

Topic : Non Linear Partial Differential Equation(Some Standard Forms)

Q.1 Solve : $p + q = pq$

Q.2 Solve the partial differential equation : $p^2 + q^2 = 2$

Q.3 Solve the partial differential equation $z = px + qy + \sin(p+q)$

Q.4 Solve : $9(p^2z + q^2) = 4$

Q.5 Solve : $p - 3x^2 = q^2 - y$

Maths - IV

DPP-6

Topic - Non Linear Partial Differential
Equation (Some Standard Form)

Q-1: Solve $p+q = pq$

$$\text{Sol: } p+q = pq \quad \dots \textcircled{1}$$

Replace p by a and q by b

$$a+b = ab \quad \dots \textcircled{2}$$

$$a = ab - b$$

$$a = b(a-1)$$

$$b = \frac{a}{a-1}$$

Complete solution is

$$z = ax + by + c$$

$$z = ax + \frac{a}{a-1}y + c$$

Ans

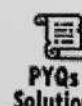
Q-2: Solve the partial differential equation: $p^2 + q^2 = 2$

$$\text{Sol: } p^2 + q^2 = 2$$

Replace p by a and q by b

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$$a^2 + b^2 = 2 \quad \text{--- (2)}$$

$$b^2 = 2 - a^2$$

$$b = \sqrt{2 - a^2}$$

Complete solution is - $z = ax + by + c$

$$z = ax + \sqrt{2 - a^2} y + c$$

Ans

Q-3: Solve the partial differential eq $Z = px + qy + \sin(p+q)$

$$Z = px + qy + \sin(p+q) \quad \text{--- (1)}$$

Replace p by a and q by b

$$z = ax + by + \sin(a+b)$$

Ans

Q-4: Solve: $g(p^2 z + q^2) = 4$

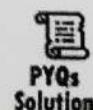
$$g(p^2 z + q^2) = 4 \quad \text{--- (1)}$$

Let $u = x + ay$ so that

$$p = \frac{dz}{du} \text{ and } q = a \frac{dz}{du}$$

$$g \left[\left(\frac{dz}{du} \right)^2 z + a^2 \left(\frac{dz}{du} \right)^2 \right] = 4$$

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$$9 \left(\frac{dz}{du} \right)^2 [z + a^2] = 4$$

$$\frac{9}{4} \left(\frac{dz}{du} \right)^2 = \frac{1}{z+a^2}$$

Square Root Both Side

$$\frac{3}{2} \left(\frac{dz}{du} \right) = \frac{1}{\sqrt{z+a^2}}$$

$$\frac{3}{2} (z+a^2)^{1/2} dz = du$$

Integrate both side

$$\frac{3}{2} \int (z+a^2)^{1/2} dz = \int du$$

$$\frac{3}{2} \frac{(z+a^2)^{3/2}}{3/2} = u + C$$

$$\frac{3}{2} \times \frac{2}{3} (z+a^2)^{3/2} = u + C$$

$$(z+a^2)^{3/2} = u + C$$

Squaring both side and put value of $u = x+ay$

$$(z+a^2)^3 = (x+ay+C)^2$$

Ans

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$$Q-5: \text{ Solve } p - 3x^2 = q^2 - y$$

$$\text{Let } p - 3x^2 = q^2 - y = a \quad - \textcircled{1}$$

$$p - 3x^2 = a$$
$$p = a + 3x^2$$

$$f_1(x) = a + 3x^2$$

$$q^2 - y = a$$
$$q^2 = a + y$$
$$q = \sqrt{a + y}$$

$$f_2(y) = \sqrt{y + a}$$

We know that

$$dz = pdx + qdy$$

Put p and q in above equation

$$dz = (a + 3x^2)dx + \sqrt{y + a} dy$$

Integrate both side

$$z = \int (a + 3x^2) dx + \int (y + a)^{1/2} dy$$

$$z = ax + x^3 + \frac{2}{3} (y + a)^{3/2} + C$$

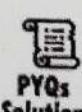
Ans.

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Topic : CHARPIT METHOD

Q.1 Solve : $2zx - px^2 - 2pxy + pq = 0$

Ans $z = ay + b(n^2 - a)$

Q.2 Solve the following equation by charpit's method : $z^2 = pqxy$

Ans $z = a^n y^b$

Q.3 Solve the following equation by charpit's method : $z = p^2x + q^2x$

Ans $\sqrt{(1+a)z} = \sqrt{an} + \sqrt{by} + b$

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DPP - 7

Topic - Charpit Method

$$Q-1: \text{Solve: } 2zx - px^2 - 2qxy + pq = 0$$

$$2zx - px^2 - 2qxy + pq = 0$$

$$\text{Let } f(x, y, z, p, q) = 2zx - px^2 - 2qxy + pq = 0 \quad \text{--- (1)}$$

$$\frac{\delta f}{\delta x} = 2z - 2px, \quad \frac{\delta f}{\delta y} = -2qx, \quad \frac{\delta f}{\delta z} = 2x, \quad \frac{\delta f}{\delta p} = -x^2 + q, \quad \frac{\delta f}{\delta q} = -2xy + p$$

$$\frac{\delta f}{\delta x} = 2z - 2px, \quad \frac{\delta f}{\delta y} = -2qx, \quad \frac{\delta f}{\delta z} = 2x, \quad \frac{\delta f}{\delta p} = -x^2 + q, \quad \frac{\delta f}{\delta q} = -2xy + p$$

Charpit's Auxiliary Equation :-

$$\frac{dx}{-\frac{\delta f}{\delta p}} = \frac{dy}{-\frac{\delta f}{\delta q}} = \frac{dz}{-\frac{\delta f}{\delta x} - \frac{p}{\delta p} \frac{\delta f}{\delta q}} = \frac{dp}{\frac{\delta f}{\delta x} + p \frac{\delta f}{\delta z}} = \frac{dq}{\frac{\delta f}{\delta y} + q \frac{\delta f}{\delta z}}$$

$$\frac{dx}{x^2 - q} = \frac{dy}{2xy - q} = \frac{dz}{px^2 - 2pq + 2qxy} = \frac{dp}{2z - 2qy} = \frac{dq}{0}$$

$$\frac{dq}{q} = 0$$

$$\boxed{q = a}$$

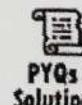
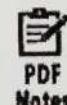
$$\text{Put } q = a \text{ in eq (1)}$$

$$2zx - px^2 - 2axy + pa = 0$$

$$p(a - x^2) = 2axy - 2zx$$

$$p = \frac{2x(ay - z)}{a - x^2}$$

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$$P = \frac{2x(z-ay)}{x^2-a}$$

We know that, $dz = pdx + qdy$

$$dz = \frac{2x(z-ay)}{x^2-a} dx + ady$$

$$dz - ady = \frac{2x(z-ay)}{x^2-a} dx$$

$$\int \frac{dz - ady}{z-ay} = \int \frac{2x}{x^2-a} dx$$

$$x^2 - a = t$$
$$2x dx = dt$$

$$\log(z-ay) = \int \frac{1}{t} dt$$

$$\log(z-ay) = \log t + \log b$$

$$\log(z-ay) = \log(x^2-a) + \log b$$

$$z-ay = (x^2-a)b$$

$$z = ay + b(x^2-a)$$

Ans

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Q-2: Solve the following equation by Charpit's method

$$z^2 = pqxy$$

Sol: $z^2 - pqxy = 0$

Let $f(x, y, z, p, q) = z^2 - pqxy = 0 \quad \dots \text{①}$

~~$$\frac{\delta f}{\delta x} = -pqy, \quad \frac{\delta f}{\delta y} = -pqx, \quad \frac{\delta f}{\delta z} = 2z$$~~

~~$$\frac{\delta f}{\delta p} = -qxy, \quad \frac{\delta f}{\delta q} = -pxy$$~~

Charpit's Auxiliary Equation :-

~~$$\frac{dx}{-\frac{\delta f}{\delta p}} = \frac{dy}{-\frac{\delta f}{\delta q}} = \frac{dz}{-\frac{p\delta f}{\delta p} - q\frac{\delta f}{\delta q}} = \frac{dp}{\frac{\delta f + p\delta f}{\delta x}} = \frac{dq}{\frac{\delta f + q\delta f}{\delta y}}$$~~

~~$$\frac{dx}{qxy} = \frac{dy}{pxy} = \frac{dz}{pqxy + pqxy} = \frac{dp}{-pqy + 2pz} = \frac{dq}{-pqx + 2qz}$$~~

No fraction pair formed, so taking x and p as a multipliers in first and fourth fraction. Similarly taking q and y as a multipliers in second and last fraction then equating :-

$$\frac{x dp + pdx}{-pqxy + 2pxz + pqxy} = \frac{y dq + qdy}{-pqxy + 2qyz + pqxy}$$

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$$\frac{x dp + pdx}{xpz} = \frac{y dq + qdy}{qyz}$$

$$\frac{x dp + pdx}{px} = \frac{y dq + qdy}{qy}$$

Now Integrate Both Side

$$\log xp = \log yq + \log c$$

$$\log xp - \log yq = \log c$$

$$\frac{xp}{yq} = c$$

$$P = \frac{cyq}{x} \quad \text{--- (2)}$$

Put value of P in equation ①

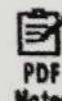
$$Z^2 = pqxy$$

$$Z^2 = cy^2 q^2$$

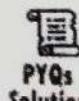
$$q^2 = \frac{Z^2}{cy^2}$$

$$q = \frac{Z}{\sqrt{cy}}$$

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Again put value of q in eq ② -

$$P = \frac{cy}{x} + \frac{z}{\sqrt{c}y}$$

$$P = \frac{z\sqrt{c}}{x}$$

$$\text{Let } \sqrt{c} = b$$

$$\therefore P = \frac{zb}{x}, \quad q = \frac{z}{by}$$

We know that,

$$dz = pdx + qdy$$

$$dz = \frac{zb}{x} dx + \frac{z}{by} dy$$

$$\frac{dz}{z} = b \frac{dx}{x} + \frac{1}{b} \frac{dy}{y}$$

On Integration :-

$$\log z = b \log x + \frac{1}{b} \log y + \log a$$

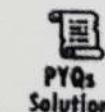
$$\log z = \log x^b + \log y^{1/b} + \log a$$

$$\log z = \log (ax^b y^{1/b})$$

$$z = ax^b y^{1/b}$$

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Q-3: Solve: $z = p^2x + q^2y$

$$z - p^2x - q^2y = 0 \quad \dots \quad (1)$$

Let $f(x, y, z, p, q) = z - p^2x - q^2y = 0$

$$\frac{\delta f}{\delta x} = -p^2, \quad \frac{\delta f}{\delta y} = -q^2, \quad \frac{\delta f}{\delta z} = 1$$

$$\frac{\delta f}{\delta p} = -2px, \quad \frac{\delta f}{\delta q} = -2qy$$

Charpit Auxiliary Equation:

$$\frac{dx}{-sf} = \frac{dy}{-sf} = \frac{dz}{-psf - qsf} = \frac{dp}{sf + psf} = \frac{dq}{sf + qsf}$$

$$\frac{dx}{2px} = \frac{dy}{2qy} = \frac{dz}{2p^2x + 2q^2y} = \frac{dp}{-p^2 + p} = \frac{dq}{q - q^2}$$

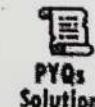
Multiply by p^2 and $2px$ as multipliers in first and fourth fraction, Similarly q^2 and $2qy$ in second and last fraction then equating them -

$$\frac{p^2 dx + 2px dp}{2p^3x - 2p^2x + 2p^2x} = \frac{q^2 dy + 2qy dq}{2q^3y + 2q^2y - 2q^2y}$$

$$\frac{p^2 dx + 2px dp}{7p^2x} = \frac{q^2 dy + 2qy dq}{7q^2y}$$

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$$\frac{P^2 dx + 2px dp}{P^2 x} = \frac{q^2 dy + 2qy dq}{q^2 y}$$

Now Integrate Both Side

$$\log(P^2 x) = \log(q^2 y) + \log a$$

$$P^2 x = a q^2 y \quad \text{--- (2)}$$

Putting value of $P^2 x$ in eq (1)

$$aq^2 y + q^2 y = z$$

$$q = \sqrt{\frac{z}{(1+a)y}}$$

Now, again put value of q in eq (2)

$$P^2 x = \frac{az}{(1+a)y}$$

$$P^2 x = \frac{az}{1+a}$$

$$P = \sqrt{\frac{az}{x(1+a)}}$$

We know that,

$$dz = pdx + qdy$$

$$dz = \left[\frac{az}{x(1+a)} \right]^{1/2} dx + \left[\frac{z}{(1+a)y} \right]^{1/2} dy$$

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$$\sqrt{1+a} \frac{dz}{z^{1/2}} = a^{1/2} \frac{dx}{x^{1/2}} + \frac{dy}{y^{1/2}}$$

$$\sqrt{1+a} \frac{z^{1/2}}{1/2} = \sqrt{a} \frac{x^{1/2}}{1/2} + \frac{y^{1/2}}{1/2} + b$$

$$2\sqrt{(1+a)z} = 2[\sqrt{ax} + \sqrt{y} + b]$$

$$\boxed{\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b}$$

Ans

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PYQs
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Lectures

**Topic : Homogeneous Linear Partial Differential Equation with constant coefficient
(Complementary function)**

Q.1 Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = 0$

Ans $z = f_1(y+3n) + f_2(y-2n)$

Q.2 Solve: $\frac{\partial^3 z}{\partial x^3} - 3 \frac{\partial^3 z}{\partial x^2 \partial y} + 2 \frac{\partial^3 z}{\partial x \partial y^2} = 0$

Ans $z = f_1(y) + f_2(y+n) + f_3(y+2n)$

Q.3 Solve: $4r - 12s + 9t = 0$

Ans $z = f_1(y + \frac{3}{2}n) + n f_2(y + \frac{3}{2}n)$

Q.4 Solve: $(D^3 D' - 4D^2 D'^2 + 4DD'^3)z = 0$

Ans $z = f_1(y) + f_2(n) + f_3(y+2n) + n f_4(y+2n)$

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DPP-8

Maths - IV

Topic - Homogeneous Linear Partial
Differential Equation with
Constant coefficient (CF)

$$Q-1: \text{ Solve } \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = 0$$

$$\text{Sol: } \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = 0$$

$$(D^2 - DD' - 6D'^2)z = 0$$

$$\text{Put } D = m, D' = 1$$

$$(m^2 - m - 6)z = 0$$

Aux Equation -

$$m^2 - m - 6 = 0$$

$$m^2 - 3m + 2m - 6 = 0$$

$$m(m-3) + 2(m-3) = 0$$

$$(m+2)(m-3) = 0$$

$$m = -2, 3$$

$$CF = f_1(y-2x) + f_2(y+3x)$$

$$PI = 0$$

Complete solution

$$z = CF + PI$$

$$z = f_1(y-2x) + f_2(y+3x)$$

Ans

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$$Q-2 \div \text{Solve } \frac{\cancel{s^3}z}{\cancel{s}x^3} - 3 \frac{\cancel{s^3}z}{\cancel{s}x^2 \cancel{s}y} + 2 \frac{\cancel{s^3}z}{\cancel{s}x \cancel{s}y^2} = 0$$

$$(D^3 - 3D^2 D' + 2D D'^2) z = 0$$

$$D(D^2 - 3DD' + 2D'^2) z = 0$$

Aux Equation :-

$$D(D^2 - 3DD' + 2D'^2) = 0$$

$$D = 0$$

$$D^2 - 3DD' + 2D'^2 = 0$$

$$\text{Put } D = m, D' = 1$$

$$CF = f_1(y)$$

$$m^2 - 3m + 2 = 0$$

$$(m-2)(m-1) = 0$$

$$m = 1, 2$$

$$CF = f_2(y+x) + f_3(y+2x)$$

$$PI = 0$$

Complete Solution

$$Z = CF + PI$$

$$Z = f_1(y) + f_2(y+x) + f_3(y+2x)$$

Ans

$$Q-3 \div \text{Solve} - 4r - 12s + 9t = 0$$

$$\frac{4s^2} {s x^2} z - 12 \frac{s^2} {s x s y} z + 9 \frac{s^2} {s y^2} z = 0$$

$$(4D^2 - 12DD' + 9D'^2) z = 0$$

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$$\text{Auxiliary Equation} \Rightarrow 4m^2 - 12m + 9 = 0$$

$$(2m-3)^2 = 0$$

$$m = \frac{3}{2}, \frac{3}{2}$$

$$C.F. = f_1 \left(y + \frac{3x}{2} \right) + x f_2 \left(y + \frac{3x}{2} \right)$$

$$P.I. = 0$$

$$\text{Complete Solution} \Rightarrow z = C.F. + P.I.$$

$$z = f_1 \left(y + \frac{3x}{2} \right) + x f_2 \left(y + \frac{3x}{2} \right)$$

Ans

$$Q-4: (D^3 D' - 4D^2 D'^2 + 4DD'^3)z = 0$$

$$DD' (D^2 - 4DD' + 4D'^2)z = 0$$

$$\text{Auxiliary Equation:}$$

$$DD' (D^2 - 4DD' + 4D'^2) = 0$$

$$D = 0$$

$$D' = 0$$

$$(D^2 - 4DD' + 4D'^2) = 0$$

$$\text{P.W. } D = m, D' = 1$$

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

$$C.F. = f_3 (y+2x) + x f_4 (y+2x)$$

$$P.I. = 0$$

$$\text{Complete solution} \Rightarrow z = C.F. + P.I.$$

$$z = f_1(y) + f_2(x) + f_3(y+2x) + x f_4(y+2x)$$

Ans

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Topic : Homogeneous Linear Partial Differential Equation with constant coefficient



Particular Integral(PI) : Case-I

Q.1 Solve the linear partial differential equation $\frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = x + y.$

$$z = f_1(y-n) + f_2(y-2n) + \frac{(n+y)^3}{36}$$

Q.2 Solve : $\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$ (MTU-2013)

$$z = f_1(y+n) + nf_2(y+n) - \sin n$$

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Maths - IV

DPP - 9

Topic - Particular Integral
(PI) Case - 1

$$Q-1: \frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = x+y$$

$$(D^2 + 3DD' + 2D'^2)z = x+y$$

Put $D = m$, $D' = 1$

$$(m^2 + 3m + 2)z = x+y$$

Auxiliary Equation : $m^2 + 3m + 2 = 0$

$$m^2 + 2m + m + 2 = 0$$

$$m(m+2) + 1(m+2) = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

$$CF = f_1(y-x) + f_2(y-2x)$$

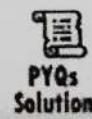
$$PI = \frac{1}{D^2 + 3DD' + 2D'^2} (x+y)$$

Put $D = 1$, $D' = 1$, $x+y = u$

$$PI = \frac{1}{1+3+2} \iint u du du$$

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$$PI = \frac{1}{6} \int \int u du du$$

$$PI = \frac{1}{6} \int \frac{u^2}{2} du$$

$$PI = \frac{1}{6} \frac{u^3}{2 \times 3}$$

$$PI = \frac{u^3}{36}$$

$$PI = \frac{(x+y)^3}{36}$$

Complete solution $\Rightarrow z = CF + PI$

$$z = f_1(y-x) + f_2(y-2x) + \frac{(x+y)^3}{36}$$

Ans

Q-2- Solve $\frac{s^2 z}{s x^2} - 2 \frac{s^2 z}{s x s y} + \frac{s^2 z}{s y^2} = \sin x$

Sol: $\frac{s^2 z}{s x^2} - 2 \frac{s^2 z}{s x s y} + \frac{s^2 z}{s y^2} = \sin x$

$$(D^2 - 2 D D' + D'^2) z = \sin x$$

Put $D = m$, $D' = 1$

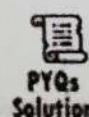
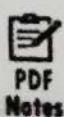
$$(m^2 - 2m + 1) z = \sin x$$

Auxiliary Equation -

$$m^2 - 2m + 1 = 0$$

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$$(m-1)^2 = 0$$

$$m = 1, 1$$

$$C.F. = f_1(y+x) + x f_2(y+x)$$

$$P.I. = \frac{1}{D^2 - 2DD' + D'^2} \sin x$$

$$P.I. = \frac{1}{D^2 - 2DD' + D'^2} \sin(x+0y)$$

$$\text{Put } D=1, D'=0, x+0y=u$$

$$P.I. = \frac{1}{1} \int \int \sin u du$$

$$P.I. = \int -\cos u du$$

$$P.I. = -\sin u$$

$$P.I. = -\sin(x+0y)$$

$$P.I. = -\sin x$$

Complete Solution :- $Z = C.F. + P.I.$

$$Z = f_1(y+x) + x f_2(y+x) - \sin x$$

Ans

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-I (Type-2)

Q.1 Solve the linear partial differential equation $\frac{\partial^3 z}{\partial x^3} - 4 \frac{\partial^3 z}{\partial x^2 \partial y} + 4 \frac{\partial^3 z}{\partial x \partial y^2} = 4 \sin(2x + y).$

Ans $f_1(y) + f_2(y+2n) + n f_3(y+2n) - n^2 \cos(2n+y)$

Q.2 Solve : $2r - s - 3t = 5 \frac{e^x}{e^y}$

Ans $z = f_1(y-n) + f_2\left(y+\frac{3}{2}n\right) + n e^{n-y}$

Q.3 Solve : $\frac{\partial^3 z}{\partial x^3} - 7 \frac{\partial^3 z}{\partial x \partial y^2} - 6 \frac{\partial^3 z}{\partial y^3} = \sin(x + 2y) + e^{3x+y}. \quad (\text{UPTU-2014})$

Ans $z = f_1(y-n) + f_2(y-2n) + f_3(y+3n) - \frac{1}{75} \cos(n+2y) + \frac{n}{20} e^{3n+y}$

Maths - IV

DPP - 10

Topic - Particular Integral
Case - 1 (Type - 2)

Q-1: Solve the linear partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - \frac{4 \partial^3 z}{\partial x^2 \partial y} + 4 \frac{\partial^3 z}{\partial x \partial y^2} = 4 \sin(2x+y)$$

$$(D^3 - 4D^2D' + 4DD'^2)z = 4 \sin(2x+y)$$

$$D(D^2 - 4DD' + 4D'^2)z = 4 \sin(2x+y)$$

Auxiliary Equation

$$D = 0$$

$$D^2 - 4DD' + 4D'^2 = 0$$

Replace $D = m$ and $D' = 1$

$$C.F. = f_1(y)$$

$$m^2 - 4m + 4 = 0$$

$$(m-2)^2 = 0$$

$$m = 2, 2$$

$$C.F. = f_2(y+2x) + x f_3(y+2x)$$

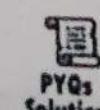
$$C.F. = f_1(y) + f_2(y+2x) + x f_3(y+2x)$$

$$P.I. = \frac{1}{D^3 - 4D^2D' + 4DD'^2} 4 \sin(2x+y)$$

$$\text{Put } D = 2, D' = 1$$

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$$PI = \frac{1}{2^3 - 4x^4x_1 + 4x_2x_1} 4 \sin(2x+y)$$

$$PI = \frac{1}{8-16+8} 4 \sin(2x+y)$$

{ Case of failure }

Differentiate w.r.t 'D' and multiply with x.

$$PI = \frac{x}{3D^2 - 8DD' + 4D'^2} 4 \sin(2x+y)$$

$$\text{Put } D=2, D'=1$$

$$PI = \frac{x}{3x^4 - 8x^2 + 4} 4 \sin(2x+y)$$

$$PI = \frac{x}{12-16+4} 4 \sin(2x+y)$$

{ Again Case of failure }

Differentiate w.r.t 'D' and multiply with x.

$$PI = \frac{x^2}{6D - 8D'} 4 \sin(2x+y)$$

$$\text{Put } D=2, D'=1, 2x+y=u$$

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$$PI = 4x^2 x \frac{1}{12-8} \int \sin u du$$

$$PI = 4x^2 x \frac{1}{4} x - \cos u$$

$$PI = -x^2 \cos u$$

$$PI = -x^2 \cos(2x+y)$$

Complete solution $\Rightarrow Z = CF + PI$.

$$Z = f_1(y) + f_2(y+2x) + x f_3(y+2x) - x^2 \cos(2x+y)$$

$$Q-2: \text{ Solve } 2r-s-3t = 5 \frac{e^x}{e^y}$$

$$2r-s-3t = 5 e^{x-y}$$

$$2 \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 3 \frac{\partial^2 z}{\partial y^2} = 5 e^{x-y}$$

$$(2D^2 - DD' - 3D'^2)z = 5 e^{x-y}$$

$$\text{Auxiliary Equation} \Rightarrow 2D^2 - DD' - 3D'^2 = 0$$

Put $D = m$, $D' = 1$

$$2m^2 - m - 3 = 0$$

$$2m^2 - 3m + 2m - 3 = 0$$

$$m(2m-3) + 1(2m-3) = 0$$

$$(m+1)(2m-3) = 0$$

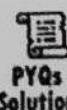
$$m = -1, 3/2$$

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$$C.F. = f_1(y-x) + f_2\left(y + \frac{3}{2}x\right)$$

$$P.I. = \frac{1}{2D^2 - DD' - 3D'^2} 5e^{x-y}$$

Put $D=1$ and $D'=-1$

$$P.I. = \frac{1}{2+1-3} 5e^{x-y}$$

{ Case of failure }

Differentiate w.r.t D and multiply with x

$$P.I. = x \times \frac{1}{4D - D'} 5e^{x-y}$$

Put $D=1$, $D'=-1$, $u=x-y$

$$P.I. = 5x \times \frac{1}{4+1} \int e^u du$$

$$P.I. = 5x \times \frac{1}{5} \times e^u$$

$$P.I. = xe^u$$

$$P.I. = xe^{x-y}$$

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Complete solution :- $Z = CF + PI$

$$Z = f_1(y-x) + f_2\left(\frac{y+3x}{2}\right) + xe^{3x+y}$$

Q-3 :- Solve : $\frac{s^3 z}{sx^3} - 7 \frac{s^3 z}{sx sy^2} - 6 \frac{s^3 z}{sy^3} = \sin(x+2y) + e^{3x+y}$

Sol :- $(D^3 - 7DD^2 - 6D^3)z = \sin(x+2y) + e^{3x+y}$

Auxiliary Equation :- $D^3 - 7DD^2 - 6D^3 = 0$
Put $D = m$, $D' = 1$

$$\begin{aligned}m^3 - 7m^2 - 6m &= 0 \\(m+1)(m+2)(m-3) &= 0 \\m &= -1, -2, 3\end{aligned}$$

$$CF = f_1(y-x) + f_2(y-2x) + f_3(y+3x)$$

$$PI = \frac{1}{D^3 - 7DD^2 - 6D^3} [\sin(x+2y) + e^{3x+y}]$$

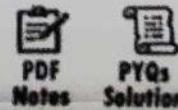
$$PI = \frac{1}{D^3 - 7DD^2 - 6D^3} \sin(x+2y) + \frac{1}{D^3 - 7DD^2 - 6D^3} e^{3x+y}$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{D^3 - 7DD^2 - 6D^3} \sin(x+2y)$$

$$\text{Put } D = 1, D' = 2, \sin(2x+2y) = u$$

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$$P_1 = \frac{1}{(1)^3 - 7(1)(2)^2 - 6(2)^3} \quad \left. \int \int \int \sin u \ du \ du \ du \right|$$

$$P_1 = \frac{-1}{75} \cos u$$

$$P_1 = \frac{-1}{75} \cos(x+2y)$$

$$P_2 = \frac{1}{D^3 - 7DD^2 - 6D^3} e^{3x+y}$$

$$\text{Put } D = 3, D' = 1$$

$$P_2 = \frac{1}{27 - 21 - 6} e^{3x+y}$$

{ Case of failure }

Differentiate w.r.t 'D' and multiply with x

$$P_2 = x \times \frac{1}{3D^2 - 7D^2} e^{3x+y}$$

$$\text{Put } D = 3, D' = 1, 3x+y = u$$

$$P_2 = x \times \frac{1}{3 \times 9 - 7 \times 1} e^{3x+y}$$

$$P_2 = x \times \frac{1}{20} \quad \left. \int \int e^u \ du \ du \right|$$

$$P_2 = \frac{x}{20} e^u$$

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$$P_2 = \frac{x}{20} e^{3x+y}$$

$$PI = P_1 + P_2$$

$$PI = -\frac{1}{75} \cos(x+2y) + \frac{x}{20} e^{3x+y}$$

Complete solution is

$$Z = CF + PI$$

$$Z = f_1(y-x) + f_2(y-2x) + f_3(y+3x) - \frac{1}{75} \cos(x+2y) + \frac{x}{20} e^{3x+y}$$

Ans.

Gateway Classes

PDE : DPP-11

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-II and Case-III

Q.1 Solve $[D^2 + (a+b)DD' + ab D'^2]z = xy$ (GBTU-2011 & 2013)

$$z = f_1(y - a\lambda) + f_2(y - b\lambda) + \frac{1}{6} \lambda^3 y - \frac{(a+b)}{24} \lambda^4 y$$

Q.2 Solve : $(D^3 + 2D^2D' - DD'^2 - 2D'^3)z = (y+2)e^x.$

$$z = f_1(y + \lambda) + f_2(y - \lambda) + f_3(y - 2\lambda) + ye^{\lambda}$$

Q.3 Solve : $(D^2 + DD' - 6 D'^2)z = y \sin x.$ (MTU-2011 & 2012)

$$z = f_1(y + 2\lambda) + f_2(y - 3\lambda) - y \sin \lambda - \cos \lambda$$

GATEWAY CLASSES

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DPP-11

Topic - Particular Integral (PI)

Case-II and Case-III

$$Q-1: \text{Solve } [D^2 + (a+b)DD' + abD'^2]z = xy$$

$$[D^2 + (a+b)DD' + abD'^2]z = xy$$

$$\text{Put } D = m, D' = 1$$

$$[m^2 + (a+b)m + ab]z = xy$$

$$\text{Auxiliary Equation} \Rightarrow m^2 + (a+b)m + ab = 0$$

$$m(m+b) + a(m+b) = 0$$

$$(m+a)(m+b) = 0$$

$$m = -a, -b$$

$$\text{C.F.} = f_1(y-ax) + f_2(y-bx)$$

$$\text{PI} = \frac{1}{D^2 + (a+b)DD' + abD'^2} xy$$

$$\text{PI} = \frac{1}{D^2 \left[1 + (a+b)\frac{D'}{D} + ab\frac{D'^2}{D^2} \right]} x xy$$

$$\text{PI} = \frac{1}{D^2} \left[1 + \left((a+b)\frac{D'}{D} + ab\frac{D'^2}{D^2} \right) \right]^{-1} x xy$$

Using expansion

$$\text{PI} = \frac{1}{D^2} \left[1 - \left((a+b)\frac{D'}{D} + ab\frac{D'^2}{D^2} \right) + \left((a+b)\frac{D'}{D} + ab\frac{D'^2}{D^2} \right)^2 - \dots \right] xy$$

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$$PI = \frac{1}{D^2} \left[1 - (a+b) \frac{D'}{D} \right] xy$$

$$PI = \frac{1}{D^2} \left[xy - (a+b) \frac{D'}{D} (xy) \right]$$

$$PI = \frac{1}{D^2} \left[xy - (a+b) \frac{x}{D} \right]$$

$$PI = \frac{1}{D^2} \left[xy - (a+b) \frac{x^2}{2} \right]$$

$$PI = \frac{1}{D} \left[\int [xy - (a+b) \frac{x^2}{2}] dx \right]$$

$$PI = \frac{1}{D} \left[\int xy dx - \int (a+b) \frac{x^2}{2} dx \right]$$

$$PI = \frac{1}{D} \left[\frac{x^2 y}{2} - (a+b) x \frac{x^3}{3} \right]$$

$$PI = \int \frac{x^2 y}{2} dx - \int \frac{(a+b)x^3}{6} dx$$

$$PI = \frac{x^3 y}{6} - \left(a+b \right) \frac{x^4}{24}$$

Complete Solution $\Rightarrow z = CF + PI$

$$z = f_1(y-ax) + f_2(y-bx) + \frac{x^3 y}{6} - \left(a+b \right) \frac{x^4}{24}$$

GATEWAY CLASSES

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$$Q-2: \text{ Solve } (D^3 + 2D^2D' - DD'^2 - 2D'^3)Z = (y+2)e^x$$

$$(D^3 + 2D^2D' - DD'^2 - 2D'^3)Z = (y+2)e^x$$

$$\text{Put } D = m, D' = 1$$

$$(m^3 + 2m^2 - m - 2)Z = (y+2)e^x$$

$$\text{Auxiliary Equation} \Rightarrow m^3 + 2m^2 - m - 2 = 0$$

$$(m-1)(m+1)(m+2) = 0$$

$$m = 1, -1, -2$$

$$C.F. = f_1(y+x) + f_2(y-x) + f_3(y-2x)$$

$$P.I. = \frac{1}{D^3 + 2D^2D' - DD'^2 - 2D'^3} (y+2)e^x$$

$$P.I. = \frac{1}{(D-D')(D+D')(D+2D')} (y+2)e^x$$

$$P.I. = \frac{1}{(D-D')(D+2D')} \left[\frac{1}{D+D'} (y+2)e^x \right]$$

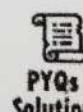
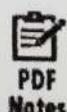
$$\text{Put } y = C + x$$

$$P.I. = \frac{1}{(D-D')(D+2D')} \int (C+x+2)e^x dx$$

$$P.I. = \frac{1}{(D-D')(D+2D')} \left[(C+x+2)e^x - 1xe^x \right]$$

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$$PI = \frac{1}{(D-D')(D+2D')} [(c+x+2-1)e^x]$$

$$PI = \frac{1}{(D-D')(D+2D')} [(c+x+1)e^x]$$

$$PI = \frac{1}{(D-D')(D+2D')} (y+1)e^x$$

$$PI = \frac{1}{D+2D'} \left[\frac{1}{D-D'} (y+1)e^x \right]$$

$$\text{Put } y = c-x$$

$$PI = \frac{1}{D+2D'} \int \underset{\text{I}}{(c-x+1)} e^x dx \underset{\text{II}}{}$$

$$PI = \frac{1}{D+2D'} \left[(c-x+1)e^x - (-1)e^x \right]$$

$$PI = \frac{1}{D+2D'} \left[(c-x+1+1)e^x \right]$$

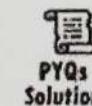
$$PI = \frac{1}{D+2D'} (c-x+2)e^x$$

$$PI = \frac{1}{D+2D'} (y+2)e^x$$

$$\text{Put } y = c+2x$$

$$PI = \int \underset{\text{I}}{(c+2x+2)} e^x dx \underset{\text{II}}{}$$

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$$P.I. = (C + 2x + 2)e^x - 2e^x$$

$$P.I. = (C + 2x + 2 - 2)e^x$$

$$P.I. = (C + 2x)e^x$$

$$P.I. = ye^x$$

Complete Solution $\Rightarrow z = C.F. + P.I.$

$$z = f_1(y+2x) + f_2(y-x) + f_3(y-2x) + ye^x$$

Q-3: Solve $(D^2 + DD' - 6D'^2)z = y \sin x$

$$(D^2 + DD' - 6D'^2)z = y \sin x$$

$$\text{Put } D = m \quad D' = 1$$

$$(m^2 + m - 6)z = y \sin x$$

Auxiliary Equation $\Rightarrow m^2 + m - 6 = 0$

$$m^2 + 3m - 2m - 6 = 0$$

$$m(m+3) - 2(m+3) = 0$$

$$(m-2)(m+3) = 0$$

$$m = 2, -3$$

$$C.F. = f_1(y+2x) + f_2(y-3x)$$



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$$PI = \frac{1}{(D+3D')(D-2D')} y \sin x$$

$$PI = \frac{1}{D+3D'} \left[\frac{1}{D-2D'} y \sin x \right]$$

$$PI = \frac{1}{D+3D'} \int_{I}^{II} (c-2x) \sin x dx$$

$$PI = \frac{1}{D+3D'} \left[(c-2x)(-\cos x) - \int (-2)(-\cos x) \right]$$

$$PI = \frac{1}{D+3D'} \left[-(c-2x)\cos x - \int 2\cos x \right]$$

$$PI = \frac{1}{D+3D'} \left[-(c-2x)\cos x - 2\sin x \right]$$

$$PI = \frac{1}{D+3D'} \left[-ycosx - 2sinx \right]$$

$$PI = \int [(c+3x)\cos x - 2\sin x] dx$$

$$PI = - \int_{I}^{II} (c+3x)\cos x dx - 2 \int \sin x dx$$

$$PI = - \left[(c+3x)\sin x - \int 3\sin x \right] - 2(-\cos x)$$

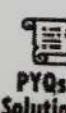
$$PI = - \left[(c+3x)\sin x - 3(-\cos x) \right] + 2\cos x$$

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$$PI = - \left[(c+3x) \sin x + 3 \cos x \right] + 2 \cos x$$

$$PI = -(c+3x) \sin x - 3 \cos x + 2 \cos x$$

$$PI = -(c+3x) \sin x - \cos x$$

$$PI = -y \sin x - \cos x$$

Complete solution $\Rightarrow Z = CF + PI$

$$Z = f_1(y+2x) + f_2(y-3x) - y \sin x - \cos x$$

Ans

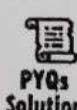
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PDE : DPP-12

Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient
Complementary Function (CF)

Q .1 Solve $2s + t - 3q = 0$

Q .2 Solve $DD'(2D + D' + 11)^2 z = 0$

DPP - 12

Maths - IV

Unit - 1

Topic - Complementary Function (CF)

Q-1: Solve $2s+t-3q=0$

$$\text{Sol: } 2s^2z + s^2z - 3sz = 0$$

$$s_x s_y \quad s_y^2 \quad s_y$$

$$(2D^2 + D^2 - 3D)z = 0$$

$$D^2 \left(D + \frac{1}{2}D - \frac{3}{2} \right) z = 0.$$

CF corresponding to $D^2 = f_1(x)$

~~CF corresponding to $\left(D + \frac{1}{2}D - \frac{3}{2} \right) = e^{\frac{3x}{2}} f_2(y - \frac{x}{2})$~~

$$PI = 0$$

Complete solution $\Rightarrow Z = CF + PI$

$$Z = f_1(x) + e^{\frac{3x}{2}} f_2 \left(y - \frac{x}{2} \right)$$

Q-2: Solve $DD^2 (2D+D^2+11)^2 z = 0$

$$DD^2 \left(D + \frac{D^2}{2} + \frac{11}{2} \right)^2 z = 0$$

CF corresponding to $D = f_1(y)$

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CF corresponding to $D' = f_2(x)$

$$\text{CF corresponding to } \left(D + \frac{D'}{2} + \frac{11}{2}\right)^2 = e^{-11x/2} f_3\left(y - \frac{x}{2}\right) + x e^{-11x/2} f_4\left(y - \frac{x}{2}\right)$$

PI = 0

Complete solution $\Rightarrow Z = CF + PI$

$$Z = f_1(y) + f_2(x) + e^{-11x/2} f_3\left(y - \frac{x}{2}\right) + x e^{-11x/2} f_4\left(y - \frac{x}{2}\right)$$

Gateway Classes



➤ Topic: Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-1 and Case-2

Q.1 Solve: $(D^3 - 3DD' + D' + 4)z = e^{2x+y}$.

Q.2 Solve $(D^2 - D'^2 - 3D + 3D')z = e^{x-2y}$

Q.3 Find the particular integral of $2s+t-3q = 5 \cos(3x-2y)$.

Q.4 Solve the following partial differential equations: $(D^2 - DD' + D' - 1)z = \cos(x+2y)$

DPP-13

Maths - IV

Unit - 1

Topic - Non Homogeneous Partial Differential Equation
with constant coefficient

$$Q-1: \text{Solve } (D^3 - 3DD' + D' + 4)Z = e^{2x+y}$$

$$(D^3 - 3DD' + D' + 4)Z = e^{2x+y}$$

$$(D^3 - 3DD' + D' + 4)Z = 0 \quad \textcircled{1}$$

Here equation $\textcircled{1}$ cannot be resolved into linear factors in D and D' .

Consider a trial solution

$$Z = Ae^{hx+ky}$$

From eq $\textcircled{1}$

$$(D^3 - 3DD' + D' + 4)Ae^{hx+ky} = 0$$

$$A [D^3 e^{hx+ky} - 3DD' e^{hx+ky} + D' e^{hx+ky} + 4e^{hx+ky}] = 0$$

$$A [h^3 e^{hx+ky} - 3hk e^{hx+ky} + ke^{hx+ky} + 4e^{hx+ky}] = 0$$

$$Ae^{hx+ky} [h^3 - 3hk + k + 4] = 0$$

$$Ae^{hx+ky} \neq 0$$

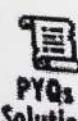
$$\therefore h^3 - 3hk + k + 4 = 0$$

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$$C.F. = \sum A e^{hx+ky} , \text{ where } h^3 - 3hk + k + 4 = 0$$

$$PI = \frac{1}{D^3 - 3DD' + D' + 4} e^{2x+y}$$

Replace $D=2, D'=1$

$$PI = \frac{1}{8 - 6 + 1 + 4} e^{2x+y}$$

$$PI = \frac{1}{7} e^{2x+y}$$

Complete solution is :-

$$Z = CF + PI$$

$$Z = \sum A e^{hx+ky} + \frac{1}{7} e^{2x+y}, \text{ where } h^3 - 3hk + k + 4 = 0$$

$$Q-2: \text{ Solve } (D^2 - D'^2 - 3D + 3D') Z = e^{x-2y}$$

$$(D^2 - D'^2 - 3D + 3D') Z = e^{x-2y}$$

$$[(D-D')(D+D') - 3(D-D')] = e^{x-2y}$$

$$(D-D')(D+D'-3) = e^{x-2y}$$

$$C.F. = f_1(y+x) + e^{3x} f_2(y-x)$$

$$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x-2y}$$

Put $D = 1, D' = -2$

$$PI = \frac{1}{1 - 4 - 3 - 6} e^{x-2y}$$

$$PI = \frac{-1}{12} e^{x-2y}$$

Complete solution - $Z = CF + PI$

$$Z = f_1(y+x) + e^{3x} f_2(y-x) - \frac{1}{12} e^{x-2y}$$

Q-3: Find the Particular Integral of $2s+t-3q = 5\cos(3x-2y)$

$$2s+t-3q = 5\cos(3x-2y)$$

$$(2DD' + D' - 3D')Z = 5\cos(3x-2y)$$

$$PI = \frac{1}{2DD' + D'^2 - 3D'} 5\cos(3x-2y)$$

Replace $D^2 = -9, D'^2 = -4, DD' = 6$

$$PI = \frac{1}{2(6) - 4 - 3D'} 5\cos(3x-2y)$$

$$PI = \frac{1}{8-3D'} 5 \cos(3x-2y)$$

Now multiply by conjugate.

$$PI = \frac{1}{8-3D'} \times \frac{8+3D'}{8+3D'} 5 \cos(3x-2y)$$

$$PI = \frac{8+3D'}{64-9D'^2} 5 \cos(3x-2y)$$

Again put $D'^2 = -4$

$$PI = \frac{8+3D'}{64-(-4)} 5 \cos(3x-2y)$$

$$PI = \frac{5}{100} (8+3D') \cos(3x-2y)$$

$$PI = \frac{1}{20} [8 \cos(3x-2y) + 3D' \cos(3x-2y)]$$

$$PI = \frac{1}{20} [8 \cos(3x-2y) + 3(-2)(-\sin(3x-2y))]$$

$$PI = \frac{1}{10} [4 \cos(3x-2y) + 3 \sin(3x-2y)]$$

Ans

Q-4: Solve the following partial differential equation

$$(D^2 - DD' + D' - 1)z = \cos(x+2y)$$

Sol:- $(D^2 - DD' + D' - 1)z = \cos(x+2y)$

$$[D^2 - 1 - D'(D-1)]z = \cos(x+2y)$$

$$[(D+1)(D-1) - D'(D-1)]z = \cos(x+2y)$$

$$(D-1)[D - D' + 1]z = \cos(x+2y)$$

$$C.F. = e^x f_1(y) + e^{-x} f_2(y+x)$$

$$P.I. = \frac{1}{D^2 - DD' + D' - 1} \cos(x+2y)$$

$$\text{Put } D^2 = -1, D'^2 = -4, DD' = -2$$

$$P.I. = \frac{1}{-1 - (-2) + D' - 1} \cos(x+2y)$$

$$P.I. = \frac{1}{-1 + x + D' - 1} \cos(x+2y)$$

$$P.I. = \frac{1}{D'} \cos(x+2y)$$

$$P.I. = \frac{\sin(x+2y)}{2}$$

GATEWAY CLASSES

Empowering learners, transforming futures

Complete solution $\Rightarrow z = CF + PI$

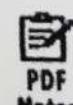
$$z = e^x f_1(y) + e^{-x} f_2(y+x) + \frac{1}{2} \sin(x+2y)$$

Ans

~~Gateway Classes 145698384~~

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➤ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

Q.1 Solve the linear partial differential equation : $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$. (GBTU-2011)

Q.2 Solve $(D + D' - 1)^2 z = xy$ (GBTU-2011)

Q.3 Solve : $(D - 3D' - 2)^3 z = 6e^{2x} \sin(3x + y)$.

Q.4 Solve : $r - 4s + 4t + p - 2q = e^{x+y}$.

GATEWAY CLASSES

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DPP-14

Maths - IV

Unit - 1

Topic - Particular Integral (PI): Case 3 and Case 4

Q-1 - Solve the linear partial differential equation
 $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$

Sol - $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$

$$[(D+D')(D-D') - 3(D-D')]z = xy + e^{x+2y}$$

$$(D-D')[D+D'-3]z = xy + e^{x+2y}$$

$$[C \cdot F \cdot = f_1(y+x) + e^{3x}f_2(y-x)]$$

$$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} \cancel{xy + e^{x+2y}}$$

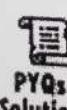
$$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} \cancel{xy} + \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x+2y}$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{(D-D')(D+D'-3)} \cancel{xy}$$

$$P_1 = \frac{1}{D\left(\frac{1-D'}{D}\right) - 3\left(1 - \left(\frac{D+D'}{3}\right)\right)} \cancel{xy}$$

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$$P_1 = \frac{-1}{3D} \left(1 - \frac{D'}{D} \right)^{-1} \left[1 - \left(\frac{D}{3} + \frac{D'}{3} \right) \right]^{-1}$$

$$P_1 = \frac{-1}{3D} \left(1 + \frac{D'}{D} + \frac{D'^2}{D^2} + \dots \right) \left(1 + \frac{D}{3} + \frac{D'}{3} + \left(\frac{D}{3} + \frac{D'}{3} \right)^2 + \dots \right) (xy)$$

$$P_1 = \frac{-1}{3D} \left(1 + \frac{D'}{D} + \frac{D'^2}{D^2} + \dots \right) \left(1 + \frac{D}{3} + \frac{D'}{3} + \frac{D^2}{9} + \frac{D'^2}{9} + \frac{2DD'}{9} + \dots \right) (xy)$$

$$P_1 = \frac{-1}{3D} \left(1 + \frac{D}{3} + \frac{D'}{3} + \frac{2DD'}{9} + \frac{D'D'}{3D} + \frac{D'^2}{3D} + \frac{2DD'^2}{9D} + \frac{D'}{D} \right) (xy)$$

$$P_1 = \frac{-1}{3D} \left(1 + \frac{D}{3} + \frac{D'}{3} + \frac{2DD'}{9} + \frac{D'}{D} + \frac{D'}{3} \right) (xy)$$

$$P_1 = \frac{-1}{3D} \left[xy + \frac{D}{3}(xy) + \frac{D'}{3}(xy) + \frac{2DD'}{9}(xy) + \frac{D'}{D}(xy) \right]$$

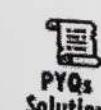
$$+ \frac{D'}{3}(xy)$$

$$P_1 = -\frac{1}{3D} \left[\frac{D}{3}y + \frac{y}{3} + \frac{x}{3} + \frac{2}{9} + \frac{D^2}{2} + \frac{D}{3} \right]$$

$$P_1 = -\frac{1}{3D} \left[xy + \frac{y}{3} + \frac{2x}{3} + \frac{D^2}{2} + \frac{2}{9} \right]$$

$$P_1 = -\frac{1}{3} \left[\frac{x^2y}{2} + \frac{xy}{3} + \frac{x^2}{3} + \frac{x^3}{6} + \frac{2x}{9} \right]$$

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$$P_2 = \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x+2y}$$

Put $D = 1, D' = 2$

$$P_2 = \frac{1}{1 - 4 - 3 + 6} e^{x+2y}$$

{ Case of failure }

Now multiply with x and differentiate w.r.t D

$$P_2 = \frac{x}{2D - 3} e^{x+2y}$$

$$P_2 = -x e^{x+2y}$$

$$PI = -\frac{1}{3} \left[\frac{x^2 y}{2} + \frac{xy}{3} + \frac{x^2}{3} + \frac{x^3}{6} + \frac{2x}{9} \right] - x e^{x+2y}$$

Complete solution $\Rightarrow Z = CF + PI$

$$Z = f_1(y+x) + e^{3x} f_2(y-x) - \frac{1}{3} \left[\frac{x^2 y}{2} + \frac{xy}{3} + \frac{x^2}{3} + \frac{x^3}{6} + \frac{2x}{9} \right] - x e^{x+2y}$$

Ans

Q-2: Solve: $(D+D'-1)^2 z = xy$

$$(D+D'-1)^2 z = xy$$

$$CF = e^x f_1(y-x) + x e^x f_2(y-x)$$

$$PI = \frac{1}{(D+D'-1)^2} xy$$

$$PI = \frac{1}{[1-(D+D')]} xy$$

$$PI = [1-(D+D')]^{-2} xy$$

Using expansion:-

$$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$PI = [1 + 2(D+D') + 3(D+D')^2 + \dots] xy$$

$$PI = [1 + 2D + 2D' + 3D^2 + 3D'^2 + 6DD' + \dots] xy$$

$$PI = [xy + 2D(xy) + 2D'(xy) + 6DD'(xy)]$$

$$PI = xy + 2y + 2x + 6$$

Complete solution $\Rightarrow z = CF + PI$

$$z = e^x f_1(y-x) + x e^x f_2(y-x) + xy + 2y + 2x + 6$$

Que-3: Solve: $(D - 3D' - 2)^3 z = 6e^{2x} \sin(3x + y)$

$$(D - 3D' - 2)^3 z = 6e^{2x} \sin(3x + y)$$

$$C.F. = e^{2x} f_1(y+3x) + x e^{2x} f_2(y+3x) + x^2 e^{2x} f_3(y+3x)$$

$$P.I. = \frac{1}{(D - 3D' - 2)^3} 6e^{2x} \sin(3x + y)$$

Replace D by $D + 2$

$$P.I. = 6e^{2x} \frac{1}{(D+2 - 3D' - 2)^3} \sin(3x + y)$$

$$P.I. = 6e^{2x} \frac{1}{(D - 3D')^3} \sin(3x + y)$$

Put $D \rightarrow 3, D' \rightarrow 1$

{Case of failure}

$$P.I. = 6e^{2x} x x x \frac{1}{3(D - 3D')^2} \sin(3x + y)$$

Again Put $D \rightarrow 3, D' \rightarrow 1$

Again case of failure

$$P.I. = 6e^{2x} x x^2 x \frac{1}{6(D - 3D')} \sin(3x + y)$$

Put $D \rightarrow 3, D' \rightarrow 1$
 { Again case of failure }
 $P.I. = \frac{1}{6} e^{2x} x^3 \sin(3x+y)$

$$P.I. = x^3 e^{2x} \sin(3x+y)$$

Complete solution $\Rightarrow z = C.F. + P.I.$

$$z = e^{2x} f_1(y+3x) + x e^{2x} f_2(y+3x) + x^2 e^{2x} f_3(y+2x) + x^3 e^{2x} \sin(3x+y)$$

Q-4 : Solve $x - 4s + 4t + p - 2q = e^{x+y}$

$$x - 4s + 4t + p - 2q = e^{x+y}$$

$$(D^2 - 4DD' + 4D'^2 + D - 2D') z = e^{x+y}$$

$$[(D - 2D')^2 + (D - 2D')] z = e^{x+y}$$

$$(D - 2D')(D - 2D' + 1) z = e^{x+y}$$

$$C.F. = f_1(y+2x) + e^{-x} f_2(y+2x)$$

$$P.I. = \frac{1}{D^2 - 4DD' + 4D'^2 + D - 2D'} e^{x+y}$$

Put $D \rightarrow 1, D' \rightarrow 1$

GATEWAY CLASSES

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{ Case of failure }

$$PI = x \times \frac{1}{2D - 4D' + 1} e^{x+y}$$

$$D \rightarrow 1, D' \rightarrow 1$$

$$PI = x \times \frac{1}{2 - 4 + 1} e^{x+y}$$

$$PI = -xe^{x+y}$$

Complete solution $\Rightarrow z = CF + PI$

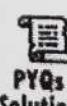
$$z = f_1(y+2x) + e^{-x}(y+2x) - xe^{x+y}$$

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PDF
Notes



PYQs
Solution



Recorded
Lectures

➤ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

Q.1 Solve the linear partial differential equation : $(x^2D^2 + 2xyDD' + y^2D'^2)z = x^m y^n$. AKTU-2020-21

DPP-15

Maths - IV

Unit - 1

Topic →

$$Q-1: \text{ Solve } (x^2 D^2 + 2xy DD' + y^2 D'^2)z = x^m y^n$$

$$\text{Let } x = e^u, y = e^v$$

$$u = \log x, v = \log y$$

$$x^2 \frac{\delta^2 z}{\delta x^2} = D(D-1)z$$

$$y^2 \frac{\delta^2 z}{\delta y^2} = D'(D'-1)z$$

$$xy \frac{\delta^2 z}{\delta x \delta y} = DD'z$$

Given equation reduced into

$$[D(D-1) + 2DD' + D'(D'-1)]z = e^{mu+nv}$$

$$(D^2 - D + 2DD' + D'^2 - D')z = e^{mu+nv}$$

$$[(D+D')^2 - (D+D')]z = e^{mu+nv}$$

$$(D+D')(D+D'-1)z = e^{mu+nv}$$

$$C.F. = f_1(v-u) + e^u f_2(v-u)$$

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$$\begin{aligned} C.F. &= f_1(\log y - \log x) + x f_2 (\log y - \log x) \\ &= f_1 \left(\log \frac{y}{x} \right) + x f_2 \left(\log \frac{y}{x} \right) \end{aligned}$$

$$C.F. = g_1 \left(\frac{y}{x} \right) + x g_2 \left(\frac{y}{x} \right)$$

$$P.I. = \frac{1}{(D+D') (D+D'-1)} e^{mu+nv}$$

$$P.I. = \frac{1}{D+D'} \left[\frac{1}{D+D'-1} e^{mu+nv} \right]$$

$$P.I. = \frac{1}{D+D'} \left[\frac{1}{m+n-1} e^{mu+nv} \right]$$

$$P.I. = \frac{1}{(m+n)(m+n-1)} e^{mu+nv}$$

$$P.I. = \frac{x^m y^n}{(m+n)(m+n-1)}$$

Complete Solution $\Rightarrow Z = C.F + P.I$

$$Z = g_1 \left(\frac{y}{x} \right) + x g_2 \left(\frac{y}{x} \right) + \frac{x^m y^n}{(m+n)(m+n-1)}$$