Concordia University
Department of Electrical and Computer Engineering
Real–Time Computer Control Systems (ELEC6061)
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Project 1: Discrete–Time Controller Design For Pitch Channel

Due: Tuesday Mar. 15, 2016
(The report is to be submitted in class or in the instructor's mailbox in EV5.175.)

- This project may be done in groups of **two**.

Consider the Two–Degree–of–Freedom Helicopter by Quansar (shown on page 4). This system which simulates two–dimensional flight is equipped with two propellers driven by DC motors. The front propeller (pitch propeller) is used to control the pitch angle θ and the back propeller (yaw propeller) is used to control the yaw angle ψ. A video demonstration is available on Quansar's website `http://www.quanser.com/Products/2dof_helicopter`.

The (linearized) state–space equations describing the system are

$$\begin{aligned} \underline{\dot{x}}(t) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.7451 & -0.2829 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.2701 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 & 0 \\ 37.2021 & 3.5306 \\ 0 & 0 \\ 2.3892 & 7.461 \end{bmatrix} \underline{u}(t) \qquad (1) \\ \underline{y}(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underline{x}(t) \end{aligned}$$

where

$$\underline{x} = \begin{bmatrix} \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix}, \qquad \underline{u} = \begin{bmatrix} v_p \\ v_y \end{bmatrix}, \qquad \underline{y} = \begin{bmatrix} \theta \\ \psi \end{bmatrix}$$

Here v_p and v_y are the pitch and yaw motor voltages (in volts). The angles and angle rates are in deg and deg/sec.

It follows from eq. (1) that

$$\begin{aligned} \theta(s) &= \frac{37.2021}{s^2 + 0.2830s + 2.7452} V_p(s) + \frac{3.5306}{s^2 + 0.2830s + 2.7452} V_y(s) \\ \psi(s) &= \frac{2.3892}{s(s + 0.2701)} V_p(s) + \frac{7.461}{s(s + 0.2701)} V_y(s) \end{aligned}$$

Note that the yaw propeller affects the pitch angle and the pitch propeller affects the yaw angle. In other words, the dynamics of pitch and yaw channels are coupled.

In this project, the objective is to design a (single–input–single–output) discrete–time controller for the pitch angle. For the design of pitch channnel, we ignore the coupling and treat the yaw voltage as disturbance. In the next project, we design the yaw controller and study the effects of coupling in simulations. Following the above approach, the block diagram for the pitch channel will be as in Fig. 1.

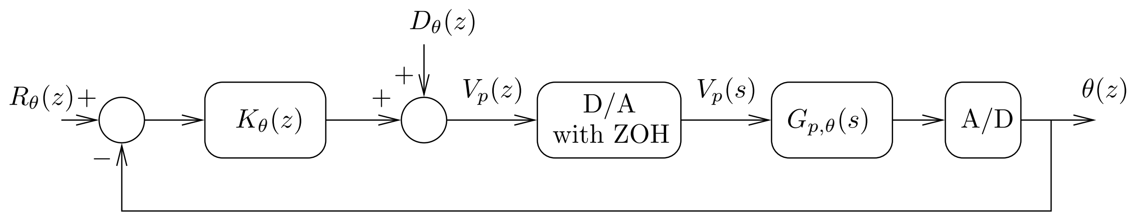


Figure 1: Pitch channel.

where $G_{p,\theta}(s) = \theta(s)/V_p(s) = 37.2021/(s^2 + 0.2830s + 2.7452)$. $D_\theta(z)$ is disturbance. The design specifications for the pitch channel are as follows.

(DS1) Percentage of overshoot for step reference input $\leq 20\%$

(DS2) Settling time of step response ≤ 18 sec.

(DS3) Rise time of step response ≤ 3 sec.

(DS4) Steady–state error for step reference input $= 0$

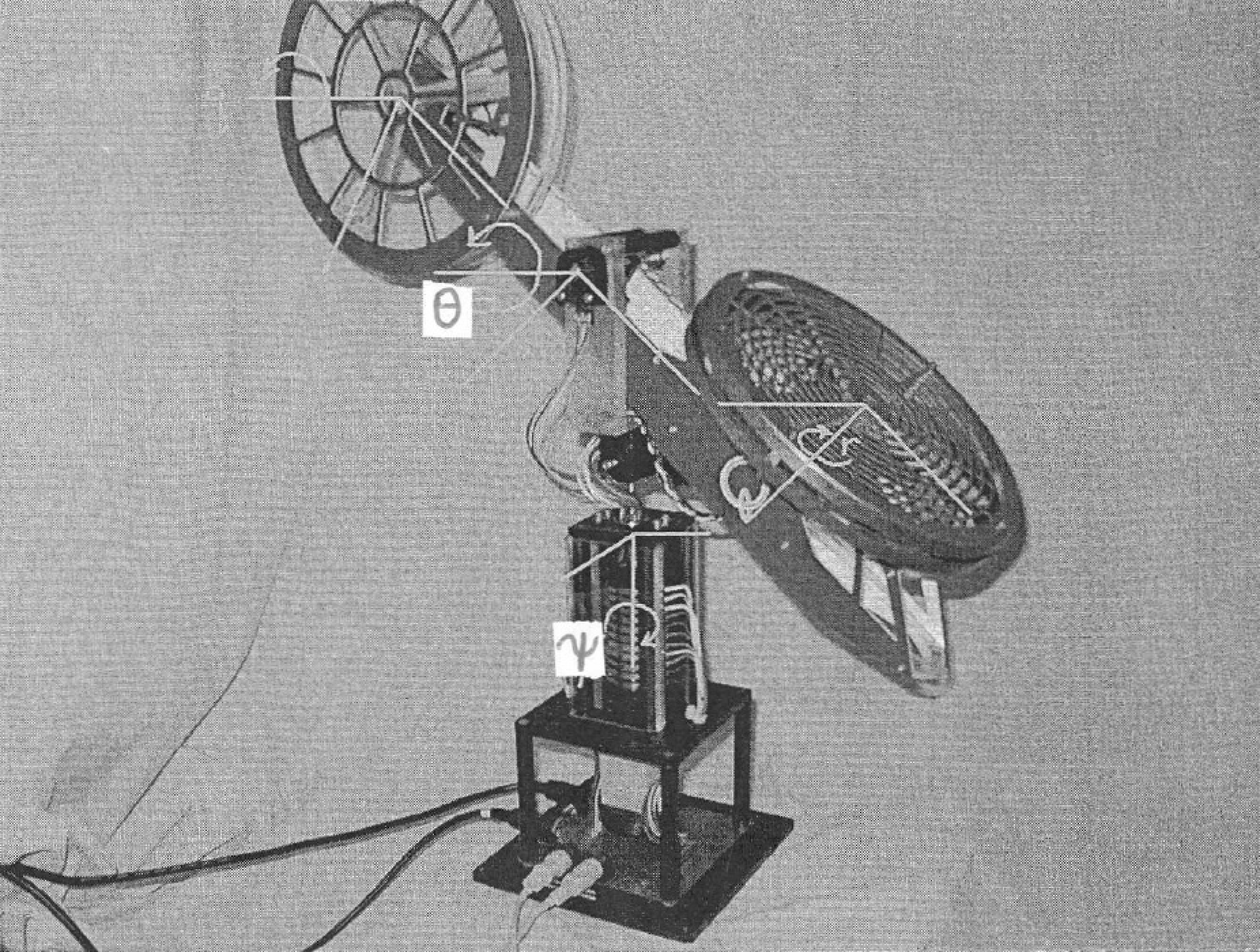
(DS5) Steady–state output in response to step disturbance $= 0$.

(A) Choose a reasonable (not too high, not too low) sampling rate, obtain the zero–order–hold discrete equivalent of the plant and design controller for the pitch channel. The design should be based on root locus or frequency response (Bode plots) techniques. Provide:

1. the details of your design calculations,
2. MATLAB commands (m-files),
3. the response of the closed–loop system ($\theta[n]$) to unit step reference input ($r[n] = 1[n]$ and $d_\theta[n] = 0$).
4. the response of the closed–loop system ($\theta[n]$) to unit step disturbance ($r[n] = 0$ and $d_\theta[n] = 1[n]$).

(B) Obtain motor voltage $v_p[n]$ in response to step reference input in the pitch channel. Submit the graph. Suppose the DC motor saturates if its input voltage exceed 8 volts. What is the maximum size of step reference input that does not result in motor saturation?

All design and simulations have to be done using MATLAB's Control System Toolbox and the m-files must be submitted. The students may wish to use Siso Design Tool or Simulink additionally but these results do not replace Control System Toolbox simulations.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