AKTU

B. Tech II-Year

Gateway Classes

GATEWAY CLASSES

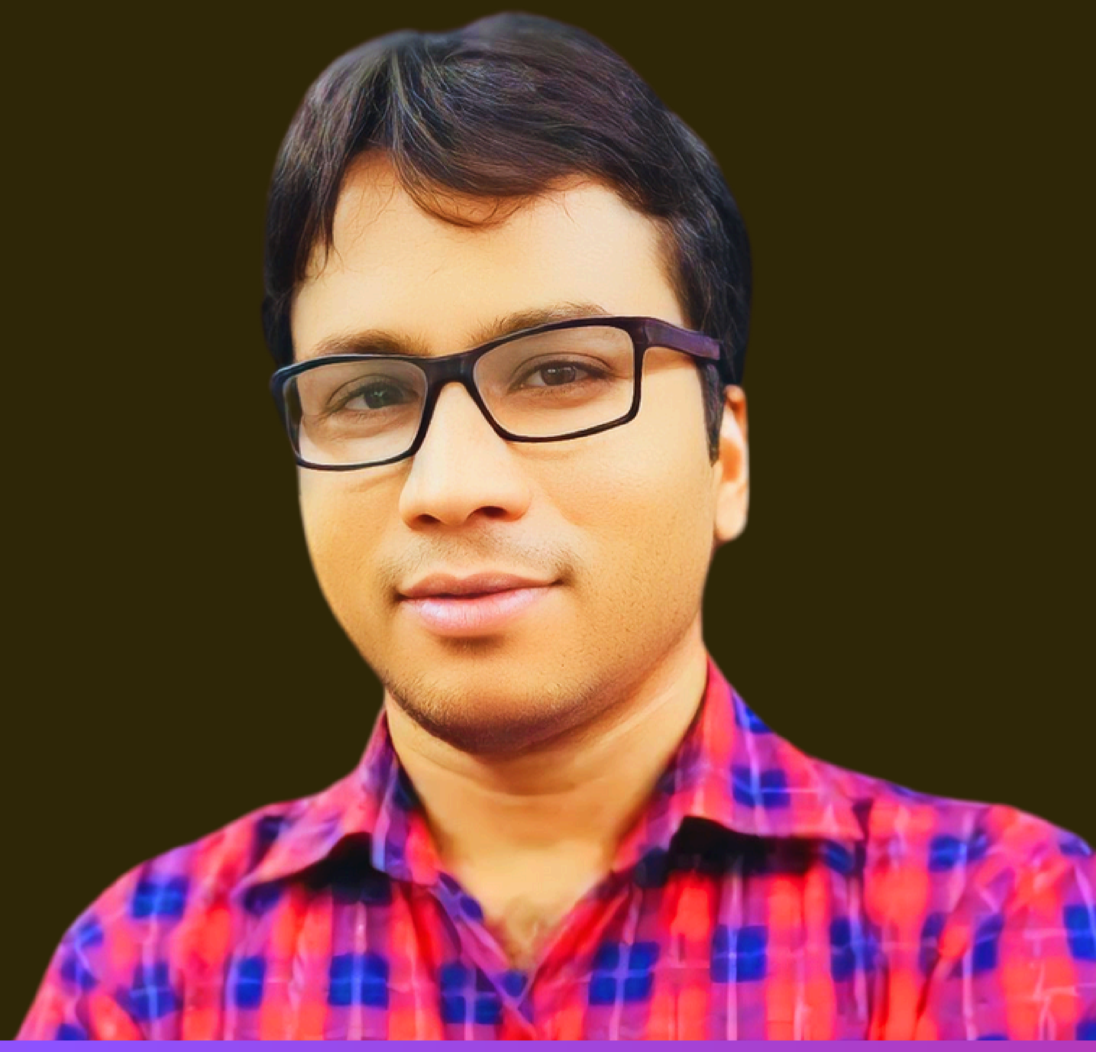
Semester -IV

ENGG.Mathematics-IV

BAS-403 ENGG- Mathematics-IV

UNIT-4 : ONE SHOT

Statistical Techniques II



Gateway Series for Engineering

- Topic Wise Entire Syllabus
- Long - Short Questions Covered
- AKTU PYQs Covered
- DPP
- Result Oriented Content

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AKTU

B. Tech II-Year

Gateway Classes

BAS-403 ENGG. MATHEMATICS-IV

Unit-4-ONE SHOT

Introduction to Statistical Techniques II

Syallbus

Module IV: Statistical Techniques II Overview of Probability Random variables (Discrete and Continuous Random variable) Probability mass function and Probability density function, Expectation and variance, Discrete and Continuous Probability distribution: Binomial, Poission and Normal distributions.


Engineering Mathematics IV
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UNIT-4
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Gateway Classes official AKTU

UNIT-4 : *Statistical Techniques – II*

Lec-01

Today's Target

- *Random variables(PART-1)*
- PYQ
- DPP

UNIT – 4 : *Statistical Technique – II* (Probability)

Aktu Syllabus

- ***Random variables***

(i) Discrete random variables

- ***Probability Mass Function***
- ***Expectation and Variance***

(ii) Continous random variables

- ***Probability Density Function***
- ***Expectation and Variance***

- ***Probability Distribution***

(i) Binomial $\longrightarrow$ ***Based on Discrete random variable***

(ii) Poisson $\longrightarrow$ ***Based on Discrete random variable***

(iii) Normal $\longrightarrow$ ***Based on Continous random variable***

Probability

Probability is a concept which numerically meansures the degree of uncertainity and certanity of the occurrence of events.

$\boxed{0 \leq P \leq 1}$

$$\boxed{Probability = \frac{Favourable\ Cases}{Total\ number\ of\ mutually\ exclusive\ and\ equally\ likely\ cases}}$$

Random variable

It is a real valued function whose domain is the sample space of a random experiment

It is denoted by X

$$X = Number\ of\ things$$

$$\underset{\substack{\uparrow \\ Domain}}{X : S} \longrightarrow \underset{\substack{\uparrow \\ codomain}}{R}$$

$X(s) = x$

Where x is any real number

Types of Random variable

(1) Discrete random variable

(2) Continous random variable

Discrete random variable

A random variable which can assume only a finite number of values is called a discrete random variable.

Example: –The number of tails in 3 tosses of a coin is a discrete random variable (X)

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

X = Number of Tails

$X(HHH) = 0$

$X(HHT, HTH, THH) = 1$

$X(TTH, THT, HTT) = 2$

$X(TTT) = 3$

Hence

$X = 0, 1, 2, 3$

Probabilities

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

$P(X=0) + P(X=1) + P(X=2) + P(X=3) =$

$\frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

$= \frac{1+3+3+1}{8}$

$= \frac{\cancel{8}}{\cancel{8}} = 1$

Probability Distribution

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Prabability Mass Function

Let $x_1, x_2, x_3, \ldots\ldots$ be the values of a discrete random variable X and let $p_1, p_2, p_3, \ldots\ldots$ be the corresponding probabilities.

A function $p(x)$ defined by

$$p(X = x_i) = p(x) = \begin{cases} p(x_i) \text{ or } p_i & i = 1, 2, 3, 4 \ldots\ldots \\ 0, & \text{otherwise} \end{cases}$$

is called the probability mass function of the discrete random variable X where

(i) $p(x_i) \geq 0$

(ii) $\sum p(x_i) = 1$

v. imp

Probability distribution of a discrete random variable

X	x_1	x_2	x_3	………	x_n
$p(x)$	p_1	p_2	p_3	………	p_n

2/3/4....Balls (withdrawn)

Successively

With replacement (Independent events)

Without Replacement (Dependent events)

Simultaneously

(Dependent

$$\boxed{{}^nC_r = \frac{n!}{r!\,(n-r)!}}$$

Q. 1 *A coin is tossed three times. if X is a random variable giving the number of ~~tails~~ Heads that appear, make a table showing the probability distribution of X.*

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

X = Number of Heads

$X(TTT) = 0$

$X(TTH, THT, HTT) = 1$

$X(HHT, HTH, THH) = 2$

$X(HHH) = 3$

Hence

$X = 0, 1, 2, 3$

Probabilities corresponding to random variables

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Q. 2 *A random variable X has the following probability distribution:*

Value of X,	x:	0	1	2	3	4	5	6	7
	$p(x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) K (ii) $P(X<6)$ (iii) $P(X\geq 6)$ (iv) $P(3<X\leq 6)$

(v) Minimum value of x for which $P(X\leq x)>\frac{1}{2}$

Solution

(i) We know that

$$\sum P(x_i)=1$$

$$0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$$

$$9K+10K^2=1$$

$$10K^2 + 9K - 1 = 0$$

$$\underline{10K^2 + 10K} - K - 1 = 0$$

$$10K(K+1) - (K+1) = 0$$

$$(K+1)(10K-1) = 0$$

$K = -1$ (Neglect)

$$\boxed{K = \frac{1}{10}}$$

$$P(X<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= 0 + K + 2K + 2K + 3K + K^2$$

$$= K^2 + 8K$$

$$= \left(\frac{1}{10}\right)^2 + 8 \times \frac{1}{10} = \frac{1}{100} + \frac{8}{10}$$

$$\boxed{P(X<6) = \frac{81}{100}}$$

(iii) $P(X \geq 6) = P(X=6) + P(X=7)$

$$= 2K^2 + 7K^2 + K$$

$$P(X \geq 6) = 9K^2 + K$$

$$= \frac{9}{100} + \frac{1}{10}$$

$$= \frac{9 + 10}{100}$$

$$\boxed{P(X \geq 6) = \frac{19}{100}}$$

(IV) $P(3 < X \leq 6) =$

$$= P(X=4) + P(X=5) + P(X=6)$$

$$= 3K + K^2 + 2K^2$$

$$P(3 < X \leq 6) = 3K^2 + 3K = \frac{3}{100} + \frac{3}{10}$$

$$\boxed{P(3 < X \leq 6) = \frac{33}{100}}$$

(V) $P(X \leq 1) = 0 + K$

$$P(X \leq 1) = K$$

$$P(X \leq 1) = \frac{1}{10} < \frac{1}{2}$$

$$P(X \leq 3) = 0 + K + 2K + 2K$$

$$P(X \leq 3) = 5K$$

$$P(X \leq 3) = \frac{5}{10} = \frac{1}{2}$$

$$P(X \leq 4) = 0 + K + 2K + 2K + 3K$$

$$P(X \leq 4) = 8K$$

$$P(X \leq 4) = \frac{8}{10} > \frac{1}{2}$$

Hence Minimum value of x is 4

Q.3 *Five defective bulbs are accidentally mixed with twenty good ones. It is not possible to just look at a bulb and tell wheter or not it is defective. Find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.*

Given

Number of Defective bulbs = 5

Number of good bulbs = 20

Total number of bulbs = 25

X = Number of Defective bulbs

$X = 0, 1, 2, 3, 4$

Probabilities

$\boxed{\begin{array}{l} 5D \\ 20G \end{array}}$

$$P(X=0) = \frac{{}^{20}C_4}{{}^{25}C_4}$$

$$P(X=0) = \frac{20!}{4! \times 16!} \times \frac{4! \times 21!}{25!}$$

$$= \frac{\cancel{20}^{\cancel{4}\,2} \times 19 \times \cancel{18}^{3} \times 17 \times \cancel{16!} \times \cancel{21!}}{\cancel{16!} \times \cancel{25}_{5} \times \cancel{24}_{\cancel{4}} \times 23 \times \cancel{22}_{11} \times \cancel{21!}}$$

$$P(X=0) = \frac{19\times 3\times 17}{5\times 2\times 11\times 23}$$

$$P(X=0) = \frac{969}{2530}$$

$$P(X=1) = \frac{{}^{5}C_1 \times {}^{20}C_3}{{}^{25}C_4}$$

$$P(X=1) = \frac{1140}{2530}$$

$$P(X=2) = \frac{{}^{5}C_2 \times {}^{20}C_2}{{}^{25}C_4}$$

$$P(X=2) = \frac{380}{2530}$$

$$P(X=3) = \frac{{}^{5}C_3 \times {}^{20}C_1}{{}^{25}C_4}$$

$$P(X=3) = \frac{40}{2530}$$

$$P(X=4) = \frac{{}^{5}C_4}{{}^{25}C_4}$$

$$P(X=4) = \frac{1}{2530}$$

X	0	1	2	3	4
P(X)	$\frac{969}{2530}$	$\frac{1140}{2530}$	$\frac{380}{2530}$	$\frac{40}{2530}$	$\frac{1}{2530}$

Q.4 *A random variable X takes values 1, 2, 3, ... with probability mass function $\frac{\lambda^r}{r!}$, $r = 1, 2, 3 \ldots \infty$. Find the value of λ.*

Given

$$P(x_i) = \frac{\lambda^r}{r!}$$

$$\sum P(x_i) = 1$$

$$\sum \frac{\lambda^r}{r!} = 1$$

$$\frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots = 1$$

$$1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots = 1 + 1$$

$$e^\lambda = 2$$

Taking log both side

$$\lambda \log_e e = \log_e 2$$

$$\boxed{\lambda = \log_e 2}$$

Engineering Mathematics by Gulshan Sir

DPP- 01

Topic : *Random variables (PART-1)*

Q.1 A box has 5 Blue and 3 Red balls. If 2 balls are to be drawn at random without replacement and X denotes the number of Blue balls, find the probability distribution for X.

Q.2 A random variable X has the following probability distribution:

Value of X,										
	x:	0	1	2	3	4	5	6	7	8
	$p(x)$:	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(i) Determine the value of a.

(ii) Find $P(X < 3), P(X \geq 3), P(2 \leq X < 5)$

(iii) What is the smallest value of x for which $P(X \leq x) > 0.5$?

ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-02

Today's Target

- *Random variables (PART-2)*
- PYQ
- DPP

Types of Random variable

(1) Discrete random variable

(2) Continous random variable

Discrete random variable

A random variable which can assume only a finite number of values is called a discrete random variable.

- Probability Mass Function
- Probability Distribution
- Expectation and Variance

Expectation and Variance

Mean

If X is a discrete random variable with probability distribution

$X:$	x_1	x_2	x_3	… … … . .	x_n
$p(X):$	$p(x_1)$	$p(x_2)$	$p(x_3)$	… … … . .	$p(x_n)$

Expectation

$$E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + \ldots\ldots\ldots + x_n p(x_n)$$

$$\boxed{E(x) = \sum x\, p(x)}$$

$$E(x) = \bar{x} = \mu = \mu_1' = \sum x\, p(x)$$

Expectation or Mathematical *Expectation* or *Mean* or *First moment about origin*

Important points:

$E(x) = \sum x\, p(x) = \mu_1'$ (*First moment about origin*)

$E\left(x^2\right) = \sum\left(x^2\right)p(x) = \mu_2'$ (*Second moment about origin*)

$E\left(x^3\right) = \sum\left(x^3\right)p(x) = \mu_3'$ (*Third moment about origin*)

$E\left(x^r\right) = \sum\left(x^r\right)p(x) = \mu_r'$ (r^{th} *moment about origin*)

Properties of Expectation:

(i) $E(c) = c$ $\quad E(1) = 1$

(ii) $E(ax) = a\,\underline{E(x)}$

(iii) $E(x \pm y) = E(x) \pm E(y)$

(iv) $E(xy) = E(x).E(y)$

GW
GATEWAY CLASSES

Variance

r^{th} moment about mean

$$\boxed{\mu_r = \frac{\sum p(x-\bar{x})^r}{\sum pi}}$$

But $\sum p_i = 1$

$$\boxed{\mu_r = \sum p(x-\bar{x})^r}$$

$$\mu_1 = \sum p(x-\bar{x})$$

$$\mu_1 = E(x-\bar{x})$$

$$\mu_1 = E(x) - E(\bar{x})$$

$$\mu_1 = E(x) - \bar{x}$$

$$\mu_1 = \cancel{\bar{x}} - \cancel{\bar{x}}$$

$$\boxed{\mu_1 = 0}$$

$$\mu_2 = \sum p(x-\bar{x})^2$$

$$\mu_2 = \sum p(x-\mu)^2$$

$$\boxed{\text{Variance} = \sum p(x-\mu)^2}$$

$$\mu_2 = \sum p(x-\bar{x})^2$$

$$\mu_2 = E(x-\bar{x})^2$$

$$\mu_2 = E(x^2+\bar{x}^2-2x\bar{x})$$

$$\mu_2 = E(x^2)+E(\bar{x}^2)-E(2x\bar{x})$$

$$\mu_2 = E(x^2)+\bar{x}^2-2\bar{x}E(x)$$

$$\mu_2 = E(x^2)+\bar{x}^2-2\bar{x}\,\bar{x}$$

$$\mu_2 = E(x^2)+\bar{x}^2-2\bar{x}^2$$

$$\mu_2 = E(x^2)-(\bar{x})^2$$

$$\boxed{\mu_2 = E(x^2)-[E(x)]^2}$$

Variance
OR
Second moment about mean

Impotant Formulae

1. *Expectation* (*Mean or First moment about origin*)

$$\boxed{E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + \dots\dots + x_n p(x_n)}$$

$$E(x) = \sum x\, p(x)$$

2. *Variance* (*Second moment about mean*)

$$E(x^2) = x_1^2 p(x_1) + x_2^2 p(x_2) + \dots + x_n^2 p(x_n)$$

$$Variance = \mu_2 = \sum p\,(x - \mu)^2$$

$$\boxed{Variance = \mu_2 = E(x^2) - [E(x)]^2}$$

3. *Standard Deviation*

$$\boxed{S.D = \sqrt{Variance}}$$

Q.1 Find the Mean, Variance and Standard deviation of the number of heads when three coins are tossed.

$S = \{HHH, HHT, HTH, THH, TTT, TTH, THT, HTT\}$

X = Number of heads

$X(TTT) = 0$

$X(TTH, THT, THH) = 1$

$X(HHT, HTH, THH) = 2$

$X(HHH) = 3$

$X = 0, 1, 2, 3$

Probabilities

$P(X=0) = \frac{1}{8}$

$P(X=1) = \frac{3}{8}$

$P(X=2) = \frac{3}{8}$

$P(X=3) = \frac{1}{8}$

Probability Distribution

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean

$$E(X) = \bar{x} = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$$

$$\bar{x} = 0 + \frac{3}{8} + \frac{6}{8} + \frac{3}{8}$$

$$\bar{x} = \frac{12}{8}$$

$$\boxed{\bar{x} = \frac{3}{2}}$$

Variance

$$E(x^2) = 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8}$$

$$E(x^2) = 0 + \frac{3}{8} + \frac{12}{8} + \frac{9}{8}$$

$$E(x^2) = \frac{3 + 12 + 9}{8} = \frac{\cancel{24}}{\cancel{8}} = 3$$

$$\text{Variance} = E(x^2) - [E(x)]^2$$

$$= 3 - \left(\frac{3}{2}\right)^2 = 3 - \frac{9}{4}$$

$$\boxed{\text{Variance} = \frac{3}{4}}$$

$$\boxed{SD = \frac{\sqrt{3}}{2}}$$

Q.2 Two cards are drawn simultaneously from a well shuffled pack of 52 cards. Find the Expectation and Variance of the number of kings.

Let X = Number of kings

$X = 0, 1, 2$

$$P(X=0) = \frac{{}^{48}C_2}{{}^{52}C_2}$$

$$= \frac{48!}{\cancel{2!} \times 46!} \times \frac{\cancel{2!} \times 50!}{52!}$$

$$= \frac{48 \times 47 \times \cancel{46!} \times \cancel{50!}}{\cancel{46!} \times 52 \times 51 \times \cancel{50!}}$$

$$\boxed{P(X=0) = \frac{188}{221}}$$

$$\boxed{{}^{n}C_r = \frac{n!}{r!\,(n-r)!}}$$

$$P(X=1) = \frac{{}^{4}C_1 \times {}^{48}C_1}{{}^{52}C_2}$$

$$\boxed{P(X=1) = \frac{32}{221}}$$

$$P(X=2) = \frac{{}^{4}C_2}{{}^{52}C_2}$$

$$\boxed{P(X=2) = \frac{1}{221}}$$

Probality Distribution

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Expectation

$$E(X) = 0 \times \frac{188}{221} + 1 \times \frac{32}{221} + \frac{2}{221}$$

$$\boxed{E(X) = \frac{2}{13}}$$

$$E(X^2) = 0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221}$$

$$E(X^2) = \frac{36}{221}$$

Varianu

$$Var(X) = E(X^2) - [E(X)]^2$$

$$= \frac{36}{221} - \left(\frac{2}{13}\right)^2$$

$$\boxed{Var(X) = 0.1392}$$

Standard deviation

$$S.D = \sqrt{0.1392}$$

$$\boxed{S.D = 0.37}$$

Q.3 Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the Expectation of X

X = sum of numbers on two dice

X	2	3	4	5	6	7	8	9	10	11	12
$P(X)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Expectation

$$E(X) = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$$

$$\boxed{E(X) = 7}$$

Q.4 A random variable X has the following distribution

X:	-2	-1	0	1	2	3
$p(X)$:	0.1	k	0.2	$2k$	0.3	k

Determine (i) k (ii) Mean (iii) Variance

(i) We know that

$\sum p(x) = 1$

$0.1 + K + 0.2 + 2K + 0.3 + K = 1$

$4K + 0.6 = 1$

$4K = 0.4$

$\boxed{K = 0.1}$

Mean

$\bar{x} = -0.2 - K + 0 + 2K + 0.6 + 3K$

$\bar{x} = -0.2 - 0.1 + 0.2 + 0.6 + 0.3$

$\boxed{\bar{x} = 0.8}$

Varianu

$E(x^2) = (-2)^2 \times 0.1 + 1^2 \times K + 0 + 1^2 \times 2K + 2^2 \times 0.3 + 3^2 \times K$

$$E(x^2) = 0.4 + K + 2K + 1.2 + 9K$$

$$= 0.4 + 0.1 + 0.2 + 1.2 + 0.9$$

$$E(x^2) = 2.8$$

$$\text{Varianu} = E(x^2) - [E(x)]^2$$

$$\text{Varianu} = 2.8 - (0.8)^2$$

$$= 2.8 - 0.64$$

$$\boxed{Var(X) = 2.16}$$

DPP- 02

Topic :*Random variables* (PART-2)

$Q.1$ *Let X denote the number of heads when a pair of coin is tossed. What is the expected value?*

$Q.2$ $Two\ dice\ are\ thrown\ simultaneously.\ if\ X\ denote\ the\ number\ of\ sixes, find\ the\ expectation\ of\ X$

ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-03

Today's Target

- *Random variables (PART-3)*
- PYQ
- DPP

Types of Random variable

(1) Discrete random variable

(2) Continous random variable

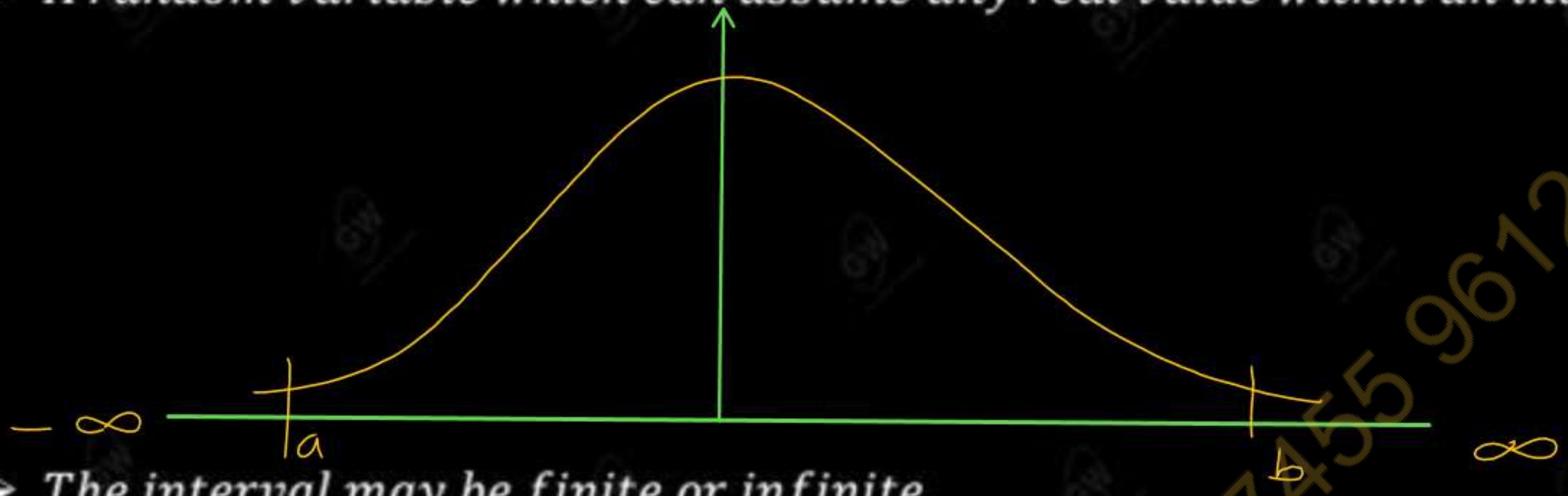
Discrete random variable

A random variable which can assume only a finite number of values is called a discrete random variable.

- Probability Mass Function
- Probability Distribution
- Expectation and Variance

Continous random variable

- A random variable which can assume any real value within an interval



- The interval may be finite or infinite

Probability Density Function

Let X be a continous random variable, then in the interval $[a, b]$, the PDF is defined by

$$\boxed{P(a \leq X \leq b) = \int_a^b f(x)\,dx}$$

Such that

(i) $f(x) \geq 0, \quad \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Note: $P((a \le X \le b) = f(x) \text{ or } p(x) = \begin{cases} 0 & x < a \\ \emptyset(x) & a \le x \le b \\ 0 & x > b \end{cases}$

$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$

Q.1 *If the function $f(x)$ is defined by $f(x) = ce^{-x}, 0 \le x \le \infty$, find the value of c which changes $f(x)$ to a probability density function.*

$f(x) = ce^{-x}, \quad 0 \le x \le \infty$

$f(x)$ is a PDF if

(i) $f(x) \geq 0, \ \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$

$0 + \int_{0}^{\infty} ce^{-x}\,dx = 1$

$\left(-c\,e^{-x}\right)_0^{\infty} = 1$

$-c\left[e^{-\infty} - e^{0}\right] = 1$

$-c[0 - 1] = 1$

$\boxed{c = 1}$

$f(x) = ce^{-x}$

$e^{-x} \geq 0$

$c \geq 0$

$\Rightarrow f(x) \geq 0$

Q.2 *If $f(x)$ has probability density $cx^2, 0 < x < 1$, determine c and find the probability that $\frac{1}{3} < x < \frac{1}{2}$ i.e., $P\left(\frac{1}{3} < x < \frac{1}{2}\right)$.*

$f(x) = cx^2, \quad 0<x<1$

Since $f(x)$ is a PDF

$\Rightarrow$ (i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

From (ii)

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{\infty} f(x)\,dx = 1$

$0 + \int_{0}^{1} cx^2\,dx + 0 = 1$

$c\left(\frac{x^3}{3}\right)_0^1 = 1$

$c\left(\frac{1}{3} - 0\right) = 1$

$\frac{c}{3} = 1$

$\boxed{c = 3}$

$f(x) = 3x^2$

$$P\left(\frac{1}{3} < x < \frac{1}{2}\right) = \int_{1/3}^{1/2} f(x)\,dx$$

$$= \int_{1/3}^{1/2} 3x^2\,dx$$

$$= \cancel{3}\left(\frac{x^3}{\cancel{3}}\right)_{1/3}^{1/2}$$

$$= \frac{1}{8} - \frac{1}{27}$$

$$= \frac{27-8}{216}$$

$$\boxed{P\left(\frac{1}{3} < x < \frac{1}{2}\right) = \frac{19}{216}}$$

Expectation and variance

If x is a continous random variable, then the expectation of x is

$$\boxed{E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx}$$

Note : $E(x) = \bar{x} = \mu = \mu_1{}' = \int_{-\infty}^{\infty} x\, f(x)\, dx$

Expectation or mean or first moment about origin

Also

- $E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$ **(First moment about origin)**
- $\boxed{E(x^2) = \int_{-\infty}^{\infty} x^2\, f(x)\, dx}$ **(Second moment about origin)**
- $E(x^r) = \int_{-\infty}^{\infty} x^r\, f(x)\, dx$ **(r^{th} moment about origin)**

Variance

$$\text{Var}(x) = E\left(x^2\right) - [E(x)]^2$$

Second moment about origin

Standard Deviation

$$S.D = \sqrt{Variance}$$

Q.3 *If the probability density function of a random variable is given by*

AKTU 2021

$$f(x) = \begin{cases} k(1-x^2), & 0 \le x \le 1 \\ 0, & elsewhere \end{cases}$$

Find (i) k (ii) Mean (iii) Varience (iv) Probability between 0.1 and 0.2

(v) Probability greater than 0.5

Since $f(x)$ is a PDF

$\therefore$ (i) $f(x) \geq 0 \quad \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

From (ii)

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = 1$$

$$0 + \int_{0}^{1} k(1-x^2) + 0 = 1$$

$$k\left(x - \frac{x^3}{3}\right)_0^1 = 1$$

$$k\left[1 - \frac{1}{3}\right] = 1$$

$$\frac{2}{3}k = 1$$

$$k = \frac{3}{2}$$

$$\boxed{k = 1.5}$$

(ii) Mean $= \int_{-\infty}^{\infty} x f(x)\,dx$

$$E(x) = \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{1} x f(x)\,dx + \int_{1}^{\infty} x f(x)\,dx$$

$$E(x) = 0 + \int_{0}^{1} x K(1-x^2)\,dx + 0$$

$$E(x) = K\int_{0}^{1} (x - x^3)\,dx$$

$$E(x) = K\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$E(x) = K\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$E(x) = K \times \frac{1}{4}$$

$$E(x) = \frac{3}{2} \times \frac{1}{4}$$

$$\boxed{E(x) = \frac{3}{8}}$$

(iii) $E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$E(x^2) = \int_{-\infty}^{0} x^2 f(x)\,dx + \int_{0}^{1} x^2 f(x)\,dx + \int_{1}^{\infty} x^2 f(x)\,dx$$

$$E(x^2) = 0 + \int_0^1 x^2 k(1-x^2)\,dx + 0$$

$$E(x^2) = k\int_0^1 (x^2 - x^4)\,dx$$

$$E(x^2) = k\left(\frac{x^3}{3} - \frac{x^5}{5}\right)_0^1$$

$$E(x^2) = k\left(\frac{1}{3} - \frac{1}{5}\right]$$

$$E(x^2) = k\left(\frac{5-3}{15}\right)$$

$$E(x^2) = \frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{15}\;5} = \frac{1}{5}$$

$$\boxed{E(x^2) = \frac{1}{5}}$$

Variance $= E(x^2) - [E(x)]^2$

$$= \frac{1}{5} - \left(\frac{3}{8}\right)^2 = \frac{1}{5} - \frac{9}{64}$$

$$= \frac{64-45}{320}$$

$$\boxed{Var(x) = \frac{19}{320} = 0.0594}$$

GW GATEWAY CLASSES

(IV) $P(0.1 < x < 0.2) = \int_{0.1}^{0.2} f(x)\,dx$

$$= \int_{0.1}^{0.2} K(1-x^2)\,dx$$

$$= K\left(x - \frac{x^3}{3}\right)_{0.1}^{0.2}$$

$$= \frac{3}{2}\left[0.2 - \frac{0.008}{3} - \left(0.1 - \frac{0.001}{3}\right)\right]$$

$$= \frac{3}{2}\left[0.1 - \frac{0.008}{3} + \frac{0.001}{3}\right]$$

$$\boxed{P(0.1 < x < 0.2) = 0.1466}$$

(V) $P(0.5 < x < 1) = \int_{0.5}^{1} f(x)\,dx$

$$= K\int_{0.5}^{1} (1-x^2)\,dx$$

$$= \frac{3}{2}\left[x - \frac{x^3}{3}\right]_{0.5}^{1}$$

$$\boxed{= 0.3125}$$

Q.4 *A continous random variable X has the probability density function*

$$f(x) = \begin{cases} kx & for\ 0 \le x < 2 \\ 2k & for\ 2 \le x < 4 \\ -kx + 6k & for\ 4 \le x \le 6 \\ 0 & elsewhere \end{cases}$$

Find (i)k (ii)Mean

Since $f(x)$ is PDF

$\therefore$ (i) $f(x) \geq 0 \ \forall x$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx + \int_{4}^{6} f(x)\,dx + \int_{6}^{\infty} f(x)\,dx = 1$$

$$0 + k\int_{0}^{2} x\,dx + 2k\int_{2}^{4} dx + k\int_{4}^{6} (-x+6)\,dx + 0 = 1$$

$$k\left(\frac{x^2}{2}\right)_0^2 + 2k(x)_2^4 + k\left(-\frac{x^2}{2}+6x\right)_4^6 = 1$$

$$k(2-0) + 2k(4-2) + k\left[-18+36-(-8+24)\right] = 1$$

$$2k + 4k + k(18+8-24) = 1$$

$$2k + 4k + 2k = 1$$

$$8k = 1$$

$$\boxed{k = \frac{1}{8}}$$

(ii) Mean

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$E(x) = \cancel{\int_{-\infty}^{0} x\, f(x)\, dx} + \int_0^2 x\, f(x)\, dx + \int_2^4 x\, f(x)\, dx$$

$$+ \int_4^6 x\, f(x)\, dx + \cancel{\int_6^{\infty} x\, f(x)\, dx}$$

$$E(x) = k\int_0^2 x^2 dx + 2k\int_2^4 x\,dx + k\int_4^6 (-x^2+6x)dx$$

$$= k\left(\frac{x^3}{3}\right)_0^2 + 2k\left(\frac{x^2}{2}\right)_2^4 + k\left(-\frac{x^3}{3} + \frac{\cancel{6}^{3} x^2}{\cancel{2}}\right)_4^6$$

$$= k\left[\frac{8}{3} - 0 + 2(8-2) + \left(-72 + 3\times 36 + \frac{64}{3} - 16\times 3\right)\right.$$

$$= \frac{1}{8}\left[\frac{8}{3} + 12 + \left(-72 + 108 + \frac{64}{3} - 48\right)\right]$$

$$\boxed{E(x) = 3}$$

DPP- 03

Topic : *Random variables (PART-3)*

Q.1 Find the constant k so function f(x) defined as follows be a density function:

$$\begin{cases} 1/k, & a \le x \le b \\ 0, & elsewhere \end{cases}$$

Q.2 A continous random variable X has

$$f(x) = \begin{cases} \frac{1}{2}(x+1), & for\ -1 \le x \le 1 \\ 0, & elsewhere \end{cases}$$

represent the density . Find mean, variance and standard deviation.

ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-04

Today's Target

- ***Binomial Probability Distribution***
- PYQ
- DPP

Probability Distribution

Discrete probability distribution

(Based on Discrete random variable)

(i) Binomial probability distribution 2L

(ii) Poisson probability distribution 2L

Continous probability distribution

(Based on continous radom variable)

(i) Normal probability Distribution 2L

Binomial probability Distribution

Bernoulli's Distribution

In Binomial distribution :

- Number of trials are finite (n)
- All trials are independent
- Each trial has only two outcomes (success or failure)

Binomial probability Distribution

Let a random variable X denote the number of successes in n independent trials in an experiment.

Let p be the probability of success and q be the probablity of failure in a single trial so that

$$p + q = 1$$

r successes can be obtained in n trials in n_{c_r} ways.

Probability of r successes in n trials

$$P(X = r) = n_{c_r} \underbrace{P(S)\,P(S) \ldots\ldots\ldots.P(S)}_{r\text{ times}}\ \underbrace{P(F)\,P(F) \ldots\ldots\ldots P(F)}_{(n-r)\text{ times}}$$

$$P(X = r) = n_{c_r}\, p\ p\ p \ldots\ldots\ldots\ldots p\ q\ q\ q \ldots\ldots\ldots\ldots q$$

$$\boxed{P(X = r) = n_{c_r}\, p^r q^{n-r}}$$

p = prob. of sucess

q = prob. of failure

n = No. of trials

r = No. of sucess

Where $p + q = 1$

And $r = 0,\ 1,\ 2,\ 3\ \ldots$

$$P(X = 0) = n_{c_0}\, q^n$$

$$P(X = 1) = n_{c_1}\, p\, q^{n-1}$$

$$P(X = 2) = n_{c_2}\, p^2 q^{n-2}$$

$$\vdots \qquad\qquad \vdots$$

$$P(X = n) = n_{c_n}\, p^n$$

$\boxed{n_{c_0}\, q^n\ ,\ n_{c_1}\, p\, q^{n-1}\ ,\ n_{c_2}\, p^2 q^{n-2}\ \ldots\ldots\ldots\ldots\ n_{c_n}\, p^n \text{ are successive terms of binomial expansion } (q+p)^n}$

- If $\boxed{\text{n independent trials constitute one experiment}}$ and this experiment is repeated N times.
- The frequuency of success is

$$N\; n_{c_r}\, p^r\, q^{n-r}$$

And Binomial Distribution is $\boxed{N(q+p)^n}$

Mean, variance and standard deviation of Binomial distribution

(i) **Mean of Binomial Distribution,**

$$\boxed{\mu = np}$$

First moment about origin

(i) **Variance of Binomial Distribution,**

$$\boxed{\text{Variance} = npq}$$

Second moment about mean

(i) ***Standard Deviation,***

$$\sigma = \sqrt{npq}$$

$$p(r+1) = \frac{n-r}{r+1} \; \frac{p}{q} \, p(r)$$

Q. 1 *Comment on the following statement : For a Binomial distribution, mean is 6 and variance is 9.*

Mean

$np = 6$ — (1)

variance

$npq = 9$ — (2)

Divide (2) by (1)

$$\frac{\cancel{np}q}{\cancel{np}} = \frac{\cancel{9}^{3}}{\cancel{6}_{2}}$$

$$q = \frac{3}{2}$$

$q = 1.5$ (impossible)

Since, $0 \le q \le 1$

Given Statement is False

Q. 2 *Ten coins are tossed simultaneously. Find the probalitity of getting at least seven heads.*

$P(H) = \frac{1}{2}$

$p = \frac{1}{2}$

$P(T) = \frac{1}{2}$

$q = \frac{1}{2}$

$n = 10$

$P(X=r) = {}^nC_r\, p^r q^{n-r}$

$P(r \geq 7) = P(r=7) + P(r=8) + P(r=9) + P(r=10)$

$= {}^{10}C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10}$

$= \left(\frac{1}{2}\right)^{10} \left[{}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} \right]$

$= \left(\frac{1}{2}\right)^{10} \left[\frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4}}{\cancel{3} \times \cancel{2}} + \frac{\cancel{10}^{5} \times 9}{\cancel{2}} + 10 + 1 \right]$

$= \left(\frac{1}{2}\right)^{10} \times \left[120 + 45 + 10 + 1\right]$

$= 176 \times \left(\frac{1}{2}\right)^{10}$

$$\boxed{P(r \geq 7) = \frac{11}{64}}$$

Q.3 *A die is thrown five times. If getting an odd number is a success, find the probability of getting at least four successes.*

$n = 5$

Getting an odd number is a sucess

$S = \{1, 2, 3, 4, 5, 6\}$

$P = \frac{\cancel{3}}{\cancel{6}\,2}$

$\boxed{p = \frac{1}{2}}$ $\boxed{q = \frac{1}{2}}$

$P(r \geq 4) = P(r=4) + P(r=5)$

$= {}^5C_4 \left(\frac{1}{2}\right)^4 \times \frac{1}{2} + {}^5C_5 \left(\frac{1}{2}\right)^5$

$= \left(\frac{1}{2}\right)^5 \left[{}^5C_4 + {}^5C_5\right]$

$= \frac{1}{32}(5 + 1)$

$= \frac{1}{\cancel{32}\,16} \times \cancel{6}\,3$

$\boxed{P(r \geq 4) = \frac{3}{16}}$

Q. 4 *A die is tossed thrice. A success is getting 1 or 6 on a toss. Find the mean and variance of the number of success.*

$S = \{1, 2, 3, 4, 5, 6\}$

Prob. of Sucess

$P = \frac{\cancel{2}}{\cancel{6}\,3}$

$\boxed{P = \frac{1}{3}}$

Prob. of failure

$\boxed{q = \frac{2}{3}}$

$n = 3$

Mean

$\mu = np = \cancel{3} \times \frac{1}{\cancel{3}}$

$\boxed{\mu = 1.}$

Variance

$Var(r) = npq = \cancel{3} \times \frac{1}{\cancel{3}} \times \frac{2}{3}$

$\boxed{Var(r) = \frac{2}{3}}$

Q.5 *A binomial variable X satisfies the relation* $\boxed{9P(X=4)=P(X=2)}$ *when* $n=6$.

Find the value of the parameter p and $P(X=1)$.

$n=6$

We know that

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

$$P(X=4) = {}^6C_4\, p^4 q^2$$

$$P(X=2) = {}^6C_2\, p^2 q^4$$

According to question

$$9P(X=4) = P(X=2)$$

$$9 \times {}^6C_4 \times p^{\cancel{4}\,2}\, \cancel{q^2} = {}^6C_2 \times \cancel{p^2}\, q^{\cancel{4}\,2}$$

$$9 \times \frac{\cancel{6!}}{\cancel{2!\times 4!}} \times p^2 = \frac{\cancel{6!}}{\cancel{4!\times 2!}} \times q^2$$

$$9p^2 = q^2$$

$$9p^2 = (1-p)^2$$

$$9p^2 = (1-p)^2$$

$$9p^2 = 1 + p^2 - 2p$$

$$8p^2 + 2p - 1 = 0$$

$$8p^2 + 4p - 2p - 1 = 0$$

$$4p(2p+1) - 1(2p+1) = 0$$

$$(4p-1)(2p+1) = 0$$

$$\boxed{p = \frac{1}{4}}$$

$$p = -\frac{1}{2} \text{ (Reject)}$$

$$q = 1 - p$$

$$q = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\boxed{q = \frac{3}{4}}$$

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$P(X=1) = {}^{6}C_{1} \left(\frac{1}{4}\right)^{1} \times \left(\frac{3}{4}\right)^{5}$$

$$P(X=1) = \cancel{6}^{\,3} \times \frac{1}{\cancel{4}_{\,2}} \times \frac{3^5}{4^5}$$

$$P(X=1) = \frac{729}{2048}$$

$$\boxed{P(X=1) = 0.3559}$$

Q.6 ***Fit a binomial distribution to the following frequency data :***

x:	0	1	2	3	4
f:	24	41	28	5	2

x	f	fx
0	24	0
1	41	41
2	28	56
3	5	15
4	2	8
	$\Sigma f = 100$	$\Sigma fx = 120$

$\Sigma f = N$

Mean

$$\mu = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{12\cancel{0}}{10\cancel{0}}$$

$$\boxed{\mu = 1.2}$$

For Binomial Distribution

$$\mu = np$$

$$1.2 = 4 \times p$$

$$p = 0.3$$

$$q = 0.7$$

Binomial Distribution

$$N(q+p)^n = 100(0.7+0.3)^4$$

DPP- 04

Topic : *Binomial probability distribution*

Q.1 *If the mean of a binomial distribution is 3 and the variabce is $\frac{3}{2}$, find the probability of obtaining at least 4 successes.*

Q.2 *The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped, find the probability that (i)exactly two will strike the target, (ii)at least two will trike the target.*

Q.3 *Fit a binimial distribution to the following frequency data:*

x:	0	1	2	3	4
f:	30	62	46	10	2

ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-05

Today's Target

- $Binomial\ probability\ distribution (Part - II)$
- PYQ
- DPP

Binomial probability Distribution

Let a random variable X denote the number of successes in n independent trials in an experiment. Let p be the probability of success and q be the probablity of failure in a single trial so that

$$p + q = 1$$

r successes can be obtained in n trials in n_{c_r} ways.

Probability of r successes in n trials

$$P(X = r) = n_{c_r}\, P(S)\, P(S) \ldots \ldots \ldots . P(S) \;\; P(F)\, P(F) \ldots \ldots \ldots P(F)$$

$$P(X = r) = n_{c_r}\, \mathrm{p\ p\ p} \ldots \ldots \ldots \ldots \mathrm{p\ \ q\ q\ q} \ldots \ldots \ldots \ldots \mathrm{q}$$

$$\boxed{P(X = r) = n_{c_r}\, p^r\, q^{n-r}}$$

- *If n independent trials constitute one experiment and this experiment is repeated N times.*
- *The frequuency of success is*

$$\boxed{N\ n_{c_r}\ p^r\ q^{n-r}}$$

And Binomial Distrivution is $N(q+p)^n$

Mean, variance and standard deviation of Binomial distribution

(i) **Mean of Binomial Distribution,**

$$\boxed{\mu = np}$$

(i) **Variance of Binomial Distribution,**

$$\boxed{\text{Variance} = npq}$$

(i) ***Standard Deviation,***

$$\boxed{\sigma = \sqrt{npq}}$$

Third moment about Mean $= npq(q-p)$

Q. 1 *If 10% of bolts produced by a machine are defective, determine the probability that out of 10 bolts choosen at random. (i) 1 (ii) None (iii) at most 2 bolts will be defective.*

$$P(\text{defective}) = \frac{\cancel{10}}{\cancel{100}_{10}}$$

$$\boxed{p = \frac{1}{10}}$$

$$P(\text{Non Defective}) = 1 - \frac{1}{10}$$

$$\boxed{q = \frac{9}{10}}$$

$$n = 10$$

$$\text{(i) } P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$P(X=1) = {}^{10}C_{1} \left(\frac{1}{10}\right)^{1} \times \left(\frac{9}{10}\right)^{9}$$

$$P(X=1) = 1\cancel{0} \times \frac{1}{1\cancel{0}} \times (0.9)^{9}$$

$$\boxed{P(X=1) = 0.3874}$$

(ii) $P(X=r) = {}^nC_r \, p^r q^{n-r}$

$$P(X=0) = {}^{10}C_0 \left(\frac{1}{10}\right)^0 \times \left(\frac{9}{10}\right)^{10}$$

$$P(X=0) = 1 \times 1 \times (0.9)^{10}$$

$$\boxed{P(X=0) = 0.3487}$$

(iii) $P(r \leq 2) = P(X=0) + P(X=1) + P(X=2)$ — ①

$$P(X=2) = {}^{10}C_2 \, p^2 q^8$$

$$P(X=2) = \frac{10!}{8! \times 2!} \times \left(\frac{1}{10}\right)^2 \times \left(\frac{9}{10}\right)^8$$

$$= \frac{\cancel{10}^{5} \times 9 \times \cancel{8!}}{\cancel{8!} \times \cancel{2}} \times \frac{1}{100} \times (0.9)^8$$

$$\boxed{P(X=2) = 0.1937}$$

Required Probability

$$P(r \leq 2) = 0.3487 + 0.3874 + 0.1937$$

$$\boxed{P(r \leq 2) = 0.9298}$$

Q.2 *If the probability of hitting a target is 10% and 10 shots are fired independently. What is the probability that the target will be hit at least once?* **AKTU 2019**

$$P(\text{Hitting a target}) = \frac{10}{100}$$

$$p = \frac{1}{10}$$

$$P(\text{Not Hitting target}) = 1 - \frac{1}{10}$$

$$q = \frac{9}{10}$$

$$n = 10$$

$$P(r \geq 1) = 1 - P(X = 0)$$

$$= 1 - {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$= 1 - {}^{10}C_{0} \left(\frac{1}{10}\right)^{0} \times \left(\frac{9}{10}\right)^{10}$$

$$= 1 - 1 \times 1 \times (0.9)^{10}$$

$$\boxed{P(r \geq 1) = 0.6513}$$

Q. 3 In 800 *families with* 5 *children each, how many families would be expected to have (i)* 3*boys and* 2 *girls, (ii)*2 *boys and* 3 *girls, (iii) no girl (iv) at the most two girls. Assume probabilities for boys and girls to be equal.* **AKTU-2017**

V. imp

$N = 800$

$n = 5$

$P(\text{Having a girl}) = \frac{1}{2}$

$$\boxed{p = \frac{1}{2}}$$

$P(\text{Having a boy}) = \frac{1}{2}$

$$\boxed{q = \frac{1}{2}}$$

(i) Expected number of families having 3 boys and 2 girls

$$N\,P(X=2) = N\,{}^{n}C_{r}\,p^{r}q^{n-r}$$

$$= 800 \times {}^{5}C_{2}\left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right)^{3}$$

$$= 800 \times \frac{5!}{3!\,2!} \times \left(\frac{1}{2}\right)^{5}$$

$$= \overset{50}{\cancel{800}} \times \frac{5 \times \cancel{4} \times \cancel{3!}}{\cancel{3!} \times \cancel{2}} \times \frac{1}{\cancel{32}\,\cancel{2}}$$

$$\boxed{N(X=2) = 250}$$

(ii) Number of families having 2 boys and 3 Girls

$$NP(X=3) = N\,{}^{n}C_{r}\,p^{r}q^{n-r}$$

$$= 800 \times {}^{5}C_{3}\left(\tfrac{1}{2}\right)^{3} \times \left(\tfrac{1}{2}\right)^{2}$$

$$= 800 \times \frac{5!}{3! \times 2!} \times \left(\tfrac{1}{2}\right)^{5}$$

$$\boxed{NP(X=3) = 250}$$

(iii) Number of families having No girl

$$NP(X=0) = N\,{}^{n}C_{r}\,p^{r}q^{n-r}$$

$$= 800 \times {}^{5}C_{0}\left(\tfrac{1}{2}\right)^{0} \times \left(\tfrac{1}{2}\right)^{5}$$

$$= \cancel{800}^{\cancel{50}\ 25} \times 1 \times \frac{1}{\cancel{32}_{\cancel{2}}}$$

$$\boxed{NP(X=0) = 25}$$

(iv) Number of families having atmost two Girls

$$NP(r \le 2) = N[P(X=0) + P(X=1) + P(X=2)]$$

$$P(X=1) = {}^5C_1\, p^r q^{n-r}$$

$$= 5 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^4$$

$$= 5 \times \frac{1}{25}$$

$$= \frac{5}{32}$$

$$NP(r \le 2) = NP(X=0) + NP(X=1) + NP(X=2)$$

$$= 25 + \cancel{800}^{25} \times \frac{5}{\cancel{32}} + 250$$

$$= 25 + 125 + 250$$

$$\boxed{NP(r \le 2) = 400}$$

Q. 4 *A student is given a true false examination with 8 questions. If he correct at least 7 questions, he passes the examination. Find the probability that he will pass given that he guesses all questions.*

$n = 8$

$p = \frac{1}{2}$

$q = \frac{1}{2}$

Prob. that he will pass

$$P(X \geq 7) = P(X=7) + P(X=8)$$

$$= {}^8C_7 \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^1 + {}^8C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^0$$

$$P(X \geq 7) = 8 \times \left(\frac{1}{2}\right)^8 + 1 \times \left(\frac{1}{2}\right)^8$$

$$= \left(\frac{1}{2}\right)^8 (8+1)$$

$$= \frac{9}{2^8}$$

$$\boxed{P(X \geq 7) = 0.035}$$

Q.5 ***Six dies are thrown 729 times. How many times do you expect at least three dice to show a five or six?***

$N = 729$

$n = 6$

$S = \{1, 2, 3, 4, 5, 6\}$

$P(\text{having } 5 \text{ or } 6) = \frac{\cancel{2}}{\cancel{6}\,3}$

$\boxed{p = \frac{1}{3}}$

$P(\text{having Not } 5 \text{ or } 6) = \frac{\cancel{4}\,2}{\cancel{6}\,3}$

$\boxed{q = \frac{2}{3}}$

$NP(r \geq 3) = N[P(X=3) + P(X=4) + P(X=5) + P(X=6)]$

$= 729\left[{}^6C_3\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + {}^6C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2 + {}^6C_5\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^1 + {}^6C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^0\right]$

$NP(r \geq 3) = 729\left[{}^6C_3 \times \frac{8}{3^6} + {}^6C_4 \times \frac{4}{3^6} + {}^6C_5 \times \frac{2}{3^6} + {}^6C_6 \times \frac{1}{3^6}\right]$

$$NP(r \geq 3) = \frac{729}{3^6}\left[8\,{}^6C_3 + 4\,{}^6C_4 + 2\,{}^6C_5 + {}^6C_6\right]$$

$$= \frac{\cancel{729}}{\cancel{729}}\left[8 \times \frac{6!}{3! \times 3!} + 4 \times \frac{6!}{4! \times 2!} + 2 \times 6 + 1\right]$$

$$= \left[\frac{8 \times \cancel{6} \times 5 \times 4 \times \cancel{3!}}{\cancel{3 \times 2 \times 1} \times \cancel{3!}} + \frac{\overset{2}{\cancel{4}} \times 6 \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2 \times 1}} + 12 + 1\right]$$

$$= 160 + 60 + 12 + 1$$

$$\boxed{NP(r \geq 3) = 233}$$

DPP- 05

Topic : *Binomial probability distribution (Part – II)*

Q.1 *Out of 800 families with 4 children each, how many families would be expected to have*

(i) 2 boys and 2 girls (ii) at least one boy (iii) no girl (iv) almost two girls?

Assume equal probabilities for boys and girls. UPTU 2014

Q.2 *A bag contains 5 white, 7 red and 8 black balls. If four balls are drawn, one by one, with replacement. what is the probability that*

(i) none is white *(ii) all are white*

(iii) at least on is white *(iv) only 2 are white?*

Q.3 *Five cards are drawn successively with repelacement from a wellshuffled deck of 52 cards. What is the probability that*

(i) All the five cards are spades

(ii) only three are spades?

(iii) none is spade?

ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-06

Today's Target

- $\boxed{\boldsymbol{Poisson\ \ Probability\ Distribution(Part - I)}}$
- PYQs
- DPPs

Poisson Probability Distribution

- *Poisson Distribution is a limiting case of binomial distribution*

$$\boxed{P(X=r)=\frac{\lambda^r e^{-\lambda}}{r\,!}} \qquad (r=0,1,2,3,\dots)$$

- *Recurrence formula for the Poisson Distribution*

$$\boxed{P(r+1)=\frac{\lambda}{(r+1)}\,P(r)} \qquad (r=0,1,2,3,\dots.)$$

- *Mean and Variance of the Poisson Distribution*

$$\text{Mean}=\lambda$$

$$\text{Variance}=\lambda$$

Important Derivations:

1. Show that Poisson Distribution is a limiting case of binomial distribution (A.K.T.U. 2021)

In Binomial Distribution

n = number of trials

p = Prob. of sucess

- If n is very large and p is very small then Binomial distribution become complicated
- If we assume that $n \to \infty$ and $p \to 0$ such that $\boxed{np = \lambda}$ is finite

Then we get poisson approximation to binomial distribution.

$$\boxed{P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}}$$

Proof:

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$P(X=r) = {}^{n}C_{r}\, p^{r}(1-p)^{n-r} \qquad \{p+q=1\}$$

$$P(X=r) = \frac{n!}{(n-r)!\,r!}\, p^{r} \times (1-p)^{n}(1-p)^{-r}$$

$$P(X=r) = \frac{n!}{(n-r)!\,r!} \times \left(\frac{\lambda}{n}\right)^{r} \times \frac{\left(1-\frac{\lambda}{n}\right)^{n}}{\left(1-\frac{\lambda}{n}\right)^{r}}$$

$$P(X=r) = \frac{\lambda^{r} \times n(n-1)(n-2)\cdots\cdots(n-r+1)\,\cancel{(n-r)!}\,\left(1-\frac{\lambda}{n}\right)^{n}}{r! \times \cancel{(n-r)!} \times n^{r} \times \left(1-\frac{\lambda}{n}\right)^{r}}$$

$$P(X=r) = \frac{\lambda^{r}}{r!} \times \frac{n}{n} \times \left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\cdots\cdots\left(\frac{n-r+1}{n}\right)\frac{\left(1-\frac{\lambda}{n}\right)^{n}}{\left(1-\frac{\lambda}{n}\right)^{r}}$$

$$P(X=r) = \frac{\lambda^{r}}{r!} \times 1 \times \left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\cdots\left(1-\frac{r}{n}+\frac{1}{n}\right)\frac{\left(1-\frac{\lambda}{n}\right)^{n}}{\left(1-\frac{\lambda}{n}\right)^{r}}$$

$$P(X=r) = \lim_{n\to\infty} \frac{\lambda^r}{r!} \underbrace{\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\cdots\left(1-\frac{r}{n}+\frac{1}{n}\right)} \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^r}$$

$$= \frac{\lambda^r}{r!}(1-0)(1-0)\cdots\cdots(1-0+0)\frac{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n}{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^r}$$

$$= \frac{\lambda^r}{r!} \times 1 \times 1 \cdots\cdots \times 1 \times \frac{e^{-\lambda}}{1}$$

$$= \frac{\lambda^r}{r!} \times e^{-\lambda}$$

$$\boxed{P(X=r) = \frac{\lambda^r e^{-\lambda}}{r!}}$$

Poisson Distribution

$$\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^n = e^{-x}$$

$$\lim_{n\to\infty}\left(1-\frac{x}{n}\right)^r = 1$$

Poisson Distribution

$$\boxed{P(X = r) = \frac{\lambda^r e^{-\lambda}}{r!}} \qquad (r = 0, 1, 2, 3, \dots)$$

- *λ is called the parameter of the distribution.*

λ = np (Finite)

- *The sum of the Probability $P(r)$ for $r = 0, 1, 2, 3, \dots \dots$ is 1*

$$\sum P(r) = 1$$

P(X=0) + P(X=1) + P(X=2) + P(X=3) . . . . = 1

2. Recurrence formula for the Poisson Distribution

By Poisson Distribution

$$P(r) = \frac{\lambda^r e^{-\lambda}}{r!} \quad \text{—— (1)}$$

Replace r by $r+1$

$$P(r+1) = \frac{\lambda^{r+1} e^{-\lambda}}{(r+1)!}$$

$$P(r+1) = \frac{\lambda^r \times \lambda^1 \times e^{-\lambda}}{(r+1) \times r!} \quad \text{—— (2)}$$

Divide (2) by (1)

$$\frac{P(r+1)}{P(r)} = \frac{\cancel{\lambda^r} \times \lambda \times \cancel{e^{-\lambda}}}{(r+1) \times \cancel{r!}} \times \frac{\cancel{r!}}{\cancel{\lambda^r} \times \cancel{e^{-\lambda}}}$$

$$\frac{P(r+1)}{P(r)} = \frac{\lambda}{r+1}$$

$$\boxed{P(r+1) = \frac{\lambda}{r+1} P(r)}$$

3. Mean and Variance of the Poisson Distribution

By Poisson Distribution

$$p(r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

Mean

$$\mu = E(r) = \sum_{r=0}^{\infty} r\, p(r)$$

$$\mu = \sum_{r=0}^{\infty} \frac{r e^{-\lambda} \lambda^r}{r!}$$

$$\mu = e^{-\lambda} \sum_{r=1}^{\infty} \frac{\cancel{r} \lambda^r}{\cancel{r}(r-1)!}$$

$$\mu = e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^r}{(r-1)!}$$

$$\mu = e^{-\lambda}\left(\frac{\lambda}{1} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \frac{\lambda^4}{3!} + \cdots\right)$$

$$\mu = e^{-\lambda} \lambda\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \cdots\right)$$

$$\text{Mean} = e^{-\lambda} \times \lambda \times e^{\lambda} = \lambda \times e^0$$

$$\boxed{\text{Mean} = \lambda}$$

Variance

$$\sigma^2 = E(r^2) - [E(r)]^2$$

$$\sigma^2 = \sum_{r=0}^{\infty} r^2 P(r) - (\lambda)^2$$

$$\sigma^2 = \sum_{r=0}^{\infty} r^2 \frac{e^{-\lambda}\lambda^r}{r!} - \lambda^2$$

$$\sigma^2 = e^{-\lambda} \sum_{r=1}^{\infty} \frac{r^2 \lambda^r}{r!} - \lambda^2$$

$$\sigma^2 = e^{-\lambda}\left(\frac{1^2 \times \lambda}{1!} + \frac{2^2 \times \lambda^2}{2!} + \frac{3^2 \times \lambda^3}{3!} + \cdots\right) - \lambda^2$$

$$\sigma^2 = e^{-\lambda} \times \lambda\left(1 + \frac{2\lambda}{1!} + \frac{3\lambda^2}{2!} + \frac{4\lambda^3}{3!} + \cdots\right) - \lambda^2$$

$$\sigma^2 = \lambda e^{-\lambda}\left[1 + \frac{\lambda + \lambda}{1!} + \frac{\lambda^2 + 2\lambda^2}{2!} + \frac{\lambda^3 + 3\lambda^3}{3!} + \cdots\right) - \lambda^2$$

$$\sigma^2 = \lambda e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{2\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{3\lambda^3}{3!} + \cdots\right) - \lambda^2$$

$$\sigma^2 = \lambda e^{-\lambda}\left[\left(1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \cdots\right) + \left(\frac{\lambda}{1!} + \frac{2\lambda^2}{2!} + \frac{3\lambda^3}{3!} + \cdots\right) - \lambda^2\right.$$

$$\sigma^2 = \lambda e^{-\lambda}\left[\left(1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\cdots\right)+\lambda\left(1+\frac{2\lambda}{2!}+\frac{3\lambda^2}{3!}+\frac{4\lambda^3}{4!}+\cdots\right)\right.-\lambda^2$$

$$\sigma^2 = \lambda e^{-\lambda}\left[e^{\lambda}+\lambda\left(1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+\cdots\right)\right.-\lambda^2$$

$$\sigma^2 = \lambda e^{-\lambda}\left[e^{\lambda}+\lambda e^{\lambda}\right]-\lambda^2$$

$$\sigma^2 = \lambda e^{-\lambda}e^{\lambda}+\lambda^2 e^{-\lambda}e^{\lambda}-\lambda^2$$

$$\sigma^2 = \lambda+\cancel{\lambda^2}-\cancel{\lambda^2}$$

$$\sigma^2 = \lambda$$

$$\boxed{\text{Varianc} = \lambda}$$

Q. 1 *If a random variable X follow a poisson distribution such that* $P(X=2)=9P(X=4)+90P(X=6)$. *Find mean and variance of X.* **(A.K.T.U. 2023)**

For poisson Distribution

$$P(X=r)=\frac{e^{-\lambda}\lambda^{r}}{r!}$$

$$P(X=2)=9P(X=4)+90P(X=6)$$

$$\frac{e^{-\lambda}\lambda^{2}}{2!}=9\times\frac{e^{-\lambda}\lambda^{4}}{4!}+\frac{90\times e^{-\lambda}\lambda^{6}}{6!}$$

$$\frac{\cancel{e^{-\lambda}}\lambda^{2}}{2!}=\cancel{e^{-\lambda}}\left(\frac{9\lambda^{4}}{4!}+\frac{90\lambda^{6}}{6!}\right)$$

$$\frac{\cancel{\lambda^{2}}}{2}=\cancel{\lambda^{2}}\left(\frac{\overset{3}{\cancel{9}}\lambda^{2}}{4\times\cancel{3}\times2\times1}+\frac{\overset{\cancel{30}}{\cancel{90}}\lambda^{4}}{\cancel{6\times5}\times4\times\cancel{3}\times2\times1}\right)$$

$$\frac{1}{2}=\frac{3\lambda^{2}}{8}+\frac{\lambda^{4}}{8}$$

$$\frac{1}{2}=\frac{3\lambda^{2}+\lambda^{4}}{8}$$

$$4=3\lambda^{2}+\lambda^{4}$$

$$\lambda^{4}+3\lambda^{2}-4=0$$

Put $\lambda^2 = x$

$$x^2 + 3x - 4 = 0$$

$$x^2 + 4x - x - 4 = 0$$

$$x(x+4) - 1(x+4) = 0$$

$$(x-1)(x+4) = 0$$

$x = 1$	$x = -4$
$\lambda^2 = 1$	$\lambda^2 = -4$
$\lambda = \pm 1$	(Reject)
$\lambda = 1$	

Hence

$$\lambda = 1$$

Mean = 1

Variance = 1

Q.2 *If the variance of the Poisson distribution is 2, find the probabilities for $r = 1, 2, 3, 4$ from the recurrence relation of the Poisson distribution. Also, find $P(r \geq 4)$.*

(M.T.U. 2013)

Given

$\lambda = 2$

By poisson Distribution

$$P(r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

$$P(0) = \frac{e^{-2} \times \lambda^{0}}{0!}$$

$$P(0) = e^{-2}$$

$$P(0) = 0.1353$$

By Recurrence Relation

$$P(r+1) = \frac{\lambda}{r+1} P(r)$$

$$P(0+1) = \frac{2}{0+1} P(0)$$

$$P(1) = 2\,P(0)$$

$$\boxed{P(1) = 0.2706}$$

$$P(1+1) = \frac{2}{1+1} P(1)$$

$$P(2) = P(1)$$

$$\boxed{P(2) = 0.2706}$$

$$P(2+1) = \frac{2}{2+1} P(2)$$

$$P(3) = \frac{2}{3} \times 0.2706$$

$$\boxed{P(3) = 0.1804}$$

$$P(3+1) = \frac{2}{3+1} P(3)$$

$$P(4) = \frac{1}{2} \times 0.1804$$

$$\boxed{P(4) = 0.0902}$$

$$P(r \geq 4) = 1 - P(r < 4)$$

$$P(r \geq 4) = 1 - [P(0) + P(1) + P(2) + P(3)]$$

$$P(r \geq 4) = 1 - 0.8569$$

$$\boxed{P(r \geq 4) = 0.1431}$$

Q.3 *Using Poisson distribution, find the probability that the ace of spades will be drawn from a pack of well – shuffled cards at least once in 104 consecutive trials.* **(U.P.T.U. 2015)**

$P(\text{ace of spades}) = \frac{1}{52}$

$p = \frac{1}{52}$

$n = 104$

$\lambda = np$

$\lambda = \cancel{104}^{2} \times \frac{1}{\cancel{52}}$

$\boxed{\lambda = 2}$

By Poisson Distribution

$P(r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$

$P(r \geq 1) = 1 - P(X=0)$

$= 1 - \frac{e^{-2} \times \lambda^{0}}{0!} = 1 - e^{-2}$

$\boxed{P(r \geq 1) = 0.8647}$

DPP- 06

Topic : *Poisson Probability Distribution (Part – I)*

Q.1 In a poisson distribution, $P(r)$ for $r = 0$ is 10%, find the mean.

Q.2 If X poisson variate such that $P(X = 1) = P(X = 2)$. Find (i) Mean (ii) $P(X = 4)$

ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-07

Today's Target

- $\boldsymbol{Poisson\ \ Probability\ Distribution(Part-II)}$
- PYQs
- DPPs

Poisson Probability Distribution

- *Poisson Distribution is a limiting case of binomial distribution*

$$\boxed{P(X = r) = \frac{\lambda^r e^{-\lambda}}{r\,!}} \qquad (r = 0, 1, 2, 3, \dots)$$

- *Recurrence formula for the Poisson Distribution*

$$\boxed{P(r + 1) = \frac{\lambda}{(r+1)} P(r)} \qquad (r = 0, 1, 2, 3, \dots.)$$

- *Mean and Variance of the Poisson Distribution*

$$\boxed{\begin{aligned} Mean &= \lambda \\ Variance &= \lambda \end{aligned}}$$

$$\boxed{\lambda = np}$$

Q.1 *It is given that 2% of the electric bulbs manufactured by a company are defective. Using Poisson distribution, find the probability that a sample of 200 bulbs will contain (i) no defective bulb (ii) two defective bulbs (iii) at the most three defective bulbs.* (G.B.T.U. 2011)

Prob. of Defective bulbs

$$p = \frac{2}{100}$$

$$p = 0.02$$

$$n = 200$$

$$\boxed{\begin{aligned}\lambda &= 0.02 \times 200\\ \lambda &= 4\end{aligned}}$$

By poisson distribution

$$P(X=r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

(i) Prob. of no defective bulb $(r=0)$

$$P(X=0) = \frac{e^{-4} \times (4)^0}{0!} = e^{-4}$$

$$\boxed{P(X=0) = 0.018315}$$

(ii) Prob. of two defective $(r=2)$

$$P(X=2) = \frac{e^{-4}(4)^2}{2!} = \frac{\cancel{16}^{\,8}}{\cancel{2}}e^{-4}$$

$P(X=2) = 8e^{-4}$

$P(X=2) = 0.146525$

(iii) Prob. of atmost 3 defective bulbs $(r \leq 3)$

$P(r \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= e^{-4} + \frac{e^{-4} \times 4}{1!} + 8e^{-4} + \frac{e^{-4}(4)^3}{3!}$

$= e^{-4} + 4e^{-4} + 8e^{-4} + \frac{32}{3}e^{-4}$

$P(r \leq 3) = e^{-4}\left(13 + \frac{32}{3}\right)$

$= e^{-4}\left(\frac{39+32}{3}\right)$

$= \frac{71}{3}e^{-4}$

$\boxed{P(r \leq 3) = 0.43347}$

Q.2 *A manufacturer knows that the condensors he makes contain on an average 1% of defectives. He packes them in boxes of 100. What is the probability that a box picked at random will contain 4 or more faulty condensors?*

Prob. of defective condensors

$$p = \frac{1}{100}$$

$$\boxed{p = 0.01}$$

$$n = 100$$

$$\lambda = np = 100 \times 0.01$$

$$\boxed{\lambda = 1}$$

By poisson distribution

$$P(x = r) = \frac{e^{-\lambda} \lambda^{r}}{r!}$$

Prob. of 4 or more faulty condensor $(r \geq 4)$

$$P(r \geq 4) = 1 - P(r < 4)$$

$$P(r \geq 4) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - \left[\frac{e^{-1}(1)^0}{0!} + \frac{e^{-1} \times (1)^1}{1!} + \frac{e^{-1} \times 1^2}{2!} + \frac{e^{-1} \times 1^3}{3!}\right]$$

$$= 1 - \left[e^{-1} + e^{-1} + \frac{1}{2}e^{-1} + \frac{1}{6}e^{-1}\right]$$

$$= 1 - e^{-1}\left(2e^{-1} + \frac{e^{-1}}{2} + \frac{e^{-1}}{6}\right)$$

$$= 1 - e^{-1}\left(\frac{12e^{-1} + 3e^{-1} + e^{-1}}{6}\right)$$

$$= \left(1 - \frac{8}{3}e^{-1}\right) = 0.018988$$

$$\boxed{P(r \geq 4) = 0.018988}$$

Q.3 *In a certain factory turning out razor blades, there is a small chance of* 0.002 *for any blade to be defective. The blades are supplied in packets of* 10*. Calculate the approximate number of packets containing no defective, one defective and two defective blades in a consignment of* $10,000$ *packets.* (*Given*: $e^{-0.02} = 0.9802$) **(A.K.T.U. 2018)**

Prob. of defective blade

$$P = 0.002$$

$$n = 10$$

$$N = 10,000$$

$$\lambda = nP = 0.002 \times 10$$

$$\boxed{\lambda = 0.02}$$

By Poisson Distribution

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

(i) Number of packets containing no defective blades

$$P(X=0) = \frac{e^{-0.02} \times (0.02)^0}{0!} = e^{-0.02}$$

$P(X=0) = e^{-0.02}$

$P(X=0) = 0.9802$

Number of packets

$= 10,000 \times 0.9802$

$= 9802$

(ii) $P(X=1) = \dfrac{e^{-0.02} \times (0.02)^1}{1!}$

$P(X=1) = 0.0196$

Number of packets having 1 defective blades

$= 0.0196 \times 10000$

$= 196$

(iii) $P(X=2) = \dfrac{e^{-0.02} \times (0.02)^2}{2!}$

$= 0.000196$

Number of packets having 2 defective blades

$= 0.000196 \times 10,000 = 1.96$

$= 2$

Q.4 *The number of accidents in a year involving taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with (i) no accidents (ii) more than 3 accidents in a year.* **(A.K.T.U. 2021)**

Given

$\lambda = 3$

$N = 1000$

By Poisson Distribution

$$P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

(i) $P(X=0) = \frac{e^{-3}(3)^0}{0!} = e^{-3}$

$P(X=0) = e^{-3} = 0.04978$

Number of drivers having no accidents

$= 0.04978 \times 1000$

$= 49.7$

$$\boxed{= 50}$$

(iii) $P(r>3) = 1-[P(X=0)+P(X=1)+P(X=2)+P(X=3)]$

$$= 1-\left[e^{-3} + \frac{e^{-3}\times(3)^1}{1!} + \frac{e^{-3}\times(3)^2}{2!} + \frac{e^{-3}\times(3)^3}{3!}\right]$$

$$= 1-\left[e^{-3} + 3e^{-3} + \frac{9}{2}e^{-3} + \frac{9}{2}e^{-3}\right]$$

$$= 1-e^{-3}(4+4.5+4.5)$$

$$= 1-13e^{-3} = 0.3527$$

Number of drivers having more than 3 accidents

$$= 0.3527\times1000 = 352.7$$

$$\boxed{=353}$$

Q.5 *The following table gives the no. of days in a 50 day period during which automobile accidents occured in a city.*

No. of accidents	:	0	1	2	3	4
No. of days (f)	:	21	18	7	3	1

Fit a Poisson distribution to the data. **(A.K.T.U. 2022)**

x	f	fx	Theoritical frequencies
0	21	0	20
1	18	18	18
2	7	14	8
3	3	9	3
4	1	4	1
	$\Sigma f = 50$	$\Sigma fx = 45$	

$$\text{Mean} = \frac{\Sigma f x}{\Sigma f}$$

$$\lambda = \frac{45}{50}$$

$$\boxed{\lambda = 0.9}$$

$$N = 50$$

Poisson Distribution $= N\dfrac{e^{-\lambda}\lambda^{r}}{r!}$

$$\boxed{= \frac{50 \times e^{-0.9}(0.9)^{r}}{r!}}$$

When $r = 0$

$= 20.32 = 20$

When $r = 0$

$= 18.29 = 18$

When $r = 2$

$= 8.2 = 8$

When $r = 3$

$= 2.46$

$= 3$

When $r = 4$

$= 0.555$

$= 1$

DPP- 07

Topic : *Poisson Probability Distribution (Part – II)*

Q. 1 *In a certain factory manufacturing razor blades, there is a small chance of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use suitable distribution to calculate the approximate number of packets containing no defective, one defective and two defective blades respectively in a consignment of 50,000 packets.* (AKTU 2018)

Q. 2 *The distribution of the number of road accidents per day in a city is Poisson with mean 4. Find the number of days out of 100 days when there will be (i) no accident (ii) at least 2 accidents (iii) at most 3 accidents (iv) between 2 and 5 accidents.*

Q. 3 *Fit a Poisson distribution to the following data and calculate theoretical frequencies.*

Deaths:	0	1	2	3	4
Frequencies:	122	60	15	2	1

Q. 4 *Fit a Poisson distribution to the following data given the number of yeast cells per square for 400 squares:*

No. of cells per sq.:	0	1	2	3	4	5	6	7	8	9	10
No. of squares:	103	143	98	42	8	4	2	0	0	0	0

ENGINEERING MATHEMATICS

UNIT-4 : *Statistical Techniques – II*

Lec-08

Today's Target

- $\boxed{\textit{Normal Probability Distribution}}(Part - I)$
- PYQs
- DPPs

Normal Distribnution

- *The normal distribution is a continous distribution based on continous random variable*
- *A continous random variable X is said to have a normal distribution with parameters μ(Mean) and σ^2(Variance) if the probability density function is given by*

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)dx = 1.$

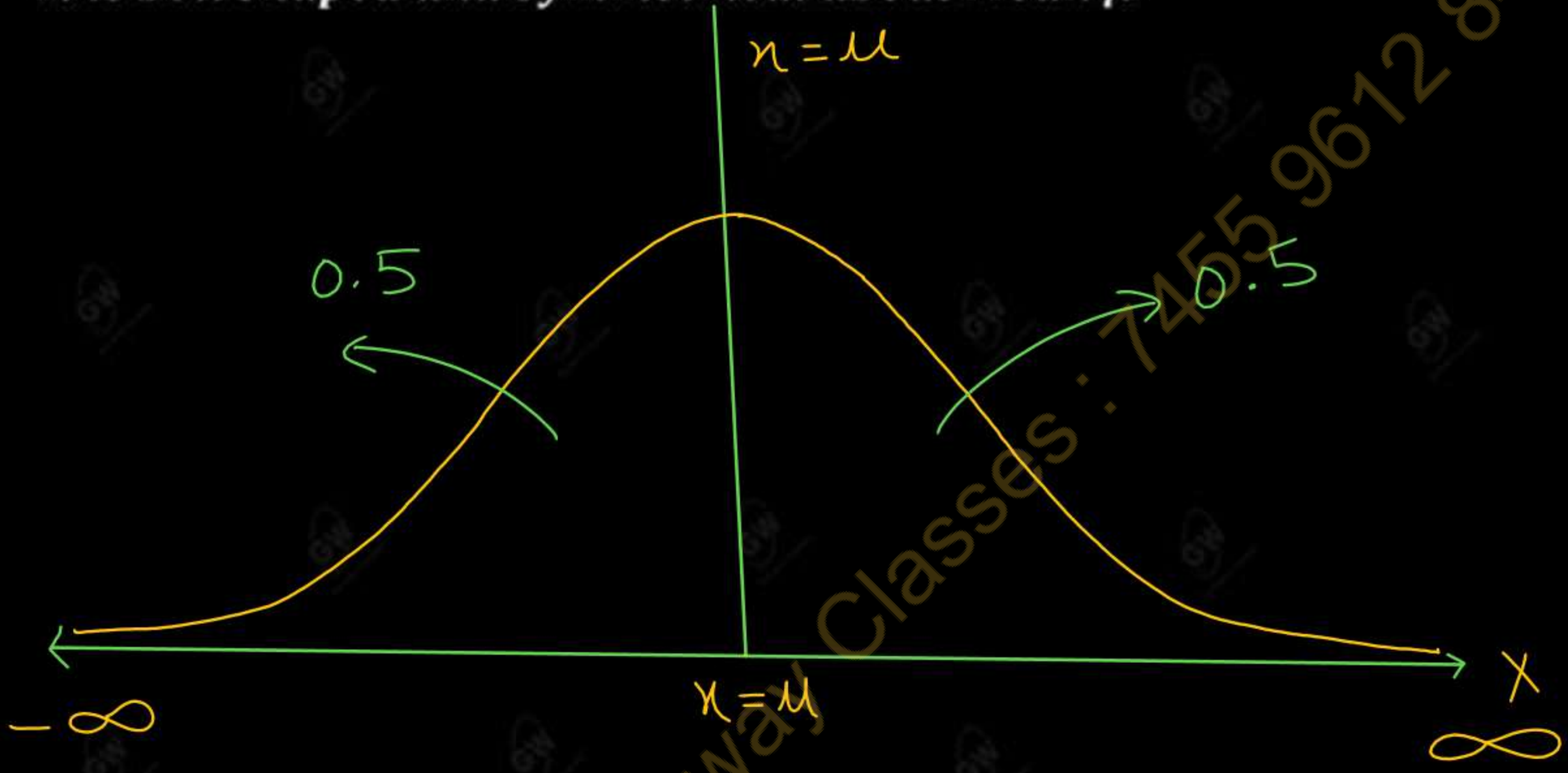
$-\infty < x < \infty$

μ = Mean

σ = Standard deviation

Normal Curve

- The graph of normal *Distribnution is called normal curve.*
- *It is bell shaped and symmetrical about Mean μ.*



- *In normal distribution Mean, median and mode coincide.*
- *Total area under the curve above x − axis is 1*

Standard Normal Distribution

- If X is a Normal random variable with mean μ and standard deviation σ, then a random Variable Z with Mean = 0 and standard deviation = 1 is called standard normal random variable.

$$\boxed{z = \frac{x-\mu}{\sigma},}$$

v.imp

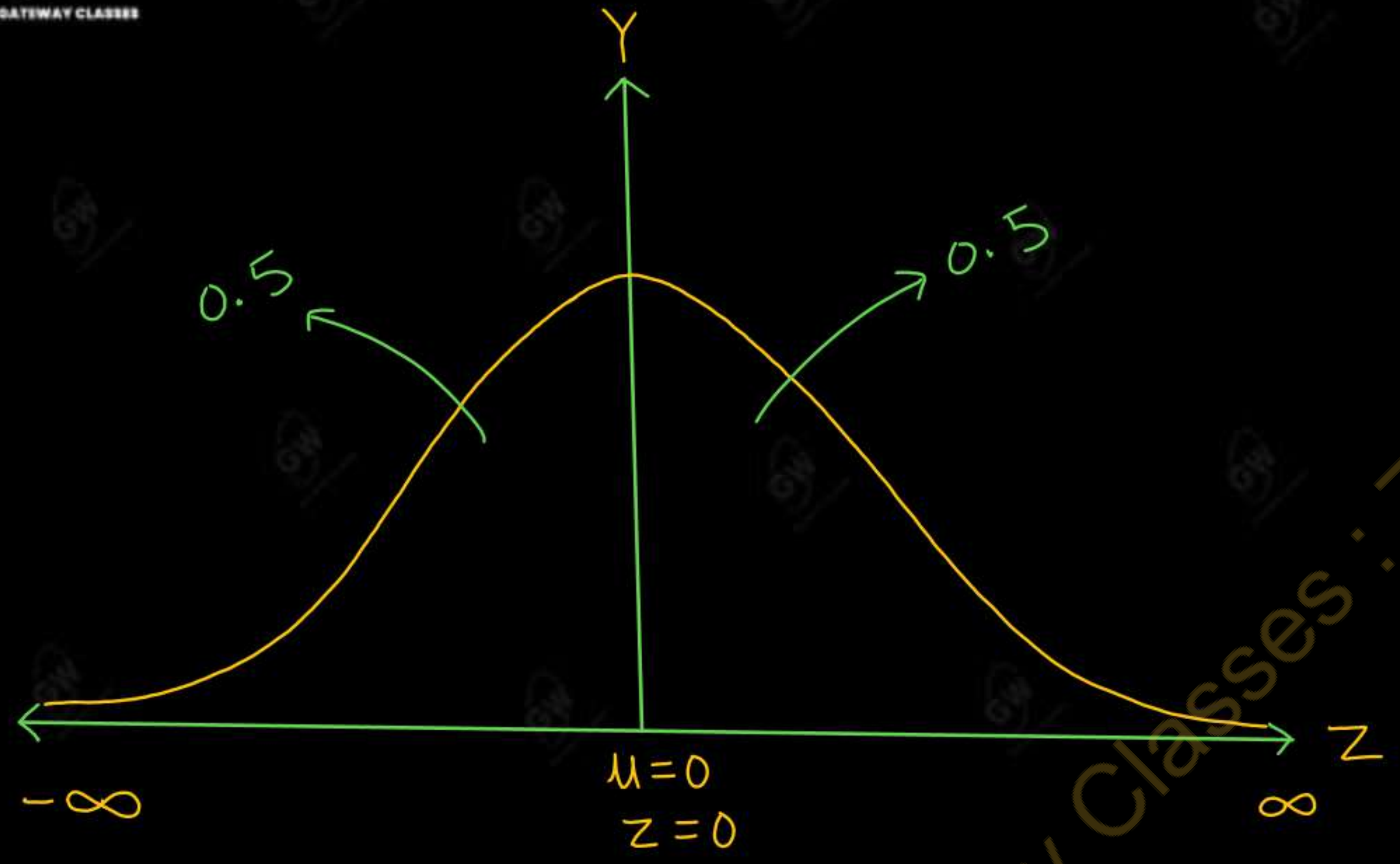
- The probability density function for the normal distribution in standard form is given by

$$\boxed{f(Z) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}(Z)^2}}$$

- Probability

$$P(z_1 < z < z_2) = \int_{z_1}^{z_2} f(z)dz$$

Standard Normal Curve



Probability

$P(z_1 < z < z)$ = Area under standard normal curve between $z = z_1$ and $z = z_2$

Q.1 *If X is a normal variate with mean* 80 *and standard deviation* 10 *, find probability* $P(X \leq 100)$.

Given

$\mu = 80$

$\sigma = 10$

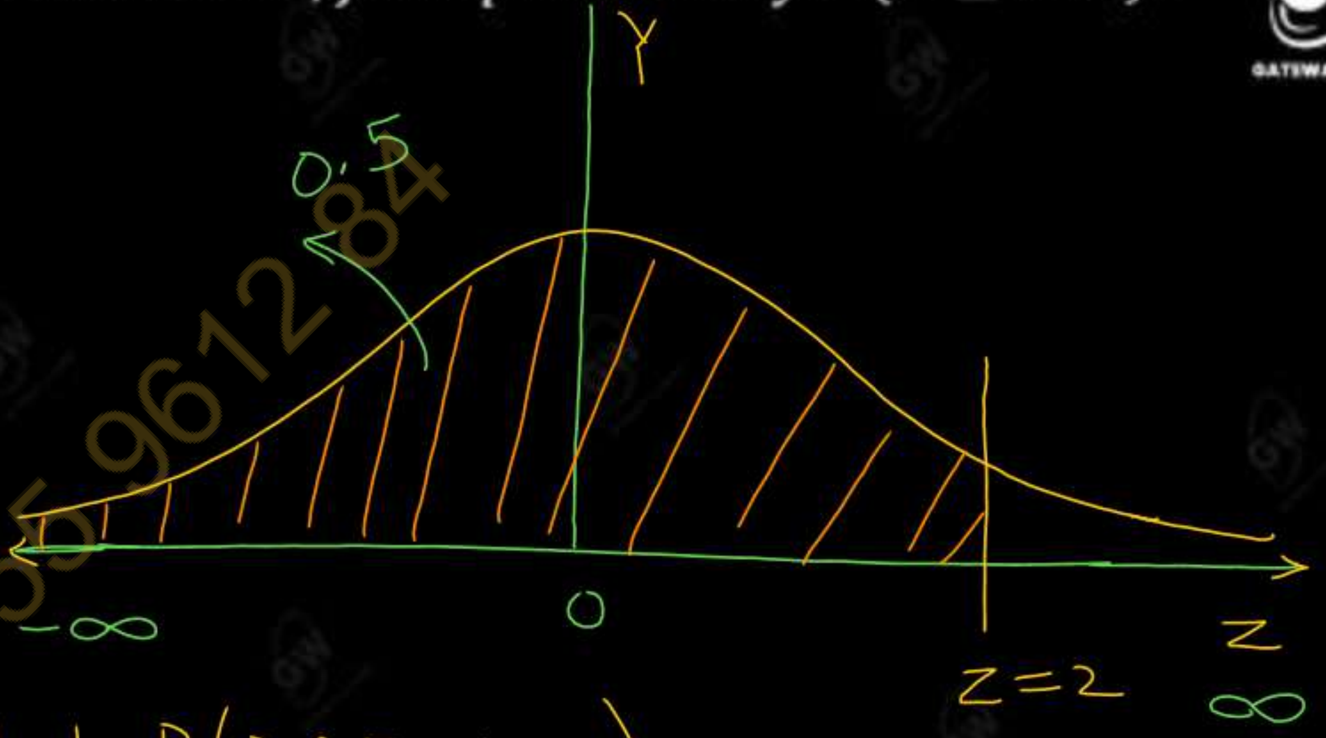
$x = 100$

$$z = \frac{x-\mu}{\sigma}$$

$$z = \frac{100-80}{10}$$

$$\boxed{z = 2}$$

$$P(X \leq 100) = P(Z \leq 2)$$



$$= P(-\infty < Z < 0) + P(0 < Z \leq 2)$$

$$= 0.5 + P(0 < Z \leq 2)$$

$$= 0.5 + 0.4772$$

$$\boxed{P(X \leq 100) = 0.9772}$$

Q.2 *The life of army shoes is normally distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are insured, how many pairs would be expected to need replacement after 12 months?* $\left[\text{Given that } P(z \geq 2) = 0.0228 \text{ and } z = \frac{x-\mu}{\sigma}\right]$. **(A.K.T.U. 2018)**

Given

$\mu = 8$

$\sigma = 2$

$N = 5000$

Let $x = 12$

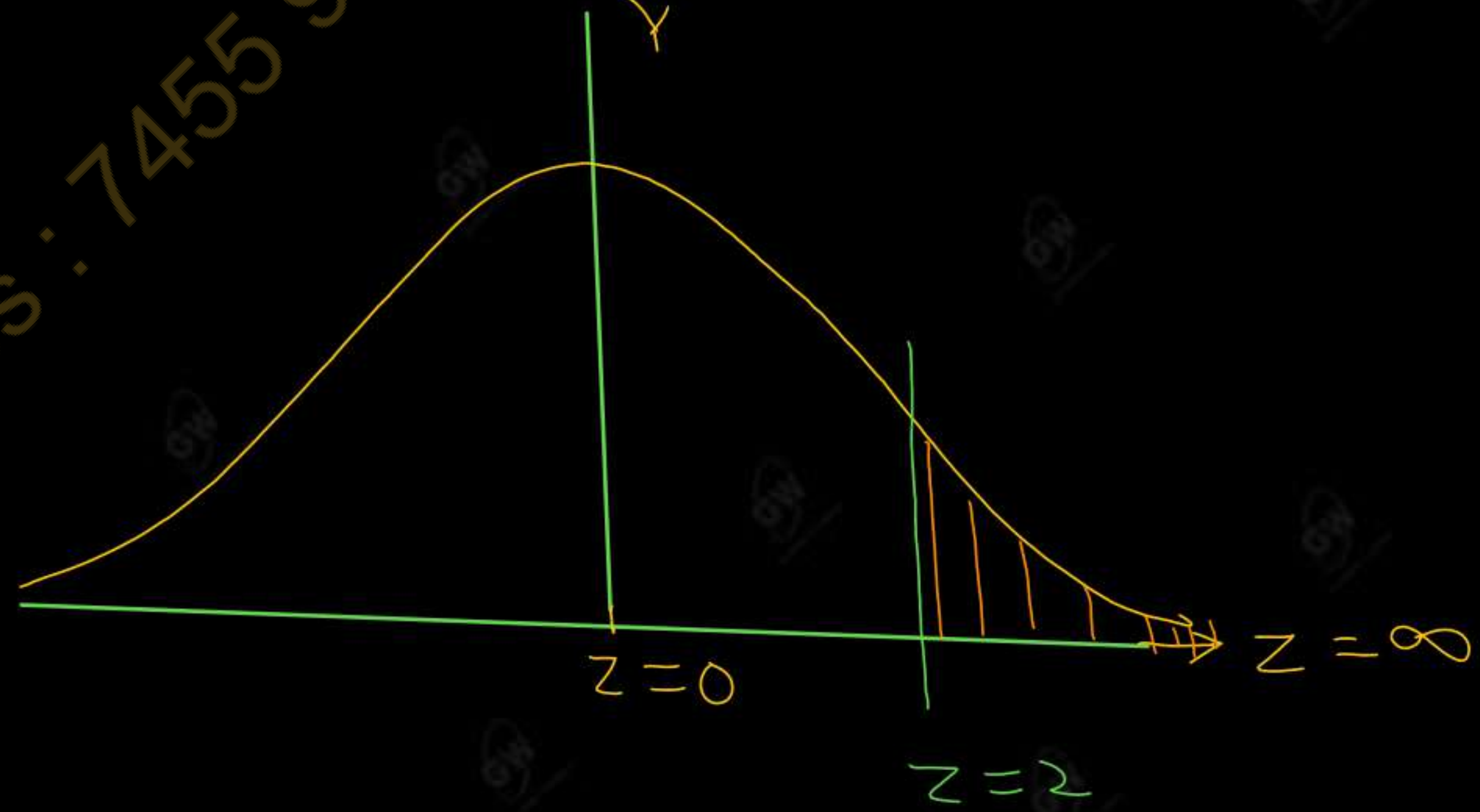
$P(x \geq 12) = ?$

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{12-8}{2}$$

$$\boxed{z = 2}$$

$P(x \geq 12) = P(z \geq 2)$



$P(x \geq 12) = P(0 < z < \infty) - P(0 < z < 2)$

$P(x \geq 12) = 0.5 - 0.4772$

$= \underbrace{0.0228}$

Number of pairs having life more then 12 months

$= 5000 \times 0.0228$

$= 114$

No. of pairs replaced after 12 months $= 5000 - 114$

$= 4886$

Q. 3 *Assume mean height of soldiers to be 68.22 inches with a variance of 10.8 inches square. How many soldiers in a regiment of 1,000 would you expect to be over 6 feet tall, given that the area under the standard normal curve between $z = 0$ and $z = 0.35$ is 0.1368 and between $z = 0$ and $z = 1.15$ is 0.3746.* **[AKTU (C.O.) 2021**

Given

$\mu = 68.22$

$\sigma^2 = 10.8$

$\sigma = \sqrt{10.8}$

$N = 1000$

$x = 6$ feet

$x = 6 \times 12$

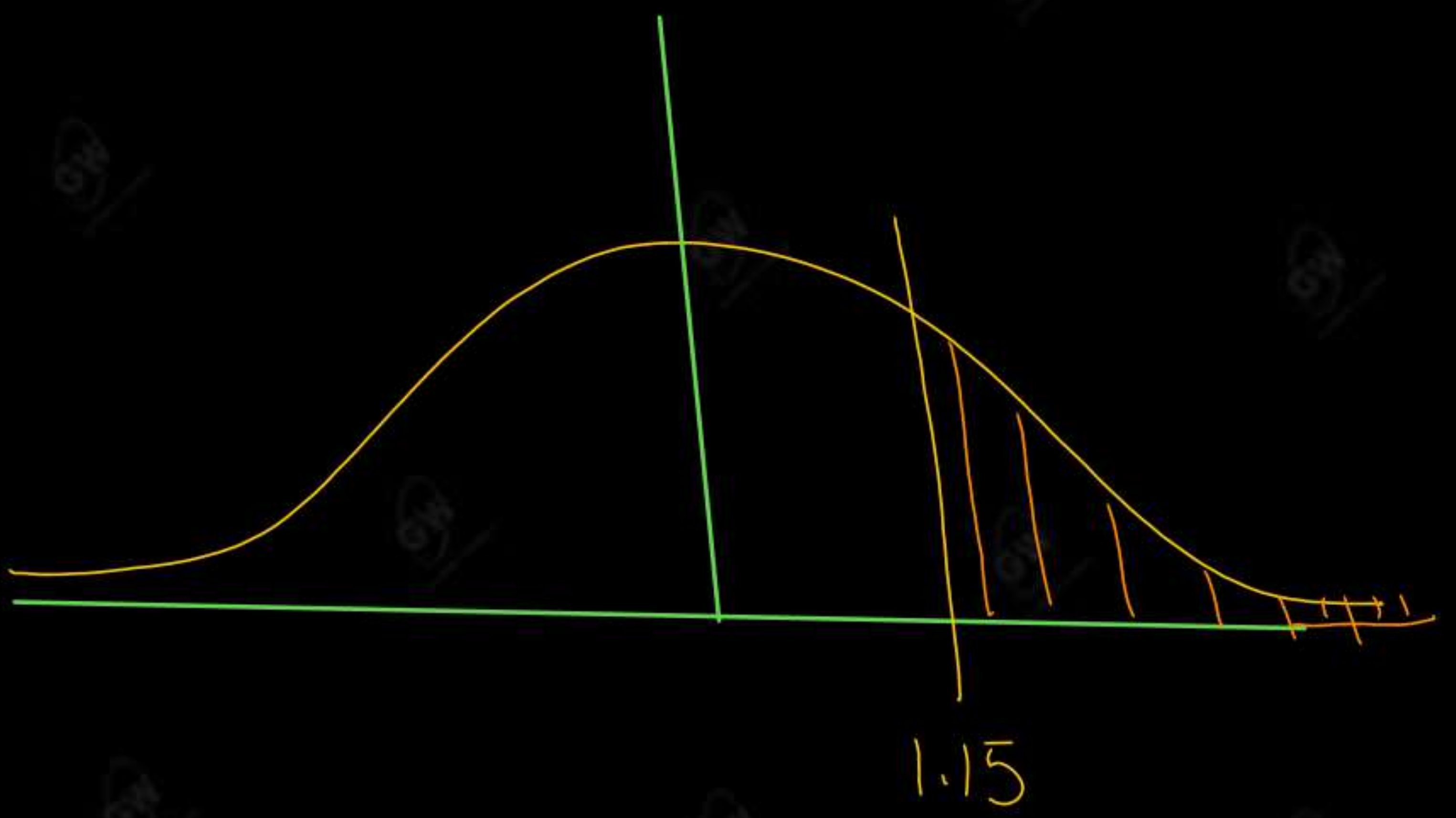
$x = 72$ inches

$z = \frac{x - \mu}{\sigma}$

$z = \frac{72 - 68.22}{\sqrt{10.8}}$

$z = 1.15$

$P(x > 72) = P(z > 1.15)$



$P(x>72) = P(z>1.15)$

$= P(0<z<\infty) - P(0<z<1.15)$

$= 0.5 - 0.3746$

$= 0.1254$

No. of soldiers $= 1000 \times 0.1254$

$= 125$ (approx)

Q.4 *The daily wages of 1000 workers are distributed around a mean of Rs. 140 and with a standard deviation of Rs. 10. Estimate the number of workers whose daily wages*

(i) between 140 and 144 *(ii) less than 126* *(iii) more than 160.* **(AKTU. 2021)**

Given

$N = 1000$

$\mu = 140$

$\sigma = 10$

$z = \dfrac{x-\mu}{\sigma}$

$z = \dfrac{x-140}{10}$ ——— (1)

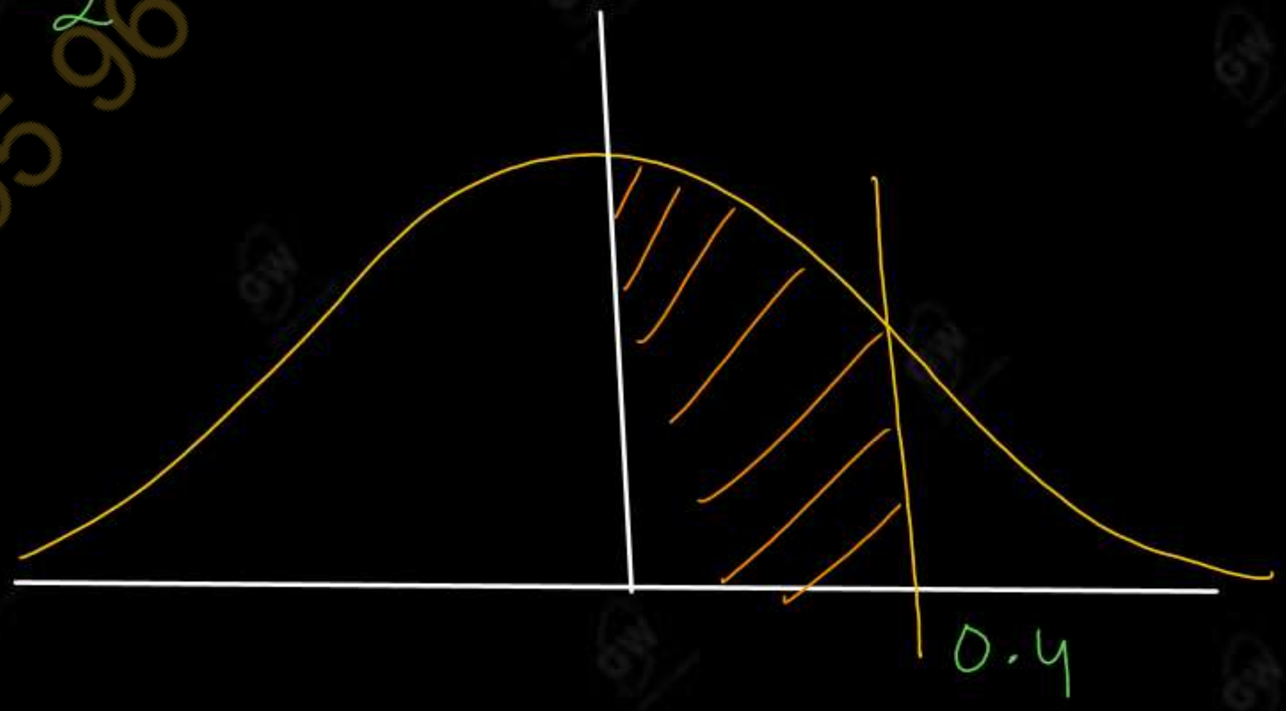
(i) $x_1 = 140$, $x_2 = 144$

$z_1 = \dfrac{140-140}{10} = 0$

$z_1 = 0$

$z_2 = \dfrac{144-140}{10} = \dfrac{4}{10}$

$z_2 = 0.4$



$P(140 < x < 144) = P(0 < z < 0.4)$

$= 0.1554$

Expected number of workers

$= 0.1554 \times 1000$

$= 155.4$

$= 155$

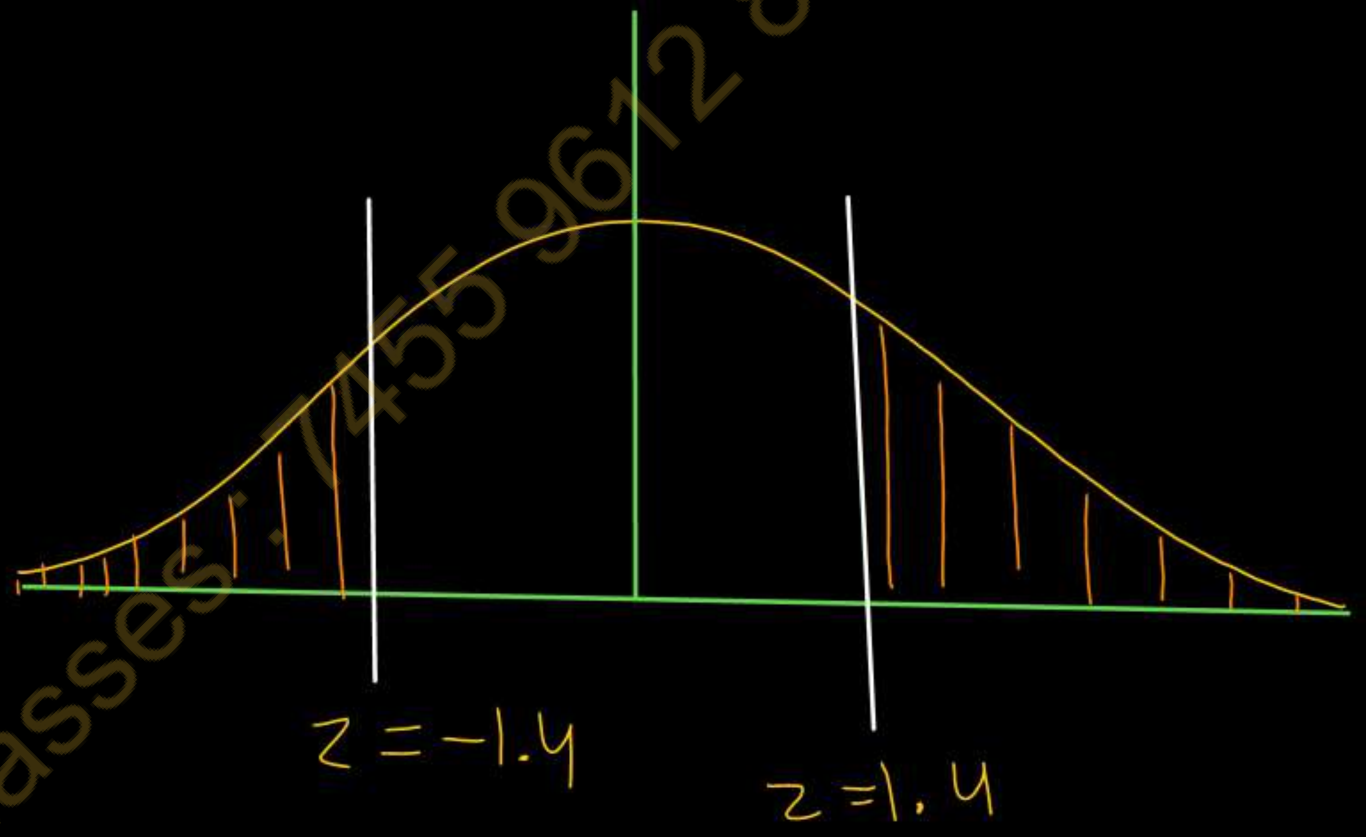
(ii) $x = 126$

$$Z = \frac{x - 140}{10}$$

$$= \frac{126 - 140}{10}$$

$z = -1.4$

$P(x < 126) = P(z < -1.4)$



$P(x < 126) = P(z > 1.4)$

$= P(0 < z < \infty) - P(0 < z < 1.4)$

$P(x<126) = 0.5 - 0.4192$

$= 0.0808$

Expected no. of workers

$= 1000 \times 0.0808$

$= 80.8$

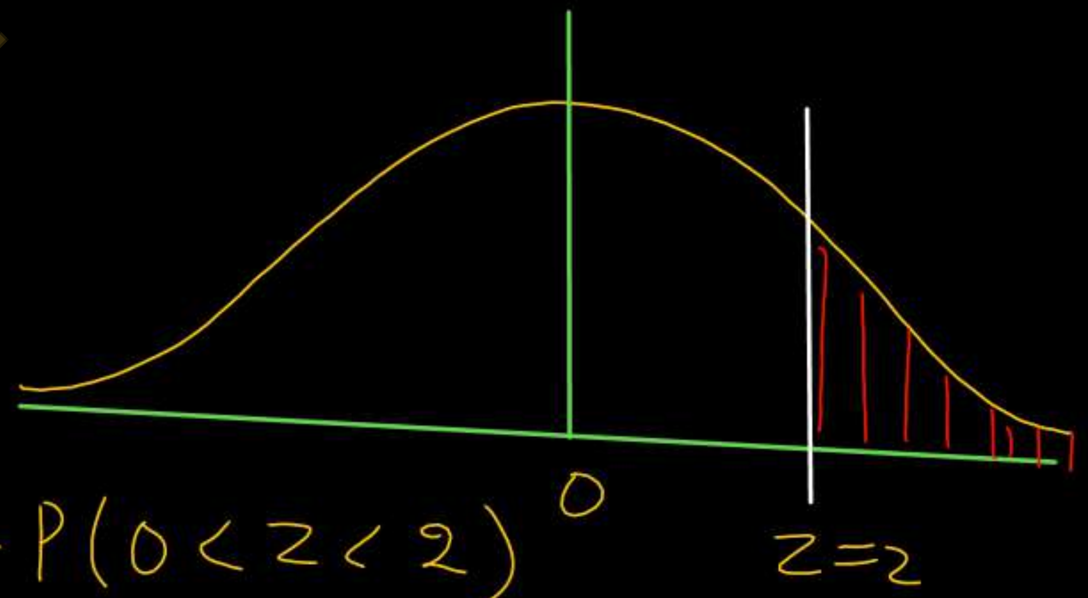
$= 81$ (approx)

(iii) $x = 160$

$$z = \frac{160-140}{10} = \frac{\cancel{20}}{\cancel{10}} = 2$$

$z = 2$

$P(x>160) = P(z>2)$



$= P(0<z<\infty) - P(0<z<2)$

$= 0.5 - 0.4772 = 0.0228$

Expected no. of workers

$= 0.0228 \times 1000 = 22.8$

$= 23$

DPP- 08

Topic : *Normal Probability Distribution* (*Part – I*)

Q.1 Define:

(i) *Binomial distribution*

(ii) *Poisson distribution*

(iii) *Normal distribution.* (*AKTU 2018*)

Q.2 If X is a normal variate with mean 30 and S.D. 5, find the probabilities that

(i) $26 \leq X \leq 40$

(ii) $X \geq 45$ *and*

(iii) $[X - 30 \mid > 5$

Ans. 2(*i*) 0.7653 (*ii*) 0.00135 (*iii*) 0.3174

Q.3 If Z is a standard normal variable, find the following probabilities:

(i) $P(Z < 1.2)$

(ii) $P(Z > -1.2)$

(iii) $P(-1.2 < Z < 1.3)$.

Ans. 3(*i*) 0.8849 (*ii*) 0.8849 (*iii*) 0.7881

ENGINEERING MATHEMATICS

UNIT-4 : ***Statistical Techniques – II***

Lec-09

Today's Target

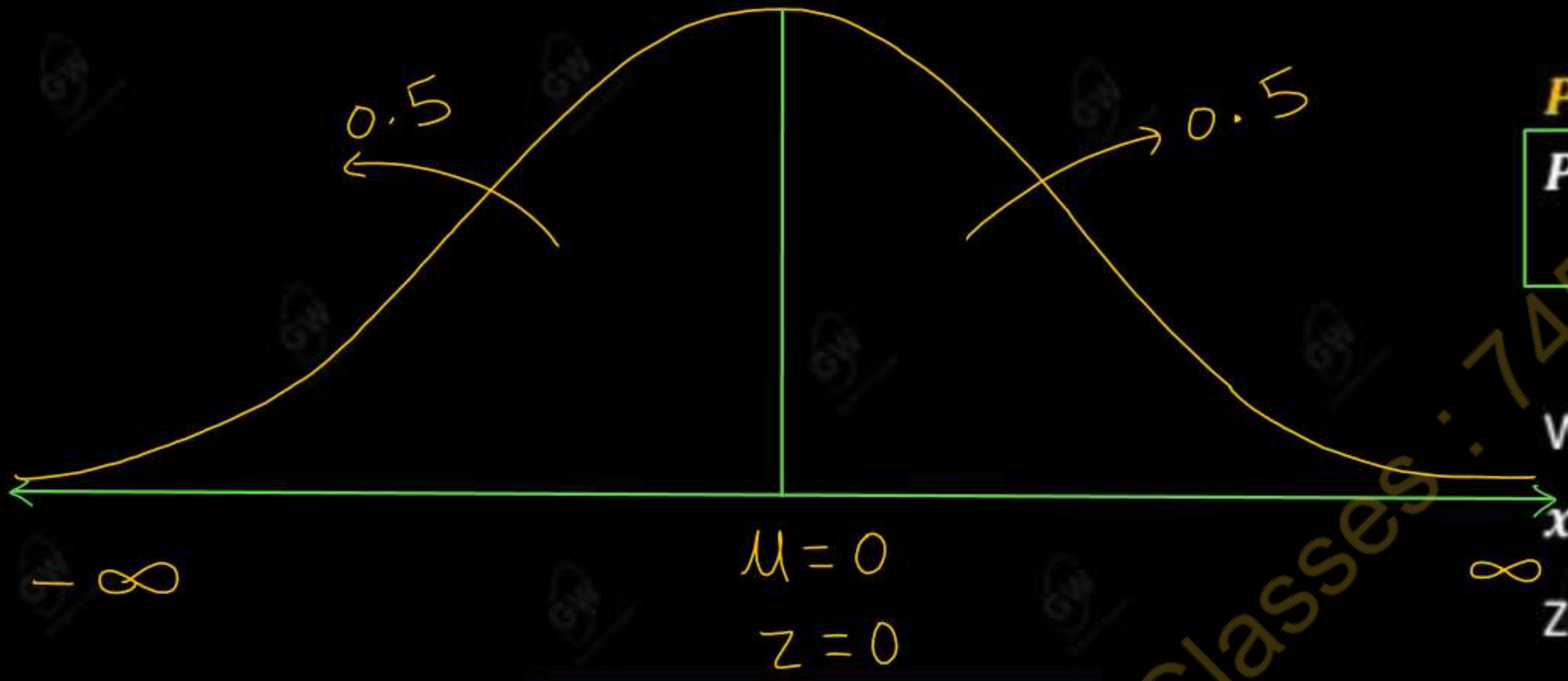
- ➢ $\boldsymbol{Normal\ Probability\ Distribution(Part-II)}$
- ➢ PYQs
- ➢ DPPs

Standard Normal Curve

$-\infty < z < 0$

$0 < z < \infty$

From Normal table



$-\infty < z < \infty$

$$z = \frac{x-\mu}{\sigma},$$

Total Area under curve $= 1$

Probability

$P(z_1 < z < z_2) =$ *Area under standard normal Curve between $z = z_1$ and $z = z_2$*

Where

x = Normal random variable

Z = Standard normal random variable

μ = Mean

σ = Standard deviation

Q. 1 *A sample of 100 dry battery cells tested to find the length of life produced the following results: $\bar{x} = 12$ hours, $\sigma = 3$ hours. Assuming the data to be normally distributed, what % of battery cells are expected to have life (i) more than 15 hours (ii) less than 6 hours (iii) between 10 and 14 hours?* **(A.K.T.U. 2018, 2021)**

Given

$N = 100$

$\mu = 12$

$\sigma = 3$

$$z = \frac{x - \mu}{\sigma}$$

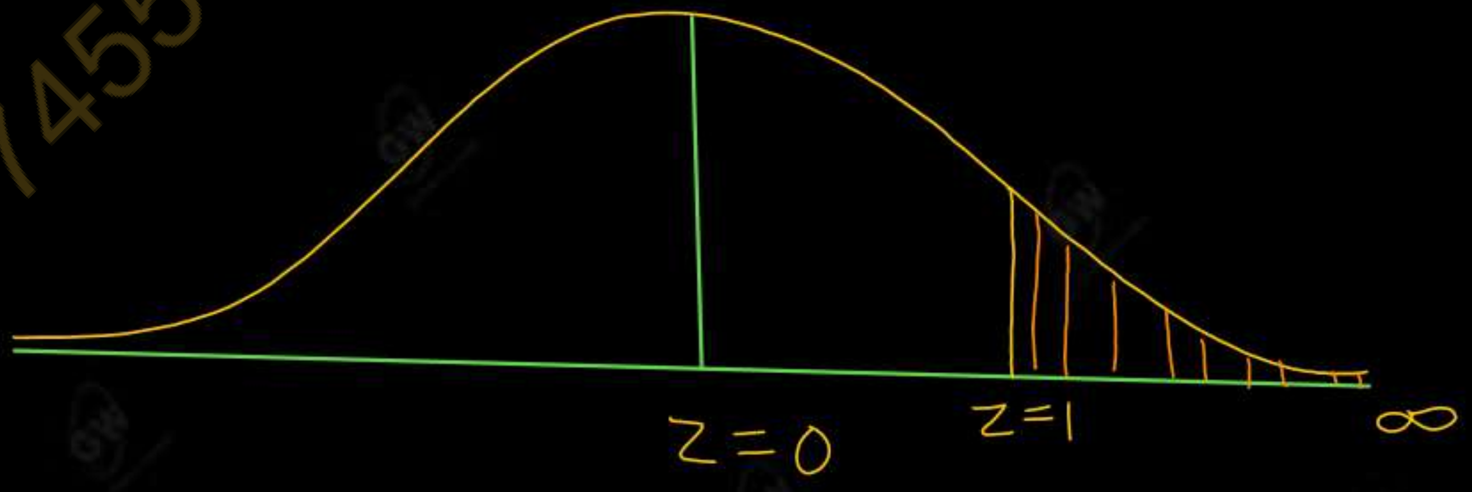
$$z = \frac{x - 12}{3} \quad \text{---} (1)$$

(i) $x = 15$

$$z = \frac{15 - 12}{3} = \frac{3}{3}$$

$z = 1$

$P(x > 15) = P(z > 1)$



$P(x > 15) = P(0 < z < \infty) - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

$$P(x>15) = 0.1587 \times 100$$

$$= 15.87\%$$

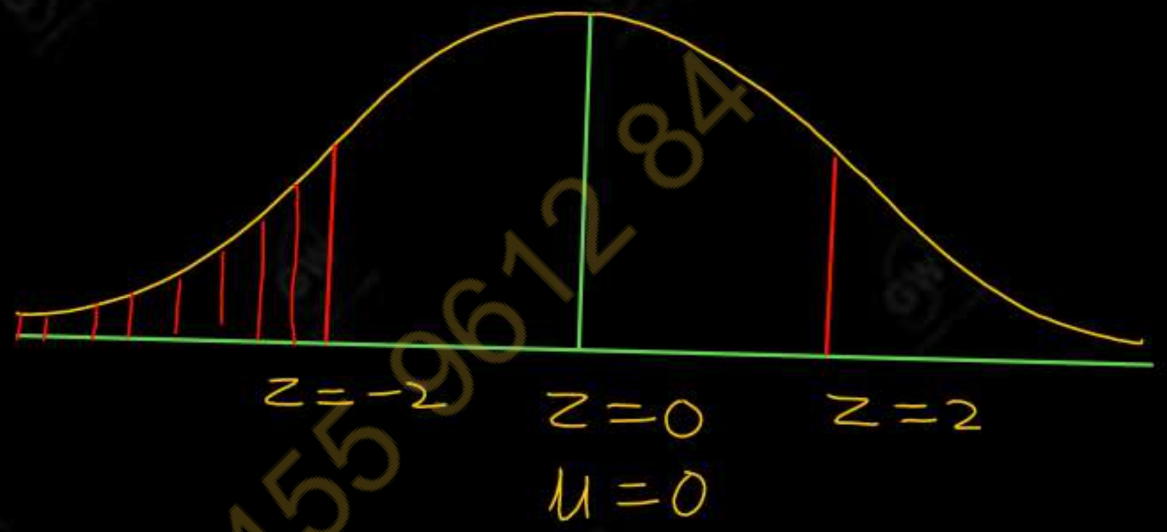
(ii) $x = 6$

$$z = \frac{x-12}{3} = \frac{6-12}{3}$$

$$z = -2$$

$$P(x<6) = P(z<-2)$$

$$= P(z>2)$$



$$P(x<6) = P(0<z<\infty) - P(0<z<2)$$

$$= 0.5 - 0.4772$$

$$= 0.0228$$

$$P(x<6) = 0.0228 \times 100 = 2.28\%$$

$P(x<6) = 2.28\%$

(iii) $x_1 = 10$

$x_2 = 14$

$$z_1 = \frac{x_1 - 12}{3}$$

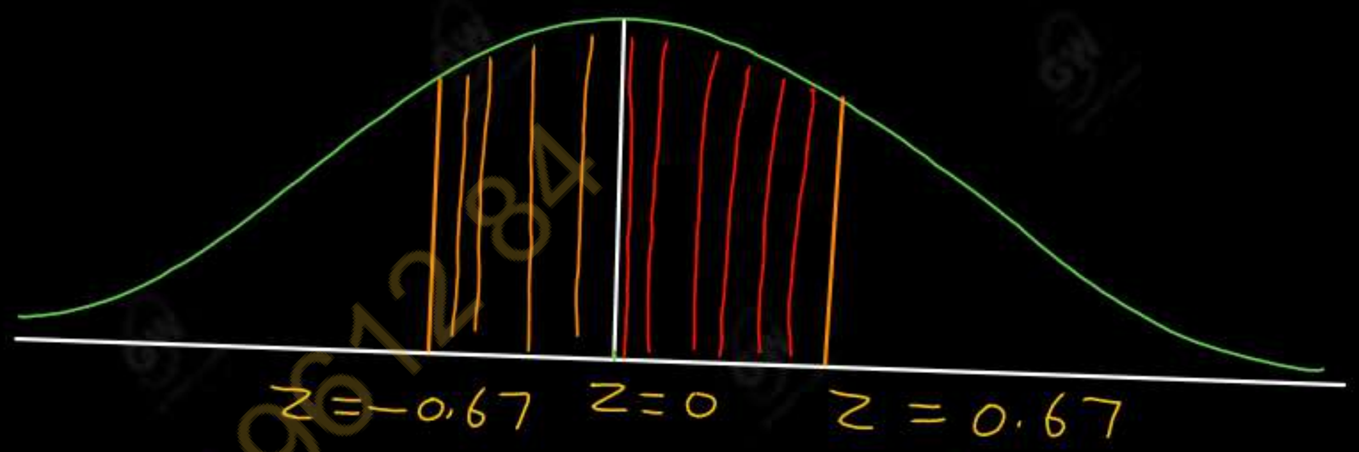
$$z_1 = \frac{10-12}{3}$$

$$z_1 = -0.67$$

$$z_2 = \frac{x_2 - 12}{3}$$

$$z_2 = \frac{14-12}{3}$$

$$z_2 = 0.67$$



$$P(10 < x < 14) = P(-0.67 < z < 0.67)$$

$$= 2P(0 < z < 0.67)$$

$$= 2 \times 0.2485 = 0.497$$

$$\boxed{P(10 < x < 14) = 49.7\%}$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2485	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4255	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4930	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4999	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993

Q. 2 *In a sample of 1000 cases, the mean of a certain test is 14 and S.D. is 2.5.* **(A.K.T.U. 2020)**
Assuming the distribution to be normal, find

(i) how many students score between 12 and 15?

(ii) how many score above 18?

(iii) how many score below 8?

(iv) how many score 16?

Given

$\mu = 14$

$\sigma = 2.5$

$N = 1000$

$z = \dfrac{x-\mu}{\sigma}$

$z = \dfrac{x-14}{2.5}$ —— (1)

(i) $x_1 = 12$, $x_2 = 15$

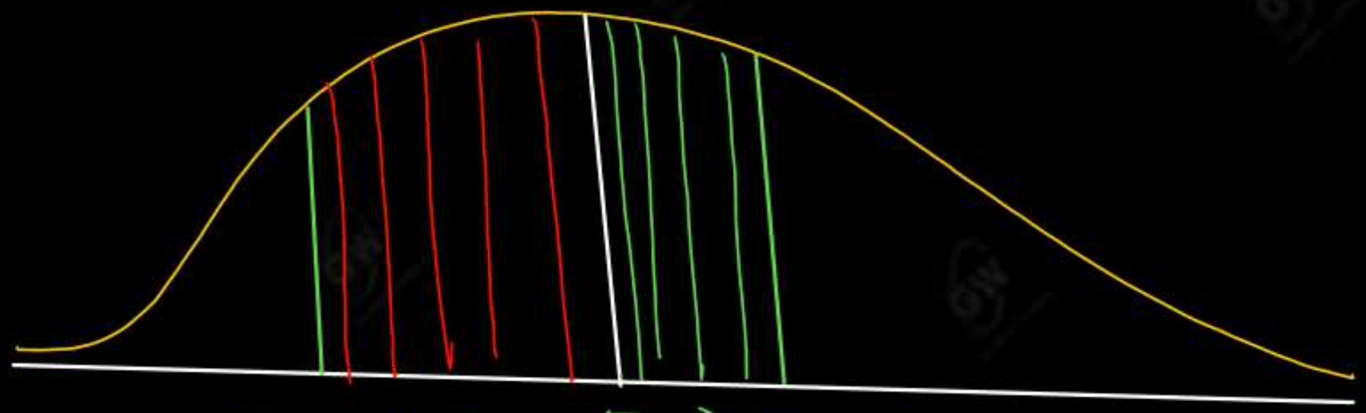
$z_1 = \dfrac{x_1 - 14}{2.5} = \dfrac{12-14}{2.5}$

$z_1 = -0.8$

$z_2 = \dfrac{x_2 - 14}{2.5} = \dfrac{15-14}{2.5}$

$z_2 = 0.4$

$P(12 < x < 15) = P(-0.8 < z < 0.4)$



$$P(12 < x < 15) = P(0 < z < 0.4) + P(0 < z < 0.8)$$

$$= 0.1554 + 0.2881$$

$$= 0.4435$$

$$\text{Number of students} = 1000 \times 0.4435$$

$$= 443.5$$

$$\boxed{= 444}$$

(ii) $x = 18$

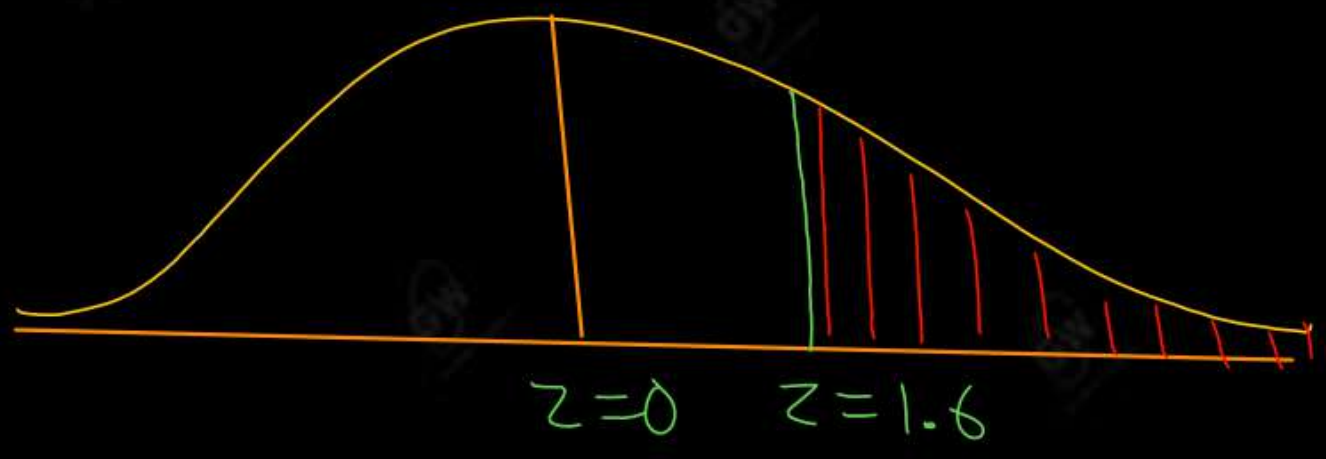
$$z = \frac{x - 14}{2.5} = \frac{18 - 14}{2.5} = \frac{4}{2.5}$$

$$z = 1.6$$

$$P(x > 18) = P(z > 1.6)$$

$$= P(0 < z < \infty) - P(0 < z < 1.6)$$

$$= 0.5 - 0.4452$$



$P(x>18) = 0.0548$

Number of students $= 1000 \times 0.0548$

$= 54.8$

$\boxed{= 55}$

(iii) $x = 8$

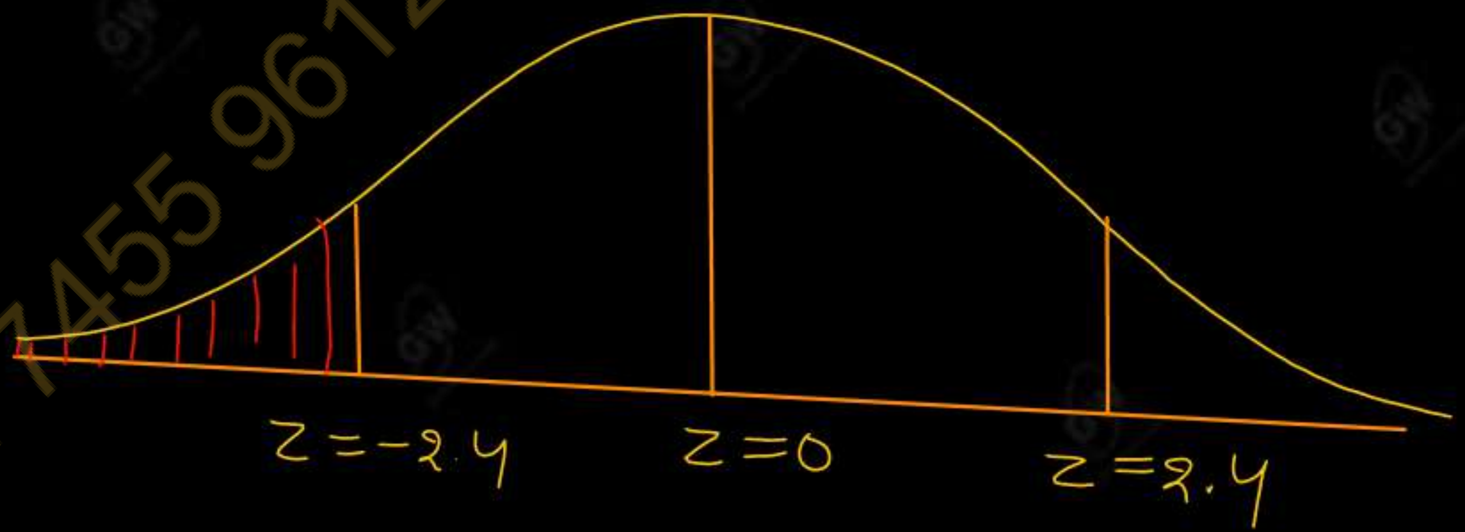
$$z = \frac{x-14}{2.5}$$

$$z = \frac{8-14}{2.5}$$

$z = -2.4$

$P(x < \infty) = P(z < -2.4)$

$= P(z > 2.4)$



$P(x<8) = P(0<z<\infty) - P(0<z<2.4)$

$= 0.5 - 0.4918$

$= 0.0082$

Number of students $= 0.0082 \times 1000$

$= 8.2$

$\boxed{= 8}$

(IV) $x = 16$

$x_1 = 15.5, \quad x_2 = 16.5$

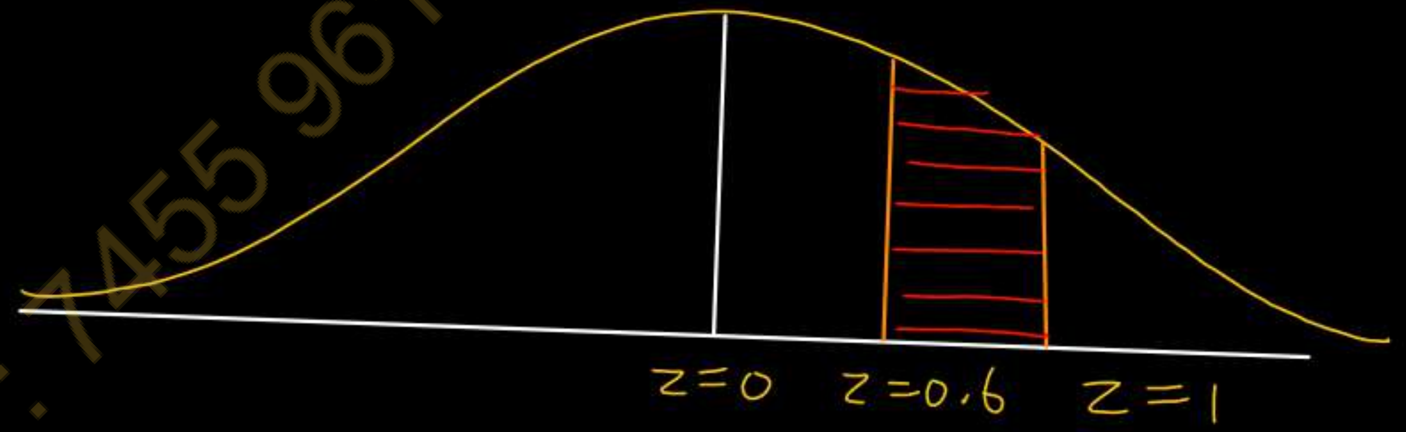
$$z_1 = \frac{x_1 - 14}{2.5} = \frac{15.5 - 14}{2.5}$$

$\boxed{z_1 = 0.6}$

$$z_2 = \frac{x_2 - 14}{2.5} = \frac{16.5 - 14}{2.5}$$

$z_2 = 1$

$P(15.5 < x < 16.5) = P(0.6 < z < 1)$



$P(15.5 < x < 16.5) = P(0 < z < 1) - P(0 < z < 0.6)$

$= 0.3413 - 0.2257$

$= 0.1156$

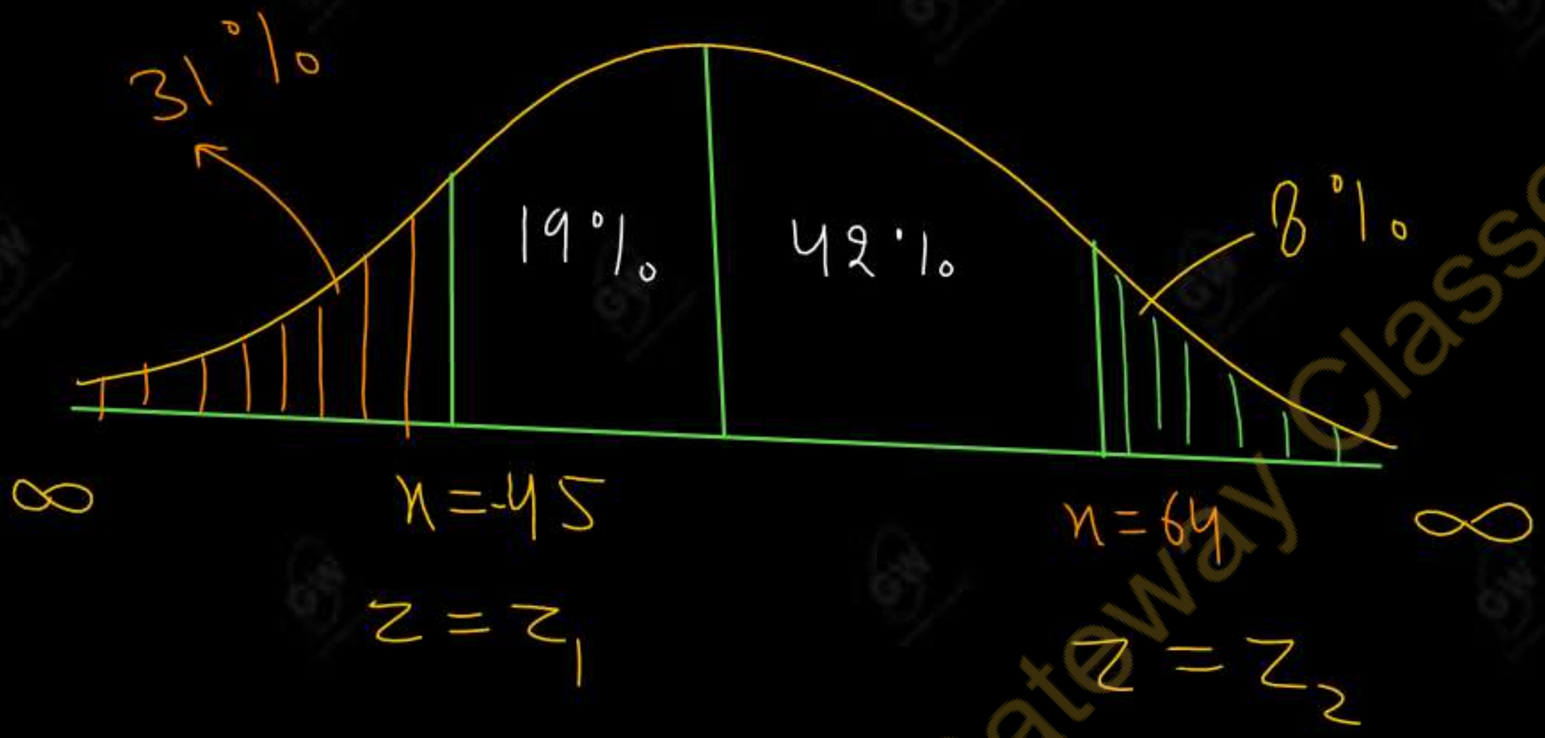
Number of students $= 0.1156 \times 1000$

$= 116$

Q.3 *In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. It is given that that if*

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_0^t e^{-\frac{1}{2}x^2} dx \text{ then } f(0.5) = 0.19 \text{ and } f(1.4) = 0.42. \qquad (A.K.T.U.\ 2020)$$

Let mean $= \mu$, Standard deviation $= \sigma$



When $x_1 = -45$, $P(-45 < x < 0) = \frac{19}{100}$

$= 0.19$

$z_1 = -0.5$

When $x_2 = 64$, $P(0 < x < 64)$

$= \frac{42}{100} = 0.42$

$z_2 = 1.4$

$$z = \frac{x-\mu}{\sigma}$$

$$z_1 = \frac{x_1-\mu}{\sigma}$$

$$-0.5 = \frac{45-\mu}{\sigma}$$

$$-0.5\sigma = 45-\mu$$

$$-0.5\sigma + \mu = 45 \quad \text{--- } (1)$$

$$z_2 = \frac{x_2-\mu}{\sigma}$$

$$1.4 = \frac{64-\mu}{\sigma}$$

$$1.4\sigma = 64-\mu$$

$$1.4\sigma + \mu = 64 \quad \text{--- } (2)$$

Subtract (1) and (2)

$$-0.5\sigma + \cancel{\mu} = 45$$

$$\underset{-}{1.4\sigma} \; \underset{-}{+} \cancel{\mu} = \underset{-}{64}$$

$$-1.9\sigma = -19$$

$$\boxed{\sigma = 10}$$

$$\boxed{\mu = 50}$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2485	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4255	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4930	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4999	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993

DPP- 09

Topic : *Normal Probability Distribution (Part – II)*

Q.1 In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours, estimate the number of bulbs likely to burn for

(A.K.T.U. 2021)

(i) *more than 2150 hours*
(ii) *less than 1950 hours*
(iii) *more than 1920 hours but less than 2160 hours.*

Ans. 1 (i) 67 (ii) 134 (iii) 1909

Q.2 The marks X obtained in Mathematics by 1000 students are normally distributed with meas 78% and standard deviation 11% Determine:

Ans. 2(i) 138 (ii) 63.92% (iii) Between 60 and 96

(i) *how many students got marks above 90%?*
(ii) *What was the highest marks obtained by the lowest 10% of students?*
(iii) *Within what limits did the middle 90% of the students lie?*

Q.3 In a normal distribution exactly normal, 7% of the items are under 35 and 89% areunder 63. What are the mean and standard deviation of the distribution?

Ans. 3 50.3 and 10.33

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Thank You