Assignment 1

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In [1]:

```
import cv2
import numpy as np
import matplotlib.pyplot as plt
import random
import heapq
import copy
from scipy import linalg
```

In [2]:

```
%matplotlib inline
```

URL of Input Images:

[https://drive.google.com/open?id=1yJxq5h2McYWXCox-JhZzMxTmO9VOfRqo (https://drive.google.com/open?id=1yJxq5h2McYWXCox-JhZzMxTmO9VOfRqo)](https://drive.google.com/open?id=1yJxq5h2McYWXCox-JhZzMxTmO9VOfRqo)

1.1 Direct Linear Transform

In this part we use all the points to estimate the projection matrix P . In the given picture there are 48 points of interest. For every real world poitns there is a corresponding image point. Using this mapping of points we can arrive at a set of equations which will solve the problem of finding P.
Let (u_i, v_i) be the image points corresponding to a real world point (X_i, Y_i, Z_i) . Then representing both in homogenous coordinate system.

$$\begin{bmatrix} x_i \\ y_i \\ w_i \end{bmatrix} = P \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

where $(u_i, v_i) = (x_i/w_i, y_i/w_i)$. If $P_{11}, P_{12}, P_{13} \ldots$ represent the elements of the P matrix we can formulate the problem into a set of equation by observing that.

$$u_i = \frac{x_i}{w_i} = \frac{P_{11}X_i + P_{12}Y_i + P_{13}Z_i + P_{14}}{P_{31}X_i + P_{32}Y_i + P_{33}Z_i + P_{34}}$$

and similarly

$$v_i = \frac{y_i}{w_i} = \frac{P_{21} X_i + P_{22} Y_i + P_{23} Z_i + P_{24}}{P_{31} X_i + P_{32} Y_i + P_{33} Z_i + P_{34}}$$

The rewriting everything into a set of equations we get.

$$\begin{bmatrix} X_i & Y_i & Z_i & 1 & 0 & 0 & 0 & 0 & -u_i X_i & -u_i Y_i & -u_i Z_i & -u_i \\ 0 & 0 & 0 & 0 & X_i & Y_i & Z_i & 1 & -v_i X_i & -v_i Y_i & -v_i Z_i & -v_i \end{bmatrix} \begin{bmatrix} P_{11} \\ P_{12} \\ P_{13} \\ . \\ . \\ . \\ P_{34} \end{bmatrix} = 0$$

$$MP = 0$$

Since there are 12 unknowns and each mapping of real points to image points gives us 2 equations we need at least 6 of these points to find a non trivial solution to the system of equations. Since the system of equations is a degenerate one we need to find a vector of minimum euclidean norm that lies in the null space of M . We do this by svd decomposition of $M = USV^T$ and than taking the last column of V^T as the required vector.

Once we have found the required P matrix we scale it by dividing every element by P 34 . Now we have

$$P = [KR|KRT]$$

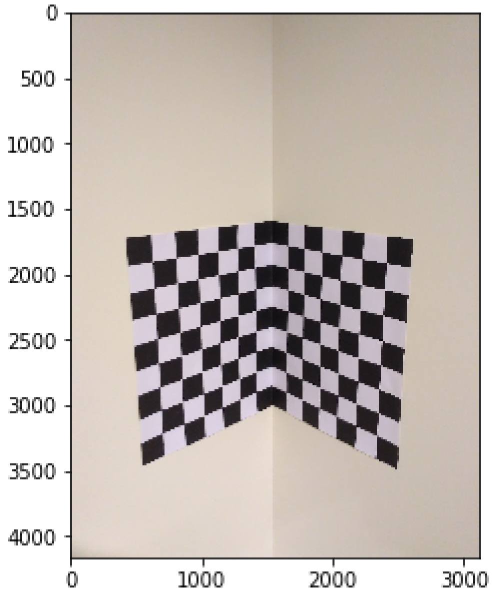
where K is the internal camera matrix(intrinsic parameters), R is the rotation matrix and T is the translation vector of the camera from the world origin.

$$K = \begin{bmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$

$f_x, f_y =$ focal length in x and y directions
s is skew
x_0, y_0 is displacement from top left corner of the image.
Thus the first three columns of $P‘= KR$. We can get K by RQ decomposition of the first three columns of P.
Then, $R = K^{-1}P'$ and $T = (KR)^{-1}P'$.
Or we can use the for $(KR)(KR)^T = PP^T$ and derive K from K^2.

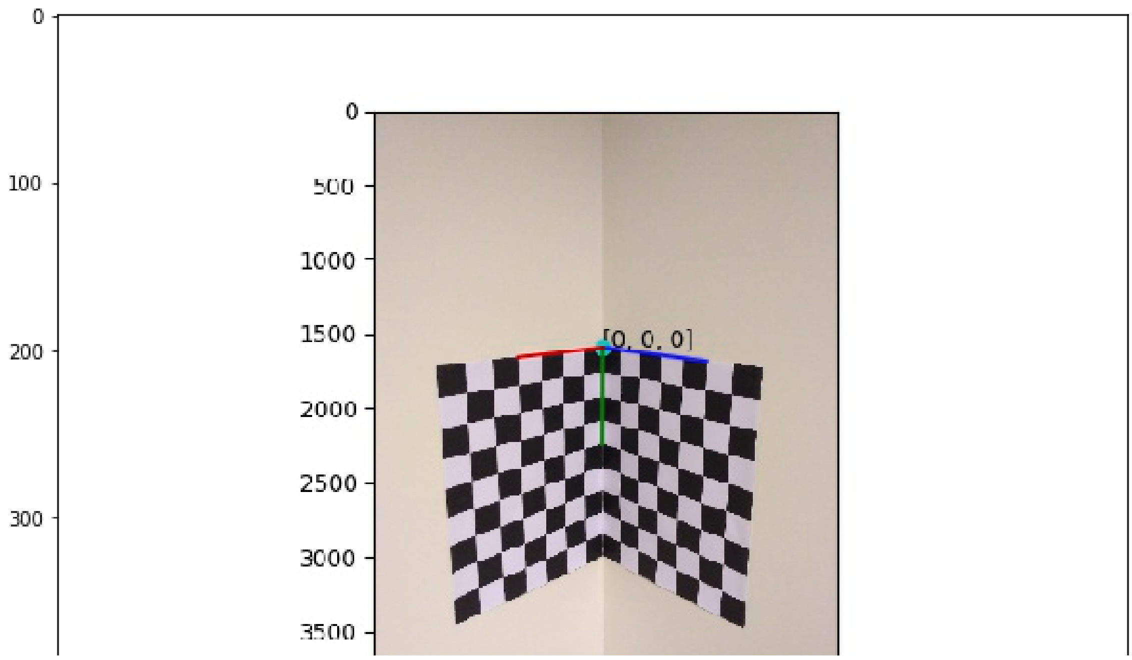
In [3]:

```
img = cv2.imread('./Camera_calibration_data/calib-object.jpg')
plt.figure(figsize = (5, 5))
plt.imshow(img[...,::-1])
plt.show()
```



In [4]:

```
legend_img = cv2.imread('./Camera_calibration_data/calib-object-legend.jpg')
plt.figure(figsize = (10, 10))
plt.imshow(legend_img)
plt.show()
```



3D (x, y, z) coordinates (in mm) of the calibration object

In [5]:

```
world_points = [[x,-28,0] for x in range(6*28,0,-56)] #row 1x
world_points += [[x,-56,0] for x in range(6*28,0,-56)] #row 2x
world_points += [[x,-84,0] for x in range(6*28,0,-56)] #row 3x
world_points += [[x,-112,0] for x in range(6*28,0,-56)] #row 4x
world_points += [[x,-140,0] for x in range(6*28,0,-56)] #row 5x
world_points += [[x,-168,0] for x in range(6*28,0,-56)] #row 6x
world_points += [[x,-196,0] for x in range(6*28,0,-56)] #row 7x
world_points += [[x,-224,0] for x in range(6*28,0,-56)] #row 8x
world_points += [[0,-28,z] for z in range(6*28,0,-56)] #row 1z
world_points += [[0,-56,z] for z in range(6*28,0,-56)] #row 2z
world_points += [[0,-84,z] for z in range(6*28,0,-56)] #row 3z
world_points += [[0,-112,z] for z in range(6*28,0,-56)] #row 4z
world_points += [[0,-140,z] for z in range(6*28,0,-56)] #row 5z
world_points += [[0,-168,z] for z in range(6*28,0,-56)] #row 6z
world_points += [[0,-196,z] for z in range(6*28,0,-56)] #row 7z
world_points += [[0,-224,z] for z in range(6*28,0,-56)] #row 8z
```

2D (u, v) coordinates (in pixels) of the image

In [6]:

```
image_points = [[2411, 1916], [2075, 1858], [1792, 1805],
                [2401, 2117], [2068, 2046], [1787, 1981],
                [2390, 2317], [2059, 2231], [1783, 2153],
                [2380, 2508], [2056, 2411], [1780, 2322],
                [2370, 2697], [2052, 2583], [1780, 2485],
                [2362, 2882], [2048, 2757], [1776, 2648],
                [2352, 3058], [2044, 2923], [1777, 2805],
                [2343, 3232], [2041, 3088], [1778, 2960],
                [629, 1904], [977, 1849], [1279, 1801],
                [642, 2106], [983, 2036], [1280, 1976],
                [654, 2301], [989, 2219], [1285, 2147],
                [665, 2493], [997, 2397], [1286, 2314],
                [678, 2680], [1002, 2572], [1288, 2480],
                [687, 2861], [1011, 2745], [1291, 2639],
                [696, 3037], [1020, 2914], [1294, 2798],
                [706, 3212], [1026, 3078], [1299, 2953]]
```

In [7]:

```
def getK(P):
    p = P[:, :3]
    p = np.matmul(p, p.T)
    k = np.zeros([3, 3])
    k[2, 2] = (p[2, 2])**0.5
    k[0, 2] = x = p[0, 2] / k[2, 2]
    k[1, 2] = y = p[1, 2] / k[2, 2]
    try:
        k[1, 1] = fy = (p[1, 1] - y**2)**0.5
    except:
        k[1, 1] = fy = p[1, 1]**0.5

    k[0, 1] = s = (p[1, 0] - x*y) / fy
    k[0, 0] = fx = (p[0, 0] - s**2 - x**2)**0.5
    return k


def DLTCalibrate(world_points, image_points, num_points = 6):    # min. no. of poin
    point_indices = [i for i in range(len(world_points))]
    random.shuffle(point_indices)
    point_indices = point_indices[:num_points]

    world_coords = []
    pixel_coords = []
    for i in point_indices:
        world_coords.append(world_points[i])
        pixel_coords.append(image_points[i])

    world_coords = np.asarray(world_coords)
    pixel_coords = np.asarray(pixel_coords)

    A = []
    # convert the points into a corresponding 12 X 12 vector
    for i in range(num_points):
        x, y, z = world_coords[i,0], world_coords[i,1], world_coords[i,2]
        u, v = pixel_coords[i,0], pixel_coords[i, 1]

        A.append([x, y, z, 1, 0, 0, 0, 0, -u*x, -u*y, -u*z, -u])
        A.append([0, 0, 0, 0, x, y, z, 1, -v*x, -v*y, -v*z, -v])

    A = np.asarray(A)
    U, S, Vh = np.linalg.svd(A)

    P = Vh[-1:]    # The rightmost column of v is the desired solution (min. eigenv
    P = P.reshape(3, 4)    # Camera projection matrix
    P = P / P[-1,-1]

    K = getK(P)
    R = np.matmul(np.linalg.inv(K), P[:, :3])
    T = np.matmul(np.linalg.inv(P[:, :3]), P[:, 3])
    return P, K/K[-1, -1], R, T
```

In [8]:

```
P, K, R, T = DLTCalibrate(world_points, image_points, 48)
print("Projection Matrix\n", P)
print("Intrinsic Parameters\n", K)
print("Rotation Matrix\n", R)
print("Translation Vector\n", T)
```

```
Projection Matrix
 [[ 2.40004318e+00 -5.85923571e-01 -6.18287023e+00  1.53756610e+03]
 [-1.31526603e+00 -6.64719377e+00 -1.31502247e+00  1.59226430e+03]
 [-1.17825387e-03 -3.67235933e-04 -1.14356038e-03  1.00000000e+00]]
Intrinsic Parameters
 [[3.63040169e+03 2.11366945e+01 1.57470672e+03]
 [0.00000000e+00 3.61427016e+03 1.94095952e+03]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00]]
Rotation Matrix
 [[ 0.6957447   0.0044318  -0.71827549]
 [ 0.15978683 -0.97587939  0.14875342]
 [-0.700291   -0.21826537 -0.67967105]]
Translation Vector
 [-429.37595574  -73.79284316 -408.36211509]
```

1.2 RANSAC

Random selection of points from the set of all points, inliers and outliers are differntiated using a threshold values on the euclidean distance between the actual image point and the point computed from the projectection matrix determined by the randomly selected points. The best solution with the most inliers is selected

In [9]:

```
def getInliersPercentage(P, world_coords, pixel_coords, threshold):    # threshold
    num_inliers = 0
    for i in range(len(pixel_coords)):
        pc = np.matmul(P, world_coords[i] + [1])
        if np.linalg.norm(pixel_coords[i] - (pc/pc[2])[:2]) < threshold:
            num_inliers += 1

    return num_inliers / len(pixel_coords)


def RANSACCalibrate(world_points, image_points, threshold=250,
                    random_set_size=8, num_iterations=100):
    best_matrices = DLTCalibrate(world_points, image_points, random_set_size)
    best_percentage = getInliersPercentage(best_matrices[0], world_points, image_po
    while num_iterations > 0:
        temp_matrices = DLTCalibrate(world_points, image_points, random_set_size)
        temp_percentage = getInliersPercentage(best_matrices[0], world_points, imag
        if temp_percentage > best_percentage:
            best_matrices = temp_matrices
            best_percentage = temp_percentage

        num_iterations -= 1

    return best_matrices

P, K, R, T = RANSACCalibrate(world_points, image_points, random_set_size=48)
print("Projection Matrix\n", P)
print("Intrinsic Parameters\n", K)
print("Rotation Matrix\n", R)
print("Translation Vector\n", T)
```

```
Projection Matrix
 [[ 2.40004318e+00 -5.85923571e-01 -6.18287023e+00  1.53756610e+03]
 [-1.31526603e+00 -6.64719377e+00 -1.31502247e+00  1.59226430e+03]
 [-1.17825387e-03 -3.67235933e-04 -1.14356038e-03  1.00000000e+00]]
Intrinsic Parameters
 [[3.63040169e+03 2.11366947e+01 1.57470672e+03]
 [0.00000000e+00 3.61427016e+03 1.94095952e+03]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00]]
Rotation Matrix
 [[ 0.6957447   0.0044318  -0.71827549]
 [ 0.15978683 -0.97587939  0.14875342]
 [-0.700291   -0.21826537 -0.67967105]]
Translation Vector
 [-429.3759557   -73.79284315 -408.36211505]
```

1.3 Radial Distortion

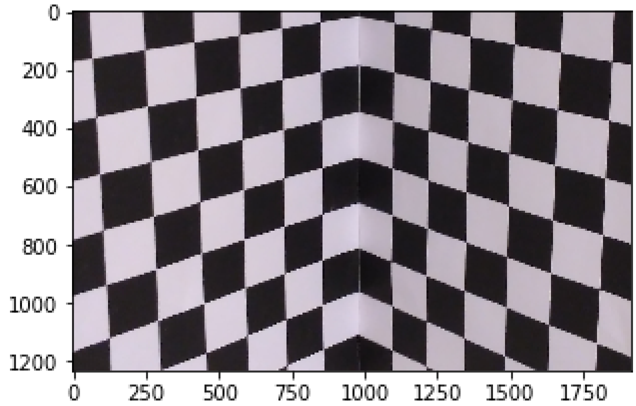
Radial distortion is caused by physical defects in the camera lens which shifts pixels from their original position. The two common type of distortions are barrel distortion and pin-cushion distortion. Both these types of distortions are quatraditc in nature i.e. the amount of distortions increase as we go further from the centre. Opencv uses the following function to calibrate for distorted images.

$$x_{corrected} = x_{actual}(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$
$$y_{corrected} = y_{actual}(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

where ***r*** is the euclidean distance of the distorted point from the distortion center. Opencv provides the cameraCalibration method which uses a set of point correspondenses on a line to determine the distortions parameters.

In [10]:

```
Fig1 = cv2.imread('./Camera_calibration_data/Fig1.png')
plt.figure(figsize = (5, 5))
plt.imshow(Fig1[...,::-1])
plt.show()
```



In [12]:

```
world_points2 = [[x, 0, 0] for x in range(6*28,0,-56)] #row 1x
world_points2 += [[x,-28,0] for x in range(6*28,0,-56)] #row 2x
world_points2 += [[x,-56,0] for x in range(6*28,0,-56)] #row 3x
world_points2 += [[x,-84,0] for x in range(6*28,0,-56)] #row 4x
world_points2 += [[x,-112,0] for x in range(6*28,0,-56)] #row 5x
world_points2 += [[x,-140,0] for x in range(6*28,0,-56)] #row 6x
world_points2 += [[0, 0, z] for z in range(6*28,0,-56)] #row 1z
world_points2 += [[0,-28,z] for z in range(6*28,0,-56)] #row 2z
world_points2 += [[0,-56,z] for z in range(6*28,0,-56)] #row 3z
world_points2 += [[0,-84,z] for z in range(6*28,0,-56)] #row 4z
world_points2 += [[0,-112,z] for z in range(6*28,0,-56)] #row 5z
world_points2 += [[0,-140,z] for z in range(6*28,0,-56)] #row 6z
```

In [13]:

```
image_points2 = [[1853, 177], [1516, 118], [1234, 68],
                 [1843, 378], [1508, 306], [1230, 240],
                 [1831, 575], [1501, 489], [1225, 413],
                 [1821, 768], [1497, 669], [1221, 580],
                 [1812, 956], [1491, 846], [1220, 746],
                 [1802, 1141], [1489, 1018], [1217, 909],
                 [70, 164], [418, 110], [718, 62],
                 [83, 364], [425, 296], [723, 236],
                 [95, 563], [431, 479], [724, 410],
                 [107, 751], [437, 657], [727, 575],
                 [118, 940], [444, 832], [730, 739],
                 [129, 1121], [452, 1002], [732, 897]]
```

In [14]:

```
def radialCorrection(world_points, image_points, image):
    world_coords = np.zeros([len(world_points),3],np.float32)
    for p in range(len(world_points)):
        world_coords[p] = [world_points[p][0], world_points[p][1], world_points[p][
    image_coords = np.zeros((len(image_points),2),np.float32)
    for p in range(len(image_points)):
        image_coords[p] = [image_points[p][0], image_points[p][1]]

    ret, K, Dist = cv2.calibrateCamera([world_coords], [image_coords], (image.shape
    K2, roi = cv2.getOptimalNewCameraMatrix(K, Dist, (image.shape[1], image.shape[0
    undistored_image = cv2.undistort(image, K, None, K2)
    return undistored_image, Dist
```

In [15]:

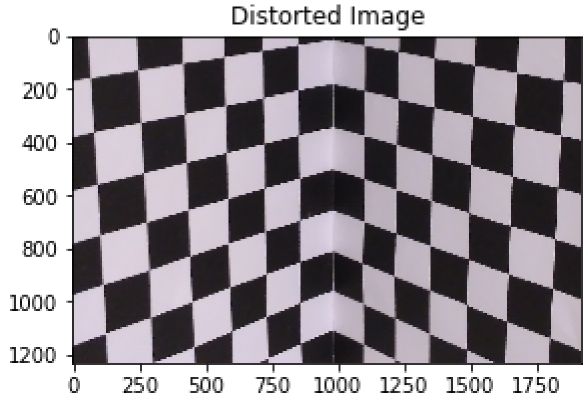
```
undistored_img, Dist = radialCorrection(world_points2[:18], image_points2[:18], Fig
print("Distortion Coefficients")
print("k1 = ", Dist[0][0], "k2 = ", Dist[0][1], "k3 = ", Dist[0][4])
```

```
Distortion Coefficients
k1 =  3.434009813731926 k2 =  -325.5958321050537 k3 =  7845.0234856028
75
```

In [17]:

```
plt.figure(figsize=(10,5))
plt.figure(figsize = (10, 10))
plt.subplot(1,2,1)
plt.imshow(Fig1[...,::-1])
plt.title("Distorted Image")

plt.figure(figsize = (10, 10))
plt.subplot(1,2,2)
plt.title("Undistorted Image")
plt.imshow(undistored_img[...,::-1])
plt.show()
```

```
<Figure size 720x360 with 0 Axes>
```

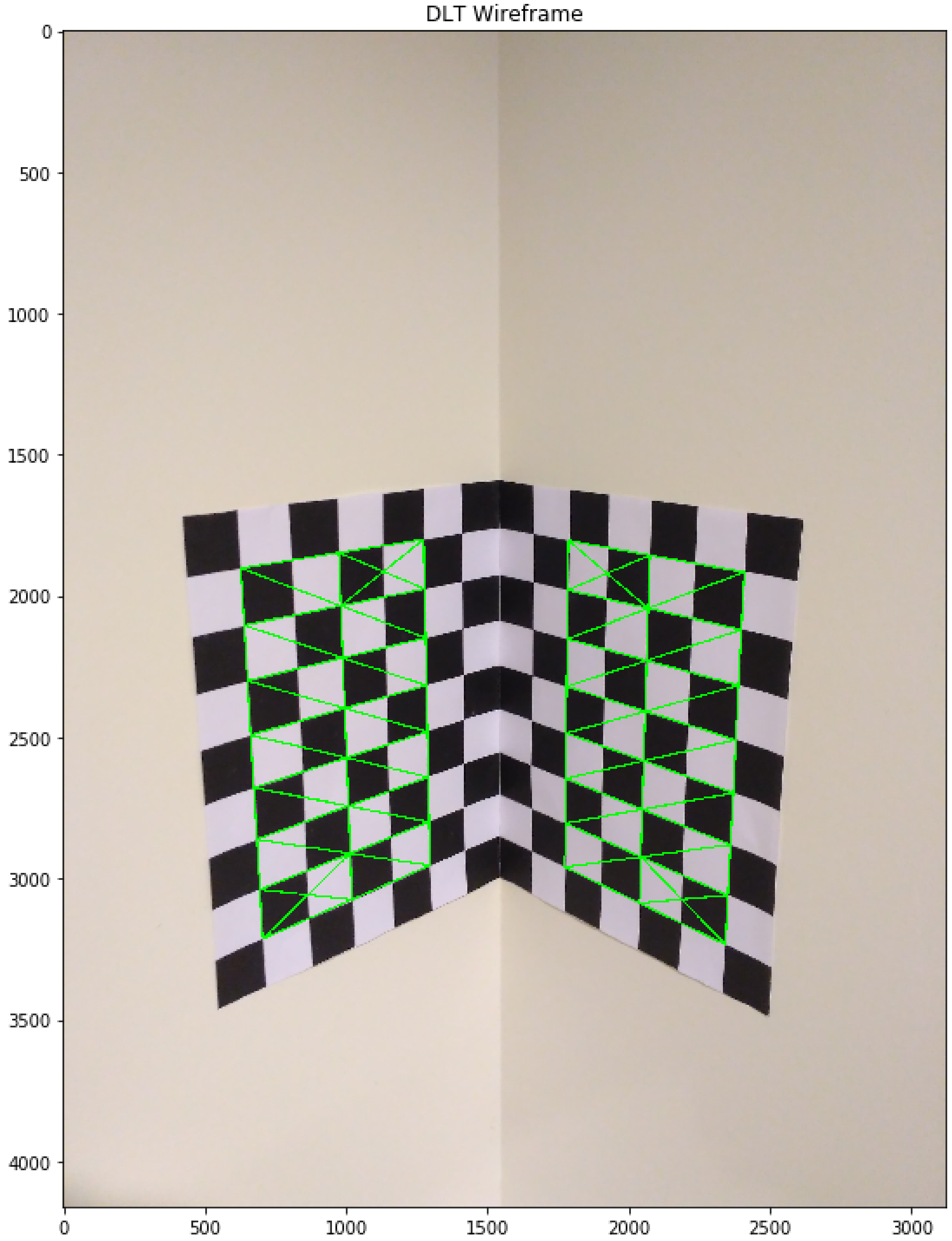


Wireframe Model

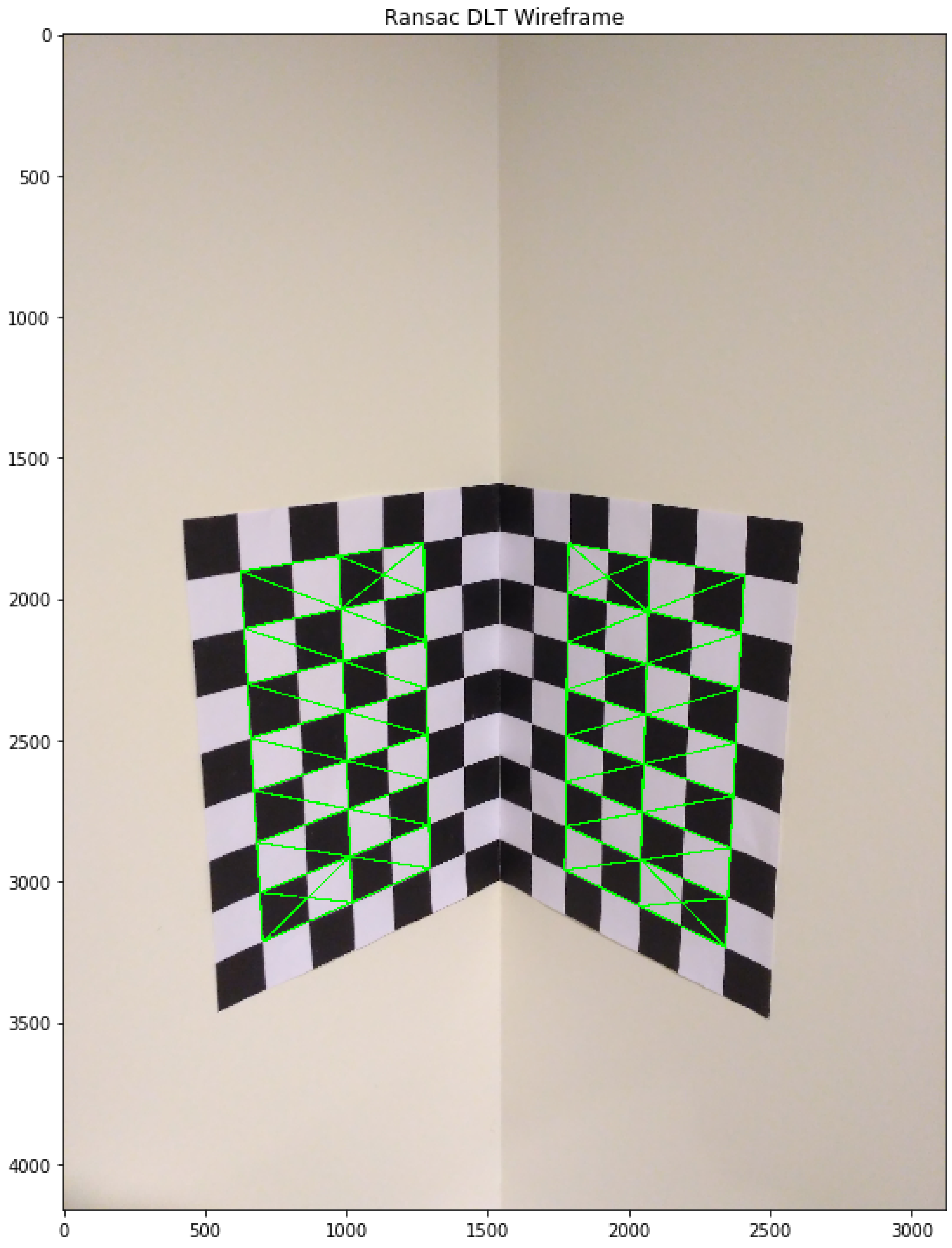
The wireframe model lets us visualize the set mapping between the real points and image points. If we have correctly estimated the intrinsic P matrix and the intrinsic parameters we should be able to get the corresponding image frame from the real world points using the P matrix alone. Overlaying the wire frame model also helps to visually identify how close our parameters are to the actual parameters of the camera by seeing how well the wirefram fits the acutal points in the image. The overlay was created by joining a point to its four nearest points.

In [18]:

```
# wireframe representaion of image points projected from world points
# w : width of line
# c : color of line
def wireframe(image_points, image, line_width=50, line_color=(0,255,0), close=5):
    px = image.shape[0]
    py = image.shape[1]
    # get 4 closest points to a given point based on euclidean distance to construc
    for i in image_points:
        npoints = heapq.nsmallest(close, image_points, key = lambda x:((x[0] - i[0]
        for j in npoints[1:]:
            cv2.line(image, (int(j[0]),int(j[1])), (int(i[0]),int(i[1])), line_colo

image1 = cv2.imread("./Camera_calibration_data/calib-object.jpg")
image2 = image1.copy()

# wireframe for image points computed from DLT transformation
P, K, R, T = DLTCalibrate(world_points, image_points, 48)
ips = []
for wp in world_points:
    ip_ = np.matmul(P, wp + [1])
    ips += [(ip_/ip_[2])]

wireframe(ips, image1, line_width=5)
plt.figure(figsize = (20,20))
plt.subplot(1,2,1)
plt.imshow(image1[...,::-1])
plt.title("DLT Wireframe")
plt.show()

# wireframe for image points computed from random sampling dlt transformation
P, K, R, T = RANSACCalibrate(world_points, image_points, random_set_size=48)
ips = []
for wp in world_points:
    ip_ = np.matmul(P, wp + [1])
    ips += [(ip_/ip_[2])]

wireframe(ips, image2, line_width=5)
plt.figure(figsize = (20,20))
plt.subplot(1,2,2)
plt.imshow(image2[...,::-1])
plt.title("Ransac DLT Wireframe")
plt.show()
```



DLT Wireframe
0
500
1000
1500
2000
2500
3000
3500
4000
0
500
1000
1500
2000
2500
3000



2.1 Zhangs Camera Calibration using OpenCV

In [19]:

```
def ZhangsCalibrate(images, grid, square_size = 29):
    criteria = (cv2.TERM_CRITERIA_EPS + cv2.TERM_CRITERIA_MAX_ITER, 300, 0.001)
    #world points
    wp = np.zeros((grid[0]*grid[1], 3), np.float32)
    for i in range(grid[1]):
        for j in range(grid[0]):
            wp[i*grid[0] + j] = (i*square_size, j*square_size, 0)

    world_points = []
    image_points = []
    for image in images:
        image = cv2.cvtColor(image,cv2.COLOR_BGR2GRAY)
        ret,corners = cv2.findChessboardCorners(image, grid, None)
        if ret:
            cv2.cornerSubPix(image,corners,(11,11),(-1,-1),criteria)
            world_points.append(wp)
            image_points.append(corners)

    return (wp,cv2.calibrateCamera(world_points,image_points,(images[0].shape[1], i
```

In [20]:

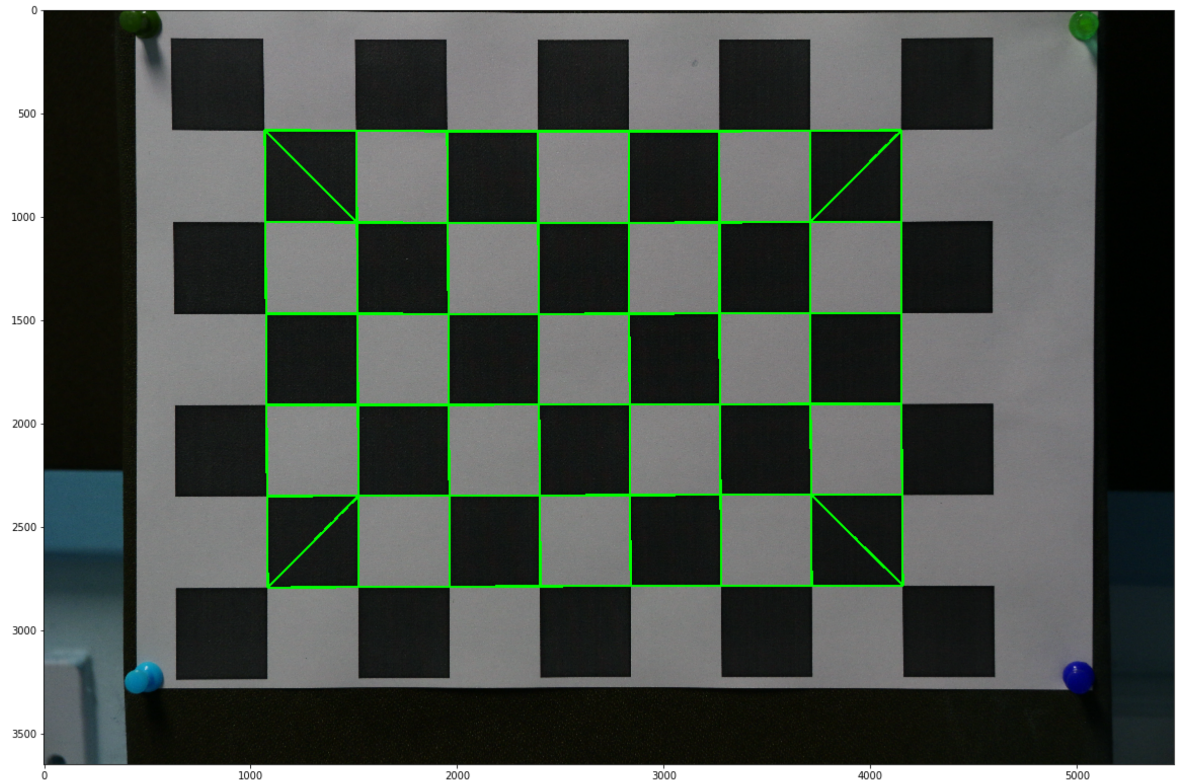
```
chessImgSet = [cv2.imread("./Camera_calibration_data/IMG_54" + str(i) + ".JPG") for
points,(ret,kmat,dist,rmat,tvec) = ZhangsCalibrate(chessImgSet, (6, 8), square_size
print("Camera Matrix using zhangs")
print(kmat)
```

```
Camera Matrix using zhangs
[[1.36415093e+04 0.00000000e+00 3.31635833e+03]
 [0.00000000e+00 1.36632517e+04 1.50037364e+03]
 [0.00000000e+00 0.00000000e+00 1.00000000e+00]]
```

2.2 Wireframe representation of checkerboard

In [21]:

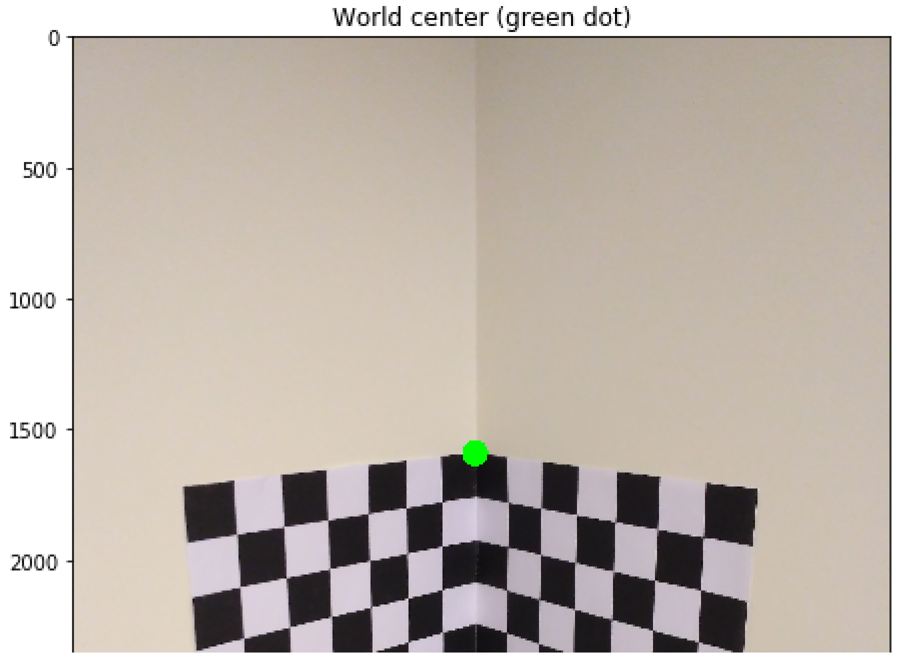
```
wfp = cv2.projectPoints(points,rmat[0],tvec[0],kmat,dist)
ips = []
for i in wfp[0]:
    ips += [i[0].tolist()]

image = cv2.imread("./Camera_calibration_data/IMG_5456.JPG")
wireframe(ips,  image,line_width=10, line_color=(0,255,0), close=4)
plt.figure(figsize = (20,20))
plt.imshow(image)
plt.show()
```



2.3 Finding the origin

In [22]:

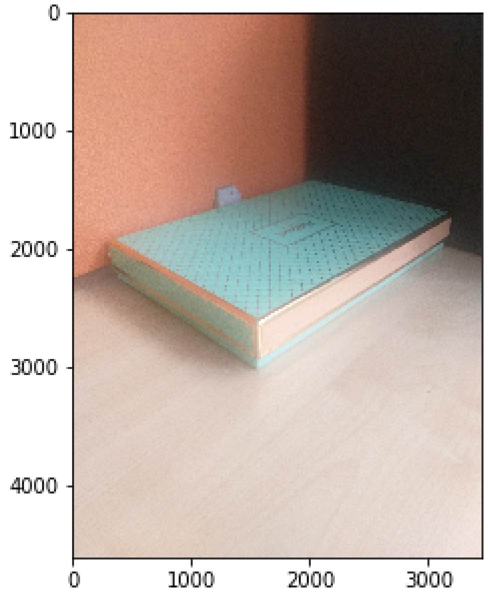
```
image = cv2.imread("./Camera_calibration_data/calib-object.jpg")
P, K, R, T = DLTCalibrate(world_points, image_points, 48)
cv2.circle(image, (int(P[0,3]), int(P[1, 3])), 50, (0, 255, 0), -1)
plt.figure(figsize = (10, 10))
plt.imshow(image[..., ::-1])
plt.title("World center (green dot)")
plt.show()
```



3.1 Own Camera Calibration

In [23]:

```
Fig2 = cv2.imread('./Camera_calibration_data/20200212_152612.jpg')
plt.figure(figsize = (5, 5))
plt.imshow(Fig2[...,::-1])
plt.show()
```



In [24]:

```
world_points3 = [[4, 0, -4], [4, 0, -25], [296, 0, -25], [296, 0, -4],
                 [0, 4, -4], [0, 4, -25], [0, 196, -25], [0, 196, -4],
                 [8, 8, 0], [56, 8, 0], [104, 8, 0], [180, 8, 0],
                 [296, 8, 0], [14, 50, 0], [52, 50, 0], [100, 50, 0],
                 [148, 50, 0], [285, 56, 0], [112, 78, 0], [192, 78, 0],
                 [192, 126, 0], [112, 126, 0], [300, 208, 0], [4, 208, 0]]
```

In [25]:

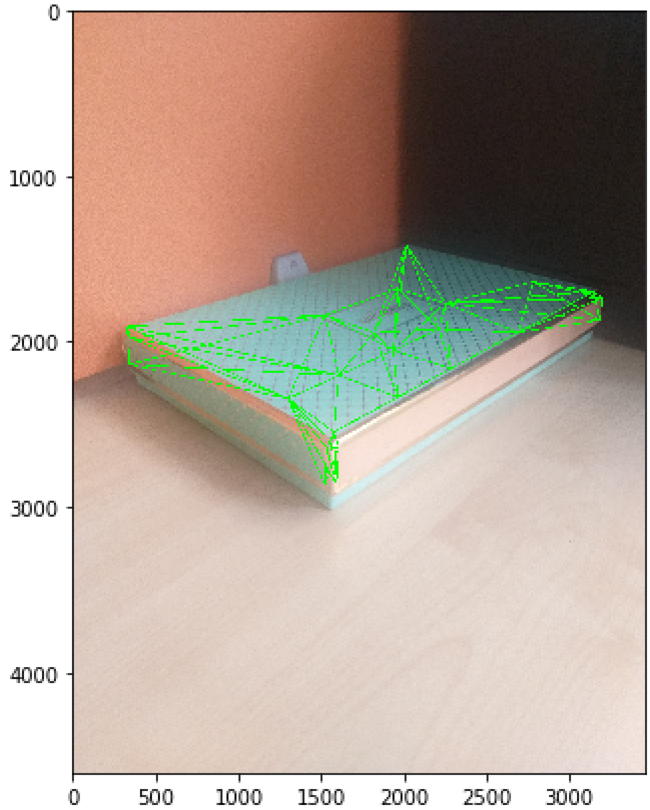
```
image_points3 = [[1605, 2655], [1619, 2868], [3179, 1879],
                 [3201, 1730], [1516, 2654], [1523, 2864],
                 [342, 2129], [324, 1961], [1563, 2550],
                 [1945, 2339], [2263, 2164], [2710, 1920],
                 [3139, 1685], [1292, 2348], [1666, 2182],
                 [1982, 2041], [2253, 1912], [2786, 1644],
                 [1803, 1969], [2219, 1781], [1944, 1699],
                 [1517, 1858], [2028, 1413], [360, 1909]]
```

DLT

In [26]:

```
P, K, R, T = DLTCalibrate(world_points3, image_points3, 24)
print ("Camera Matrix")
print (K)
image = cv2.imread("./Camera_calibration_data/20200212_152612.jpg")
image = cv2.cvtColor(image,cv2.COLOR_BGR2RGB)
ips = []
for i in range(len(world_points3)):
    ip_ = np.matmul(P, world_points3[i] + [1])
    ips += [ip_/ip_[2]]

wireframe(ips,image, line_width=5, line_color=(0,255,0),close=6)
plt.figure(figsize=(10,7))
plt.imshow(image)
plt.show()
```

```
Camera Matrix
[[ 4.06440768e+03 -2.18817437e+02  1.84644077e+03]
 [ 0.00000000e+00  3.55143145e+03  6.45406934e+02]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00]]
```



RANSAC DLT

RANSAC DLT

In [27]:

```
P, K, R, T = RANSACCalibrate(world_points3, image_points3, random_set_size=24)
print("Camera Matrix")
print(K)
image = cv2.imread("./Camera_calibration_data/20200212_152612.jpg")
image = cv2.cvtColor(image,cv2.COLOR_BGR2RGB)
ips = []
for i in range(len(world_points3)):
    ip_ = np.matmul(P, world_points3[i] + [1])
    ips += [ip_/ip_[2]]

wireframe(ips,image, line_width=5, line_color=(0,255,0),close=6)
plt.figure(figsize=(10,7))
plt.imshow(image)
plt.show()
```

```
Camera Matrix
[[ 4.06440768e+03 -2.18817437e+02  1.84644077e+03]
 [ 0.00000000e+00  3.55143145e+03  6.45406934e+02]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00]]
```