Concordia University
Department of Electrical and Computer Engineering
Real–Time Computer Control Systems (ELEC6061)
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Project 2: Discrete–Time Controller Design For a Two–DOF Helicopter

Due: Thursday April 21, 2016
(The report may be submitted in the instructor's mailbox in EV5.175.)

- This project may be done in groups of **two**.

This project is a followup to the previous one. As discussed in Project 1, the Two–Degree–of–Freedom Helicopter by Quansar is equipped with two propellers driven by DC motors. The front propeller (pitch propeller) is used to control the pitch angle θ and the back propeller (yaw propeller) is used to control the yaw angle ψ. The objective in this project is to design two (single–input–single–output) discrete–time controllers for the pitch and yaw angles.

Open–loop System (Plant)

The (linearized) state–space equations describing the open–loop system are

$$\begin{aligned} \underline{\dot{x}}(t) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.7451 & -0.2829 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -0.2701 \end{bmatrix} \underline{x}(t) + \begin{bmatrix} 0 & 0 \\ 37.2021 & 3.5306 \\ 0 & 0 \\ 2.3892 & 7.461 \end{bmatrix} \underline{u}(t) \qquad (1) \\ \underline{y}(t) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underline{x}(t) \end{aligned}$$

where

$$\underline{x} = \begin{bmatrix} \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \end{bmatrix}, \qquad \underline{u} = \begin{bmatrix} v_p \\ v_y \end{bmatrix}, \qquad \underline{y} = \begin{bmatrix} \theta \\ \psi \end{bmatrix}$$

Here v_p and v_y are the pitch and yaw motor voltages (in volts). The angles and angle rates are in deg and deg/sec.

It follows from eq. (1) that

$$\begin{aligned} \theta(s) &= \frac{37.2021}{s^2 + 0.2830s + 2.7452} V_p(s) + \frac{3.5306}{s^2 + 0.2830s + 2.7452} V_y(s) \\ \psi(s) &= \frac{2.3892}{s(s + 0.2701)} V_p(s) + \frac{7.461}{s(s + 0.2701)} V_y(s) \end{aligned}$$

Note that the yaw propeller affects the pitch angle and the pitch propeller affects the yaw angle. In other words, the dynamics of pitch and yaw channels are coupled.

Controller Design

In this project, the objective is to design two single–input–single–output discrete–time controllers, $K_\theta(z)$ and $K_\psi(z)$, for the pitch and yaw angles (Fig. 1).

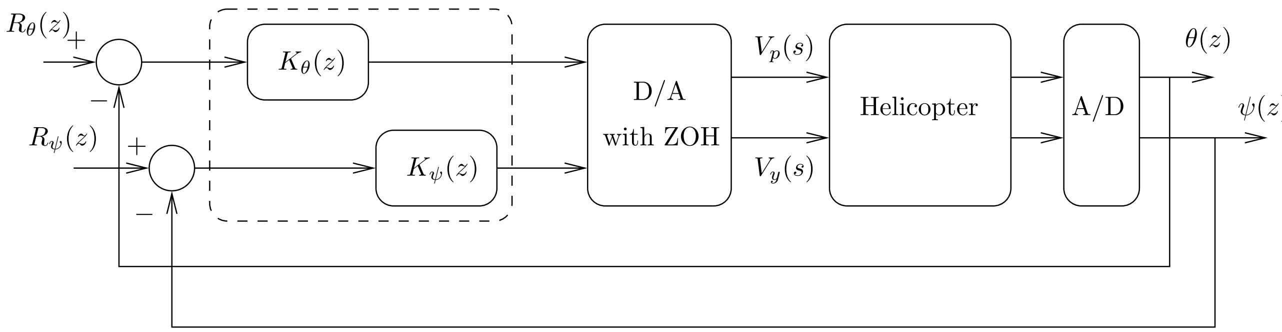

Figure 1: Closed–loop system.

For the design of pitch controller ($K_\theta(z)$), we ignore the coupling and treat the yaw voltage as disturbance. Similarly for the yaw controller ($K_\psi(z)$), the pitch voltage is treated as disturbance. Later, using computer simulation, we investigate the effects of coupling.

Pitch channel: Following the above approach, the block diagram for the design of controller for the pitch channel will be as in Fig. 2 where

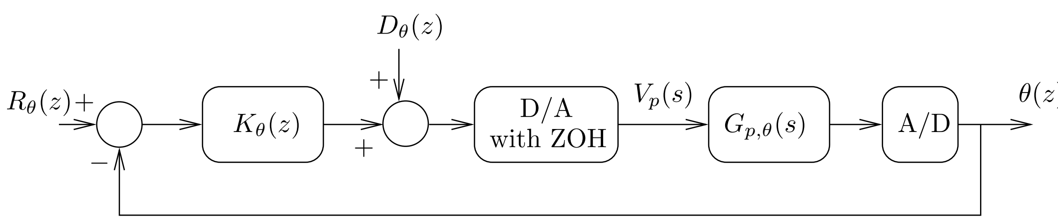

Figure 2: Pitch channel.

$$G_{p,\theta}(s) = \frac{\theta(s)}{V_p(s)} = \frac{37.2021}{s^2 + 0.2830s + 2.7452}$$

$D_\theta(z)$ is disturbance.

The design specifications for the pitch channel are the same as in Project 1:

(DS1) Percentage of overshoot for step reference input $\leq 20\%$

(DS2) Settling time of step response ≤ 18 sec.

(DS3) Rise time of step response ≤ 3 sec.

(DS4) Steady–state error for step reference input = 0

(DS5) Steady–state output in response to step disturbance = 0.

Yaw channel: The block diagram of the yaw channel is shown in Fig. 3 where

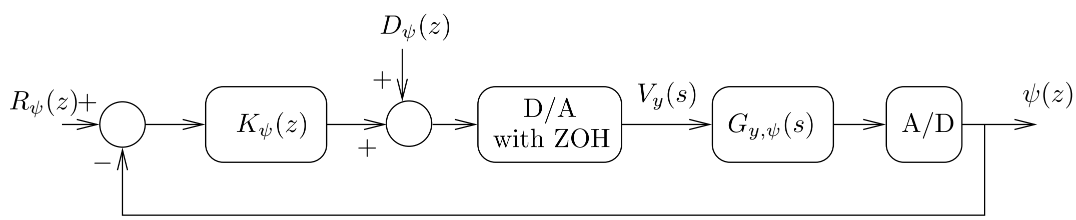

Figure 3: Yaw channel.

$$G_{p,\psi}(s) = \frac{\psi(s)}{V_y(s)} = \frac{7.461}{s(s+0.2701)}$$

$D_\psi(z)$ is disturbance. The design specifications for the yaw channel are the same as those of the pitch channel.

Design and Simulation

(A) Choose a reasonable (not too high, not too low) sampling rate, obtain the zero–order–hold discrete equivalent of the plant and design controllers for the pitch and yaw channels. The design should be based on root locus or frequency response (Bode plots) techniques. For each channel provide:

1. the details of your design calculations,
2. MATLAB commands (m-files),
3. the response of the closed–loop system to unit step reference input, with disturbance set to zero.
4. the response of the closed–loop system to unit step disturbance, with reference input set to zero.

For the pitch channel you can use your design from Project 1.

(B) In part(A), each channel was examined separately. Now we simulate the entire system (Fig. 1) and investigate the effects of the coupling of pitch and yaw channels.

To simulate the closed–loop system, you may first obtain state–space realizations for your controllers ($K_\theta(z)$, $K_\psi(z)$) (using `ss`) and then `append` them to obtain a state–space model for your controller. Next, form the `series` (cascade) connection of the controller and the ZOH discrete equivalent of the plant (eq. (1)), and form the closed-loop system (using `feedback`). Finally, simulate the closed–loop system. For simulation, obtain pitch θ and yaw ψ in response to a step input in pitch reference input ($r_\theta[n] = 1[n]$, $r_\psi[n] = 0$). Next, repeat the simulation to obtain the system response to unit step in the yaw reference input ($r_\theta[n] = 0$, $r_\psi[n] = 1[n]$). Provide MATLAB commands (m-files) and the outputs for both simulations (four graphs).

Compare the step responses in (A) and (B) and comment on the effects of cross-coupling. Is the cross-coupling observed in simulations acceptable? Justify your answer. Start by choosing a criterion to assess the effects of cross-coupling.

Note: All design and simulations have to be done using MATLAB's Control System Toolbox and the m-files must be submitted. The students may wish to use Siso Design Tool, Linear System Analyzer app or Simulink *additionally* but these results do not replace Control System Toolbox simulations.