Complex Integration

Q.1) Evaluate $\int_0^{1+i} (x + iy^2)\, dz$ along the parabola $x = y^2$.

Q.2) Evaluate $\int_0^{1+i} (x - y + ix^2)\, dz$ along the parabola $y^2 = x$.

[MAY 24]

Q.3) Evaluate $\int_0^{1+i} z^2\, dz$ along

i) the parabola $x = y^2$. ii) the line $y = x$. **[MAY 16], [MAY 18]**

Q.4) Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along

i) the parabola $y = x^2$. ii) the line $y = x$. **[JUN 21], [MAY 22]**

Q.5) Evaluate $\int_0^{1+i} (x^2 + iy)\, dz$ along

i) the parabola $y = x^2$. ii) the line $y = x$. **[MAY 14], [DEC 22]**

Q.6) Integrate the function $f(z) = x^2 + ixy$ from $A(1,1) to\ B(2,4)$ along $y = x^2$ **[DEC 24]**

Q.7) Evaluate $\int_c \frac{\sin \pi z^2 + cos\pi z^2}{(z-2)(z-1)}\, dz$, where C is circle $|z| = 3$. **[MAY 22]**

Q.8) Evaluate $\int_c \frac{cos\ z}{z} dz$, Where C is the ellipse $9x^2 + 4y^2 = 1$.

[MAY 19]

Q.9) Evaluate $\int_c \frac{Sin^6 z}{(z-\frac{\pi}{2})^3} dz$, where C is $|z| = 2$. **[MAY 14]**

Q.10) Evaluate $\int_c \frac{z^2+7}{(z-2)(z-1)} dz$ when C is $|z| = 1.5$ using Cauchy's integral formula.

Q.11) Evaluate $I = \int_c \frac{z+8}{z^2+5z+6} dz$ when C is $|z| = 5$ using Cauchy's integral formula. **[DEC 24]**

Q.12) Evaluate $\int_c \frac{3z^3+z}{(z-1)^4} dz$ when C is $|z| = 2$, using Cauchy's integral formula.

Q.13) Show that $\int_c \log z\, dz = 2\pi i$ here C is the unit circle in the z-plane. **[MAY 17], [MAY 18]**

Q.14) Evaluate the following integral using Cauchy's Residue Theorem $\int_c \frac{z+2}{z^3-2z^2} dz$, where C is the circle $|z-2-i| = 2$

Q.15) Evaluate the following integral using Cauchy-Residue theorem.
$I = \int_C \frac{z^2+3z}{\left(z+\frac{1}{4}\right)^2 (z-2)}$ where c is the circle $\left|z-\frac{1}{2}\right| = 1.$ **[JUN 21]**

Q.16) Evaluate the following integral using Cauchy-Residue theorem.
$I = \int_C \frac{4z^2+1}{(2z-3)(z+1)^2} dz$ where c is the circle $|z| = 4.$ **[MAY 22]**

Q.17) Evaluate the following integral using Cauchy-Residue theorem.
$I = \int_C \frac{1-2z}{z(z-1)(z-2)} dz$, where c is the circle $|z| = 1.5.$ **[DEC 22]**

Q.18) Evaluate the following integral using Cauchy-Residue theorem.
$I = \int_C \frac{2z-1}{z(2z+1)(z+2)} dz$, where c is the circle $|z| = 1$ **[MAY 24]**

Q.19) Obtain Laurent's series expansion of $f(z) = \frac{2}{(Z-1)(Z-2)}$ about $z = 0.$

Q.20) Obtain all Taylor's and Laurent's series expansions of function $\frac{(z+1)(z+4)}{(z-2)(z+2)}$ about $z = 0.$

Q.21) If $f(z) = \frac{z-1}{(z-3)(z+1)}$, obtain Taylor's and Laurent's series expansions of f(z) in the domain $|z| < 1$ & $1 < |z| < 3$ respectively. **[JUN 21]**

Q.22) If $f(z) = \frac{4z+3}{z(z-3)(z+2)}$, obtain Laurent's series expansions of f(z) in the domain $2 < |z| < 3.$ **[MAY 24]**

Q.23) If $f(z) = \frac{1}{z^2+4z+3}$, obtain Laurent's series expansions of f(z) in the domain i) $|z| < 1$ ii) $1 < |z| < 3$ iii) $|z| > 3.$ **[DEC 22]**

Q.24) Integrate the function $f(z) = z^2$ from $A(0,0)\ to\ B(1,1)$ along straight-line AB. **[DEC 23]**

Q.25) Evaluate $\int \frac{z^2dz}{(z-1)(z-2)}$;Where C is a circle $|z - 1| = 1$. **[DEC 23]**

Q. 26) Find all possible Laurent's series expansions of the function $f(z) = \frac{1}{(z-1)(z+2)}$, about z = 0 indicating the region of convergence in each case. **[DEC 23]**

Q. 27) Find all possible Laurent's series expansions of the function $f(z) = \frac{1}{(z+1)(z-2)}$, about z = 0 indicating the region of convergence in each case. **[DEC 24]**