4.12.8

AI25BTECH11003 - Bhavesh Gaikwad

Question: Distance of the point (α, β, γ) from y-axis is
a) β
b) $|\beta|$
c) $|\beta + \gamma|$
d) $\sqrt{\alpha^2 + \gamma^2}$

Solution:
Let $\mathbf{A} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$
Let $\mathbf{B}$ be the point on y-axis which is nearest to $\mathbf{A}$.

$$\text{Equation of y-axis: } \mathbf{r} = \lambda \mathbf{e_2}, \textit{ where } e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{0.1}$$

Since point $\mathbf{B}$ satisfies the equation,

$$\mathbf{B} = \lambda \mathbf{e_2} \tag{0.2}$$

$(\mathbf{B} - \mathbf{A})$ is perpendicular to the y-axis as its the shortest distance between a line and a point $\mathbf{A}$ or $\mathbf{B} - \mathbf{A}$ is perpendicular to $\mathbf{B}$ as it lies on the y-axis itself.

$$\therefore\ (\mathbf{B} - \mathbf{A})^\top \mathbf{B} = 0 \tag{0.3}$$

$$\mathbf{A}^\top \mathbf{B} - \|\mathbf{B}\|^2 = 0 \implies \mathbf{A}^\top \mathbf{B} = \|\mathbf{B}\|^2 \tag{0.4}$$

From Equations 0.2 and 0.4,

$$\mathbf{A}^\top (\lambda \mathbf{e_2}) = \lambda^2 \tag{0.5}$$

$$\therefore\ \lambda = \mathbf{e_2}^\top \mathbf{A} \quad \Rightarrow \lambda = \beta \tag{0.6}$$

From Equation 0.6,

$$\mathbf{B} = \beta \mathbf{e_2} \textit{ OR } \mathbf{B} = \beta \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{0.7}$$

Let d be the shortest distance between the y-axis and $\mathbf{A}$.

$$d = \|\mathbf{B} - \mathbf{A}\| = \sqrt{(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A})} \tag{0.8}$$

$$\therefore\ d = \sqrt{\alpha^2 + \gamma^2} \tag{0.9}$$

$$\boxed{\text{Therefore, Option D is Correct.}}$$