5.11.2

AI25BTECH11003 - Bhavesh Gaikwad

Question: Determine the current in each branch of the network shown in Fig.01

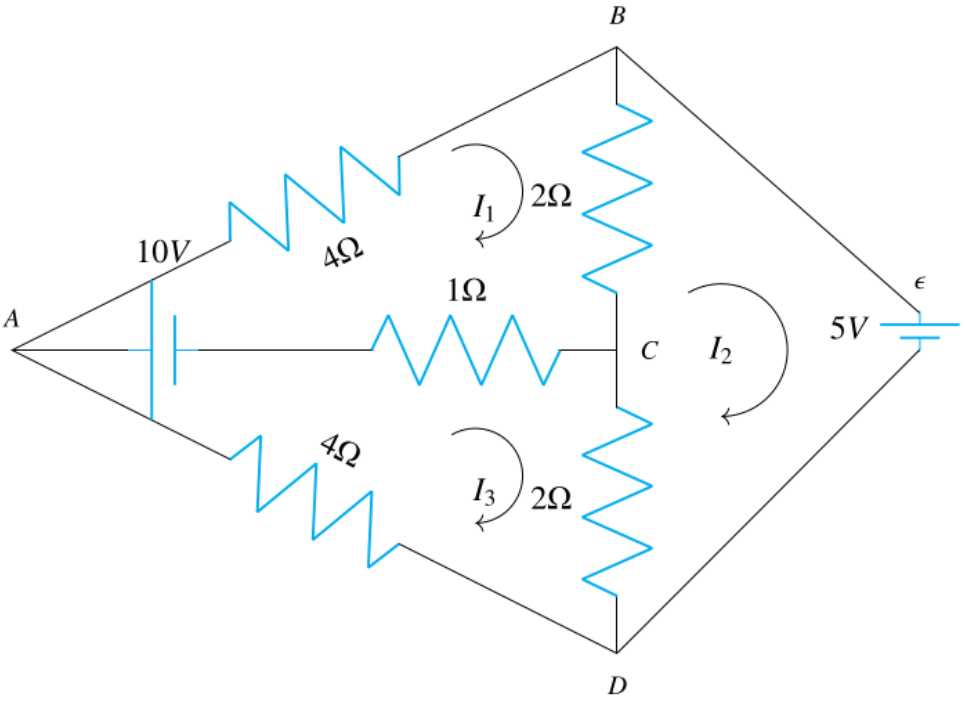

Fig. 0.1

Solution:

Using mesh current analysis with Kirchhoff's Voltage Law (KVL) to solve the question.

Applying KVL to the loop ABCA,

$$-7I_1 + 2I_2 + I_3 = -10 \tag{0.1}$$

Applying KVL to the loop CBDC,

$$2I_1 - 4I_4 + 2I_3 = -5 \tag{0.2}$$

Applying KVL to the loop ACDA,

$$-I_1 - 2I_2 + 7I_3 = 10 \tag{0.3}$$

Therefore, the Three equations are:

$$-7I_1 + 2I_2 + I_3 = -10$$

$$2I_1 - 4I_4 + 2I_3 = -5$$

$$-I_1 - 2I_2 + 7I_3 = 10$$

Let $\mathbf{M} = \begin{pmatrix} -7 & 2 & 1 \\ 2 & -4 & 2 \\ -1 & -2 & 7 \end{pmatrix}$ and $\mathbf{x} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$ and $\mathbf{T} = \begin{pmatrix} -10 \\ -5 \\ 10 \end{pmatrix}$

$$\therefore \mathbf{Mx} = \mathbf{T} \tag{0.4}$$

OR

$$\begin{pmatrix} -7 & 2 & 1 \\ 2 & -4 & 2 \\ -1 & -2 & 7 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -10 \\ -5 \\ 10 \end{pmatrix} \tag{0.5}$$

The Augmented Matrix:

$$\left(\begin{array}{ccc|c} -7 & 2 & 1 & -10 \\ 2 & -4 & 2 & -5 \\ -1 & -2 & 7 & 10 \end{array}\right) \tag{0.6}$$

Row Transformation-1: $R_3 \rightarrow R_3 + R_1$

$$\left(\begin{array}{ccc|c} -7 & 2 & 1 & -10 \\ 2 & -4 & 2 & -5 \\ -8 & 0 & 8 & 0 \end{array}\right) \tag{0.7}$$

Row Transformation-2: $R_2 \rightarrow R_2 + \frac{R_3}{4}$

$$\left(\begin{array}{ccc|c} -7 & 2 & 1 & -10 \\ 0 & -4 & 4 & -5 \\ -8 & 0 & 8 & 0 \end{array}\right) \tag{0.8}$$

Row Transformation-3: $R_3 \rightarrow R_3 - \frac{8R_1}{7} - \frac{4R_1}{7}$

$$\left(\begin{array}{ccc|c} -7 & 2 & 1 & -10 \\ 0 & -4 & 4 & -5 \\ 0 & 0 & 32/7 & -60/7 \end{array}\right) \tag{0.9}$$

$$\begin{pmatrix} -7 & 2 & 1 \\ 0 & -4 & 4 \\ 0 & 0 & 32/7 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} -10 \\ -5 \\ -60/7 \end{pmatrix} \tag{0.10}$$

$$\boxed{\therefore I_1 = 55/56A,\ I_2 = 5/8A,\ I_3 = 15/8} \tag{0.11}$$