# Heat Transfer Analysis with Critical Radius

Key Analysis

Heat Transfer Mechanism:
- Conduction through insulation (radial direction).
- Convection from the insulation's outer surface to the

surroundings. Thermal Resistances:

Conduction Resistance (insulation):
R_cond = ln(r2 / r1) / (2pikL)

Convection Resistance:
R_conv = 1 / (h 2pir2L)

Total Resistance:
R_total = R_cond + R_conv

Heat Transfer Rate:
Q = (T1 - T) / R_total

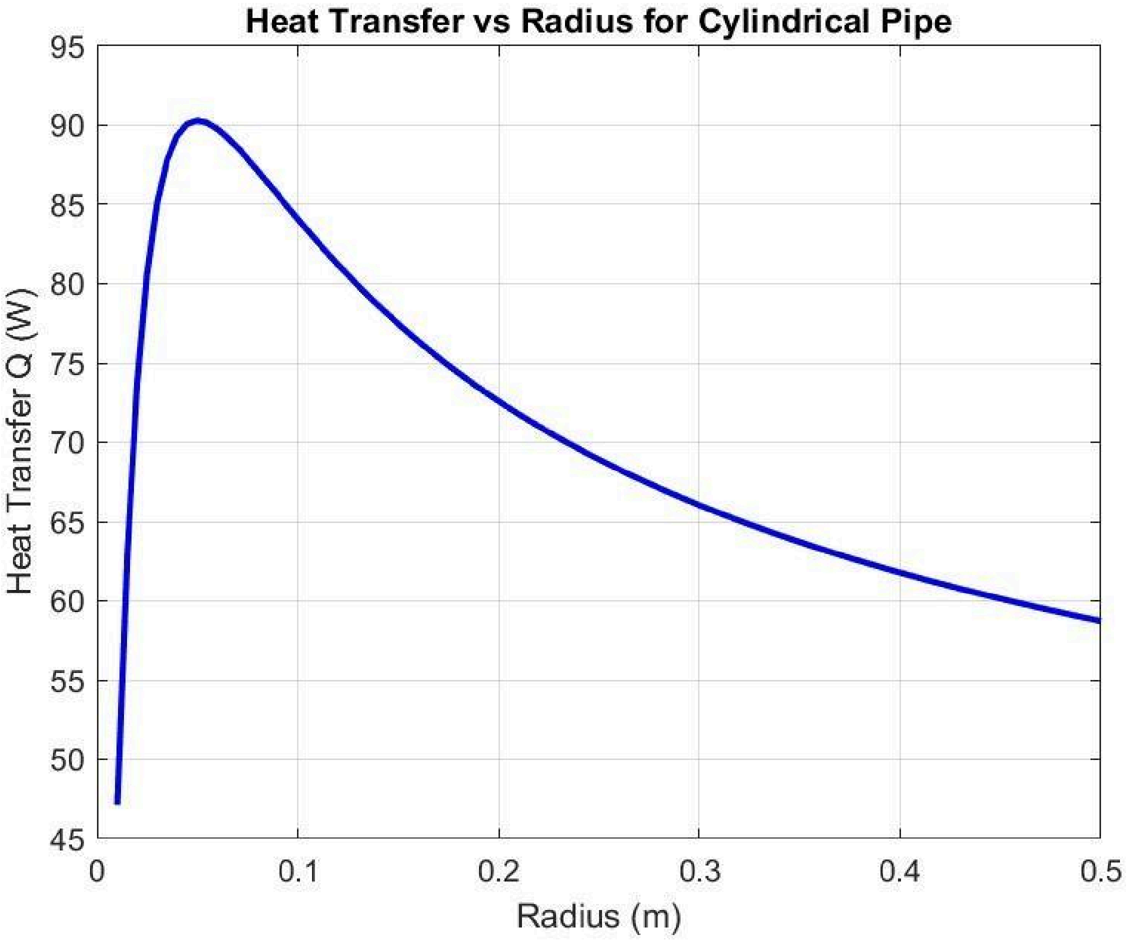
For simplicity, assume L = 1m:
Q = (2pi (T1 - T)) / [ln(r2 / r1) / k + 1 / (h r2)]
Critical Radius of Insulation:
For cylinders, critical radius rc = k / h

- If r2 < rc: Adding insulation increases heat loss.
- If r2 > rc: Adding insulation decreases

heat loss. Plotting Q vs r (for r2):

Behavior:
- Q initially decreases until r2 = rc.
- Q then increases for r2 > rc.

Conclusion:
The plot shows a U-shaped curve due to the critical radius effect. Insulation is only beneficial when r2 > rc.


Heat Transfer vs Radius for Cylindrical Pipe
Heat Transfer Q (W)
Radius (m)
95
90
85
80
75
70
65
60
55
50
45
0
0.1
0.2
0.3
0.4
0.5

### Derivation of Critical Radius

1. **Heat Transfer Rate Equation**:

   For a cylinder with insulation, the heat transfer rate $Q$ is given by:

$$Q = \frac{2\pi L(T_1 - T_{\text{amb}})}{\underbrace{\frac{\ln(r_2/r_1)}{k}}_{\text{Conduction Resistance}} + \underbrace{\frac{1}{hr_2}}_{\text{Convection Resistance}}}$$

   For simplicity, assume $L = 1\ \text{m}$:

$$Q = \frac{2\pi(T_1 - T_{\text{amb}})}{\frac{\ln(r_2/r_1)}{k} + \frac{1}{hr_2}}$$

2. **Minimize $Q$**:

   $Q$ is minimized when the denominator $D = \frac{\ln(r_2/r_1)}{k} + \frac{1}{hr_2}$ is **maximized**.

   Take the derivative of $D$ with respect to $r_2$:

$$\frac{dD}{dr_2} = \frac{1}{kr_2} - \frac{1}{hr_2^2}$$

3. **Set Derivative to Zero**:

$$\frac{1}{kr_2} - \frac{1}{hr_2^2} = 0 \quad \Rightarrow \quad \frac{1}{kr_2} = \frac{1}{hr_2^2}$$

   Simplify:

$$hr_2 = k \quad \Rightarrow \quad r_2 = \frac{k}{h}$$

The critical radius of insulation for a cylinder is given by:

$$rc=k/h$$

**MATLAB CODE**

```
% Given Data

k = 0.5;  % Thermal conductivity of insulation (W/mK)

h = 10;   % Convective heat transfer coefficient (W/m²K)

T1 = 400; % Inner temperature (K)
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```
T_inf = 300; % Ambient temperature (K)

r1 = 0.02; % Inner radius (m)


% Critical Radius Calculation

r_critical = k / h;

fprintf('Critical Radius of Insulation: %.4f m\n', r_critical);


% Range of Outer Radii

r2_values = linspace(0.01, 0.1, 100); % Outer radius from 0.01m to 0.1m

Q_values = zeros(size(r2_values)); % Initialize Q array


% Heat Transfer Calculation

for i = 1:length(r2_values)

    r2 = r2_values(i);

    if r2 > r1

        R_cond = log(r2 / r1) / (2 * pi * k * 1); % Conduction resistance

        R_conv = 1 / (h * 2 * pi * r2 * 1);       % Convection resistance

        R_total = R_cond + R_conv;

        Q_values(i) = (2 * pi * (T1 - T_inf)) / R_total; % Heat transfer rate

    else

        Q_values(i) = NaN; % Ignore invalid values

    end

end


% Plot Results

figure;
```

```
plot(r2_values, Q_values, 'b', 'LineWidth', 2);

hold on;

xline(r_critical, '--r', 'Critical Radius');

xlabel('Outer Radius r_2 (m)');

ylabel('Heat Transfer Rate Q (W)');

title('Heat Transfer vs Outer Radius');

grid on;

legend('Heat Transfer Q', 'Critical Radius');

hold off;
```