Heat Project Assignment 3

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1. Nodes Equations

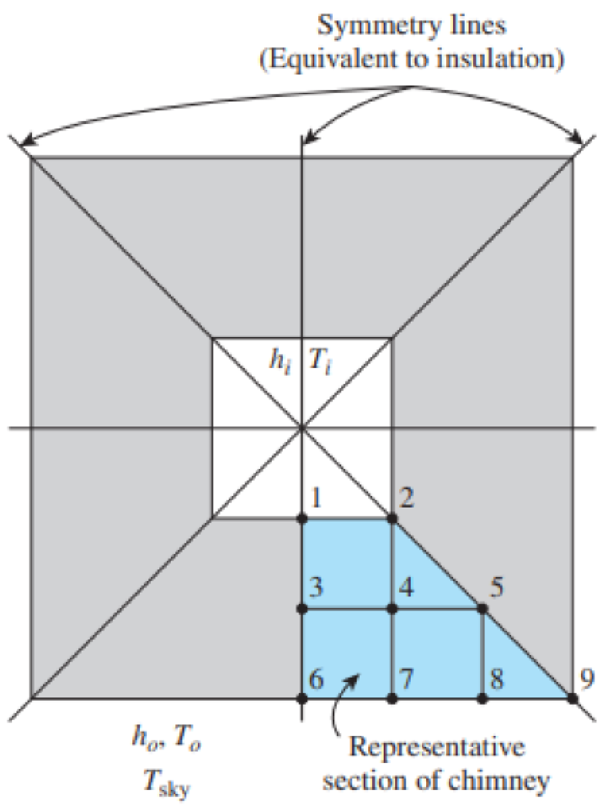

Figure 1: Schematic of the chimney and the nodal network for a representative section

(a) Node 1. On the inner boundary, subjected to convection

$$0 + h_i \frac{\Delta x}{2}(T_i - T_1) + k\frac{\Delta y}{2}T_2 - T_1 + k\frac{\Delta x}{2}T_3 - T_1 = 0 \tag{1}$$

$$-\left(2 + \frac{h_i l}{k}\right) T_1 + T_2 + T_3 = -\frac{h_i l}{k}T_i \tag{2}$$

(b) Node 2. On the inner boundary, subjected to convection

$$k\frac{\Delta y}{2}T_1 - T_2 + h_i \frac{\Delta x}{2}(T_i - T_2) + k\frac{\Delta x}{l}(T_4 - T_i) = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$T_i - \left(3 + \frac{h_i l}{k}\right) T_i + T_4 = -\frac{h_i l}{k}T_i \tag{3}$$

(c) Nodes 3, 4, and 5. (Interior nodes)

Node 3:

$$T_4 + T_1 + T_4 + T_6 - 4T_3 = 0 \tag{4}$$

Node 4:

$$T_3 + T_2 + T_5 + T_7 - 4T_4 = 0 \tag{5}$$

Node 5:

$$T_4 + T_4 + T_8 + T_8 - 4T_5 = 0 \tag{6}$$

(d) Node 6. (On the outer boundary, subjected to convection and radiation)

$$0 + k\frac{\Delta x}{2}(T_3 - T_6) + k\frac{\Delta y}{2}(T_7 - T_6) + h_o\frac{\Delta x}{2}(T_o - T_6) + \varepsilon\sigma\frac{\Delta x}{2}(T_{\text{sky}}^4 - T_6^4) = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$T_2 + T_3 - \left(2 + \frac{h_o l}{k}\right) T_6 = -\frac{h_o l}{k}T_o - \frac{\varepsilon\sigma l}{k}(T_{\text{sky}}^4 - T_6^4) \tag{7}$$

(e) Node 7. (On the outer boundary, subjected to convection and radiation)

$$k\frac{\Delta y}{2}(T_6 - T_7) + k\frac{\Delta x}{2}(T_4 - T_7) + k\frac{\Delta y}{2}(T_8 - T_7) + h_o\Delta x(T_o - T_7) + \varepsilon\sigma\Delta x(T_{\text{sky}}^4 - T_7^4) = 0$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$2T_4 + T_6 - \left(4 + \frac{2h_o l}{k}\right) T_7 + T_8 = -2\frac{h_o l}{k}T_o - 2\frac{\varepsilon\sigma l}{k}(T_{\text{sky}}^4 - T_7^4) \tag{8}$$

(f) Node 8. Same as node 7, except shift the node numbers up by 1

$$2T_5 + T_7 - \left(4 + \frac{2h_o l}{k}\right) T_8 + T_9 = -2\frac{h_o l}{k} T_o - 2\frac{\varepsilon \sigma l}{k}(T_{\text{sky}}^4 - T_8^4) \tag{9}$$

(g) Node 9. (On the outer boundary, subjected to convection and radiation)

$$k\frac{\Delta y}{2}(T_8 - T_9) + 0 + h_o \frac{\Delta x}{2}(T_o - T_9) + \varepsilon \sigma \frac{\Delta x}{2}(T_{\text{sky}}^4 - T_9^4) = 0 \tag{10}$$

Taking $\Delta x = \Delta y = l$, it simplifies to

$$T_8 - \left(1 + \frac{h_o l}{k}\right) T_9 = -\frac{h_o l}{k} T_o - \frac{\varepsilon \sigma l}{k}(T_{\text{sky}}^4 - T_9^4) \tag{11}$$

All Nodes Equations at one place

The system of nine equations for the determination of nine unknown nodal temperatures in a form suitable for use with an iteration method becomes:

$$T_1 = \frac{T_2 + T_3 + 2865}{7} \tag{12}$$

$$T_2 = \frac{T_1 + 2T_4 + 2865}{8} \tag{13}$$

$$T_3 = \frac{T_1 + 2T_4 + T_6}{4} \tag{14}$$

$$T_4 = \frac{T_2 + T_3 + T_5 + T_7}{4} \tag{15}$$

$$T_5 = \frac{2T_4 + 2T_8}{4} \tag{16}$$

$$T_6 = \frac{T_2 + T_3 + 456.2 - 0.3645 \times 10^{-9} T_6^4}{3.5} \tag{17}$$

$$T_7 = \frac{2T_4 + T_6 + T_8 + 912.4 - 0.729 \times 10^{-9} T_4^4}{7} \tag{18}$$

$$T_8 = \frac{2T_5 + T_7 + T_9 + 912.4 - 0.729 \times 10^{-9} T_4^4}{7} \tag{19}$$

$$T_9 = \frac{T_8 + 456.2 - 0.3645 \times 10^{-9} T_9^4}{2.5} \tag{20}$$

Final Expressions

The average temperature at the outer surface of the chimney weighed by the surface area is:

$$T_{\text{wall, out}} = \frac{0.5 \times T_6 + T_7 + T_8 + 0.5 \times T_9}{0.5 + 1 + 1 + 0.5}$$

Then the rate of heat loss through the 1-m-long section of the chimney can be determined approximately from:

$$Q_{\text{chimney}} = h_i A_i (T_i - T_{\text{wall, in}})$$