

12.497

AI25BTECH11003 - Bhavesh Gaikwad

Question: Consider the following simultaneous equations (with c_1 and c_2 being constants)

$$3x_1 + 2x_2 = c_1$$

$$4x_1 + x_2 = c_2$$

The characteristic equation for these simultaneous equations is

(CE 2017)

$$(a) \lambda^2 - 4\lambda - 5 = 0$$

$$(b) \lambda^2 - 4\lambda + 5 = 0$$

$$(c) \lambda^2 + 4\lambda - 5 = 0$$

$$(d) \lambda^2 + 4\lambda + 5 = 0$$

Solution:

Given:

$$3x_1 + 2x_2 = c_1 \quad (0.1)$$

$$4x_1 + x_2 = c_2 \quad (0.2)$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \text{ and } \mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

From Equation 0.1 and 0.2,

$$\mathbf{Ax} = \mathbf{c} \quad (0.3)$$

The characteristic equation is given by,

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \quad (0.4)$$

$$\left| \begin{pmatrix} 3 - \lambda & 2 \\ 4 & 1 - \lambda \end{pmatrix} \right| = 0 \quad (0.5)$$

$$(0.6)$$

$$\boxed{\lambda^2 - 4\lambda - 5 = 0} \quad (0.7)$$

Thus, Option-A is correct.