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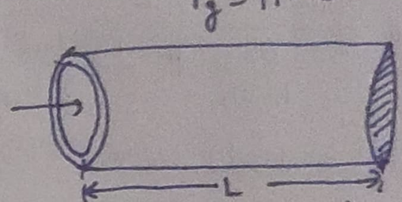
Heat Transfer Project Assignment

Q-1- Ethylene glycol is heated from 25°C to 40°C at rate of 2.5 kg/s in horizontal copper tube ($k = 386\ W/mK$) with an inner diameter of 2 cm and outer diameter of 2.5 cm. A saturated vapour ($T_g = 110°C$) condenses on outside-tube surface with an heat-transfer coefficient (in kW/m^2K) given by $9.2/(T_g - T_w)^{0.25}$, where T_w is average outside tube wall temperature. What tube length must be used? Take properties of ethylene glycol to be $\rho = 1109\ kg/m^3$, $C_p = 2428\ J/kgK$, $k = 0.253\ W/mK$, $\mu = 0.01545\ kg/m\text{-}s$ and $Pr = 148.5$.



Sol:→ $\dot{Q} = \dot{m} c_p (T_o - T_i)$

Rate of heat transfer; outlet temperature; inlet temperature

$$Q = 2.5 \times 2428 \times (40-25) = 91050\,W$$

fluid velocity: $$V = \frac{\dot{m}}{\rho A_c} = \frac{2.5}{1109 \times \left[\frac{\pi (0.02)^2}{4}\right]} = 7.176\ m/s$$

Reynolds no: $$Re = \frac{\rho V D}{\mu} = \frac{1109 \times 7.176 \times 0.02}{0.01545} = 10{,}302 > 10{,}000$$

So, we have fully developed flow & we will evaluate Nusselt number from turbulent flow relation:→

$$Nu = \frac{hD}{k} = 0.023\, Re^{0.8} Pr^{0.4} = 0.023\,(10302)^{0.8}(148.5)^{0.4}$$

$$Nu = 275.9$$

So $$h_i = \left(\frac{k}{D}\right) Nu = \frac{0.253}{0.02} \times 275.9 = 3490\ W/m^2\text{-}°C$$

Heat transfer coefficient on inner surface

Assuming wall temperature of 100°C, the heat transfer coefficient on outer surface is determined to be:-

$$h_o = 9200\,(T_g - T_w)^{-0.25} = 9200\,(110-100)^{-0.25} = 5174\ W/m^2.°C$$

Let us check if assumption for wall temperature holds:

$$h_i A_i (T_w - T_{b,avg}) = h_o A_o (T_g - T_w)$$

$$h_i \pi D_i L (T_w - T_{b,avg}) = h_o \pi D_o L (T_g - T_w)$$

$3490 \times 0.02\,(T_w - 32.5) = 5174 \times 0.025\,(110 - T_w)$

$T_w\,[3490 \times 0.02 + 5174 \times 0.025] = 5174 \times 0.025 \times 110 + 3490 \times 0.02 \times 32.5$

$$T_w = \frac{16497}{199.149} = \frac{16497}{199.149} = 82.837 \simeq 82.84\,°C$$

Now we assume a wall temperature of 80°C

$h_o = 9200\,(T_g - T_w)^{-0.25} = 9200\,(110-80)^{-0.25} = 3931\ W/m^2\text{-}°C$

Again checking: $3490 \times 0.02 \times (T_w - 30) = 3931 \times 0.025 \times (110 - T_w)$

$T_w\,[3490 \times 0.02 + 3931 \times 0.025] = 3931 \times 0.025 \times 110 + 3490 \times 0.02 \times 30$

$$T_w = \frac{12904.25}{168.075} \Rightarrow T_w = 76.8\,°C$$ close to assumed value of 80°c

Overall heat transfer coefficient based on outer surface area:

$$U_o = \frac{1}{\frac{D_o}{h_i D_i} + \frac{D_o \ln(D_2/D_1)}{2 K_{copper}} + \frac{1}{h_o}} = \frac{1}{\frac{0.025}{3490 \times 0.02} + \frac{0.025 \ln(2.5/2)}{2(386)} + \frac{1}{3931}}$$

$U_o = 1613\ W/m^2\text{-}°C$

Rate of heat transfer: $\dot{Q} = U_o A_o \Delta T_{lm}$ log mean temperature difference

$$\Delta T_{lm} = \frac{(T_g - T_e) - (T_g - T_i)}{\ln\left(\frac{T_g - T_e}{T_g - T_i}\right)} = \frac{(110-40) - (110-25)}{\ln\left(\frac{110-40}{110-25}\right)} = \frac{-15}{\ln\left(\frac{70}{85}\right)}$$

$$\Delta T_{lm} = \frac{-15}{\ln(0.823529)} = \frac{\cancel{-}15}{\cancel{-}0.194156} = 77.257 \simeq 77.26\,°C$$

$\dot{Q} = U_o A_o \Delta T_{lm}$

$91050 = 1613 \times \pi \times (0.025) \times L \times 77.26$

$$L = \frac{91050}{1613 \times 3.14 \times 0.025 \times 77.26} = \frac{91050}{9782.69983} = 9.307\ m$$

Hence, Tube length is 9.307 m.

Q-2- A single-pass cross flow heat exchanger is used to cool jacket water ($C_p = 1.0$ Btu/lbm.°F) of a diesel engine from 190°F to 140°F, using air ($C_p = 0.245$ Btu/lbm-°F) with inlet temperature of 90°F. Both air flow & water flow are unmixed. If the water and air mass flow rates are 92,000 lbm/hr and 400000 lbm/hr, respectively. Determine the log mean temperature difference for this heat exchanger.

Sol → $\dot{Q} = \dot{m}_h C_{ph} (T_{h,in} - T_{h,out}) = 92,000 \times 1.0 \times (190 - 140)$

Rate of heat transfer $= 92000 \times 50 \Rightarrow \boxed{\dot{Q} = 4600000 \text{ Btu/h}}$

Since heat transfer from hot liquid (fluid) is equal to the heat transfer to cold fluid, we have:

$$\dot{Q} = \dot{m}_c C_{pc} (T_{c,out} - T_{c,in}) \Rightarrow T_{c,out} = \frac{\dot{Q}}{\dot{m} C_{pc}} + T_{c,in}$$

$$T_{c,out} = \frac{46 \times 10^5}{4 \times 10^5 \times 0.245} + 90 = 46.9387 + 90$$

$$\boxed{T_{c,out} = 136.9387\,^\circ F}$$

Log mean temperature difference for counter-flow arrangement is:

$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{(190 - 136.9387) - (140 - 90)}{\ln[(190 - 136.9387)/(140 - 90)]}$$

$$\Delta T_{lm,CF} = \frac{53.0613 - 50}{\ln\left(\frac{53.0613}{50}\right)} = \frac{3.0613}{0.0594248} \Rightarrow \boxed{\Delta T_{lm,CF} = 51.515\,^\circ F}$$

Using Fig 11-18 c of Cengel's Book. Correction factor can be determined to be →

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{140 - 190}{90 - 190} = \frac{-50}{-100} = 0.50$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{90 - 136.9387}{140 - 190} = \frac{-46.9387}{-50} = 0.938774 \approx 0.94$$

$\boxed{F = 0.91}$

Log-mean temperature difference for cross-flow arrangement:

$$\Delta T_{lm} = F \cdot \Delta T_{lm,CF} = 0.91 \times 51.615 \Rightarrow \boxed{\Delta T_{lm} = 46.96965\,^\circ F}$$

Correlation factor which represents how closely cross-flow heat exchanger approximates counter-flow heat exchanger in terms of its logarithmic mean temperature difference.

Q-3. Saturated liquid benzene flowing at a rate of 5 kg/s is to be cooled from 75°C to 45°C by using source of cold water ($C_p = 4187$ J/kgK) flowing at 3.5 kg/s and 15°C through a 20 mm diameter tube of negligible wall thickness. The overall heat transfer coefficient of heat exchanger is estimated to be 750 W/m²K. If specific heat of liquid benzene is 1939 J/kg K and assuming that capacity ratio and effectiveness remain the same, determine the heat exchanger surface area for following four heat exchangers →

(a) parallel flow (b) counter flow
(c) shell and tube heat exchanger with 2-shell passes and 40-tube passes
(d) cross-flow heat exchanger with one fluid mixed (liquid benzene) and other fluid unmixed (water).

Sol: We have made some assumptions →
① Steady state conditions exist ② Heat exchanger is well insulated
③ Fluid properties remain constant ④ No fouling inside heat exchanger

Heat balance b/w saturated liquid benzene solution and cooling water :→ Heat lost by liquid benzene = Heat gained by water

$$\dot{m}_h C_{ph} (T_{h,in} - T_{h,out}) = \dot{m}_c C_{pc} (T_{c,out} - T_{c,in})$$

$$T_{c,out} = \frac{\dot{m}_h C_{ph}}{\dot{m}_c C_{pc}} (T_{h,in} - T_{h,out}) + T_{c,in}$$

$$T_{c,out} = \frac{10 \cancel{50} \times 1839 \times (75-45)}{7 \cancel{35} \times 4187} + 15 = \frac{551700}{29309} + 15$$

$$T_{c,out} = 18.823569 + 15 \Rightarrow \boxed{T_{c,out} = 33.824\,°C}$$

In order to use effectiveness - NTU method, we first need to determine heat capacity rates, capacity ratio and effectiveness of heat exchanger.

Heat capacity rate of cold fluid (water) is : $C_c = \dot{m}_c C_{pc}$

$$C_c = 3.5 \times 4187 \Rightarrow \boxed{C_c = 14654.5\ W/K}$$

Heat capacity rate of hot fluid (liquid benzene) is : $C_h = \dot{m}_h C_{ph}$

$$C_h = 5 \times 1839 \Rightarrow \boxed{C_h = 9195\ W/K}$$

Thus, capacity ratio is : $c = \frac{C_{min}}{C_{max}} = \frac{9195}{14654.5} = \underline{0.627}$.

(a) For parallel flow arrangement :→

$$\therefore \varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{C_h (T_{h,in} - T_{h,out})}{C_{min} (T_{h,in} - T_{c,in})} = \frac{75-45}{75-15} = \frac{30}{60} = 0.5$$

So No. of transfer units (NTU) :→ $NTU = -\frac{\ln[1 - \varepsilon(1+c)]}{1+c}$

$$NTU = -\frac{\ln[1 - 0.5(1+0.627)]}{1+0.627} = \frac{-\ln(0.1865)}{1.627} = \frac{1.6793}{1.627}$$

$$\boxed{NTU = 1.032}$$

surface area : $A_s = \frac{(NTU)\, C_{min}}{U} = \frac{1.032 \times 9195}{750} = \frac{9489.24}{750}$

$$\boxed{A_s = 12.65\ m^2}$$

So for parallel flow arrangement, with 12.65 m^2 surface area the tube length is : $L = \frac{A_s}{\pi D} = \frac{12.65}{3.14 \times 0.02} \Rightarrow \boxed{L = 201.33\ m}$

(b) for counter flow arrangement with $\varepsilon = 0.5$ from part (a).

$$NTU = \frac{1}{c-1} \ln\left(\frac{\varepsilon - 1}{c\varepsilon - 1}\right) = \frac{1}{0.627-1} \ln\left(\frac{0.5 - 1}{0.627 \times 0.5 - 1}\right) = 0.85$$

Surface area : $A_s = \frac{(NTU)\,C_{min}}{U} = \frac{0.85 \times 9195}{750} = \frac{7815.75}{750}$

$\boxed{A_s = 10.421\ m^2}$

So for counter flow arrangement with 10.42 m^2 surface area, tube length is : $L = \frac{A_s}{\pi D} = \frac{10.421}{3.14 \times 0.02} \Rightarrow \boxed{L = 165.85\ m}$

(c) For two shell passes and 40 tube passes, calculated effectiveness of 0.5 from part (a) is for 2 shell passes so $\boxed{\varepsilon_2 = 0.5}$.

$$F = \left[\frac{c\,\varepsilon_2 - 1}{\varepsilon_2 - 1}\right]^{1/2} = \left[\frac{0.627 \times 0.5 - 1}{0.5 - 1}\right]^{0.5} \Rightarrow \boxed{F = 1.172}$$

effectiveness of heat exchanger in case of one shell pass would be : $\varepsilon_1 = \frac{F-1}{F-c} = \frac{1.172-1}{1.172-0.627} \Rightarrow \boxed{\varepsilon_1 = 0.315}$

$$NTU_1 = \frac{-1}{\sqrt{1+c^2}} \ln\left[\frac{2/\varepsilon_1 - 1 - c - \sqrt{1+c^2}}{2/\varepsilon_1 - 1 - c + \sqrt{1+c^2}}\right] = \frac{-1}{\sqrt{1+0.627^2}} \ln\left[\frac{2/0.315 - 1 - 0.627 - \sqrt{1+0.627^2}}{2/0.315 - 1 - 0.627 + \sqrt{1+0.627^2}}\right]$$

$\boxed{NTU_1 = 0.433}$

Assuming that NTU is distributed equally during each pass:→

For multiple passes, $NTU_2 = n\,NTU_1$

So for two shell passes : $NTU_2 = 2\,NTU_1 = 2 \times 0.433$

$\boxed{NTU_2 = 0.866}$

Surface area, $A_s = \frac{(NTU_2)\,C_{min}}{U} = \frac{0.866 \times 9195}{750} = \frac{7962.87}{750}$

$\boxed{A_s = 10.617\ m^2}$

So for two shell and 40 tube pass flow arrangement with 10.62 m^2 surface area, tube length is : $L = \frac{A_s}{n \times \pi D} = \frac{10.617}{\pi \times (40 \times 0.02)}$

no. of tubes

$L = \frac{10.617}{\pi \times (0.8)} \Rightarrow \boxed{L = 4.224\ m}$

(d) For cross flow heat exchanger with liquid benzene (mixed) and cooling water (unmixed) and $\varepsilon = 0.5$ from part (a) we get →

$$NTU = -\frac{\ln[c\ln(1-\varepsilon)+1]}{c} = -\frac{\ln[0.627\ln(1-0.15)+1]}{0.627}$$

$$NTU = \frac{-\ln(0.565396)}{0.627} = \frac{0.5702289}{0.627} \Rightarrow \boxed{NTU = 0.90945}$$

Surface area: $A_s = \frac{(NTU)\,C_{min}}{U} = \frac{0.90945 \times 9195}{750} = \frac{8362.39275}{750}$

$$\boxed{A_s = 11.149\ m^2}$$

Q-4- During an experiment, a plate heat exchanger that is used to transfer heat from a hot water stream to a cold-water stream is tested, and following measurements are taken :→

* For Hot Water Stream :→ Inlet Temp = $T_{h,in} = 38.9°C$, Outlet Temp = $T_{h,out} = 27.0°C$
  Volume flow rate = $\dot{V}_h = 2.5\ L/min$

* For Cold Water Stream :→ Inlet Temp = $T_{c,in} = 14.3°C$, Outlet Temp = $T_{c,out} = 19.8°C$
  Volume flow rate = $\dot{V}_c = 4.5\ L/min$

The heat transfer area is calculated to be $0.040\ m^2$

(a) Calculate rate of heat transfer to cold water

(b) Calculate the overall heat transfer coefficient

(c) Determine if heat exchanger is truly adiabatic. If it is not, determine fraction of heat loss & calculate heat transfer efficiency.

(d) Determine effectiveness and NTU values of heat exchanger

Also, discuss if measured values are reasonable

Sol. - We have made some assumptions →

① Steady operating conditions exist

② Changes in kinetic and potential energies of fluid stream are negligible ③ Fluid properties are constant

Some properties :→ At average temperature : $T_{h,avg} = \frac{T_{h,in} + T_{h,out}}{2}$ } for hot water

At $T_{h,avg} = \frac{38.9+27}{2} = 32.95°C$: $\boxed{\rho_{h,water} = 994.8\ kg/m^3 \quad c_{ph} = 4178\ J/kg\cdot°C}$

At average temperature : $T_{c,avg} = \frac{T_{c,in} + T_{c,out}}{2} = \frac{14.3+19.8}{2}$ } for cold water

At $T_{c,avg} = 17.05°C$: $\boxed{\rho_{c,water} = 998.6\ kg/m^3 \quad C_{pc} = 4184\ J/kg\text{-}°C}$

Mass flow rates are :

$\dot{m}_h = \rho_h \dot{V}_h = \dfrac{994.8 \times 2.5 \times 10^{-3}}{60} = \boxed{41.45 \times 10^{-3}\ kg/sec}$

$\dot{m}_c = \rho_c \dot{V}_c = \dfrac{998.6 \times 4.5 \times 10^{-3}}{60} = \boxed{74.895 \times 10^{-3}\ kg/sec}$

Rates of heat transfer from hot water & to cold water are →

$\dot{Q}_h = [\dot{m}\, c_{ph} (T_{h,in} - T_{h,out})]_h = 41.45 \times 10^{-3} \times 4178 \times (38.9 - 27)$

$\boxed{\dot{Q}_h = 2060.819\ W}$

$\dot{Q}_c = [\dot{m}\, c_{pc} (T_{c,out} - T_{c,in})]_c = 74.895 \times 10^{-3} \times 4184 \times (19.8 - 14.3)$

$\boxed{\dot{Q}_c = 1723.483 W}$

(b) Logarithmic mean temperature difference and overall heat transfer coefficient are :→

$\Delta T_1 = T_{h,in} - T_{c,out} = 38.9 - 19.8 = 19.1\,°c$

$\Delta T_2 = T_{h,out} - T_{c,in} = 27 - 14.3 = 12.7\,°c$

$\Delta T_{lm} = \dfrac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \dfrac{19.1 - 12.7}{\ln\left(\frac{19.1}{12.7}\right)} = \dfrac{6.4}{0.408} \Rightarrow \boxed{\Delta T_{lm} = 15.68\,°c}$

$U = \dfrac{\dot{Q}_{ne,avg}}{A\, \Delta T_{lm}} = \dfrac{\dot{Q}_h + \dot{Q}_c}{2\, A\, \Delta T_{lm}} = \dfrac{(1723.483 + 2060.819)}{2 \times 0.04 \times 15.68} = \dfrac{3784.302}{1.2544}$

$\boxed{U = 3016.82\ W/m^2\text{-}°c}$

(c) Fraction of heat loss : $f_{loss} = \dfrac{\dot{Q}_h - \dot{Q}_c}{\dot{Q}_h} = \dfrac{2060.819 - 1723.483}{2060.819}$

$f_{loss} = \dfrac{337.336}{2060.819} \Rightarrow \boxed{f_{loss} = 0.16369 = 16.369\,\%}$

Heat transfer efficiency : $\eta = \dfrac{\dot{Q}_c}{\dot{Q}_h} = \dfrac{1723.483}{2060.819} = 0.8363$

$\boxed{\eta = 0.8363 = 83.63\,\%}$

(d) Heat capacity rates of hot & cold fluids are →

$C_h = \dot{m}_h c_{ph} = 41.45 \times 10^{-3} \times 4178 \Rightarrow \boxed{C_h = 173.178\ W/°c}$

$C_c = \dot{m}_c c_{pc} = 74.895 \times 10^{-3} \times 4184 \Rightarrow \boxed{C_c = 313.36\ W/°c}$

So, $\boxed{C_{min} = C_h = 173.178\ W/°c}$ {Smaller b/w C_c & C_h}

maximum heat transfer rate : $\dot{Q}_{max} = C_{min} (T_{h,in} - T_{c,in})$

$\dot{Q}_{max} = 173.178 \times (38.9 - 14.3) \Rightarrow \boxed{\dot{Q}_{max} = 4260.178\ W}$

Effectiveness of the heat exchanger is :

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{3784.302/2}{4260.178} = \frac{1892.151}{4260.178} \Rightarrow \boxed{\varepsilon = 0.444 = 44.4\%}$$

Number of transfer units : $NTU = \frac{UA}{C_{min}} = \frac{3016.82 \times 0.04}{173.178}$

$$\boxed{NTU = 0.6968}$$

A/c to me : all measured values are reasonable

Q-5 :- A heat exchanger is to be selected to cool a hot liquid chemical at specified rate to a specified temperature. Explain steps involved in the selection process.

Sol :- Steps are as follows :-

Step-1 : Calculate heat transfer rate

Step-2 : Select a suitable type of heat exchanger

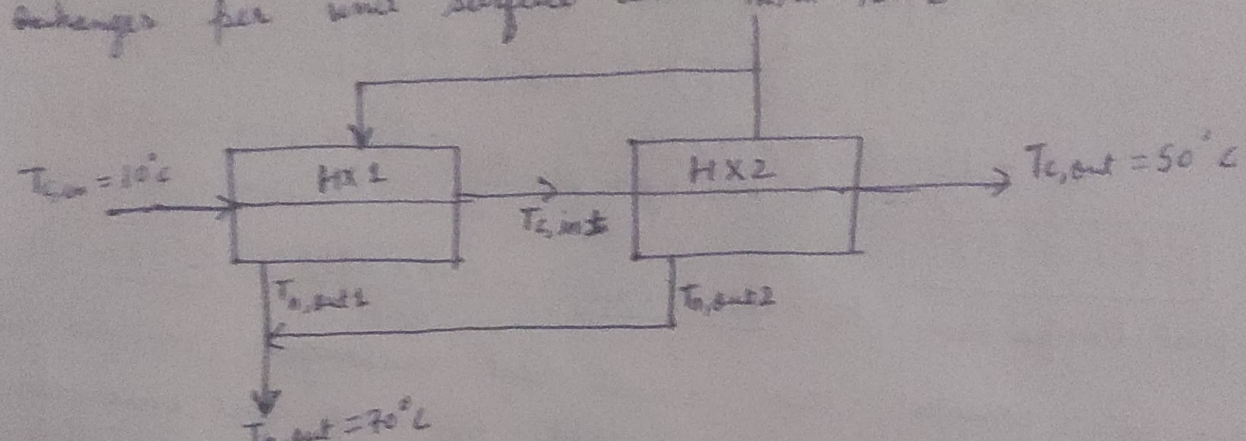
Step-3 : Select a suitable type of cooling fluid & its temperature range

Step-4 : Calculate or Select U

Step-5 : Calculate the size (surface area) of heat exchanger

Q-6 :- Heat Exchanger Design Analysis :-

In an industrial facility, a countercurrent double pipe heat exchanger is used to heat glycerin flowing at a rate of 1.5 kg/s from 10°C to 50°C by passing hot water at an inlet temperature of 120°C. The hot water experiences a temperature drop of 50°C as it flows through heat exchanger. Overall heat transfer coefficient of heat exchanger may be assumed to be 950 W/m^2K. A similar arrangement is to be installed at another location and it is proposed to use two small heat exchangers of same surface area to be arranged in series instead of one single large heat exchanger. However, it is first required to compare surface area of two small heat exchangers against that of single large heat exchanger, since available space is major constraint for proposed design. Arrangement of heat exchangers is as shown figure. Water flow rate is split b/w two heat exchangers such that 60% would go to first heat exchanger and remaining 40% would go to second heat exchanger. The overall heat transfer coefficient of two small heat exchangers is assumed to remain same as that of large heat exchanger. As a design engineer, for both cases determine the heat exchanger (a) effectiveness, (b) NTU, (c) surface area, and (d) choice of heat exchanger if construction cost of smaller heat exchangers is about 15% higher than single large heat exchanger per unit surface area.



Solution :→ **Assumptions** : ① Steady state conditions exist.

② Fluid properties remain constant

③ Heat exchanger is well-insulated

④ Negligible fouling resistance.

Properties :- Specific heat of glycerin is calculated at an inlet and exit average temperature of $(10+50)/2 = 30°C$: $\boxed{c_{pc} = 2447 J/kg K}$

Analysis : Let's first consider 'Large single heat exchanger'.

Energy balance within heat exchanger gives →

$\dot{Q} = \dot{m}_h c_{ph} (T_{h,in} - T_{h,out}) = \dot{m}_c c_{pc} (T_{c,out} - T_{c,in})$

$m_h c_{ph} (120-70) = 1.5 \times 2447 \times (50-10) = 146820 W$

$m_h c_{ph} = \frac{146820}{50} = 2936.4 W/K$

So Heat capacity rate of hot fluid : $\boxed{C_h = m_h c_{ph} = 2936.4 W/K}$

Heat capacity rate for cold fluid : $C_c = m_c c_{pc}$

$C_c = 1.5 \times 2447 \Rightarrow \boxed{C_c = 3670.5 W/K}$

Thus, $\boxed{C_{min} = 2936.4 W/K = C_h}$

(a) Effectiveness of large single heat exchanger is :-

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{min}(T_{h,in} - T_{c,in})} = \frac{3670.5 \times (50-10)}{2936.4 \times (120-10)}$$

$$\varepsilon = \frac{3670.5 \times 40}{2936.4 \times 110} = \frac{14682}{32300.4} \Rightarrow \boxed{\varepsilon = 0.4545}$$

Capacity ratio of large single heat exchanger is :-

$$c = \frac{C_{min}}{C_{max}} = \frac{2936.4}{3670.5} \Rightarrow \boxed{c = 0.8}$$

(b) From counter-flow relation :→ The number of transfer units (NTU) for single heat exchanger is determined as →

$$NTU = \frac{1}{c-1} \ln\left(\frac{\varepsilon - 1}{c\varepsilon - 1}\right) = \frac{1}{0.8-1} \ln\left(\frac{0.4545-1}{0.8 \times 0.4545 - 1}\right)$$

$$= \frac{-10}{2} \ln\left(\frac{\cancel{-}0.5455}{\cancel{-}0.6364}\right) = -5 \ln(0.857165305)$$

$$\boxed{NTU = 0.77}$$

(c) Heat transfer area : $A_s = \frac{(NTU)\, C_{min}}{U} = \frac{0.77 \times 2936.4}{950} = \boxed{2.38\, m^2}$

Now, let's consider two small heat exchangers connected in series such that flow of water is split b/w two heat exchangers such that 60% would go to first heat exchanger and remaining 40% would go to second heat exchanger. Heat capacity rate of first heat exchanger is : $C_{h1} = 0.6 \times 2936.4 \Rightarrow \boxed{C_{h,1} = 1761.8\, W/K}$

while that of second heat exchanger : $C_{h2} = 0.4 \times 2936.4$

$\Rightarrow \boxed{C_{h,2} = 1174.6\, W/K}$.

Heat capacity rate of cold side remains unchanged i.e 3670.5 W/K

Capacity ratio of first heat exchangers is : $c_1 = \frac{C_{min}}{C_{max}}$

$$c_1 = \frac{1761.8}{3670.5} \Rightarrow \boxed{c_1 = 0.48}$$

Capacity ratio of second heat exchangers is $c_2 = \frac{C_{min}}{C_{max}}$

$$c_2 = \frac{1174.6}{3670.5} \Rightarrow \boxed{c_2 = 0.32}$$

The effectiveness of first heat exchanger is :

$$\varepsilon_1 = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{min}(T_{h,in} - T_{c,in})} = \frac{3670.5 \times (T_{c,int} - 10)}{1761.8 \times (120-10)}$$

$$\varepsilon_1 = 0.0189\,(T_{c,int} - 10)$$

The effectiveness of second heat exchanger is :

$$\varepsilon_2 = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{C_c\,(T_{c,out} - T_{c,int})}{C_{min}\,(T_{h,in} - T_{c,in})} = \frac{3670.5 \times (50 - T_{c,int})}{1174.6 \times (120 - T_{c,int})}$$

$$\boxed{\varepsilon_2 = \frac{3.125\,(50 - T_{c,int})}{120 - T_{c,int}}} \quad -(2)$$

Since total water flow rate is split into 60% in first heat exchanger and 40% in second heat exchanger, the average exit temperature of hot water from energy balance is:

$$\dot{m}_{h,out\,1}\, C_{ph}\, T_{h,out1} + \dot{m}_{h,out2}\, C_{ph}\, T_{h,out2} = \dot{m}_{h,out}\, C_{ph}\, T_{h,out}$$

where, C_{ph} : constant evaluated at average inlet and outlet temperature of the hot stream.

$$0.6\, \dot{m}_{h,out}\, C_{ph}\, T_{h,out1} + 0.4\, \dot{m}_{h,out}\, C_{ph}\, T_{h,out2} = \dot{m}_{h,out}\, C_{ph}\, T_{h,out}$$

$$0.6\, T_{h,out1} + 0.4\, T_{h,out\,2} = T_{h,out} = 70^\circ C$$

Further, energy balance on first heat exchanger gives →

$$\dot{m}_h\, C_{ph,1}\, (T_{h,in} - T_{h,out1}) = \dot{m}_c\, C_{pc}\, (T_{c,out} - T_{c,int})$$

$$1761.8\,(120 - T_{h,out\,1}) = 3670.5\,(T_{c,int} - 10) \quad -(3)$$

for second heat exchanger, energy balance is :

$$\dot{m}_h\, C_{ph,1}\, (T_{h,in} - T_{h,out2}) = \dot{m}_c\, C_{pc}\, (T_{c,out} - T_{c,int}) \quad -(4)$$

Number of transfer units for first heat exchanger is :

$$NTU_1 = \frac{1}{c_1 - 1} \ln\left(\frac{\varepsilon_1 - 1}{c_1\varepsilon_1 - 1}\right) \quad -(5)$$

Number of transfer units for second heat exchanger is :

$$NTU_2 = \frac{1}{c_2 - 1} \ln\left(\frac{\varepsilon_2 - 1}{c_2\varepsilon_2 - 1}\right) \quad -(6)$$

further, since surface area of heat exchanger is same i.e $A_{s1} = A_{s2}$ we get,

$$\frac{NTU_1\, C_{ph,1}}{U} = \frac{NTU_2\, C_{ph,2}}{U} \quad -(7)$$

Solving equations (1) to (7) simultaneously in MATLAB software, the exit temperature of hot fluid in each heat exchanger and intermediate temperature of cold fluid is

$\boxed{T_{h,out1} = 70.82^\circ C}$, $\boxed{T_{h,out\,2} = 68.77^\circ C}$, $\boxed{T_{c,int} = 33.61^\circ C}$

(a) effectiveness of first and second heat exchanger is $\boxed{0.477}$ and $\boxed{0.593}$ respectively.

(b) Number of transfer units (NTU) of first and second heat exchanger is $\boxed{0.675}$ and $\boxed{1.012}$ respectively.

(c) Surface area of each small heat exchanger is $\boxed{1.252\ m^2}$

(d) Heat exchanger surface area of $1.252\ m^2$ is for one heat exchanger. Thus for two heat exchangers arranged in series the total surface area is $2.5\ m^2$. This area is about 5% higher than that of single heat exchanger. Moreover, construction cost of a small heat exchanger is about 15% higher than large heat exchanger per unit surface area. This translates to about 2% increase in the construction cost. Hence, it is recommended to use one large heat exchanger instead of two heat exchangers arranged in series.