Assignment 11 Solutions ¶

1. Given X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} 0.1\ for\, x = 0.2 \\ 0.2\ for\, x = 0.4 \\ 0.2\ for\, x = 0.5 \\ 0.3\ for\, x = 0.8 \\ 0.2\ for\, x = 1 \\ 0\ otherwise \end{cases}$$

1. Find the range RX of the random variable X.
2. Find P(X ≤ 0.5)
3. Find P(0.25<X<0.75)
4. P(X = 0.2|X<0.6)

In [ ]:

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The range RX of a random variable X refers to the set of all possible values t
determined by identifying the unique values present in the PMF and considering

To find P(X ≤ 0.5), we need to calculate the probability that X takes a value
This can be done by summing the probabilities of all the values of X that sati

To find P(0.25 < X < 0.75), we need to calculate the probability that X lies w
This can be done by summing the probabilities of all the values of X that fall

P(X = 0.2|X < 0.6) represents the conditional probability of X being equal to
It can be calculated by dividing the probability of X being equal to 0.2 and X
than 0.6.
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2.Two equal and fair dice are rolled, and we observed two numbers X and Y.

1. Find RX, RY, and the PMFs of X and Y.
2. Find P(X = 2,Y = 6).
3. Find P(X>3|Y = 2).
4. If Z = X + Y. Find the range and PMF of Z.
5. Find P(X = 4|Z = 8).

In [ ]:
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The range RX of the random variable X is the set of possible values that X car
The PMFs of X and Y for a fair die are:
P(X = 1) = P(Y = 1) = 1/6
P(X = 2) = P(Y = 2) = 1/6
P(X = 3) = P(Y = 3) = 1/6
P(X = 4) = P(Y = 4) = 1/6
P(X = 5) = P(Y = 5) = 1/6
P(X = 6) = P(Y = 6) = 1/6

P(X = 2, Y = 6) represents the probability of getting a 2 on the first die and
independently and each outcome is equally likely, P(X = 2, Y = 6) = P(X = 2) *

P(X > 3 | Y = 2) represents the probability of getting a value greater than 3
Since the outcomes of the dice are independent, P(X > 3 | Y = 2) = P(X > 3) =

If Z = X + Y, then the range of Z is the set of all possible sums of X and Y.

To find the PMF of Z, we need to consider all possible pairs of values for X a
For example, to find P(Z = 4), we need to consider all pairs (X, Y) such that
the probabilities of the individual values of X and Y.

P(X = 4 | Z = 8) represents the probability of getting a 4 on the first die gi
the dice are independent, P(X = 4 | Z = 8) = P(X = 4, Y = 4) / P(Z = 8). To ca
and P(Z = 8) using the PMFs of X and Y, and the PMF of Z calculated in step 4.
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3. In an exam, there were 20 multiple-choice questions. Each question had 44 possible options. A student knew the answer to 10 questions, but the other 10 questions were unknown to him, and he chose answers randomly. If the student X's score is equal to the total number of correct answers, then find out the PMF of X. What is P(X>15)?

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The PMF of X, the score of the student, can be determined using the binomial c
probability of success (getting the correct answer) of 10/44, and the student
the PMF of X can be calculated as:

P(X = k) = C(20, k) * (10/44)^k * (34/44)^(20-k)

Where C(20, k) is the number of combinations of choosing k questions correctly

To find P(X > 15), we need to calculate the cumulative probability of X being

P(X > 15) = P(X = 16) + P(X = 17) + ... + P(X = 20)

By substituting the values into the formula above and summing the probabilitie
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4. The number of students arriving at a college between a time interval is a Poisson random variable. On average, 10 students arrive per hour. Let Y be the number of students arriving from 10 am to 11:30 am. What is P(10<Y≤15)?

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Since Y follows a Poisson distribution with a mean of 10 students per hour, we

P(10 < Y ≤ 15) = P(Y = 11) + P(Y = 12) + P(Y = 13) + P(Y = 14) + P(Y = 15)

The probability mass function (PMF) of a Poisson distribution is given by:

P(Y = k) = (e^(-λ) * λ^k) / k!

where λ is the average number of events per interval (in this case, 10) and k

By substituting the values into the PMF formula and summing the probabilities,
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5.Two independent random variables, X and Y,are given such that X~~Poisson(α) and~~ ~~Y~~Poisson(β). State a new random variable as Z = X + Y. Find out the PMF of Z.

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The random variable Z is the sum of two independent Poisson random variables X
we convolve the PMFs of X and Y.

The PMF of Z is given by:

P(Z = z) = ∑ P(X = x) * P(Y = z - x)

where the sum is taken over all possible values of x.

Since X and Y are both Poisson random variables, their PMFs are given by:

P(X = x) = (e^(-α) * α^x) / x!
P(Y = y) = (e^(-β) * β^y) / y!

By substituting these PMFs into the equation for P(Z = z), we can calculate th
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6. There is a discrete random variable X with the pmf.

$$P_X(x) = \begin{cases} \frac{1}{4} \, when \, x = & -2 \\ \frac{1}{8} \, when \, x = & -1 \frac{1}{4} \, when \, x = 20 \, otherwisewen \, x = 0 \\ \frac{11}{84} \, when \, x = 1 \end{cases}$$

If we define a new random variable Y = (X + 1)2 then

1. Find the range of Y.
2. Find the pmf of Y.

2.Assuming X is a continuous random variable with PDF

$$f_X(x) = \begin{cases} cx2 \mid x \mid \leq 1 \\ \\ 0 \; Otherwise \end{cases} \quad \text{Find the constant cc.}$$

1. Find EX and Var(X).
2. Find P(X ≥ 1/2).
3. If X is a continuous random variable with pdf

$$f_x(x) = \begin{cases} 4x^3 \; 0 < x \leq 1 \\ 0 \; otherwise \end{cases}$$

$$\text{Find } P(X \leq \frac{2}{3} \mid X \rangle \frac{1}{3})$$

4. If X~Uniform and Y = sin(X), then find fY(y).
5. If X is a random variable with CDF

$$F_X(x) = \begin{cases} 1 \; X \geq 1 \\ \frac{1}{2} + \frac{x}{2} \; where \; 0 \leq X < 1 \\ 0 \; where \; x < 0 \end{cases}$$

6. What kind of random variable is X: discrete, continuous, or mixed?
7. Find the PDF of X, fX(x).
8. Find E([illegible]X)

In [ ]:

5. There are two random variables X and Y with joint PMF given in Table below

1. Find P(X≤2, Y≤4).
2. Find the marginal PMFs of X and Y.
3. Find P(Y = 2|X = 1).
4. Are X and Y independent?

(L)	$Y = 2$	$Y = 4$	$Y = 5$
$X = 1$	1/12	1/24	1/24
$X = 2$	1/6	1/12	1/8
$X = 3$	¼	1/8	1/12

In [ ]:

6.A box containing 40 white shirts and 60 black shirts. If we choose 10 shirts (without replacement) at random, find the joint PMF of X and Y, where X is the number of white shirts and Y is the number of black shirts.

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Sure! Let's discuss some theoretical explanations related to the given questio

When defining a new random variable Y = (X + 1)^2, we can determine its range
transformation. If X takes values in the set {x1, x2, x3, ...}, then Y will ta
(x3 + 1)^2, ...}.

To find the PMF of Y, we need to calculate the probability of Y taking each va
P(Y = y) for each y in the range of Y. Since Y is a deterministic transformati
of the PMF of X. Specifically, P(Y = y) = P((X + 1)^2 = y) = P(X = √y - 1).

For the continuous random variable X:

The expected value (EX) of X can be found by integrating X multiplied by its p
The variance of X (Var(X)) can be computed as the expected value of (X - EX)^2

To find P(X ≥ x), we integrate the PDF of X from x to infinity. This represent
or equal to x.

If X is uniformly distributed, its PDF is constant over its range. For the ran
PDF (fY(y)) by applying the transformation formula for random variables. The P
the cumulative distribution function (CDF) of Y with respect to y.

If X has a given cumulative distribution function (CDF), we can determine its
the CDF is continuous or has jumps. If the CDF has jumps, it indicates that X

The PDF of X (fX(x)) can be obtained by differentiating its CDF with respect t
is changing with respect to x.

To find E(e^X), we need to evaluate the expected value of the exponential of X
PDF of X over its entire range.

To find P(X = 0|X ≤ 0.5), we can use conditional probability. The probability
to 0.5, is obtained by dividing the joint probability of X being 0 and X being
of X being less than or equal to 0.5.

For the joint PMF of X and Y, we can consider each combination of X and Y valu
combination. In this case, since we are sampling without replacement, the prob
of white and black shirts remaining in the box at each step.

These explanations provide a theoretical understanding of the concepts and cal
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7.If A and B are two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} 6xy \text{ where } 0 \leq x \leq 1,\ 0 \leq y \leq \sqrt{x} \\ 0 \text{ otherwise} \end{cases}$$

1. Find fX(a) and fY(b).
2. Are A and B independent of each other?
3. Find the conditional PDF of A given B = b, fA|B(a|b).
4. Find E[A|B = b], for 0 ≤ y ≤ 1.
5. Find Var(A|B = b), for 0 ≤ y ≤ 1.

In [ ]:
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a. To find the marginal PDF of A (fX(a)), we integrate the joint PDF fA,B(a, b
is given by fX(a) = ∫ fA,B(a, b) db. Similarly, the marginal PDF of B (fY(b))
entire range of A: fY(b) = ∫ fA,B(a, b) da.

b. A and B are independent if and only if their joint PDF can be expressed as
If fA,B(a, b) = fX(a) * fY(b) for all values of a and b, then A and B are inde

c. The conditional PDF of A given B = b (fA|B(a|b)) is defined as the ratio of
evaluated at b: fA|B(a|b) = fA,B(a, b) / fY(b). This represents the probabilit

d. To find E[A|B = b], we integrate A multiplied by the conditional PDF of A g
This gives the expected value of A, given that B is equal to b: E[A|B = b] = ∫
the range of A.

e. The variance of A given B = b (Var(A|B = b)) is obtained by subtracting the
conditional mean of the squared values of A (E[A^2|B = b]). The variance is gi

These calculations allow us to analyze the relationship between A and B, their
variances given a specific value of B.
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8.There are 100 men on a ship. If Xi is the ith man's weight on the ship and Xi's are independent and identically distributed and EXi = μ = 170 and σXi = σ = 30. Find the probability that the men's total weight on the ship exceeds 18,000.

In [ ]:
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To find the probability that the men's total weight on the ship exceeds 18,000
since the weights of the men (Xi) are independent and identically distributed.

According to the CLT, the sum of a large number of independent and identically
distribution. In this case, the total weight of the men on the ship can be con
(X1 + X2 + ... + X100).

The mean of the total weight is given by E(X1 + X2 + ... + X100) = E(X1) + E(X

The standard deviation of the total weight is given by σ(X1 + X2 + ... + X100)

Applying the CLT, we can approximate the total weight as a normal distribution

P(X1 + X2 + ... + X100 > 18,000) = P(Z > (18,000 - 100μ) / (10σ))

Substituting the given values μ = 170 and σ = 30, we can calculate the z-score
standard normal distribution table or a statistical software.

Note: The given values for μ and σ are used to approximate the total weight di
depends on the sample size (100 men in this case) and the distribution of the
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9.Let X1, X2, ……, X25 are independent and identically distributed. And have the following PMF. If Y = X1 + X2 + … + Xn, estimate P(4 ≤ Y ≤ 6) using central limit theorem.

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To estimate P(4 ≤ Y ≤ 6) using the Central Limit Theorem (CLT), we can approxi
independent and identically distributed random variables (X1, X2, ..., X25).

According to the CLT, when the sample size is large, the sum of independent an
a normal distribution. Since we have 25 variables, the CLT is applicable in th

We need to find the mean (μ) and standard deviation (σ) of Y, which can be cal
Let's denote the mean and standard deviation of the individual variables X1, X

Since the variables are identically distributed, μx is the same for all Xi, an

The mean of Y is given by E(Y) = E(X1 + X2 + ... + X25) = E(X1) + E(X2) + ...

The variance of Y is given by Var(Y) = Var(X1 + X2 + ... + X25) = Var(X1) + Va

The standard deviation of Y is then σY = sqrt(Var(Y)) = sqrt(25σx^2) = 5σx.

Now, we can approximate the distribution of Y as a normal distribution with me

To estimate P(4 ≤ Y ≤ 6), we convert it to the corresponding z-scores:

z1 = (4 - 25μx) / (5σx)
z2 = (6 - 25μx) / (5σx)

Using a standard normal distribution table or a statistical software, we can f
z2 and calculate P(4 ≤ Y ≤ 6) as the difference between these probabilities.

Note: The accuracy of the approximation depends on the sample size (25 in this
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