

Course Code: A4BS02

# MLR INSTITUTE OF TECHNOLOGY

(An Autonomous Institute)

I B.Tech. I Semester Supplementary Examination September-2023

## LINEAR ALGEBRA AND CALCULUS

(Common to CSE & IT)

Time: 3 Hours.

Max. Marks: 70

Note: 1. This question paper contains two parts A and B.

2. Part -A is Compulsory which carries 20 marks. Answer all Questions in part A.

3. Part -B consists 5units. Answer any one question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

### PART- A

10 x 2M = 20Marks

|    |    |  |    |     |     |
|----|----|--|----|-----|-----|
| 1. | a) | Find Integrating factor of $\frac{dy}{dx} - \frac{y}{x} = x$   | 2M | CO1 | BL3 |
|    | b) | Solve $(D^2 + 4)y = \sin 2x$   | 2M | CO1 | BL3 |
|    | c) | Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$  | 2M | CO2 | BL4 |
|    | d) | Test for convergence of the series $\sum_{n=1}^{\infty} ne^{-n^2}$   | 2M | CO2 | BL4 |
|    | e) | Find the value of k such that the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2.           | 2M | CO3 | BL1 |
|    | f) | Define Orthogonal matrix and Unitary matrix.   | 2M | CO3 | BL1 |
|    | g) | Explain, Why the set $S = \{(2,1,-2), (-2,-1,2), (4,2,-4)\}$ is not a basis of $R^3$ .   | 2M | CO4 | BL2 |
|    | h) | Find the nullity of the matrix $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \end{pmatrix}$                              | 2M | CO4 | BL2 |
|    | i) | Find a unit vector which is orthogonal to the vector $\alpha = (2, -1, 6)$ of $V_3(R)$ with respect to standard inner product. | 2M | CO5 | BL2 |
|    | j) | Find the eigen values of A, Where $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$   | 2M | CO5 | BL1 |

### PART- B

5 x 10M = 50Marks

|    |    |   |     |     |    |
|----|----|---|-----|-----|----|
| 2  | a) | Solve $(2x - y + 1)dx + (2y - x - 1)dy = 0$   | 5M  | CO1 | L3 |
|    | b) | Solve $(D^2 + a^2)y = \tan ax$ by method of variation of parameters.  | 5M  | CO1 | L3 |
| OR |    |   |     |     |    |
| 3  |    | Solve $D^2 + 3D + 2)y = e^x \sin x$ .   | 10M | CO1 | L3 |
| 4  | a) | Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1^2 \cdot 5^2 \cdot 9^2 \dots (4n-3)^2}{4^2 \cdot 8^2 \cdot 12^2 \dots (4n)^2}$ | 5M  | CO2 | L4 |
|    | b) | Test the convergence of the series $\sum_{n=1}^{\infty} (1 + \frac{1}{\sqrt{n}})^{-n^2}$  | 5M  | CO2 | L4 |
| OR |    |   |     |     |    |
| 5  | a) | Test the convergence of the series $\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} \dots$  | 5M  | CO2 | L4 |
|    | b) | Show that $1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \dots$ is absolutely convergent.  | 5M  | CO2 | L4 |

|           |    |  |     |     |    |
|-----------|----|--|-----|-----|----|
| 6         | a) | If $A = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ then show that matrix A is Hermitian and matrix $iA$ is Skew-Hermitian.  | 5M  | CO3 | L3 |
|           | b) | Convert the given matrix into Normal form and hence find it's rank where $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$  | 5M  | CO3 | L5 |
| <b>OR</b> |    |  |     |     |    |
| 7         | a) | Solve the following system of equations<br>$x + y - 3z + 2w = 0$ , $2x - y + 2z - 3w = 0$ ,<br>$3x - 2y + z - 4w = 0$ , $-4x + y - 3z + w = 0$ .   | 5M  | CO3 | L3 |
|           | b) | Test for consistency and solve the system of equations<br>$x - 2y - 3z = 1$ , $3x + 4y - 6z = -2$ , $4x + 2y - 3z = 5$ .   | 5M  | CO3 | L4 |
|           |    |  |     |     |    |
| 8         |    | The linear transformation $T: V_2 \rightarrow V_3$ is defined as<br>$T(x, y) = (x, x + y, y)$ then find the Range, Kernel, Rank and Nullity of T. and also find whether T is one-one or not?   | 10M | CO4 | L3 |
| <b>OR</b> |    |  |     |     |    |
| 9         |    | Find the matrices of the linear transformation T on $V_3(R)$ defined as $T(a, b, c) = (2b + c, a - 4b, 3a)$ with respect to standard ordered basis $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ and ordered basis $B^1 = \{(1,1,1), (1,1,0), (1,0,0)\}$ . | 10M | CO4 | L4 |
|           |    |  |     |     |    |
| 10        |    | Diagonalize the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$   | 10M | CO5 | L3 |
| <b>OR</b> |    |  |     |     |    |
| 11        |    | Apply Gram-Schmidt orthogonalization process to the vectors $\beta_1 = (1,0,1)$ , $\beta_2 = (1,0,-1)$ , $\beta_3 = (0,3,4)$ to obtain an orthonormal basis $(\alpha_1, \alpha_2, \alpha_3)$ for $R^3$ With Standard inner product.              | 10M | CO5 | L3 |

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