

4.7.64

AI25BTECH11003 - Bhavesh Gaikwad

Question: Find the distance between the point $\mathbf{P}(6, 5, 9)$ and the plane determined by the points $\mathbf{A}(3, -1, 2)$, $\mathbf{B}(5, 2, 4)$ and $\mathbf{C}(-1, -1, 6)$.

Solution: Let the Equation of the Plane be $\mathbf{n}^\top \mathbf{x} = 1$.

Since, \mathbf{A} , \mathbf{B} , \mathbf{C} lie on the Plane.

$$\therefore \mathbf{n}^\top \mathbf{A} = 1, \mathbf{n}^\top \mathbf{B} = 1, \mathbf{n}^\top \mathbf{C} = 1 \quad (0.1)$$

OR

$$\mathbf{A}^\top \mathbf{n} = 1, \mathbf{B}^\top \mathbf{n} = 1, \mathbf{C}^\top \mathbf{n} = 1 \quad (0.2)$$

Let $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$

From Equation 0.2,

$$(\mathbf{A} \ \mathbf{B} \ \mathbf{C})^\top \mathbf{n} = 1 \quad (0.3)$$

$$\begin{pmatrix} 3 & -1 & 2 \\ 5 & 2 & 4 \\ -1 & -1 & 6 \end{pmatrix} \mathbf{n} = 1 \quad (0.4)$$

From Equation 0.4,

$$\therefore \text{The Equation of the Plane is: } \frac{1}{19} \begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \mathbf{x} = 1 \quad (0.5)$$

The Distance of Point from a Plane is given by the formula:

$$d = \frac{\|\mathbf{n}^\top \mathbf{Q} - 1\|}{\|\mathbf{n}\|} \quad (0.6)$$

Let d be the distance between Point \mathbf{P} and the Plane.

$$\text{Then, } d = \frac{\|\mathbf{n}^\top \mathbf{P} - 1\|}{\|\mathbf{n}\|} \quad (0.7)$$

$$d = \frac{\left\| \frac{18-20+27}{19} - 1 \right\|}{\left(\frac{\sqrt{34}}{19} \right)} \quad (0.8)$$

$$\therefore d = \frac{3\sqrt{34}}{17} \quad (0.9)$$

The Distance between the Plane and **P** is $\frac{3\sqrt{34}}{17}$ units.

(0.10)

Distance between Point and Plane

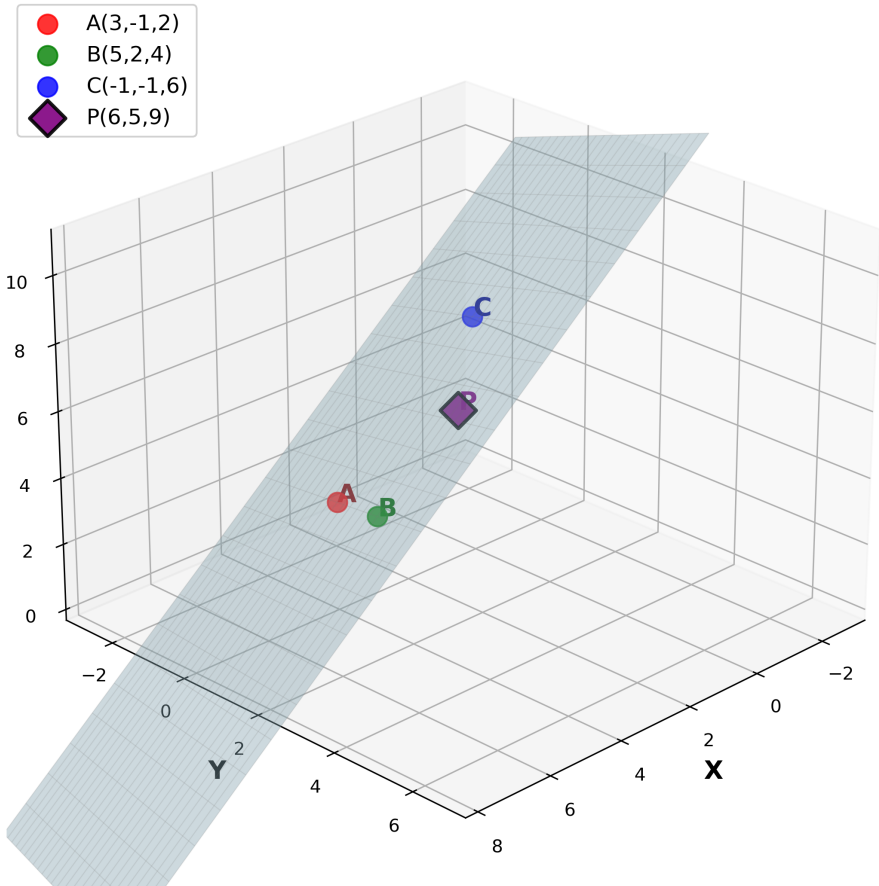


Fig. 0.1: Plane