

5.5.20

AI25BTECH11003 - Bhavesh Gaikwad

Question: Using elementary row transformations, find the inverse of the matrix

$$\begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

Solution:

$$\text{Let } \mathbf{A} = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

Augment the matrix \mathbf{A} with the identity

$$[\mathbf{A} | \mathbf{I}] = \left(\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \quad (0.1)$$

Row Transformation-1: $R_1 \rightarrow \frac{R_1}{3}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \quad (0.2)$$

Row Transformation-2: $R_2 \rightarrow R_2 - 2R_1$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 3 & \frac{2}{3} & -\frac{2}{3} & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \quad (0.3)$$

Row Transformation-3: $R_2 \rightarrow \frac{R_2}{3}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array} \right) \quad (0.4)$$

Row Transformations 4 and 5: Replace $R_1 \rightarrow R_1 + \frac{1}{3}R_2$ And $R_3 \rightarrow R_3 - 4R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{7}{27} & \frac{7}{27} & \frac{1}{9} & 0 \\ 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 8 & -4 & 9 \end{array} \right) \quad (0.5)$$

Row Transformation-6: $R_3 \rightarrow 9R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{7}{27} & \frac{7}{27} & \frac{1}{9} & 0 \\ 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 8 & -4 & 9 \end{array} \right) \quad (0.6)$$

Row Transformations 7 and 8: $R_1 \rightarrow R_1 + \frac{7}{27}R_3$ And $R_2 \rightarrow R_2 - \frac{2}{9}R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{3} & -\frac{25}{27} & \frac{7}{3} \\ 0 & 1 & 0 & -2 & \frac{11}{9} & -2 \\ 0 & 0 & 1 & 8 & -4 & 9 \end{array} \right) \quad (0.7)$$

The Inverse Matrix of **A**:

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{7}{3} & -\frac{25}{27} & \frac{7}{3} \\ -2 & \frac{11}{9} & -2 \\ 8 & -4 & 9 \end{pmatrix} \quad (0.8)$$