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Date: ____ Page: ____

ASSIGNMENT - 1

Question-1 :-

1.1. Picture a scenario where you place a pot of water on a heated stove. As the water starts to boil, discuss the primary mode of heat transfer responsible for heating the water. Is it conduction, convection, or radiation? Provide reasons for your choice.

Solution : The primary mode of heat transfer responsible for heating the water in this scenario is 'convection'.

Reasons :- * Convection involves transfer of heat through movement of fluids. Here, as the water heats up, molecules near bottom of pot gain energy, becomes less dense, and rise. This creates a circulation of hot water, and cooler water moves down to replace it.

* Process of hot water rising and cold water sinking sets up a natural convection current, leading to an efficient transfer of heat throughout water in pot.
* Here conduction would not be effective as it relies on direct contact, but here water molecules at top are not in direct contact with heated stove.

1.2 Imagine you are holding a metal rod over a flame, and after some time, you feel the other end of rod getting hot. explain heat transfer mechanism involved in this situation and identify whether it is conduction convection or radiation. Justify your answer.

Solution :- The heat transfer mechanism involved in this situation is 'Conduction'.

Reasons :-

* Conduction is transfer of heat through direct contact b/w particles within the material. Here, metal rod conducts heat from hot end (closest to flame) to the cooler end (farther from flame).
* As metal rod is held over flame, the molecules at hot end gain energy and vibrate more rapidly. This increased kinetic energy is then transferred to the neighboring molecules through collisions.
* Heat travels along metal rod in step-by step process with each segment of rod passing heat to the next segment until it reaches the end that you feel getting hot.

Question 2 :

Consider heat loss from 200-L cylinderical hot water tank in a house to the surrounding medium. Would you consider this to be steady or transient heat transfer problem? Also, would you consider heat transfer problem to be one-, two-, or three- dimensional? explain.

Solution :- I would consider heat loss from hot water tank in a house to be a transient heat transfer problem because water temperature must be changing with time. Also I would consider heat transfer problem to be two-dimensional since temperature differences (and thus heat transfer) will exist in radial and axial directions (but there will be symmetry about center line and no heat transfer in the azimuthal direction).

Date: Page:

Question-3:

Starting with an energy balance on a cylindrical shell volume element, derive the steady one-dimensional heat conduction equation for long cylinders with constant thermal conductivity in which heat is generated at a rate of g and indicate what each variable represents.

Solution: Let us consider a thin cylindrical shell element of thickness 'Δr' in a long cylinder. Taking density of cylinder to be 'ρ', specific heat to be 'C' and length of cylinder to be 'L'. Area of cylinder to be 'A'

$A = 2\pi r L$ where r is radius of

↳ acting normal to direction of heat transfer.

Energy balance on cylinderical shell element →

Rate of heat going in − Rate of heat coming out + Rate of heat generation = Accumulation of heat

$$\dot{Q}_r - \dot{Q}_{r+\Delta r} + \dot{E}_{element} = \frac{\Delta E_{element}}{\Delta t}$$

Where $\Delta E_{element} = E_{t+\Delta t} - E_t = \overset{\text{mass}}{m} C (T_{t+\Delta t} - T_t)$

$\Delta E_{element} = \rho C A \Delta r (T_{t+\Delta t} - T_t)$

$\dot{E}_{element} = g\, V_{element}$ → Volume

↳ Heat generation rate

$\dot{E}_{element} = g \cdot A \Delta r$

Substituting, $\dot{Q}_r - \dot{Q}_{r+\Delta r} + g A \Delta r = \dfrac{\rho C A \Delta r (T_{t+\Delta t} - T_t)}{\Delta t}$

where $A = 2\pi r L$: $\dot{Q}_r - \dot{Q}_{r+\Delta r} + 2\pi g r L \Delta r = \dfrac{\rho C A \Delta r (T_{t+\Delta t} - T_t)}{\Delta t}$

Dividing by $A \Delta r$ or $(2\pi r L \Delta r)$: $-\dfrac{(\dot{Q}_{r+\Delta r} - \dot{Q}_r)}{A \Delta r} + g = \rho C \left(\dfrac{T_{t+\Delta t} - T_t}{\Delta t} \right)$ - (A)

Taking limit $\Delta r \to 0$ & $\Delta t \to 0$

So $\lim_{\Delta t \to 0} \dfrac{T_{t+\Delta t} - T_t}{\Delta t} = \dfrac{\partial T}{\partial t}$ — (1)

from fourier's law of Heat Conduction

$$\lim_{\Delta r \to 0} \frac{\dot{Q}_{r+\Delta r} - \dot{Q}_r}{\Delta r} = \frac{\partial Q}{\partial r} = \frac{\partial}{\partial r}\left(-KA\frac{\partial T}{\partial r}\right) \quad -(2) \quad \left[\because Q = -KA\frac{\partial T}{\partial r}\right]$$

Substituting 1 & 2 in A :

$$-\frac{1}{A}\frac{\partial}{\partial r}\left(-KA\frac{\partial T}{\partial r}\right) + g = \rho c \frac{\partial T}{\partial t}$$

$A = 2\pi r L$ ~~is constant~~, K : thermal conductivity → constant (given)

$$\frac{1}{A}\cdot K \frac{\partial}{\partial r}\left(A\frac{\partial T}{\partial r}\right) + g = \rho c \frac{\partial T}{\partial t}$$

$$\frac{1}{2\pi L\cdot r} K \frac{\partial}{\partial r}\left(2\pi r L\frac{\partial T}{\partial r}\right) + g = \rho c \frac{\partial T}{\partial t}$$

$$\frac{\cancel{2\pi L}\, K}{\cancel{2\pi L}\cdot r} \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + g = \rho c \frac{\partial T}{\partial t} \quad [\because 2\pi \,\&\, L \to \text{constant}]$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{g}{K} = \frac{\rho c}{K}\frac{\partial T}{\partial t} \quad [\text{dividing by } K]$$

$\because \alpha = \frac{K}{\rho c}$ → thermal diffusivity so $\frac{\rho c}{K} = \frac{1}{\alpha}$

Then, $$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{g}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$ {generalised equation}

But in question, it is mentioned that 'steady' which means no change with time so $\frac{\partial T}{\partial t} \to 0$

Hence, Steady One-dimensional heat conduction equation :

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{g}{K} = 0}$$

Question-4 :

Consider a medium in which heat conduction equation is given in its simplest form : $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

(a) Is heat transfer steady or transient ?

Date: ________ Page: ________

(b) Is heat transfer one-, two-, or three- dimensional?
(c) Is there heat generation in medium?
(d) Is the thermal conductivity of medium constant or variable?

Solution: (a) Here, $\frac{\partial T}{\partial t}$ term is present that indicates heat transfer is changing with time. So heat transfer is 'transient' here.

(b) No. of dimensions depends on no of spatial coordinates involved. Here only one coordinate i.e x is involved so heat transfer is one-dimensional.

(c) clearly, given Eq^n doesn't have a heat generation term so there is no heat generation within the medium.

(d) $\alpha = \frac{k}{\rho C_p} \Rightarrow k = \alpha \rho C$, considering α to be constant, ↳ thermal conductivity; $\rho \to$ constant, $C \to$ constant
so $k \to$ constant i.e. thermal conductivity of medium is constant

Question - 5:

Consider a large, 3 cm thick stainless steel plate in which heat is generated uniformly at a rate of 5×10^6 W/m^3. Assuming plate is losing heat from both the sides, determine the heat flux on the surface of plate during steady operation.

Solution: Considering a unit surface area, $A = 1 m^2$.
Total rate of heat generation in the plate →
$\dot{E}_{gen} = \dot{e}_{gen} V_{plate} = 5 \times 10^6 \times 1 \times 0.03$
[$\because \dot{e}_{gen} = 5 \times 10^6$ W/m^3 given, Volume = Area × ~~thickness~~, thickness = 0.03 m]
$\dot{E}_{gen} = 0.15 \times 10^6 = 15 \times 10^4$ W/$\cancel{m^3}$ × $\cancel{m^3}$ = 15×10^4 W

~~Noting~~ Since heat is getting lost from both sides of the plate, the heat flux on surface of plate during steady operation :→

$$q = \frac{\dot{E}_{gen}}{\text{Area of plate}} = \frac{15 \times 10^4}{2 \times (1)} = 7.5 \times 10^4 \text{ W/m}^2$$

$$q = 75 \times 10^3 \text{ W/m}^2 \qquad [\because 1 \text{ kW} = 10^3 \text{ W}]$$

$$\boxed{q = 75 \text{ kW/m}^2} \qquad \{\text{Heat flux}\}$$