

12.601

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Question

The matrix $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$, one of the eigen values is 1. The eigen vectors corresponding to the eigen value 1 are: (CS 2016)

- a) $\alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix}, \alpha \neq 0, \alpha \in \mathbb{R}$
- b) $\alpha \begin{pmatrix} -4 & 2 & 1 \end{pmatrix}, \alpha \neq 0, \alpha \in \mathbb{R}$
- c) $\alpha \begin{pmatrix} -2 & 0 & 1 \end{pmatrix}, \alpha \neq 0, \alpha \in \mathbb{R}$
- d) $\alpha \begin{pmatrix} 2 & 0 & 1 \end{pmatrix}, \alpha \neq 0, \alpha \in \mathbb{R}$

Theoretical Solution

Given: $\lambda = 1$, Let $\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

Let \mathbf{v} be the corresponding eigenvector.

$$\Rightarrow \mathbf{A}\mathbf{v} = (1)\mathbf{v} \quad (1)$$

$$(\mathbf{A} - \mathbf{I})\mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \quad (3)$$

Let $\mathbf{v} = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$

Theoretical Solution

Substituting value of \mathbf{v} in Equation 3,

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \quad (4)$$

$$\text{Row} - 1 \rightarrow v_2 + 2v_3 = 0 \quad (5)$$

$$\text{Row} - 2 \rightarrow 0 + 0 + 0 = 0 \text{ (Always true)} \quad (6)$$

$$\text{Row} - 3 \rightarrow v_1 + 2v_2 = 0 \quad (7)$$

Let $v_3 = \alpha$ (Free parameter)

Substituting value of v_3 in Equations 5 and 7

$$\therefore v_2 = -2\alpha \text{ \& } v_1 = 4\alpha \quad (8)$$

$$\therefore \mathbf{v} = \alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix} \quad (9)$$

Thus, Option-A is correct.