4.7.64

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Find the distance between the point **P**(6, 5, 9) and the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6).

Theoretical Solution

Let the Equation of the Plane be $\mathbf{n}^\top \mathbf{x} = 1$.

Since, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ lie on the Plane.

$$\therefore \; \mathbf{n}^\top \mathbf{A} = 1, \; \mathbf{n}^\top \mathbf{B} = 1, \; \mathbf{n}^\top \mathbf{C} = 1 \tag{1}$$

OR

$$\mathbf{A}^\top \mathbf{n} = 1, \; \mathbf{B}^\top \mathbf{n} = 1, \; \mathbf{C}^\top \mathbf{n} = 1 \tag{2}$$

Let $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$

From Equation 2,

$$(\mathbf{A} \, \mathbf{B} \, \mathbf{C})^\top \mathbf{n} = 1 \tag{3}$$

$$\begin{pmatrix} 3 & -1 & 2 \\ 5 & 2 & 4 \\ -1 & -1 & 6 \end{pmatrix} \mathbf{n} = 1 \tag{4}$$

From Equation 4,

$$\therefore \text{ The Equation of the Plane is: } \frac{1}{19} \begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \mathbf{x} = 1 \tag{5}$$

The Distance of Point from a Plane is given by the formula:

$$d = \frac{\left\| \mathbf{n}^\top \mathbf{Q} - 1 \right\|}{\|\mathbf{n}\|} \tag{6}$$

Theoretical Solution

Let d be the distance between Point $\mathbf{P}$ and the Plane.

$$\text{Then, } d = \frac{\left\| \mathbf{n}^\top \mathbf{P} - 1 \right\|}{\|\mathbf{n}\|} \tag{7}$$

$$d = \frac{\left\| \frac{18 - 20 + 27}{19} - 1 \right\|}{\left(\frac{\sqrt{34}}{19} \right)} \tag{8}$$

$$\therefore \ d = \frac{3\sqrt{34}}{17} \tag{9}$$

The Distance between the Plane and $\mathbf{P}$ is $\dfrac{3\sqrt{34}}{17}$ units. $\tag{10}$

Plane

Distance between Point and Plane