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SUBJECT	DAA
EXPERIMENT NO :	03
AIM:	To perform matrix multiplication using strassen's algorithm.
PROBLEM STATEMENT 1:	
THEORY	<p>The main reason for high time complexity in naive divide and conquer is because of 8 recursive calls. The strassen matrix reduces these recursive calls to 7. The idea is similar to naive divide and conquer i.e divide the matrix in sub-matrices of $N/2 \times N/2$ dimension until we get 2×2 matrix. We compute this matrix using formulae provided by strassens algorithm.</p> <div style="display: flex; justify-content: space-around; margin: 10px 0;"> <div style="text-align: left;"> $p1 = a(f - h)$ $p3 = (c + d)e$ $p5 = (a + d)(e + h)$ $p7 = (a - c)(e + f)$ </div> <div style="text-align: left;"> $p2 = (a + b)h$ $p4 = d(g - e)$ $p6 = (b - d)(g + h)$ </div> </div> <p>The $A \times B$ can be calculated using above seven multiplications. Following are values of four sub-matrices of result C</p> $ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \underset{A}{\times} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \underset{B}{=} \begin{bmatrix} p5 + p4 - p2 + p6 & p1 + p2 \\ p3 + p4 & p1 + p5 - p3 - p7 \end{bmatrix} \underset{C}{} $ <p> A, B and C are square metrices of size $N \times N$ a, b, c and d are submatrices of A, of size $N/2 \times N/2$ e, f, g and h are submatrices of B, of size $N/2 \times N/2$ $p1, p2, p3, p4, p5, p6$ and $p7$ are submatrices of size $N/2 \times N/2$ </p>

	<p>As there are 7 recursive calls we get following recurrence-</p> $T(N) = 7T(N/2) + O(N^2)$ <p>Solving above recurrence we get time complexity of $O(N^{2.81})$ which is less than naive approach.</p> <p>Disadvantages of strassen's algorithm-</p> <ol style="list-style-type: none"> 1.)The constants used in strassen's multiplication are high and naive approach is used for typical application. 2.)For sparse matrix there are better algorithms. 3.)The sub-matrices in recursion take extra space. 4.)Because of limited precision of computer arithmetic on non integer values, large errors accumulate in strassen's algorithm.
ALGORITHM	<ol style="list-style-type: none"> 1.)Divide the input matrices A and B into $N/2 \times N/2$ dimension. After dividing we get sub-matrices $A_{11}, A_{12}, A_{21}, A_{22}$ and $B_{11}, B_{12}, B_{21}, B_{22}$. 2.)Computing sub-matrices using following formulae- $M_1 = (A_{11} + A_{22}) * (B_{11} + B_{22})$ $M_2 = (A_{21} + A_{22}) * B_{11}$ $M_3 = A_{11} * (B_{12} - B_{22})$ $M_4 = A_{22} * (B_{21} - B_{11})$ $M_5 = (A_{11} + A_{12}) * B_{22}$ $M_6 = (A_{21} - A_{11}) * (B_{11} + B_{12})$ $M_7 = (A_{12} - A_{22}) * (B_{21} + B_{22})$ 3.)Compute the output using intermediate matrices $C_{11} = M_1 + M_4 - M_5 + M_7$ $C_{12} = M_3 + M_5$ $C_{21} = M_2 + M_4$ $C_{22} = M_1 - M_2 + M_3 + M_6$

4.)Combine the output matrices for final matrix.

PROGRAM:

```
strassen.cpp > main()
1  #include<iostream>
2
3  using namespace std;
4
5  int main(){
6      int A[2][2],B[2][2];
7      cout << "Enter 4 elements for matrix A: ";
8      for(int i=0;i<2;i++)
9          for(int j=0;j<2;j++)
10             cin >> A[i][j];
11      cout << "Enter 4 elements for matrix B: ";
12      for(int i=0;i<2;i++)
13          for(int j=0;j<2;j++)
14             cin >> B[i][j];
15      int p[7];
16      p[0] = A[0][0]*(B[0][1] - B[1][1]);
17      p[1] = (A[0][0] + A[0][1])*B[1][1];
18      p[2] = (A[1][0] + A[1][1])*B[0][0];
19      p[3] = A[1][1]*(B[1][0] - B[0][0]);
20      p[4] = (A[0][0] + A[1][1])*(B[0][0] + B[1][1]);
21      p[5] = (A[0][1] - A[1][1])*(B[1][0] + B[1][1]);
22      p[6] = (A[0][0] - A[1][0])*(B[0][0] + B[0][1]);
23
24      int C[2][2];
25      C[0][0] = p[3]+p[4]-p[1]+p[5];
26      C[0][1] = p[0] + p[1];
27      C[1][0] = p[2] + p[3];
28      C[1][1] = p[0] + p[4] - p[2] -p[6];
29
30      cout << "Multiplication of A and B using strassen's algorithm: " << endl;
31      for(int i=0;i<2;i++){
32          for(int j=0;j<2;j++){
33              cout << C[i][j] << "\t";
34          }
35          cout << endl;
36      }
37  }
```

RESULT (SNAPSHOT)

```
PROBLEMS  OUTPUT  DEBUG CONSOLE  TERMINAL

PS E:\Sem4\DAA\Exp3> cd "e:\Sem4\DAA\Exp3\" ; if ($?) { g++ strassen.cpp
Enter 4 elements for matrix A: 1 2 3 4
Enter 4 elements for matrix B: 5 6 7 8
Multiplication of A and B using strassen's algorithm:
19      22
43      50
PS E:\Sem4\DAA\Exp3> 
```

CONCLUSION:

Through this experiment I understood how to compute matrices product using strassen's algorithm. The complexity of this algorithm was found out to be $O(N^{2.81})$ which is slightly better than naive approach whose time complexity is $O(N^3)$. This can have huge impact on matrices with high dimensions.