

AI-1110 Assignment-1

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12.13.1.11 Question: A fair die is rolled. Consider events $E = \{1,3,5\}$, $F = \{2,3\}$ and $G = \{2,3,4,5\}$
Find

- 1) $P(E|F)$ and $P(F|E)$
- 2) $P(E|G)$ and $P(G|E)$
- 3) $P((E + F)|G)$ and $P((EF)|G)$

Solution: Sample space when a die is rolled = $\{1,2,3,4,5,6\}$. We have $E = \{1,3,5\}$, $F = \{2,3\}$ and $G = \{2,3,4,5\}$. So, $P(E) = 3/6$, $P(F) = 2/6$ and $P(G) = 4/6$.

$$\begin{aligned} \Rightarrow P(E) &= 1/2 & (1) \\ \Rightarrow P(F) &= 1/3 & (2) \\ \Rightarrow P(G) &= 2/3 & (3) \end{aligned}$$

1)

$$\begin{aligned} P(E|F) &= \frac{P(EF)}{P(F)} & (4) \\ EF &= \{3\} & (5) \\ \Rightarrow P(EF) &= 1/6 & (6) \\ \therefore P(E|F) &= \frac{1/6}{1/3} & (7) \\ P(E|F) &= 1/2 & (8) \end{aligned}$$

2)

$$\begin{aligned} P(F|E) &= \frac{P(EF)}{P(E)} & (9) \\ \therefore P(F|E) &= \frac{1/6}{1/2} & (10) \\ P(F|E) &= 1/3 & (11) \end{aligned}$$

3)

$$\begin{aligned} P(E|G) &= \frac{P(EG)}{P(G)} & (12) \\ EG &= \{3, 5\} & (13) \\ \Rightarrow P(EG) &= 2/6 & (14) \\ P(EG) &= 1/3 & (15) \\ \therefore P(E|G) &= \frac{1/3}{2/3} & (16) \\ P(E|G) &= 1/2 & (17) \end{aligned}$$

4)

$$P(G|E) = \frac{P(EG)}{P(E)} \quad (18)$$

$$\therefore P(G|E) = \frac{1/3}{1/2} \quad (19)$$

$$P(G|E) = 2/3 \quad (20)$$

5)

$$P((E + F)|G) = \frac{P((E + F)G)}{P(G)} \quad (21)$$

$$(E + F)G = \{2, 3, 5\} \quad (22)$$

$$\Rightarrow P((E + F)G) = 3/6 \quad (23)$$

$$\Rightarrow P((E + F)G) = 1/2 \quad (24)$$

$$\therefore P((E + F)|G) = \frac{1/2}{2/3} \quad (25)$$

$$P((E + F)G) = 3/4 \quad (26)$$

6)

$$P((EF)|G) = \frac{P((EF)G)}{P(G)} \quad (27)$$

$$(EF)G = \{3\} \quad (28)$$

$$\Rightarrow P((EF)G) = 1/6 \quad (29)$$

$$\therefore P((EF)|G) = \frac{1/6}{2/3} \quad (30)$$

$$P((EF)|G) = 1/4 \quad (31)$$

Answer:

$$P(E|F) = 1/2, \quad (32)$$

$$P(F|E) = 1/3, \quad (33)$$

$$P(E|G) = 1/2, \quad (34)$$

$$P(G|E) = 2/3, \quad (35)$$

$$P((E + F)|G) = 3/4, \quad (36)$$

$$P((EF)|G) = 1/4 \quad (37)$$