5.5.20

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Question

Using elementary row transformations, find the inverse of the matrix

$$\begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$$

Theoretical Solution

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{pmatrix}$

Augment the matrix $\mathbf{A}$ with the identity

$$[\mathbf{A} \mid \mathbf{I}] = \left(\begin{array}{ccc|ccc} 3 & 0 & -1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \tag{1}$$

Row Transformation-1: $R_1 \rightarrow \frac{R_1}{3}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2}$$

Theoretical Solution

Row Transformation-2: $R_2 \to R_2 - 2R_1$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 3 & \frac{2}{3} & -\frac{2}{3} & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \tag{3}$$

Row Transformation-3: $R_2 \to \frac{R_2}{3}$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right) \tag{4}$$

Row Transformations 4 and 5: Replace $R_1 \to R_1 + \frac{1}{3}R_2$ And $R_3 \to R_3 - 4R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{7}{27} & \frac{7}{27} & \frac{1}{9} & 0 \\ 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 8 & -4 & 9 \end{array}\right) \tag{5}$$

Theoretical Solution

Row Transformation-6: $R_3 \rightarrow 9R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{7}{27} & \frac{7}{27} & \frac{1}{9} & 0 \\ 0 & 1 & \frac{2}{9} & -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 8 & -4 & 9 \end{array}\right) \tag{6}$$

Row Transformations 7 and 8: $R_1 \rightarrow R_1 + \frac{7}{27}R_3$ And $R_2 \rightarrow R_2 - \frac{2}{9}R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{3} & -\frac{25}{27} & \frac{7}{3} \\ 0 & 1 & 0 & -2 & \frac{11}{9} & -2 \\ 0 & 0 & 1 & 8 & -4 & 9 \end{array}\right) \tag{7}$$

The Inverse Matrix of **A**:

$$\mathbf{A}^{-1} = \begin{pmatrix} \dfrac{7}{3} & -\dfrac{25}{27} & \dfrac{7}{3} \\ -2 & \dfrac{11}{9} & -2 \\ 8 & -4 & 9 \end{pmatrix} \tag{8}$$