APPLIED HEAT TRANSFER

Assignment 1

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Question 1

1.1 Convection is the primary mode of heat transfer responsible for heating the water. Initially, when the pot is kept on stove, conduction plays a major role, in heating the pot and water. But, once water starts to boil, warmer water at the bottom of the pot starts moving up (less dense) and cooler (denser) water starts going down. This leads to conduction with a lot of **fluid motion**, which means convection takes place.

1.2 Conduction is the heat transfer mechanism involved in this situation. It is because heat travels through the metal rod through molecular interactions and vibrations, transferring kinetic energy to neighboring atoms, which leads to heat transfer in the case of a rod.

Question 2

Since, the temperature inside the hot water tank would decrease with time as heat is getting lost to the surroundings, the system is in **transient state**. Assuming, that the cylinder is symmetric along its axis, we can assume it to be a 1D radial heat transfer problem. This would help in making calculations for temperature profile and heat transfer evaluation less complex.

Question 4

(a) The heat transfer is transient due to presence of $\frac{\partial T}{\partial t}$ term in the equation.

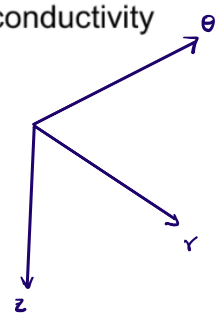
(b) The heat transfer is one – dimensional along x direction due to availability of only $\frac{\partial^2 T}{\partial x^2}$ term in the equation.

(c) There is no heat generation in the medium. The heat generation component is not present in the conduction equation, so assuming no heat generation.

(d) The thermal conductivity of the material is assumed as constant. Thereby, the equation is expressed in terms of thermal diffusivity $\alpha(=\frac{k}{\rho C_p})$.

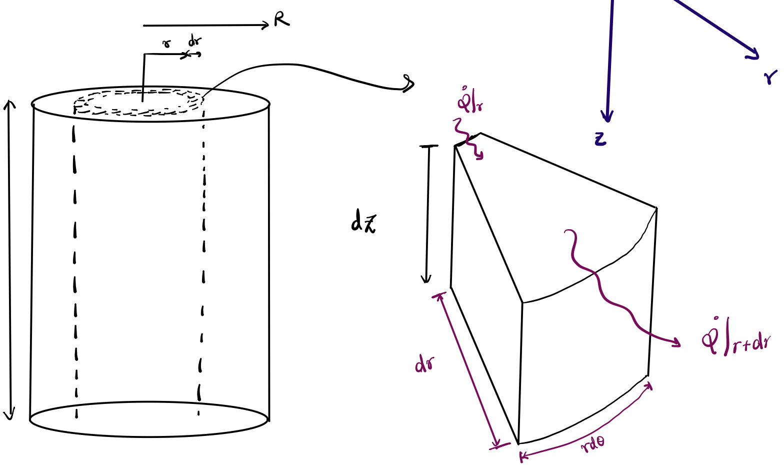
QUESTION-3

Starting with an energy balance on a cylindrical shell volume element, derive the steady one-dimensional heat conduction equation for a long cylinder with constant thermal conductivity in which heat is generated at a rate of **g** and indicate what each variable represents.



Given: A long cylinder $(L >> r)$

Hence, we can say that heat transfer occuring through radial direction contributes way more than the heat transfer through axial direction.



Hence, assuming that heat transfer occurs in radial direction only.

Area of cross section: $A_c = (rd\theta) \times (dz)$
(along r direction)

Volume of section: $V = A_c \times dr$

Energy balance:→ Accumulation = Input − Output + generation.

∵ Steady state :→ Accumulation = 0.

g is rate of heat generation.

$$\therefore \quad 0 = \dot{Q}|_r - \dot{Q}|_{r+dr} + g\,[\underbrace{(dz)(rd\theta)\,dr}_{\text{volume of section}}]$$

from Taylor series: $\dot{Q}|_{r+dr} = \dot{Q}|_r + \left[\dfrac{\partial \dot{Q}|_r}{\partial r}\right] dr$

∴ ON SUBSTITUTION;

$$0 = -\left[\frac{\partial \dot{Q}|_r}{\partial r}\right] dr + g\,[(dz)(rd\theta)\,dr]$$

Now; from fourier's law of conduction;

$$\dot{Q}|_r = -kA_c \left.\frac{dT}{dr}\right|_r$$

Given that k is constant.

$$\therefore \quad -\frac{\partial}{\partial r}\left[-kA_c\frac{dT}{dr}\right] dr + g\,[(dz)(rd\theta)\,dr] = 0$$

$$\rightarrow \quad -\frac{\partial}{\partial r}\left[-k \times [(rd\theta)\,dz]\frac{dT}{dr}\right] dr + g(rd\theta\,dz)\,dr = 0$$

$$+k\cancel{(d\theta)}\cancel{dz}\,\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right]\cancel{dr} = g\,[r\cancel{d\theta}\,\cancel{dz}\,\cancel{dr}]$$

$$\rightarrow \quad k\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] = g \cdot r$$

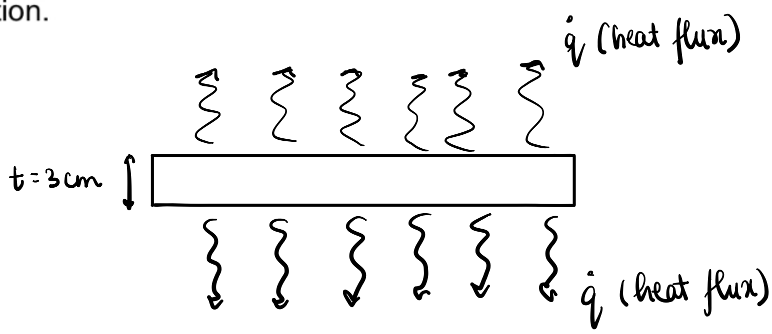
$$\rightarrow \quad \frac{k}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] = g$$

OR

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T}{\partial r}\right] = \frac{g}{k}}$$

QUESTION-5

Consider a large, 3-cm-thick stainless steel plate in which heat is generated uniformly at a rate of 5 x 10^6 W/m3 . Assuming the plate is losing heat from both sides, determine the heat flux on the surface of the plate during steady operation.



Given:→

$\dot{e}_{gen} = 5\times10^6 \; W/m^3$

thickness $(t) = 3\times10^{-2}$ m

$$\text{Total Heat flux } (\dot{q}_t) = \left(\begin{matrix}\text{Heat generated} \\ \text{per unit} \\ \text{volume}\end{matrix}\right) \times \text{thickness}$$

$$= \dot{e}_{gen} \times t$$

$$= 5\times10^6 \frac{W}{m^3} \times 3\times10^{-2} \, m$$

$$\text{Total heat flux } (\dot{q}_t) = 150000 \frac{W}{m^2}$$

$$\text{Heat flux through a surface } (\dot{q}) = \frac{\dot{q}_t}{2}$$

Since, in case of steady state operation, we assume due to symmetricity, heat flux through both the surfaces are equal.

$$\therefore \quad \dot{q} = \frac{150000}{2} = \underline{75000 \; W/m^2}$$