12.289

AI25BTECH11003 - Bhavesh Gaikwad

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Question

The approximate eigenvalue of the matrix

$$\mathbf{A} = \begin{pmatrix} 15 & 4 & 3 \\ 10 & 12 & 6 \\ 20 & 4 & 2 \end{pmatrix}$$

obtained after two iterations of Power method, with the initial vector $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^\top$ is

(MA 2012)

(a) 7.768 (b)9.468 (c)10.548 (d)19.468

Theoretical Solution

Given:

$$\mathbf{A} = \begin{pmatrix} 15 & 4 & 3 \\ 10 & 12 & 6 \\ 20 & 4 & 2 \end{pmatrix}$$

$$\text{Let the initial vector be } \mathbf{x}_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\mathbf{x}_1 = \mathbf{A}\mathbf{x}_0 = \begin{pmatrix} 22 \\ 28 \\ 26 \end{pmatrix} \tag{1}$$

Let Normalization vector: $\mathbf{y}_i = \dfrac{1}{x_{max}}\mathbf{x}_i$

where, x_{max} is the largest element in $\mathbf{x}_i$

Theoretical Solution

$$\mathbf{y}_1 = \frac{1}{28}\mathbf{x}_1 \tag{2}$$

$$\mathbf{x}_2 = \mathbf{A}\mathbf{y}_1 = \frac{1}{7}\begin{pmatrix} 260 \\ 356 \\ 302 \end{pmatrix} \tag{3}$$

$$\mathbf{y}_2 = \frac{1}{356}\mathbf{x}_2 = \frac{1}{1246}\begin{pmatrix} 130 \\ 178 \\ 151 \end{pmatrix} \tag{4}$$

Theoretical Solution

Let λ be the Dominant eigenvalue.

By Rayleigh-Quotient,

$$\lambda = \frac{\mathbf{y}_2^\top \mathbf{A}\mathbf{y}_2}{\mathbf{y}_2^\top \mathbf{y}_2} \tag{5}$$

$$\boxed{\therefore\ \lambda = 24.1453}$$

Thus, all the given options are incorrect.