12.705

AI25BTECH11003 - Bhavesh Gaikwad

Question: The minimum value of y for the equation $y = x^2 - 2x + 4$ is

(MT 2021)

a) 3
b) 1
c) 4
d) 6

Solution:
Given:

$$Parabola:\ x^2 - 2x - y + 4 = 0 \tag{0.1}$$

Parameters of the Parabola:

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -1 \\ -1/2 \end{pmatrix}, \quad f = 4 \tag{0.2}$$

Equation of Parabola:

$$\mathbf{X}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{X} + 2 \begin{pmatrix} -1 & -1/2 \end{pmatrix} \mathbf{X} + 4 = 0 \tag{0.3}$$

Let line L be parallel to the x-axis and passes through y_{min}.
Let ϕ represent the minimum value of y.

$$\therefore\ L:\ \mathbf{X} = k \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \phi \end{pmatrix} \tag{0.4}$$

Parameters of Line L:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{h} = \begin{pmatrix} 0 \\ \phi \end{pmatrix} \tag{0.5}$$

Intersection of L with Parabola:

$$\mathbf{x}_i = k_i \mathbf{m} + \mathbf{h} \tag{0.6}$$

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \tag{0.7}$$

Since it is an opening upward parabola, therefore only one possible value of y_{min} can occur.
Thus, only one value of k

$$\therefore \ [\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top\mathbf{V}\mathbf{m}) = 0 \tag{0.8}$$

$$(-1)^2 - (4-\phi)(1) = 0 \tag{0.9}$$

$$\Rightarrow \boxed{\phi = 3} \tag{0.10}$$

$$y_{min} = \phi = 3 \tag{0.11}$$

$$\boxed{\text{Option-A is Correct.}}$$

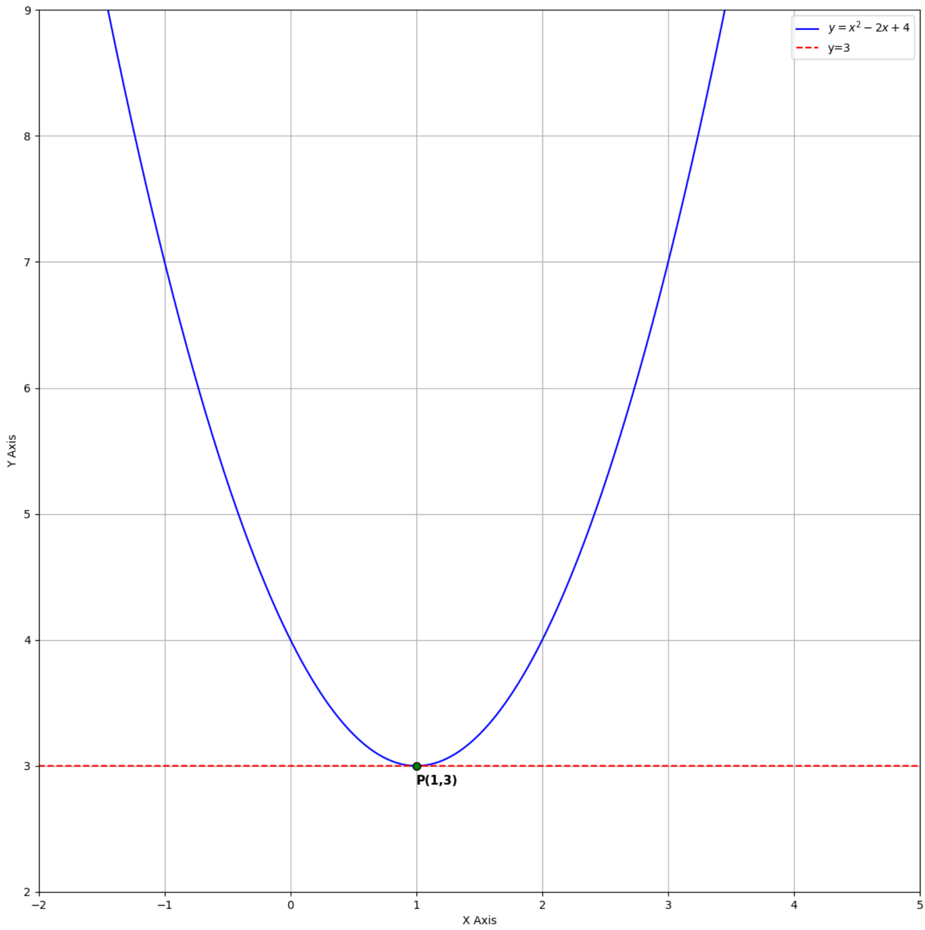

Fig. 0.1: Parabola and Line