Course Code: A5BS02

MLR INSTITUTE OF TECHNOLOGY
(An Autonomous Institute)

I B.Tech. I Semester Supplementary Examination September-2023

LINEAR ALGEBRA AND CALCULUS
(Common to AIML, CSC, CSD, CSE, CSIT, CSM & IT)

Time: 3 Hours.   **Max. Marks: 70**

Note: 1. This question paper contains two parts A and B.
   2. Part -A is Compulsory which carries 20 marks. Answer all Questions in part A.
   3. Part -B consists 5units.Answer any one question from each unit. Each question
      carries 10 marks and may have a, b, c as sub questions.

PART- A   10 x 2M = 20Marks

1.	a)	Find Integrating factor of $\frac{dy}{dx} - \frac{y}{x} = x$	2M	CO1	BL3
	b)	Solve $(D^2 + 4)y = sin\, 2x$	2M	CO1	BL3
	c)	Test for convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.	2M	CO2	BL4
	d)	Test for convergence of the series $\sum_{n=1}^{\infty} ne^{-n^2}$.	2M	CO2	BL4
	e)	Find the value of k such that the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2.	2M	CO3	BL1
	f)	Define Orthogonal matrix and Unitary matrix.	2M	CO3	BL1
	g)	Explain, Why the set $S = \{(2,1,-2),(-2,-1,2),(4,2,-4)\}$ is not a basis of R³.	2M	CO4	BL2
	h)	Find the nullity of the matrix A=$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 \end{pmatrix}$	2M	CO4	BL2
	i)	Find a unit vector which is orthogonal to the vector $\alpha = (2,-1,6)$ of $V_3(R)$ with respect to standard inner product.	2M	CO5	BL2
	j)	Find the eigen values of A, Where A=$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$	2M	CO5	BL1

PART- B   5 x 10M = 50Marks

2	a)	Solve $(2x - y + 1)dx + (2y - x - 1)dy = 0$	5M	CO1	L3
	b)	If the population of a country doubles in 50 years, in how many years will it be three times, assuming that the rate of increase is proportional to the number of inhabitants?	5M	CO1	L3
OR					
3		Solve $D^2 + 3D + 2)y = e^x sinx$.	10M	CO1	L3
4	a)	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1^2.5^2.9^2.....(4n-3)^2}{4^2.8^2.12^2......(4n)^2}$	5M	CO2	L4
	b)	Test the convergence of the series $\sum_{n=1}^{\infty}(1 + \frac{1}{\sqrt{n}})^{-n^{\frac{3}{2}}}$.	5M	CO2	L4
OR					
5	a)	Test the convergence of the series $\frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} ... ...$	5M	CO2	L4
	b)	Show that $1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \cdots ... ...$ is absolutely convergent.	5M	CO2	L4

6	a)	If $A = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ then show that matrix A is Hermitian and matrix iA is Skew-Hermitian.	5M	CO3	L3
	b)	Convert the given matrix into Normal form and hence find it's rank where $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$	5M	CO3	L5

OR

7	a)	Solve the following system of equations $x + y - 3z + 2w = 0, 2x - y + 2z - 3w = 0,$ $3x - 2y + z - 4w = 0, \ -4x + y - 3z + w = 0.$	5M	CO3	L3
	b)	Test for consistency and solve the system of equations $x - 2y - 3z = 1, \ 3x + 4y - 6z = -2, \ 4x + 2y - 3z = 5.$	5M	CO3	L4

8	The linear transformation $T:V_2 \rightarrow V_3$ is defined as $T(x,y) = (x, x+y, y)$ then find the Range, Kernel, Rank and Nullity of T. and also find whether T is one-one or not?	10M	CO4	L3

OR

9	Find the matrices of the linear transformation T on $V_3(R)$ defined as $T(a,b,c) = (2b+c, a-4b, 3a)$ with respect to standard ordered basis $B = \{(1,0,0),(0,1,0),(0,0,1)\}$ and ordered basis $B^1 = \{(1,1,1),(1,1,0),(1,0,0)\}$.	10M	CO4	L4

10	Diagonalize the matrix A=$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$	10M	CO5	L3

OR

11	Apply Gram-Schmidt orthogonalization process to the vectors β_1= (1,0,1),β_2= (1,0,-1), β_3= (0,3,4) to obtain an orthonormal basis $(\alpha_1, \alpha_2, \alpha_3,)$ for R^3 With Standard inner product.	10M	CO5	L3

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