Modern Physics PHY315

Experimental Demonstration of Level Attraction In Coupled Pendulums

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Aim of the Experiment:

To experimentally demonstrate dissipative coupling in a double pendulum system. The pendulums are coupled through electromagnetic damping employing the Lenz' effect.

Setup:

The dissipative coupled pendulum is illustrated in the given figure. This pendulum system contains two major parts: the pendulum arms (aluminum) and the coupling mechanism. Both pendulum arms are mounted on a rigid frame through a pair of rod ends that contain low friction ball bearings to achieve the system's low intrinsic damping. The pendulum on the left features a bracket made out of 3D printed polylactic acid (PLA) of radius 25mm from TA202 laboratory. The setup was assembled at the Physics Workshop IITK.

We have a coupling device to couple the two pendulums together.

The coupling mechanism contains two components, one is a conductor coil, and the other is a pair of magnets that create a magnetic field that goes through the coil. Each component is directly fastened to one of the pendulum arms. The right pendulum contains a round-shape 3D printed polylactic acid (PLA) bracket with a diameter of 25.0 mm. 150 turns of a thin copper wire are wound around it. The left pendulum has an apparatus which serves to hold the two strong magnets . This was also 3D printed in the TA202 lab.

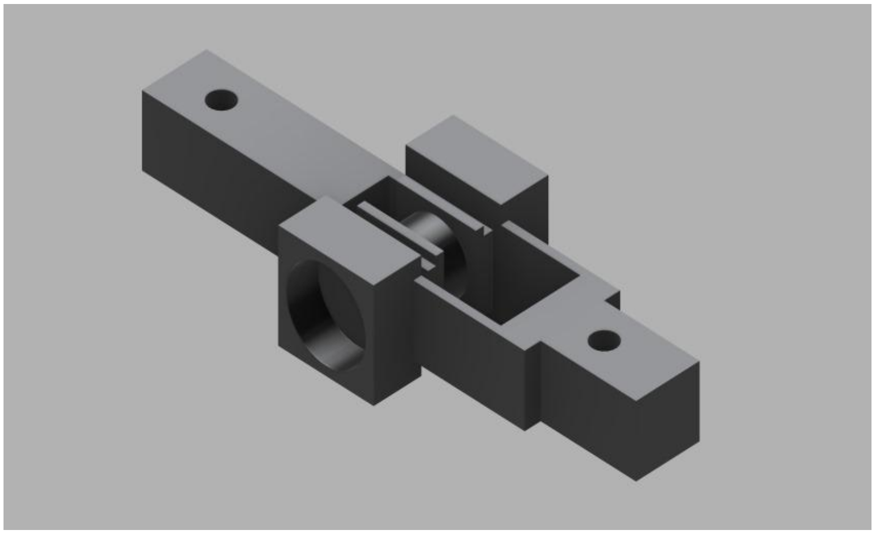


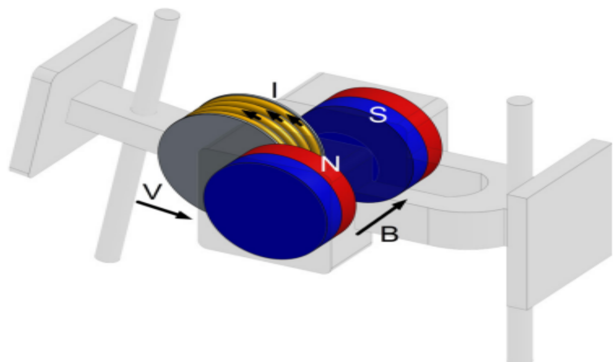
Fig: CAD model of Coupling device

The two strong neodymium magnets provide a nearly uniform magnetic field of 0.25 T measured by a gauss meter. The entire set up containing the supporting vertical and horizontal rods, ball bearings, and the two pendulum rods were made by modifying a previously made double pendulum set up to suit our requirements. The modifications were made in the Physics workshop.

To maintain the system's linearity, proper alignment of the coils and magnets is important to prevent pendulum collision and maintain consistent coupling between them throughout the experiment.

Theory and Procedure:

Dissipative coupling between the pendulums is achieved by introducing a Lenz's effect induced electromagnetic damping that allows energy to dissipate through Joule heating (see fig.)



To maintain the system's linearity, proper alignment of the coils and magnets is important to prevent pendulum collision and maintain consistent coupling between them throughout the experiment, as we limit the oscillation angle of both pendulums to be within 1.5-2 degrees. During the oscillation, the overlapping area between coupling mechanism components (coil and the magnetic field induced by parallel magnets) changes, causing a variation in the enclosed magnetic flux. This induces an emf on the coil that generates a damping-like force, opposing the change of motion. Its reaction force also drives the pendulum due to Newton's third law. The oscillation angle is small to ensure that some overlap area is always maintained. See figures:

Fig. Relative motion of the coupling components

The left pendulum's (pendulum 2: P2) natural frequency is adjustable, through a counterweight placed at different locations along its length, thereby shifting the pendulum's center of mass and effectively tuning its length. Experimentally, the center of mass/effective length can be determined by their natural frequencies. However, the difference in effective length between the pendulums results in a non-equivalent coupling effect, which manifests as a torque applied to each pendulum. Due to this circumstance, we will define coupling strength parameters $\Gamma_{1,2}$ to describe the influence of the coupling on individual pendulums .

We will also introduce intrinsic damping parameters $\gamma_{1,2}$ to characterize the non-coupling related damping such as frictions at the pendulum hanging pivots and air resistance on each pendulum. Both coupling strength parameters $\Gamma_{1,2}$ and intrinsic damping parameters $\gamma_{1,2}$ are inversely proportional to each pendulum's effective length.

The data collection of the two pendulums's location trajectory was done by video recording the pendulum dynamics on a camera and then performing the data analysis using an image processing software called ImageJ.

The left pendulum was released from a small non-zero amplitude and the right pendulum starts by hanging at its rest position. During the experiment, the natural frequency ω_2 of the right pendulum was held constant, while the natural frequency ω_1 of the left pendulum was swept through a range of values by adding counterweights to it, thereby changing its effective length and natural frequency.

Analytical Model:

Both pendulums have identical mass m. Each pendulum has a natural angular resonant frequency $\omega_{1,2} = \sqrt{g/L_{1,2}}$, where $L_{1,2}$ is the pendulum's effective length and g is the Earth's gravitational constant. Lagrangian of the system is

$$L = T - U = \frac{1}{2}mL_1\dot{\theta}_1^2 + \frac{1}{2}mL_2\dot{\theta}_2^2 - \frac{1}{2}mgL_1\theta_1^2 - \frac{1}{2}mgL_2\theta_2^2 - \frac{k}{2}(l\theta_1 - l\theta_2)^2$$

The dissipation of this system contains two terms, including intrinsic damping and dissipative coupling. Here, the dissipative coupling is because of the Lenz effect. The emf for this dissipative coupling is

$$V = NBhl(\dot{\theta}_1 - \dot{\theta}_2)$$

where N is the number of turns of the coil and B is the magnetic field strength. Due to the small angle oscillation, the overlapping area of the coil and magnets are approximated to be a rectangle with constant height h = 2.5cm.

This emf induces a current I in the coil, which has resistance R. Under a magnetic field, the current conducting wires experience a dynamic force, which is the dissipative coupling force,

$$F = \frac{Nh^2B^2l^2}{R}(\dot{\theta}_1 - \dot{\theta}_2)$$

The intrinsic damping force is characterized with first order velocity proportional to the damping rates $\gamma_{1,2}$. Now, the equations of motion can be obtained by solving Euler-Lagrange equations.

We define the coherent coupling strength : $J_i = \frac{kl^2}{2m\omega_{1,2}L_i^2}$ ......(1)

and dissipative coupling strength: $\Gamma_i = \frac{Nh^2B^2l^2}{2mRL_i^2}$ ......(2)

These parameters are not dynamical so the equations of motion can be calculated using the Euler Lagrange equations. General solutions for periodic motion are $\theta_i = A_i e^{i\omega t}$. We can solve for the complex hybridized mode frequency $\overline{\omega}$ using a standard eigenvalue problem solution approach. Note that the real part ω of eigenfrequency ϖ is the actual measurable frequency while the imaginary part corresponds to the damping of each mode.

In our case, we don't have any spring system, so k=0, which means that J_i=0.

For the purely dissipative coupling which happens in our case of electromagnetic damper coupled, the hybridized eigenmode frequencies are -

$$\tilde{\omega}_{\pm} = \frac{\omega_1 + \omega_2 - i(\Gamma_1 + \gamma_1 + \Gamma_2 + \gamma_2)}{2} \pm \frac{\sqrt{[\Delta - i(\Gamma_1 + \gamma_1 - \Gamma_2 - \gamma_2)]^2 - 4\Gamma_1\Gamma_2}}{2}. \quad \text{.........(3)}$$

where $\Delta = \omega_1 - \omega_2$, called detuning.

For a simplified stationary solution of $\gamma_1 = \gamma_2$ *and* $\Gamma_1 = \Gamma_2 = \Gamma$, and $\gamma_1 = \gamma_2 = \gamma$,

$$\omega_{\pm} = \frac{\omega_1 + \omega_2}{2} \pm \frac{\sqrt{\Delta^2 - 4\Gamma^2}}{2}.$$

The real part of the hybridized mode eigenfrequencies is degenerate in the region $-2\Gamma < \Delta < 2\Gamma$. In this range, the two eigenmodes' frequencies merge into one value (same real parts) with mismatched damping (different imaginary parts).

When the pendulums are swinging in phase, there is no relative velocity between them, so the dissipation from the coupling is not triggered and energy loss from the pendulums is minimized. When the pendulums swing in anti-phase motion, they experience the maximum relative velocity so the energy dissipation is also maximized. The high energy anti-phase mode often experiences higher damping than the lower energy in-phase mode. This makes it possible for the final energy of both modes to be equal, which leads to the mode degeneracy as earlier demonstrated in the frequency domain. This behavior is called level - attraction.

Experimental Parameters used:

- Length of pendulum 1: L_1 = 45cm
- Length of pendulum 2: L_2 = 45cm
- Material of pendulum rod: Aluminum
- Distance between pivot of pendulums 1 and 2 = 12cm
- Diameter of pendulum rods = 6mm
- Mass of pendulum rod 1 = Mass of pendulum rod 2 = m = 280± 5 g
- Total length of Copper wire used in electromagnet = 9.5m
- Diameter of Copper wire = 0.75 ± 0.1 mm
- Resistance of copper wire = 0.087 ± 0.02 Ω
- Number of turns on the electromagnet = 150 turns
- Strength of magnet used: B = 0.25 ± 0.01 T
- Diameter of magnet used = 2.5cm
- Distance between Pivot of pendulum and Coupling device: l = 35cm

Data and Observations:

I. Calculating the natural frequency of the Pendulum 2:

We needed to change the natural frequency of P2. To achieve this we added some weights to P2 and then made it oscillate freely. By adding the extra weights, we changed the effective length of the pendulum, due to which its natural frequency changed. We measured the time period using our phone stopwatch.

$$\omega = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \quad \Rightarrow L = \frac{g}{4\omega^2\pi^2}$$

Here is the data of the measured natural frequencies:

1)With one 50g weight, attached at a distance of 14 cm from the pivot:

Number of free Oscillations	Time (s)	Time period of one oscillation (s)	Frequency (Hz)
10	9.86	0.986	1.01
10	11.23	1.123	0.89
10	10.60	1.060	0.94
10	11.25	1.125	0.88
10	11.62	1.162	0.86

ω(theoretical) = 0.74Hz

Average Frequency = 0.92 Hz (clearly this is not the same as the theoretical actual natural frequency).

2) With one 50 g weight attached, at 14 cm from the pivot:

Number of free oscillations	Time (s)	Time period of one oscillation (s)	Frequency (Hz)
10	8.82	0.882	1.13
10	9.31	0.931	1.07
10	8.72	0.872	1.15
10	9.24	0.924	1.08
10	9.12	0.912	1.10

Frequency = 1.12 Hz

Effective length of pendulum L_2= 20 cm

The least count of the stop watch was 0.02sec, which corresponds to an error of 0.03 Hz in frequency.

II. Calculating the frequency of oscillations of Pendulum 1:

Now, we moved the left pendulum (pendulum 2: P2), without disturbing the other pendulum. Now, we observed the oscillations of the right pendulum (pendulum 1: P1). We recorded the video of these oscillations. We did this for three different natural frequencies of P2.

We then used ImageJ software, to break the video into frames, then converted the video to binary, and tracked the motion of the particle situated at P1. By this, we got the displacement of P1 with time, and the plot of displacement of P1 v/s time.

We then used Matlab to take the Fast Fourier Transform (fft) of this data, and got the plot of Amplitude vs frequency of the oscillations of P1.

While performing the experiment, we observed that the dissipative action was strong and the oscillations of the second pendulum died out quickly (in about 5 seconds). The amplitude of oscillations was also small for the second pendulum. This could be attributed to the fact that the area of cross section of the magnets was small, which allowed for limited coupling. Also the pendulum rods might have been too heavy to cause significant angular displacement when acted upon by the electromotive force.

Analysis and Calculations:

The values of various parameters like $\gamma_1, \gamma_2, \Gamma_1, \Gamma_2$ are calculated. We calculated Γ_1, Γ_2 analytically, by using the equation (2). The values of γ_1, γ_2
were taken as it is from the reference paper.

Parameter	Physical Meaning	Value (Hz)	Error (Hz)	Determination method used
ω_1	Pendulum 1 natural frequency	0.74 1.12	o.o3	Experimental
ω_2	Pendulum 2 natural frequency	0.74	0.03	Experimental
γ_1	Intrinsic damping parameter of pendulum 1	0.0181 0.0502	0.0003	Taken from paper
γ_2	Intrinsic damping parameter of pendulum 2	0.0031	0.0003	Taken from paper
Γ_1	Dissipative Coupling strength of pendulum 1	0.2800 14.5100	0.0012 0.0078	Analytical
Γ_2	Dissipative Coupling strength of pendulum 2	0.2800	0.0012	Analytical

Theoretical expected values of eigen frequencies are calculated using equation (3) and using the values of $\gamma_1, \gamma_2, \Gamma_1, \Gamma_2$ as given in the table above.

Case 1: No weight attached (same effective lengths):

Theoretical calculated eigenfrequencies: $\omega_{\pm} = 0.74 \pm 0.28i$

$Real(\omega_{+}) = Real(\omega_{-}) = 0.74 Hz$

Case 2 - 50g weight attached at 14 cm from the pivot, :

Theoretically calculated eigenfrequencies -

$\omega_{+} = 1.045 - 7.38i$

$\omega_{-} = 1.725 - 22.22i$

$Real(\omega_{+}) = 1.045\, Hz$

$Real(\omega_{-}) = 1.725$ Hz

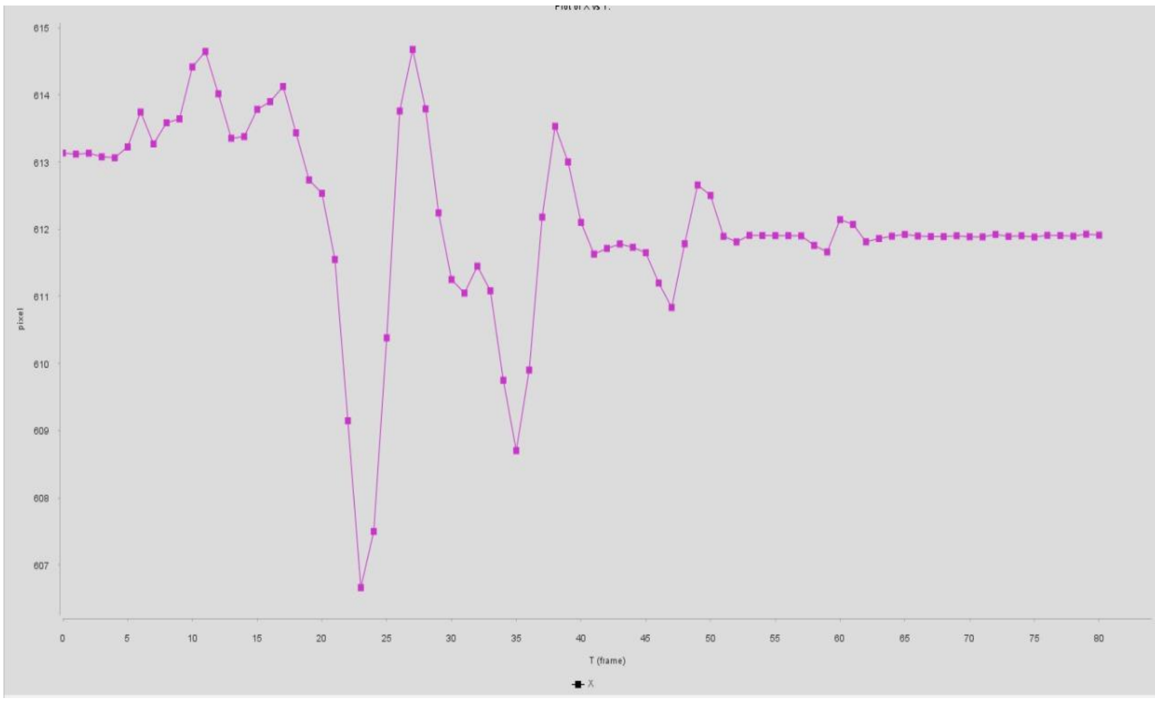
Here, the real part of the eigen frequencies correspond to the actual observed frequency, while the imaginary part corresponds to damping of each mode.

Results:

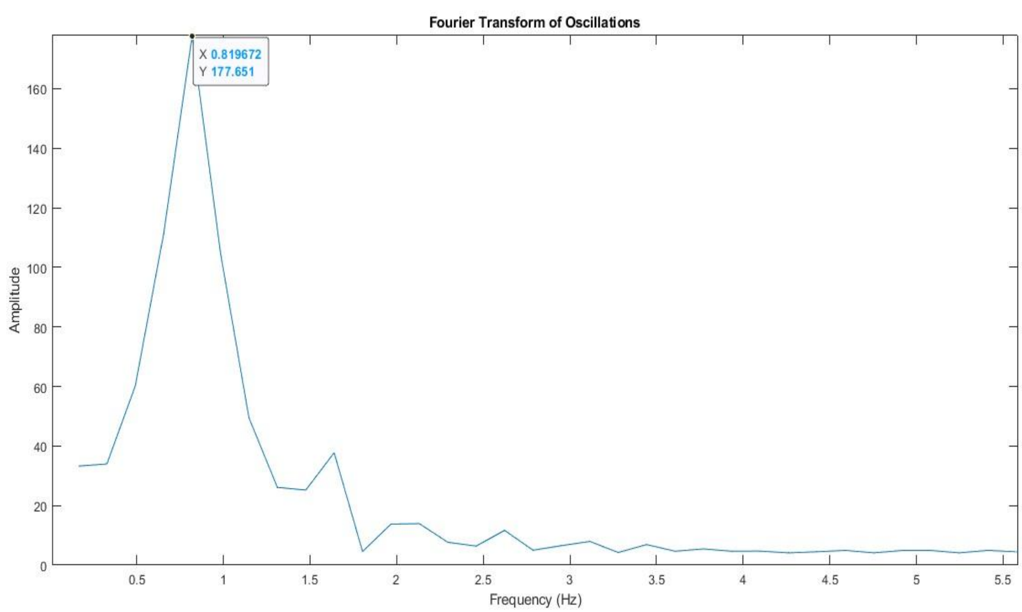
I. <u>When both the pendulums had same natural frequencies</u>

$(\omega_1 = \omega_2)$

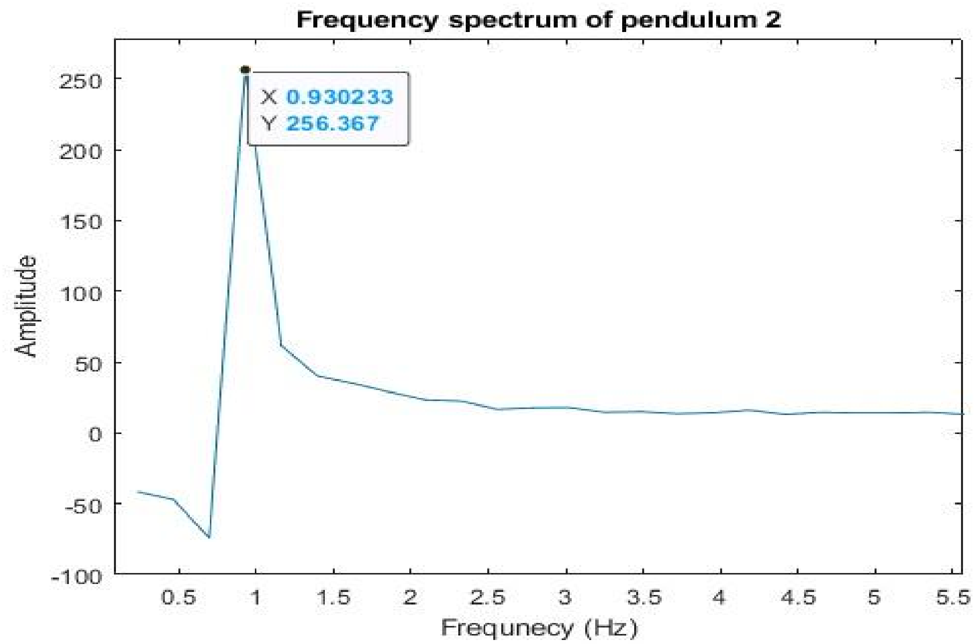
$\Delta = 0$



Amplitude of oscillation of P1 v/s Time (x 0.1 sec) of Pendulum 1



Fourier Spectrum of Pendulum 1



Frequency spectrum of Pendulum 2

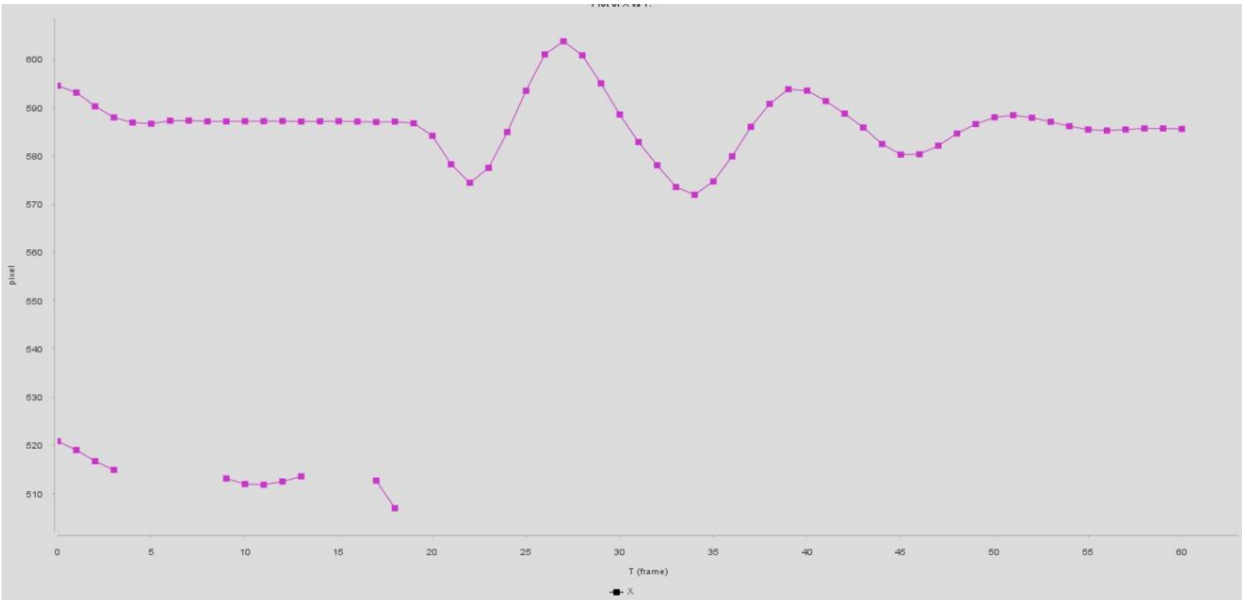
From the above frequency spectrum we clearly see that the peak occurs at a frequency of 0.81Hz for pendulum 1 and 0.93 for pendulum 2, which is near to the theoretical estimate of 0.74Hz.

Clearly, we see that when the detuning of the pendulums is 0, the two pendulums oscillate at nearly the same frequency, and the system quickly dissipates. The system only experiences intrinsic damping.

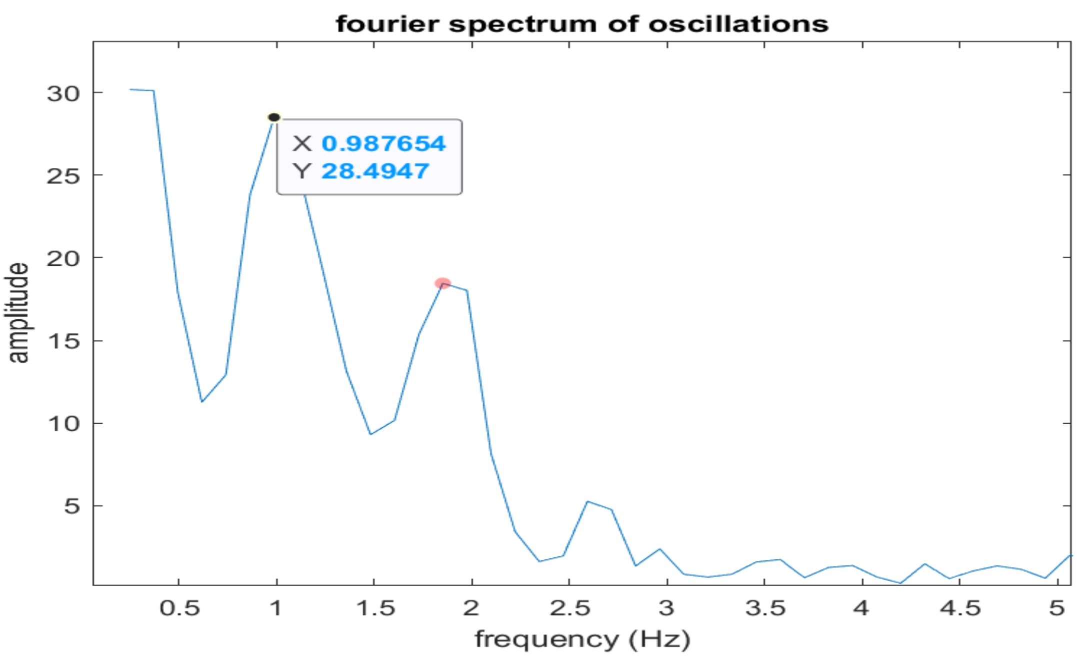
II. When both the pendulums had different natural frequencies

$(\omega_1 \neq \omega_2)$

$\Delta = 0.38$



Amplitude of oscillation of P1 v/s Time (x 0.1 sec) of Pendulum 1



Fourier Spectrum of Pendulum 1 (Fourier transform of the above data)

We see from the fourier spectrum that the peak occurs at a frequency of 0.98Hz and the second peak occurs at around 1.8Hz for pendulum 1, which is comparable to the theoretically predicted values of 1.025Hz and 1.725 Hz .

We also see many peaks in the spectrum which correspond to the different eigenmode frequencies. It shows that the system is in off degenerate state. Also the oscillations die out very quickly, as it experiences a combined damping from the coupling dissipation and intrinsic damping.

Conclusion:

In this experiment, an electro-dynamic friction coupled pendulum system was constructed to demonstrate dissipative coupling. The system's time evolution is studied: when at zero detuning, the system shows a long-lasting full synchronization pattern with minimized damping. We observe a single peak in the fourier spectrum which corresponds to merger eigenmode frequencies. This is same as the theoretically predicted model. We identify it as a perfectly degenerate state since the two oscillators are nearly identical and the oscillation pattern has no relative phase difference

When detuning is non-zero, the oscillation shows a different pattern. We see two peaks in the fourier spectrum of pendulum 1 which correspond to the different eigenmode frequencies. While pendulum 2, shows a single peak. This means that the system is non-degenerate.

There is however some uncertainty in the theoretical and experimental values. It is possible because the oscillations in our system were not very prominent, and also the oscillations died out in 5-6 secs, because of which we were not able to study the time evolution of the system properly.

Possible Sources of Errors:

- Pendulum Rod Weight: The weight of pendulum rods may have been leading to reduced angular displacement when influenced by the electromotive force, thereby affecting the oscillations.
- Small Oscillation Angle: Limited oscillation angles (1.5-2 degrees) might have resulted in a small overlap area in the coupling mechanism, influencing the accuracy of coupling strength parameters.
- Area Approximation: The approximation of the coil and magnet overlapping area as a rectangle could introduce inaccuracies in estimating the enclosed magnetic flux and, consequently, the induced electromotive force.
- Damping Measurement: The strong dissipative action causing rapid damping (about 5 seconds) might have resulted in a limited observation time and small amplitude, affecting accurate data collection.
- Magnetic Field Uniformity: Small magnets providing a nearly uniform magnetic field may have led to non-uniform field distribution, impacting the accuracy of induced current and dissipative coupling force measurements.
- Alignment Sensitivity: Precise alignment is crucial for consistent coupling; misalignment could potentially lead to pendulum collision or inconsistent coupling, influencing experimental outcomes.
- The values of experimentally observed frequency and theoretical model, do not match. This is possible because of the mass of the coupling mechanism which was was not taken into account while performing the theoretical calculations.

- Sensor Resolution: The least count of the stopwatch and potential errors in frequency measurement may have affected the accuracy of determining natural frequencies and intrinsic damping parameters.

References:

Chenyang Lu, Bentley Turner, Yongsheng Gui, Jacob Burgess, Jiang Xiao, Can-Ming Hu; An experimental demonstration of level attraction with coupled pendulums. *Am. J. Phys.* 1 August 2023; 91 (8): 585–594. https://doi.org/10.1119/5.0081906