = -1 + e^x \log(1+e^{-x}) - e^{-x}\log(1+e^x)}$$

$$y = C.F. + P.I$$

$$\boxed{y = c_1 e^x + c_2 e^{-x} - 1 + e^x \log(1+e^{-x}) - e^{-x}\log(1+e^x)}$$

$$y = c_1 e^x + c_2 e^{-x} + e^x \log\left(\frac{1+e^x}{e^x}\right) - e^{-x}\log(1+e^x) - 1$$

Answer

Que-① $\dfrac{d^2y}{dx^2} + y = \operatorname{cosec} x$ — ①

Here $P = 0$ $Q = 1$ $R = \operatorname{cosec} x$

$y = C.f. + P.I$ RHS $= P.I. = 0$

$A.E. = m^2 + 1 = 0$

$m = \pm i$

$\boxed{C.f. = c_1 \cos x + c_2 \sin x}$ $\rightarrow B(x)$

$A(x)$ $u(x)$ $v(x)$

$u = y_1 = \cos x$ $v = y_2 = \sin x$

$\boxed{P.I. = A(x)\,u(x) + B(x)\,v(x)}$

$$A(x) = \int \frac{-v \cdot R}{uv' - u'v}\,dx$$

$$B(x) = \int \frac{u \cdot R}{uv' - u'v}\,dx$$

$\therefore uv' - u'v$

$\Rightarrow \cos x \cdot \cos x + \sin x \sin x$

$\Rightarrow \cos^2 x + \sin^2 x = 1$

$$\boxed{uv' - u'v = 1}$$

$$B(x) = \int \frac{\cos x \cdot \operatorname{cosec} x}{1}\,dx$$

$$B(x) = \int \frac{\cos x}{\sin x}\,dx$$

$$B(x) = \int \cot x\,dx$$

$$\boxed{B(x) = \log|\sin x|}$$

$$A(x) = \int \frac{-\sin x \operatorname{cosec} x}{1}\,dx$$

$$A(x) = -\int \frac{\cancel{\sin x}}{\cancel{\sin x}}\,dx$$

$$\boxed{A(x) = -x}$$

why not have coefficient C after integration

$$\text{P.I.} = A(x)\,u(x) + B(x)\,v(x)$$

$$\boxed{\text{P.I.} = -x\cos x + \sin x(\log \sin x)}$$

Complete solution —

$y = \text{C.F.} + \text{P.I.}$

$y = c_1 \cos x + c_2 \sin x - x\cos x + \sin x \log|\sin x|$

<u>Que. Extra</u> $\dfrac{d^2y}{dx^2} + y = \tan x$

$\text{A.E.} = m^2 + 1 = 0 \Rightarrow m^2 = -1 = i^2$

$$\boxed{\text{AE} = m = \pm i}$$

$$\boxed{\text{C.F.} = c_1 \cos x + c_2 \sin x}$$

$A(x) = c_1 \qquad u = \cos x$

$B(x) = c_2 \qquad v = \sin x$

$$\boxed{P.I. = A(x)\, u(x) + B(x)\, v(x)}$$

$$P.I. = A(x)\cos x + B(x)\sin x$$

$$A(x) = \int \frac{-v(x)\cdot R}{uv' - u'v}\,dx$$

$\therefore uv' - u'v$

$\Rightarrow \cos x \cos x + \sin x \cdot \sin x$

$\Rightarrow \cos^2 x + \sin^2 x = 1$

$$\boxed{uv' - u'v = 1}$$

$$A(x) = -\int \frac{\sin x \cdot \tan x}{1}\,dx$$

$$A(x) = -\int \sin x \times \frac{\sin x}{\cos x}\,dx$$

$$A(x) = -\int \frac{\sin^2 x}{\cos x}\,dx$$

$$A(x) = -\int \frac{1 - \cos^2 x}{\cos x}\,dx$$

$$= \int \frac{-1}{\cos x} + \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}}\,dx$$

$$A(x) = \int \cos x - \sec x\,dx$$

$$\boxed{A(x) = \sin x - \log|\sec x + \tan x|}$$

$$B(x) = \int \cos x \cdot \tan x\,dx$$

$$B(x) = \int \sin x\,dx$$

$$\boxed{Bx = -\cos x}$$

$$P.I. = A(x)\,u(x) + B(x)\,v(x)$$

$$P.I. = (\sin x - \log|\sec x + \tan x|)\cos x + (-\cos x)\sin x$$

C.S $\Rightarrow$

$$y = C.F. + P.I$$

$$\boxed{y = c_1 \cos x + c_2 \sin x + (\sin x - \log|\sec x + \tan x|)\cos x - \cos x \sin x}$$

Que-8 $\frac{d^2y}{dx^2} + y = \sec x$

A.E $\Rightarrow m^2 + 1 = 0 \qquad m = \pm i$

$$\boxed{C.F. = c_1 \cos x + c_2 \sin x}$$

$A(x) = c_1$, $B(x) = c_2$ | $u = \cos x$, $v = \sin x$

$\Rightarrow$ $P.I. = A(x)u(x) + B(x)v(x)$

$P.I = A(x)\cos x + B(x)\sin x$

$\Rightarrow$ $A(x) = \int \frac{-v \cdot R}{uv' - u'v} \times dx \rightarrow (1)$

$= \int \frac{-\sin x \cdot \sec x}{1} dx$

$\Rightarrow$ $A(x) = -\int \frac{\sin x}{\cos x} dx$

$A(x) = -\int \tan x \, dx$

$$\boxed{A(x) = -\log|\sec x|}$$

$B(x) = \int \frac{uR}{uv' - u'v} \times dx$

$= \int \frac{\cos x \cdot \sec x}{1} dx$

$B(x) = \int \frac{\cos x}{\cos x} dx$

$$\boxed{B(x) = x}$$

$\Rightarrow$ $P.I. = A(x)u(x) + B(x)v(x)$

$P.I = [-\log(\sec x)]\cos x + x(\sin x)$

$\Rightarrow$ C.S. $\Rightarrow$ $y = C.f. + P.I$

$$\boxed{y = c_1\cos x + c_2 \sin x + (-\log(\sec x))\cos x + x(\sin x)}$$

Que-⑦ $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$

A.E. $= m^2 - 2m = 0$

$\Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{2 \pm \sqrt{4 - 4 \times 0}}{2} \Rightarrow \frac{2 \pm 2}{2}$

$m = 1+1 = 2$ $\boxed{m = 0, 2}$

$m = 1-1 = 0$

$C.F. = c_1 e^{0x} + c_2 e^{2x}$

$A(x) = c_1$ | $u = e^0 x = 1$

$B(x) = c_2$ | $v = e^{2x}$

$P.I. = A(x)\, u(x) + B(x)\, v(x)$

$P.I. = A(x)(1) + B(x)\, e^{2x}$

$\rightarrow A(x) = \int \frac{-v \cdot R}{uv' - u'v}\, dx$

नहीं करना

Normaly solve करना है।

$P.I. = \frac{1}{D^2 - 2D} \cdot e^x \cdot \sin x$

$P.I. = e^x \frac{1}{[(D+1)^2 - 2(D+1)]} \times \sin x$

$P.I. = e^x \cdot \frac{1}{[D^2 + 1 + \cancel{2D} - \cancel{2D} - 2]} \sin x$

$P.I. = e^x \cdot \frac{1}{(D^2 - 1)} \sin x$

$= e^x \frac{}{(-1^2 - 1)} \sin x$

$P.I. = e^x \frac{}{-2} \sin x$

$\boxed{P.I. = -\frac{1}{2} e^x \sin x}$

General solution is —

$y = C.F. + P.I.$ $\Rightarrow \boxed{y = c_1 + c_2 e^{2x} - \frac{1}{2} e^x \sin x}$

Que-5 $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(x+1)y = x^3$

Sol^n- $\frac{d^2y}{dx^2} - \frac{2\cancel{x}(1+x)}{x^{\cancel{2}}}\frac{dy}{dx} + \frac{2(x+1)}{x^2}y = \frac{x^{\cancel{3}}}{\cancel{x^2}}$

$$\frac{d^2y}{dx^2} - 2\left(\frac{1}{x}+1\right)\frac{dy}{dx} + 2\left(\frac{1}{x}+\frac{1}{x^2}\right)y = x \quad -(1)$$

$\therefore$ $\underline{P = -2\left(\frac{1}{x}+1\right)}$, $Q = 2\underline{\left(\frac{1}{x}+\frac{1}{x^2}\right)}$

$\Rightarrow$ $\boxed{R = x}$

$\therefore$ $P + Qx = -2\left(\frac{1}{x}+1\right) + 2\left(\frac{1}{x}+\frac{1}{x^2}\right)x$

$= -2\left(\frac{1}{x}+1\right) + 2\left(\frac{\cancel{x}}{\cancel{x}}+\frac{\cancel{x}}{x^2}\right)$

$P + Qx = -2\left(\cancel{\frac{1}{x}}+1\right) + 2\left(1+\cancel{\frac{1}{x}}\right)$

$\boxed{P + Qx = 0}$ $y = x$ is part of C.F.

to find C.F of eq (1) is -

$$\frac{d^2y}{dx^2} - 2\left(\frac{1}{x}+1\right)\frac{dy}{dx} + 2\left(\frac{1}{x}+\frac{1}{x^2}\right)y = 0 \quad -(2)$$

put $\boxed{y = u \cdot v}$ $\quad \boxed{u_1 = x}$

$y = x \cdot v$

$$\boxed{\frac{dy}{dx} = v \cdot 1 + x \cdot \frac{dv}{dx}}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(v + x \cdot \frac{dv}{dx}\right)$$

$$\frac{d^2y}{dx^2} = \frac{dv}{dx} + \frac{d}{dx}\, x \cdot \frac{dv}{dx}$$

(1) (2)

$$\Rightarrow \frac{dv}{dx} + \left[x \cdot \frac{d^2v}{dx^2} + \frac{dv}{dx} \cdot 1\right]$$

$$\boxed{\frac{d^2y}{dx^2} = x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}}$$

put $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ in eq-(2)

$$\Rightarrow \left(x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}\right) - 2\left(\frac{1}{x}+1\right)\left(v + x\frac{dv}{dx}\right) + 2\left(\frac{1}{x} + \frac{1}{x^2}\right)vx = 0$$

$$\Rightarrow x\frac{d^2v}{dx^2} + 2\frac{dv}{dx} - 2\left(\frac{1}{x}+1\right)v - 2x\left(\frac{1}{x}+1\right)\frac{dv}{dx} + 2 \cdot \frac{1}{\cancel{x}} v \cancel{x} + 2\frac{1}{x^{\cancel{2}}} v \cancel{x} = 0$$

$\Rightarrow$ ~~$x\frac{d^2v}{dx^2} + \{2x$~~

$$\Rightarrow x\frac{d^2v}{dx^2} + \left[2 - 2x\left(\frac{1}{x}+1\right)\right]\frac{dv}{dx} + \left[-2\left(\frac{1}{x}+1\right) + 2 + \frac{2}{x}\right]v = 0$$

$$\Rightarrow x\frac{d^2v}{dx^2} + \left[2 - 2\cancel{x}\frac{1}{\cancel{x}} - 2x\right]\frac{dv}{dx} + \left[-\cancel{\frac{2}{x}} - \cancel{2} + \cancel{2} + \cancel{\frac{2}{x}}\right]v = 0$$

$$\Rightarrow x\frac{d^2v}{dx^2} + \left[\cancel{2} - \cancel{2} - 2x\right]\frac{dv}{dx} + 0 = 0$$

$$x\frac{d^2v}{dx^2} - 2x\frac{dv}{dx} = 0 \Rightarrow \frac{d^2v}{dx^2} - 2\frac{dv}{dx} = 0$$

$$\frac{d^2v}{dx^2} = 2\frac{dv}{dx} \Rightarrow \frac{\frac{d}{dx}\left(\frac{dv}{dx}\right)}{\frac{dv}{dx}} = 2$$

Integrate w.r.t x

$$\int \frac{\frac{d}{\cancel{dx}}\left(\frac{dv}{dx}\right)}{\frac{dv}{dx}} \cancel{dx} = \int 2\,dx + c_1$$

$$\int \frac{d\left(\frac{dv}{dx}\right)}{\frac{dv}{dx}} = 2\int dx + c_1'$$

$$\log\left(\frac{dv}{dx}\right) = 2x + c_1' \quad \therefore c_1' = \log c''$$

$$\log\left(\frac{dv}{dx}\right) = 2x + \log c'' = 2x$$

$$\log_e \frac{\left(\frac{dv}{dx}\right)}{c''} = 2x \Rightarrow \frac{\frac{dv}{dx}}{c''} = e^{2x}$$

$$\frac{dv}{dx} = c'' e^{2x} \Rightarrow \int \frac{dv}{\cancel{dx}} \cancel{dx} = c'' \int e^{2x}\,dx + c_2$$

$$v = c'' \cdot \frac{e^{2x}}{2} + c_2$$

let $\frac{c''}{2} = c_1$: $\boxed{v = c_1 e^{2x}}$

C.f. of eq ① is $y = v x$ $\rightarrow u = x$

$$y = (c_1 e^{2x} + c_2)\, x$$

$$\boxed{\text{C.F } y = c_1 x e^{2x} + c_2 x} \rightarrow \text{C.F}$$

$u = x e^{2x}$ $\quad v = x$

P.I. $= A(x)u(x) + B(x)v(x)$

P.I. $= A(x)\, x e^{2x} + B(x)\, x.$

$$A(x) = \int \frac{-\cancel{v} \cdot R}{\cancel{uv'-u'v}} dx$$

$$A(x) = \int \frac{-V \cdot R}{uv' - u'v} \longrightarrow (w).$$

$$w = \begin{vmatrix} u & v \\ u_1 & v_1 \end{vmatrix} = \begin{vmatrix} xe^{2x} & x \\ xe^{2x} \cdot 2 + e^{2x} \cdot 1 & 1 \end{vmatrix}$$

$$w = x \cdot e^{2x} \times \begin{bmatrix} 1 & 1 \\ 2x+1 & 1 \end{bmatrix}$$

$$w = xe^{2x} \cdot [\cancel{1} - (2x+\cancel{1})]$$

$$\boxed{w = -2x^2 e^{2x}}$$

$$A(x) = \int -\frac{VR}{w} dx = \int \frac{\cancel{-} x \cdot x}{\cancel{-} 2\cancel{x^2} \cdot e^{2x}} dx$$

$$= \frac{1}{2} \int e^{-2x} dx \Rightarrow \frac{1}{2} \frac{e^{-2x}}{-2} = -\frac{1}{4} e^{-2x}$$

$$\boxed{A(x) = -\frac{1}{4} e^{-2x}}$$

$$B(x) = \int \frac{u \cdot R}{w} dx \Rightarrow \int \frac{\cancel{x} \cdot \cancel{e^{2x}} \times \cancel{x}}{-2\cancel{x^2} \cdot \cancel{e^{2x}}} dx$$

$$B(x) = -\frac{1}{2} \int 1 \, dx$$

$$\boxed{B(x) = -\frac{1}{2} x}$$

$$P.I. = -\frac{1}{4} e^{-2x} \cdot xe^{2x} + (-\frac{1}{2}x) \cdot x$$

$$P.I. = -\frac{1}{4} e^{\cancel{-2x} + \cancel{2x}} \cdot x - \frac{1}{2} x^2 \qquad e^0 = 1$$

$$\boxed{P.I = -\frac{1}{4}x - \frac{1}{2}x^2}$$

$\boxed{y = P.I + C.F.}$ $\boxed{y = c_1 x e^{2x} + c_2 x - \frac{1}{4}x - \frac{1}{2}x^2}$ Ans

Que-(6) $\frac{d^2y}{dx^2} + (1-\cot x)\frac{dy}{dx} - \cot x . y = \sin^2 x$

Sol^n The given D.E. is

$$\frac{d^2y}{dx^2} + (1-\cot x)\frac{dy}{dx} - \cot x\, y = \sin^2 x \quad -(1)$$

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Q y = R$$

$P = (1-\cot x)$ $\quad Q = -\cot x$ $\quad R = \sin^2 x$

$1 - P + Q = 1 - (1-\cot x) + (-\cot x)$

$= \cancel{1} - \cancel{1} + \cancel{\cot x} - \cancel{\cot x}$

$\boxed{1 - P + Q = 0}$

$\boxed{u = e^{-x}}$

$\Rightarrow e^{-x}$ is a part c.f.

substitute, $y = u.v$

$y = e^{-x}.v$

$$\frac{dy}{dx} = e^{-x}\frac{dv}{dx} + v.e^{-x}.(-1) = e^{-x}\frac{dv}{dx} - v e^{-x}$$

$$\frac{d^2y}{dx^2} = \left[e^{-x}\frac{d^2v}{dx^2} + \frac{dv}{dx}.e^{-x}.(-1)\right] - \left[e^{-x}.\frac{dv}{dx} + v e^{-x}.(-1)\right]$$

$$= e^{-x}\frac{d^2v}{dx^2} - e^{-x}\frac{dv}{dx} - e^{-x}\frac{dv}{dx} + v.e^{-x}$$

$$= e^{-x}\left[\frac{d^2v}{dx^2} - \frac{dv}{dx} - \frac{dv}{dx} + v\right]$$

$$\frac{d^2y}{dx^2} = e^{-x}\left[\frac{d^2v}{dx^2} - 2\frac{dv}{dx} + v\right]$$

in eq -(1).

$$e^{-x}\left[\frac{d^2v}{dx^2} - 2\frac{dv}{dx} + v\right] + [1-\cot x).e^{-x}\left[\frac{dv}{dx} - v\right) \Rightarrow$$

$\Rightarrow -\cot x \cdot e^{x} v = \sin^2 x$

$e^{-x}\left[\frac{d^2v}{dx^2} - 2\frac{dv}{dx} + v + (1-\cot x)\left(\frac{dv}{dx} - v\right) - \cot x \cdot v\right] = \sin^2 x$

$\frac{d^2v}{dx^2} - 2\frac{dv}{dx} + v + (1-\cot x)\left(\frac{dv}{dx} - v\right) - \cot x \cdot v = \frac{\sin^2 x}{e^{-x}}$

$\frac{d^2v}{dx^2} + \left[-2 + (1-\cot x)\right]\frac{dv}{dx} + \left[1 - (1-\cot x) - \cot x\right]v = e^{x}\sin^2 x$

$\frac{d^2v}{dx^2} + \left[-2+1-\cot x\right]\frac{dv}{dx} + \left[1 - 1 + \cot x - \cot x\right]v = e^{x}\sin^2 x$

$\frac{d^2v}{dx^2} + (-1-\cot x)\frac{dv}{dx} + 0 \cdot v = e^{x}\sin^2 x$

$\frac{d}{dx}\left(\frac{dv}{dx}\right) - (1+\cot x)\left(\frac{dv}{dx}\right) = e^{x}\sin^2 x$

putting $\frac{dv}{dx} = p$

$\frac{dp}{dx} - (1+\cot x) \cdot p = e^{x}\sin^2 x$

$\boxed{\frac{dy}{dx} + Py = Q \quad , \quad I.f = e^{\int P\,dx} \\ y \times I.f = \int Q \times I.f \, dx + c}$

$I.f. = e^{\int -(1+\cot x)dx} = e^{-[\int 1 dx + \int \cot x \, dx]}$

$= e^{-[x + \log \sin x]} \Rightarrow e^{-x - \log \sin x}$

$= e^{-x} \times e^{-\log \sin x}$

$I.f. = \frac{e^{-x}}{e^{\log \sin x}} = \frac{e^{-x}}{\sin x} \Rightarrow$

$p \times I.f = \int e^{x} \cdot \sin^2 x \cdot \times I.f. \, dx + c_1$

$p \times \frac{e^{-x}}{\sin x} = \int e^{x}\sin^2 x \times \frac{e^{-x}}{\sin x} \, dx + c_1$

$\frac{dv}{dx} \times \frac{e^{-x}}{\sin x} = \int \sin x \, dx + c_1$

$$\frac{e^{-x}}{\sin x}\cdot\frac{dv}{dx} = -\cos x + c_1'$$

$$\frac{dv}{dx} = \frac{-\cos x \times \sin x}{e^{-x}} + \frac{c_1 \sin x}{e^{-x}}$$

Integrating w.r.t. x

$$v = -\int e^{x} \sin x \cos x\,dx + c_1'\int e^{x} \sin x\,dx + c_2$$

$$v = -\tfrac{1}{2}\int e^{x}\cdot(2\sin x\cos x)\,dx + c_1'\int e^{x}\sin x\,dx + c_2$$

$$v = -\tfrac{1}{2}\int e^{x}\sin 2x\,dx + c_1'\int e^{x}\sin x\,dx + c_2$$

$$\left[\because \int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}\right]$$

$$v = -\tfrac{1}{2}\left[e^{x}\frac{(1\cdot\sin 2x - 2\cos 2x)}{1^2+2^2}\right] + c_1'\left[\frac{e^{x}(1\cdot\sin x - 1\cdot\cos x)}{1^2+1^2}\right] + c_2$$

$$v = -\tfrac{1}{2}\left[\frac{e^{x}(\sin 2x - 2\cos 2x)}{1+4}\right] + c_1'\frac{e^{x}(\sin x - \cos x)}{1+1} + c_2$$

or

$$v = \frac{-1}{10}e^{x}(\sin 2x - 2\cos 2x) + \frac{c_1'}{2}e^{x}(\sin x - \cos x) + c_2$$

setting $\frac{c_1'}{2} = c_1$

$$v = \frac{-1}{10}e^{x}(\sin 2x - 2\cos 2x) + c_1 e^{x}(\sin x - \cos x) + c_2$$

Therefore, solution of given D.E. is

$$y = e^{-x}\cdot v$$

$$y = e^{-x}\left[-\frac{1}{10}e^{x}(\sin 2x - 2\cos 2x) + c_1 e^{x}(\sin x - \cos x) + c_2\right]$$

$$y = -\frac{1}{10}(\sin^2 x - 2\cos 2x) + c_1(\sin x - \cos x) + c_2 e^{-x}$$

$$\frac{d^2y}{dx^2} + n^2y = \sec nx$$

$$(D^2+n^2)y = \sec nx$$

A.E $= m^2+n^2 = 0$

$m = -n^2 = \sqrt{-n^2} = \sqrt{n^2 i^2}$

$m = \pm ni$

$$\boxed{C.F. = c_1 \cos nx + c_2 \sin nx}$$

$A(x) = c_1 \qquad B(x) = c_2$

$u = \cos nx \qquad v = \sin nx$

$$\boxed{P.I. = A(x)\cos nx + B(x)\cdot \sin nx}$$

$uv' - u'v \Rightarrow \cos nx\,[\sin nx]' - [\cos nx]'\sin nx$

$\Rightarrow \cos nx\,[n\cos nx - [-n\sin nx]]\,[\sin nx]$

$\Rightarrow n\cos^2 nx + n\sin^2 nx$

$\Rightarrow n(\sin^2 nx + \cos^2 nx)$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow n(1) \qquad \boxed{uv' - u'v = n}$

~~$\Rightarrow n \cos^2 nx$~~

$$A = \int \frac{-VER(x)}{uv' - u'v}\,dx$$

$$= \int \frac{-\sin nx\,(\sec nx)}{n}\,dx$$

$$= -\frac{1}{n}\int \sin nx\left(\frac{1}{\cos nx}\right)dx$$

$$= -\frac{1}{n}\int \frac{\sin nx}{\cos nx}\,dx$$

$$= -\frac{1}{n}\int \tan nx\,dx$$

$\therefore \frac{1}{a}\log \sec ax$

or

$-\frac{1}{a}\log \cos ax$

$$A(x) = -\frac{1}{n}\left(-\frac{1}{n}\log \cos nx\right)$$

$$\boxed{A x = \frac{1}{n^2}\log \cos nx}$$

$$B(x) = \int \frac{y_1 R(x)}{y_1 y_2' - y_1' y_2}\,dx \Rightarrow \int \frac{\cos nx\,(\sec nx)}{n}\,dx$$

$$B(x) = \frac{1}{n}\int \frac{\cancel{\cos nx}}{\cancel{\cos nx}}\,dx$$

$$B(x) = \frac{1}{n}\int dx \Rightarrow \frac{1}{n}x$$

$$B(x) = \frac{x}{n}$$

$$\cancel{P.I.} = \cancel{A(x)\cos x}$$

$$P.I. = A(x)\,y_1(x) + B(x)\,y_2(x)$$

$$P.I. = \left[\frac{1}{n^2}\log(\cos nx)\right]\cos nx + \left(\frac{x}{n}\right)\sin nx$$

$$P.I. = \frac{1}{n^2}\cos nx \log(\cos nx) + \frac{x}{n}\sin nx$$

$$Y = P.I. + C.F.$$

$$\boxed{Y = c_1 \cos nx + c_2 \sin nx + \frac{1}{n^2}\cos nx \log(\cos nx) + \frac{1}{n}x \sin nx}$$