10.7.12

AI25BTECH11003 - Bhavesh Gaikwad

Question: On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are

(1999)

(a) $(\frac{2}{5}, \frac{1}{5})$
(b) $(\frac{-2}{5}, \frac{1}{5})$
(c) $(\frac{-2}{5}, \frac{-1}{5})$
(d) $(\frac{2}{5}, \frac{-1}{5})$

Solution:
Given: Ellipse: $4x^2 + 9y^2 = 1$ & Line: $8x - 9y = 0$

Parameters of Ellipse:

$$\mathbf{V} = \begin{pmatrix} 4/9 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, e = \frac{\sqrt{5}}{3}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, f = \frac{-1}{9} \tag{0.1}$$

Equation of Ellipse:

$$\mathbf{X}^\top \begin{pmatrix} 4/9 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{X} = \frac{1}{9} \tag{0.2}$$

Parameters of Given Line:

$$\mathbf{m} = \begin{pmatrix} 8 \\ -9 \end{pmatrix}, \mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.3}$$

Equation of Given Line:

$$L : \mathbf{X} = k\mathbf{m} \text{ OR } L : \mathbf{X} = k \begin{pmatrix} 8 \\ -9 \end{pmatrix} \tag{0.4}$$

Since, Tangents are parallel to L,

$$\therefore \text{The normal vector to the tangents is } \mathbf{n}_2 = \begin{pmatrix} 9 \\ 8 \end{pmatrix} \tag{0.5}$$

Let $\mathbf{q}_i$ be the points of contact. i =1,2.

$$\mathbf{q}_i = \mathbf{V}^{-1}(k_i \mathbf{n}_2 - u) \qquad \text{where, } k_i = \pm \sqrt{\frac{f_o}{\mathbf{n}_2^\top \mathbf{V}^{-1} \mathbf{n}_2}} \tag{0.6}$$

$$f_o = \mathbf{u}^\top \mathbf{V}^{-1} \mathbf{u} - f = 1/9 \tag{0.7}$$

From Equation 0.6,

$$\mathbf{q}_1 = \begin{pmatrix} \frac{-2}{5} \\ \frac{1}{5} \end{pmatrix} \qquad \& \qquad \mathbf{q}_2 = \begin{pmatrix} \frac{2}{5} \\ \frac{-1}{5} \end{pmatrix} \tag{0.8}$$

$$\boxed{\text{Thus, Option B and D are correct.}}$$

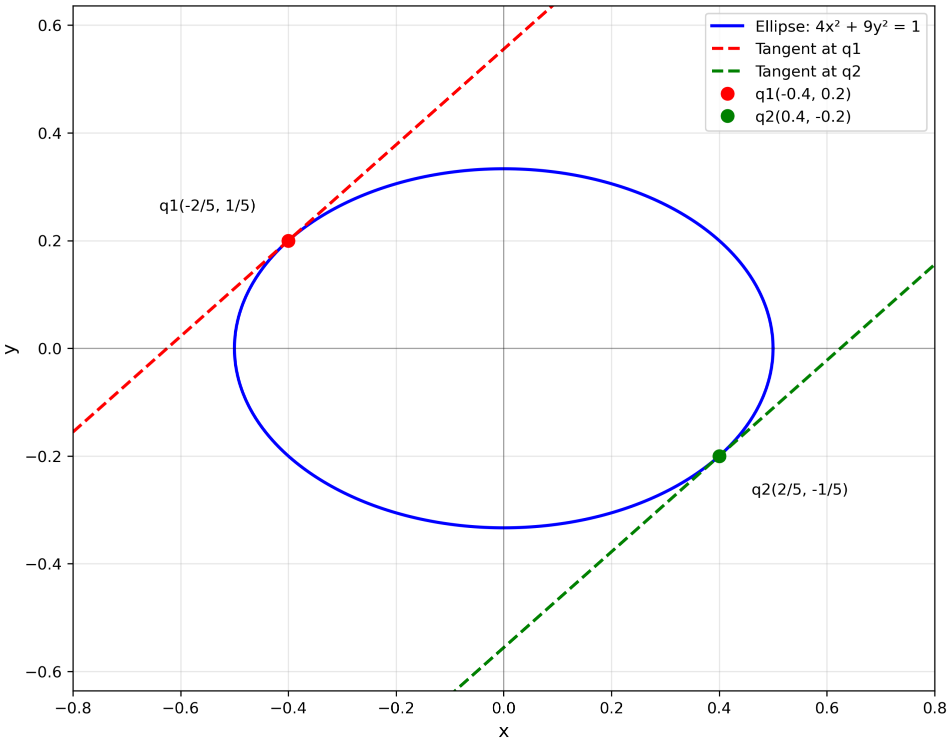

Fig. 0.1