9.2.37

AI25BTECH11003 - Bhavesh Gaikwad

Question: Find the area of the region bounded by the curve $x^2 = y$ and the lines $y = x + 2$ and the x-axis.

Solution:

Given: $y = x^2$ and $y = x + 2$

The General Equation of a Conic is:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

On comparing, we get:

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}, f = 0 \tag{0.2}$$

The General Equation of a Line:

$$\mathbf{x} = k\mathbf{m} + \mathbf{h} \tag{0.3}$$

On comparing, we get:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{h} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.4}$$

The Intersection of the given conic with the given line can be written as:

$$\mathbf{x}_i = \mathbf{h} + k_i \mathbf{m} \tag{0.5}$$

$$where,\ k_i = \left(\frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}}\right)\left(-\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(h)(\mathbf{m}^\top \mathbf{V}\mathbf{m})}\right) \tag{0.6}$$

Let $\mathbf{K} = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$

The Solution Matrix can be expressed as:

$$\mathbf{X} = \begin{pmatrix} \mathbf{h} & \mathbf{m} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{k} \end{pmatrix}^\top \tag{0.7}$$

Therefore, The points of intersection are:

$$\mathbf{x}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \& \quad \mathbf{x}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{0.8}$$

From Fig.0.1, the area bounded by the curve $y = x^2$ and line $y = x + 2$ is given by:

$$\int_{-1}^{2} [(x + 2) - (x^2)]\, dx = \int_{-1}^{2} [2 + x - x^2]\, dx \tag{0.9}$$

$$\int_{-1}^{2} [2 + x - x^2]\, dx = \frac{9}{2} = 4.5\, sq.units \tag{0.10}$$

Therefore, The Area of the region bounded between $y = x^2$ and $y = x + 2$ is 4.5 sq.units.

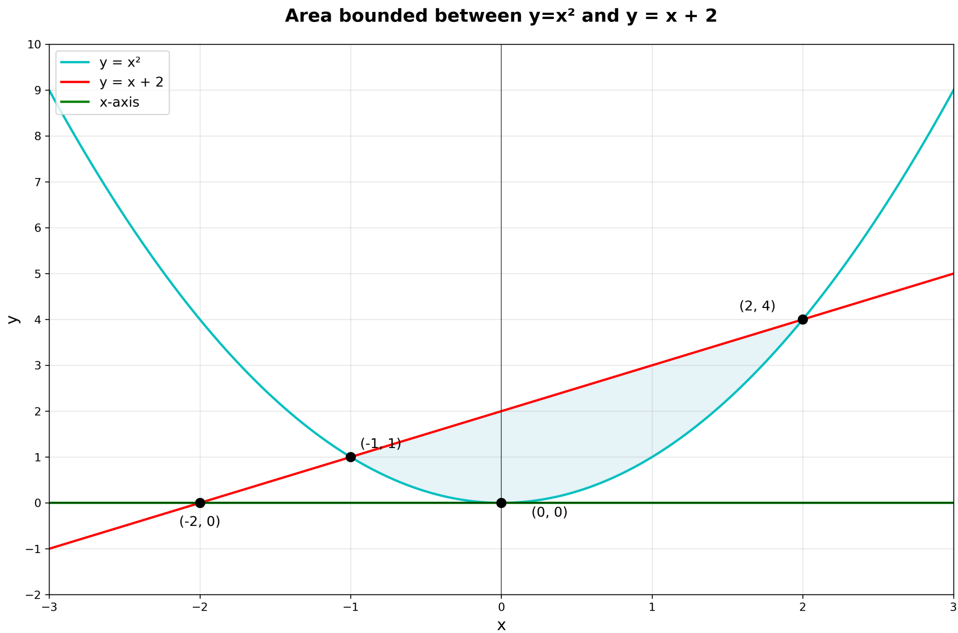

Fig. 0.1: Intersection of Conic and Line