







Prelims Question Paper



MATHEMATICS CLUB IITM

September 4, 2024

Instructions

- You will be given 45 minutes to solve the 20 questions in this paper.
- Any use of online resources / gadgets is prohibited.
- Use of calculators of any kind is prohibited.

1)
$$\int_0^1 \frac{1}{1+x^3} dx$$

2)
$$\int_0^1 \frac{1}{1+x^3} dx + \int_0^1 \frac{y^3}{y^2+y^5} dy$$

3)
$$\int_0^1 \frac{x^7 - 1}{\ln x} dx$$

4)
$$3\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{dx}{\cos^2 x (1 - \tan x)^2}$$

5)
$$I_n = \int_0^1 (1 - x^{24})^n dx$$
 Find the value of $\frac{I_{26}}{I_{27}}$.

6)
$$\int_0^1 \frac{e^{x^4} x^{11}}{(x^8 + 2x^4 + 2)^2} \ dx$$

7)
$$\int_0^{\frac{\pi}{4}} \left(\frac{3\sin^2 x}{1 + \tan^2 x} - \frac{1}{\sec^4 x} \right) dx$$

8) Solve:
$$2(dy - dx) + e^{\ln(y-1)} dy = y dx - x dy$$
; $y(1) = -1$.
Find the value of $\int_{2}^{6} (x-1) dy$.

9)
$$\frac{dy}{dx} - \frac{\tan y}{1+x} = (1+x) e^x \sec y \; ; \; y(0) = 0$$

Find the value of $\int_0^1 \sin y \; dx$.

10)
$$\phi(x) = \int \left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}\right) \exp\left(a \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}\right) da$$

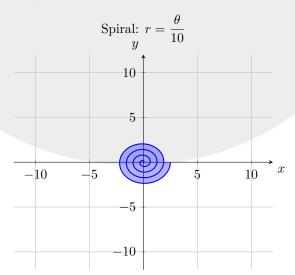
Find the value of $\phi(\pi) - \phi(0)$.

11)
$$\int_0^1 \frac{\left(\sin^{-1} x\right)^{2024} \left(\cos^{-1} x\right)^{2024}}{\sqrt{1-x^2}} \ dx$$

$$12) \quad \int_0^\pi \ln(1+\cos(x)) \ dx$$

13)
$$\lim_{n \to \infty} \frac{\log_{10} \prod_{k=1}^{n} (100n+k)}{n} - \log_{10} n \text{ (Answer in terms of } \log_{10})$$

14) Find the (shaded) area of the spiral below.



$$15) \quad \int_{-\infty}^{\infty} e^{-ex^2} e^{-iex} \ dx$$

16)
$$x'(t) = Ax(t), x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}; \text{ find } x(e).$$

17) Let:

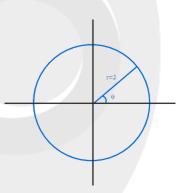
$$A = \exp\left[\left(\int_0^1 e^{-x^2} dx\right)^2\right]$$
$$B = \exp\left[-\left(\int_0^1 e^{x^2} dx\right)^2\right]$$

Which of the following is greater: $\frac{A}{e}$ or Be?

- 18) If $a = \int_0^{\frac{\pi}{2}} \sin x \, dx$ and $b = \int_0^a 2(x-1) \, dx$ Find the value of the following integral: $\int_b^a \sin(x - \sin(x -$
- 19) Integrate the function $f(x,y) = xy^2$ over the *area* of a rectangle defined by $0 \le x \le 2$ and $0 \le y \le 1$.

 Hint: You basically need to find $\int \int f(x,y) \ dA$, where dA can be written as dxdy.
- 20) Find the integral of $f(x,y) = \frac{1}{\sqrt{x^2 + y^2}}$ over the area of the circle with centre (0,0) and radius 2.

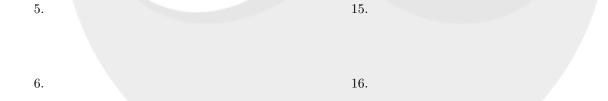
Hint: Take x and y in terms of polar coordinates, i.e, $x = r \cos \theta$ and $y = r \sin \theta$. Hence, find dA in terms of r and θ , and integrate with respect to those new variables.



Happy Solving!

Beehive Name:
Name of Bee 1: Roll no. of Bee 1:
Contact of Bee 1:
Name of Bee 2: Roll no. of Bee 2: Contact of Bee 2:





7.	17.

8.	18.