PDE : DPP-6

Topic : Non Linear Partial Differential Equation(Some Standard Forms)

Q .1 *Solve :* $p + q = pq$

Q .2 *Solve the partial differential equation :* $p^2 + q^2 = 2$

Q .3 *Solve the partial differential equation* $z = px + qy + \sin(p + q)$

Q .4 *Solve :* $9(p^2z + q^2) = 4$

Q .5 *Solve :* $p - 3x^2 = q^2 - y$

Maths - IV

DPP-6

Topic - Non Linear Partial Differential Equation (Some Standard Form)

Q-1 :- Solve $p+q=pq$

Sol :- $p+q=pq \quad -(1)$

Replace p by a and q by b

$$a+b=ab \quad -(2)$$
$$a=ab-b$$
$$a=b(a-1)$$
$$b=\frac{a}{a-1}$$

Complete solution is

$$z=ax+by+c$$

$$\boxed{z=ax+\frac{a}{a-1}y+c}$$

Ans

Q-2 :- Solve the partial differential equation : $p^2+q^2=2$

Sol :- $p^2+q^2=2$

Replace p by a and q by b

$a^2 + b^2 = 2 \quad -②$

$b^2 = 2 - a^2$

$b = \sqrt{2-a^2}$

Complete solution is - $z = ax + by + c$

$$\boxed{z = ax + \sqrt{2-a^2}\, y + c}$$

Ans

Q-3:- Solve the partial differential eq $z = px + qy + \sin(p+q)$

$z = px + qy + \sin(p+q) \quad — ①$

Replace p by a and q by b

$$\boxed{z = ax + by + \sin(a+b)}$$

Ans

Q-4:- Solve: $9(p^2 z + q^2) = 4$

$9(p^2 z + q^2) = 4 \quad — ①$

Let $u = x + ay$ so that

$p = \frac{dz}{du}$ and $q = a\frac{dz}{du}$

$$9\left[\left(\frac{dz}{du}\right)^2 z + a^2\left(\frac{dz}{du}\right)^2\right] = 4$$

$$9\left(\frac{dz}{du}\right)^2\left[z+a^2\right]=4$$

$$\frac{9}{4}\left(\frac{dz}{du}\right)^2=\frac{1}{z+a^2}$$

Square Root Both Side

$$\frac{3}{2}\left(\frac{dz}{du}\right)=\frac{1}{\sqrt{z+a^2}}$$

$$\frac{3}{2}(z+a^2)^{1/2}dz=du$$

Integrate both side

$$\frac{3}{2}\int(z+a^2)^{1/2}dz=\int du$$

$$\frac{3}{2}\frac{(z+a^2)^{3/2}}{3/2}=u+c$$

$$\frac{\cancel{3}}{\cancel{2}}\times\frac{\cancel{2}}{\cancel{3}}(z+a^2)^{3/2}=u+c$$

$$(z+a^2)^{3/2}=u+c$$

Squaring both side and put value of $u=x+ay$

$$\boxed{(z+a^2)^3=(x+ay+c)^2}$$

Ans

Q-5:- Solve : $p - 3x^2 = q^2 - y$

Let $p - 3x^2 = q^2 - y = a$ — (1)

$p - 3x^2 = a$
$p = a + 3x^2$

$q^2 - y = a$
$q^2 = a + y$
$q = \sqrt{a+y}$

$f_1(x) = a + 3x^2$

$f_2(y) = \sqrt{y+a}$

We know that

$dz = p\,dx + q\,dy$

Put p and q in above equation

$dz = (a+3x^2)dx + \sqrt{y+a}\,dy$

Integrate both side

$z = \int (a+3x^2)\,dx + \int (y+a)^{1/2}\,dy$

$$\boxed{z = ax + x^3 + \frac{2}{3}(y+a)^{3/2} + C}$$

Ans

PDE : DPP-7

Topic : CHARPIT METHOD

Q .1 *Solve :* $2zx - px^2 - 2pxy + pq = 0$

Ans $z = ay + b(x^2 - a)$

Q .2 *Solve the following equation by charpit's method :* $z^2 = pqxy$

Ans $z = ax^b y^{1/b}$

Q .3 *Solve the following equation by charpit's method :* $z = p^2x + q^2x$

Ans $\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b$

DPP - 7 Topic - Charpit Method

Q-1 : Solve: $2zx - px^2 - 2qxy + pq = 0$

$2zx - px^2 - 2qxy + pq = 0$

Let $f(x, y, z, p, q) = 2zx - px^2 - 2qxy + pq = 0$ — (1)

$\frac{\delta f}{\delta x} = 2z - 2px - 2qy$, $\frac{\delta f}{\delta y} = -2qx$

$\frac{\delta f}{\delta z} = 2x$, $\frac{\delta f}{\delta p} = -x^2 + q$, $\frac{\delta f}{\delta q} = -2xy + q$

Charpit's Auxiliary Equation :-

$$\frac{dx}{-\frac{\delta f}{\delta p}} = \frac{dy}{-\frac{\delta f}{\delta q}} = \frac{dz}{-p\frac{\delta f}{\delta p} - q\frac{\delta f}{\delta q}} = \frac{dp}{\frac{\delta f}{\delta x} + p\frac{\delta f}{\delta z}} = \frac{dq}{\frac{\delta f}{\delta y} + q\frac{\delta f}{\delta z}}$$

$$\frac{dx}{x^2 - q} = \frac{dy}{2xy - q} = \frac{dz}{px^2 - 2pq + 2qxy} = \frac{dp}{2z - 2qy} = \frac{dq}{0}$$

$$dq = 0$$

$$\boxed{q = a}$$

Put $q = a$ in eq (1)

$$2zx - px^2 - 2axy + pa = 0$$

$$p(a - x^2) = 2axy - 2zx$$

$$p = \frac{2x(ay - z)}{a - x^2}$$

$$\boxed{p = \frac{2x(z-ay)}{x^2-a}}$$

We know that, $dz = p\,dx + q\,dy$

$$dz = \frac{2x(z-ay)}{x^2-a}dx + a\,dy$$

$$dz - a\,dy = \frac{2x(z-ay)}{x^2-a}dx$$

$$\int \frac{dz - a\,dy}{z-ay} = \int \frac{2x}{x^2-a}dx$$

$$x^2 - a = t$$
$$2x\,dx = dt$$

$$\log(z-ay) = \int \frac{1}{t}dt$$

$$\log(z-ay) = \log t + \log b$$

$$\log(z-ay) = \log(x^2-a) + \log b$$

$$z - ay = (x^2-a)b$$

$$\boxed{z = ay + b(x^2-a)}$$

Ans

Q-2:- Solve the following equation by charpit's method
$z^2 = pqxy$

Sol:- $z^2 - pqxy = 0$

Let $f(x,y,z,p,q) = z^2 - pqxy = 0 \quad — ①$

$\frac{\delta f}{\delta x} = -pqy \quad , \quad \frac{\delta f}{\delta y} = -pqx \quad , \quad \frac{\delta f}{\delta z} = 2z$

$\frac{\delta f}{\delta p} = -qxy \quad , \quad \frac{\delta f}{\delta q} = -pxy$

Charpit's Auxiliary Equation :-

$$\frac{dx}{-\frac{\delta f}{\delta p}} = \frac{dy}{-\frac{\delta f}{\delta q}} = \frac{dz}{-p\frac{\delta f}{\delta p} - q\frac{\delta f}{\delta q}} = \frac{dp}{\frac{\delta f}{\delta x} + p\frac{\delta f}{\delta z}} = \frac{dq}{\frac{\delta f}{\delta y} + q\frac{\delta f}{\delta z}}$$

$$\frac{dx}{qxy} = \frac{dy}{pxy} = \frac{dz}{pqxy + pqxy} = \frac{dp}{-pqy + 2pz} = \frac{dq}{-pqx + 2qz}$$

No fraction pair formed, so taking x and p as a multipliers in first and fourth fraction. Similiary taking q and y as a multipliers in second and last fraction then equating :-

$$\frac{xdp + pdx}{-pq\cancel{xy} + 2pxz + \cancel{pqxy}} = \frac{ydq + qdy}{-pq\cancel{xy} + 2qyz + \cancel{pqxy}}$$

$$\frac{x\,dp + p\,dx}{\cancel{z}px\cancel{z}} = \frac{y\,dq + q\,dy}{\cancel{z}qy\cancel{z}}$$

$$\frac{x\,dp + p\,dx}{px} = \frac{y\,dq + q\,dy}{qy}$$

Now Integerate Both Side

$$\log xp = \log yq + \log c$$

$$\log xp - \log yq = \log c$$

$$\frac{xp}{yq} = c$$

$$\boxed{p = \frac{cyq}{x}} \quad -(2)$$

Put value of p in equation (1)

$$z^2 = pqxy$$

$$z^2 = cy^2q^2$$

$$q^2 = \frac{z^2}{cy^2}$$

$$q = \frac{z}{\sqrt{c}\,y}$$

Again put value of q in eq ② -

$$p = \frac{cy}{x} \cdot \frac{z}{\sqrt{c}\,y}$$

$$p = \frac{z\sqrt{c}}{x}$$

Let $\sqrt{c} = b$

$$\therefore \boxed{p = \frac{zb}{x}, \qquad q = \frac{z}{by}}$$

We know that,

$$dz = p\,dx + q\,dy$$

$$dz = \frac{zb}{x}dx + \frac{z}{by}dy$$

$$\frac{dz}{z} = b\frac{dx}{x} + \frac{1}{b}\frac{dy}{y}$$

On Integration :-

$$\log z = b\log x + \frac{1}{b}\log y + \log a$$

$$\log z = \log x^b + \log y^{1/b} + \log a$$

$$\log z = \log\left(a x^b y^{1/b}\right)$$

$$\boxed{z = a x^b y^{1/b}}$$

Q-3:- Solve: $z = p^2x + q^2y$

$$z - p^2x - q^2y = 0 \quad \text{—— (1)}$$

Let $f(x,y,z,p,q) = z - p^2x - q^2 = 0$

$$\frac{\delta f}{\delta x} = -p^2, \quad \frac{\delta f}{\delta y} = -q^2, \quad \frac{\delta f}{\delta z} = 1$$

$$\frac{\delta f}{\delta p} = -2px, \quad \frac{\delta f}{\delta q} = -2qy$$

Charpit Auxiliary Equation:-

$$\frac{dx}{-\frac{\delta f}{\delta p}} = \frac{dy}{-\frac{\delta f}{\delta q}} = \frac{dz}{-p\frac{\delta f}{\delta p} - q\frac{\delta f}{\delta q}} = \frac{dp}{\frac{\delta f}{\delta x} + p\frac{\delta f}{\delta z}} = \frac{dq}{\frac{\delta f}{\delta y} + q\frac{\delta f}{\delta z}}$$

$$\frac{dx}{2px} = \frac{dy}{2qy} = \frac{dz}{2p^2x + 2q^2y} = \frac{dp}{-p^2 + p} = \frac{dq}{q - q^2}$$

Multiply by p^2 and $2px$ as multipliers in first and fourth fraction, Similiarly q^2 and $2qx$ in second and last fraction then equating them -

$$\frac{p^2dx + 2px\,dp}{\cancel{2p^3x} - \cancel{2p^3x} + 2p^2x} = \frac{q^2dy + 2qy\,dq}{\cancel{2q^3y} + 2q^2y - \cancel{2q^3y}}$$

$$\frac{p^2dx + 2px\,dp}{\cancel{2}p^2x} = \frac{q^2dy + 2qy\,dq}{\cancel{2}q^2y}$$

$$\frac{p^2 dx + 2px\,dp}{p^2 x} = \frac{q^2 dy + 2qy\,dq}{q^2 y}$$

Now Integrate Both Side

$$\log(p^2x) = \log(q^2y) + \log a$$

$$\boxed{p^2x = aq^2y} \quad - (2)$$

Putting value of p^2x in eq (1)

$$aq^2y + q^2y = z$$

$$\boxed{q = \sqrt{\frac{z}{(1+a)y}}}$$

Now, again put value of q in eq (2)

$$p^2x = \frac{az\cancel{y}}{(1+a)\cancel{y}}$$

$$p^2x = \frac{az}{1+a}$$

$$\boxed{p = \sqrt{\frac{az}{x(1+a)}}}$$

We know that, $dz = p\,dx + q\,dy$

$$dz = \left[\frac{az}{x(1+a)}\right]^{1/2} dx + \left[\frac{z}{(1+a)y}\right]^{1/2} dy$$

$$\sqrt{1+a}\ \frac{dz}{z^{1/2}} = a^{1/2}\frac{dx}{x^{1/2}} + \frac{dy}{y^{1/2}}$$

Integerate both side

$$\sqrt{1+a}\ \frac{z^{1/2}}{1/2} = \sqrt{a}\ \frac{x^{1/2}}{1/2} + \frac{y^{1/2}}{1/2} + b$$

$$\cancel{2}\sqrt{(1+a)z} = \cancel{2}\left[\sqrt{ax} + \sqrt{y} + b\right]$$

$$\boxed{\sqrt{(1+a)z} = \sqrt{ax} + \sqrt{y} + b}$$

Ans

PDE : DPP-8

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient (Complementary function)

Q.1 *Solve* : $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0$

Ans $z = f_1(y+3x) + f_2(y-2x)$

Q.2 Solve $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial x^2 \partial y} + 2\frac{\partial^3 z}{\partial x \partial y^2} = 0$

Ans $z = f_1(y) + f_2(y+x) + f_3(y+2x)$

Q.3 Solve: $4r - 12s + 9t = 0$

Ans $z = f_1\left(y+\frac{3}{2}x\right) + x f_2\left(y+\frac{3}{2}x\right)$

Q.4 Solve $(D^3D' - 4D^2D'^2 + 4DD'^3)z = 0$

Ans

$z = f_1(y) + f_2(x) + f_3(y+2x) + x f_4(y+2x)$

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DPP-8 Maths - IV

Topic - Homogeneous Linear Partial Differential Equation with Constant coefficient (CF)

Q-1: Solve $\frac{\delta^2 z}{\delta x^2} - \frac{\delta^2 z}{\delta x \delta y} - 6\frac{\delta^2 z}{\delta y^2} = 0$

Sol: $\frac{\delta^2 z}{\delta x^2} - \frac{\delta^2 z}{\delta x \delta y} - 6\frac{\delta^2 z}{\delta y^2} = 0$

$(D^2 - DD' - 6D'^2)z = 0$

Put $D = m$, $D' = 1$

$(m^2 - m - 6)z = 0$

Aux Equation -

$m^2 - m - 6 = 0$

$m^2 - 3m + 2m - 6 = 0$

$m(m-3) + 2(m-3) = 0$

$(m+2)(m-3) = 0$

$m = -2, 3$

$CF = f_1(y-2x) + f_2(y+3x)$

$PI = 0$

Complete solution $z = CF + PI$

$z = f_1(y-2x) + f_2(y+3x)$

Ans

Q-2 : Solve $\frac{\delta^3 z}{\delta x^3} - 3\frac{\delta^3 z}{\delta x^2 \delta y} + 2\frac{\delta^3 z}{\delta x \delta y^2} = 0$

$(D^3 - 3D^2D' + 2DD'^2)z = 0$

$D(D^2 - 3DD' + 2D'^2)z = 0$

Aux Equation :

$D(D^2 - 3DD' + 2D'^2) = 0$

$D = 0$

$CF = f_1(y)$

$D^2 - 3DD' + 2D'^2 = 0$

Put $D = m,\ D' = 1$

$m^2 - 3m + 2 = 0$

$(m-2)(m-1) = 0$

$m = 1, 2$

$CF = f_2(y+x) + f_3(y+2x)$

$PI = 0$

Complete solution $z = CF + PI$

$z = f_1(y) + f_2(y+x) + f_3(y+2x)$

Ans

Q-3 : Solve- $4r - 12s + 9t = 0$

$4\frac{\delta^2 z}{\delta x^2} - 12\frac{\delta^2 z}{\delta x \delta y} + 9\frac{\delta^2 z}{\delta y^2} = 0$

$(4D^2 - 12DD' + 9D'^2)z = 0$

Auxiliary Equation :- $4m^2 - 12m + 9 = 0$

$$(2m-3)^2 = 0$$

$$m = \frac{3}{2}, \frac{3}{2}$$

$$C.F. = f_1\left(y + \frac{3}{2}x\right) + x f_2\left(y + \frac{3}{2}x\right)$$

$PI = 0$

Complete solution $\Rightarrow$ $z = CF + PI$

$$z = f_1\left(y + \frac{3}{2}x\right) + x f_2\left(y + \frac{3}{2}x\right)$$

Ans

Q-4-: $(D^3D' - 4D^2D'^2 + 4DD'^3)z = 0$

$DD'(D^2 - 4DD' + 4D'^2)z = 0$

Auxiliary Equation:-

$DD'(D^2 - 4DD' + 4D'^2) = 0$

$D = 0$ $\quad$ $D' = 0$ $\quad$ $(D^2 - 4DD' + 4D'^2) = 0$

Put $D = m$, $D' = 1$

$m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$m = 2, 2$

$CF = f_1(y)$ $\quad$ $CF = f_2(x)$ $\quad$ $C.F. = f_3(y+2x) + x f_4(y+2x)$

$PI = 0$

Complete solution $\Rightarrow$ $z = CF + PI$

$$z = f_1(y) + f_2(x) + f_3(y+2x) + x f_4(y+2x)$$

Ans

PDE : DPP-9

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-I

Q .1 *Solve the linear partial differential equation* $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y.$

$$z = f_1(y-x) + f_2(y-2x) + \frac{(x+y)^3}{36}$$

Q .2 *Solve :* $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x.$ (MTU-2013)

$$z = f_1(y+x) + x f_2(y+x) - \sin x$$

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Maths - IV

DPP - 9

Topic - Particular Integral (PI) Case-1

Q-1:- $\frac{\delta^2 z}{\delta x^2} + 3\frac{\delta^2 z}{\delta x \delta y} + 2\frac{\delta^2 z}{\delta y^2} = x+y$

$(D^2+3DD'+2D'^2)z = x+y$

Put $D = m$, $D' = 1$

$(m^2+3m+2)z = x+y$

Auxiliary Equation :- $m^2+3m+2=0$

$m^2+2m+m+2=0$

$m(m+2)+1(m+2)=0$

$(m+1)(m+2)=0$

$m = -1, -2$

$$\boxed{CF = f_1(y-x)+f_2(y-2x)}$$

$PI = \frac{1}{D^2+3DD'+2D'^2}(x+y)$

Put $D=1$, $D'=1$, $x+y=u$

$PI = \frac{1}{1+3+2}\iint u\,du\,du$

$$PI = \frac{1}{6} \iint u \, du \, du$$

$$PI = \frac{1}{6} \int \frac{u^2}{2} du$$

$$PI = \frac{1}{6} \frac{u^3}{2 \times 3}$$

$$PI = \frac{u^3}{36}$$

$$PI = \frac{(x+y)^3}{36}$$

Complete solution $\Rightarrow$ $z = CF + PI$

$$z = f_1(y-x) + f_2(y-2x) + \frac{(x+y)^3}{36}$$

Ans

Q-2- Solve $\frac{\delta^2 z}{\delta x^2} - 2\frac{\delta^2 z}{\delta x \delta y} + \frac{\delta^2 z}{\delta y^2} = \sin x$

Sol:- $\frac{\delta^2 z}{\delta x^2} - 2\frac{\delta^2 z}{\delta x \delta y} + \frac{\delta^2 z}{\delta y^2} = \sin x$

$$(D^2 - 2DD' + D'^2)z = \sin x$$

Put $D = m$, $D' = 1$

$$(m^2 - 2m + 1)z = \sin x$$

Auxiliary Equation -

$$m^2 - 2m + 1 = 0$$

$(m-1)^2 = 0$
$m = 1, 1$

$$\boxed{C.F. = f_1(y+x) + x f_2(y+x)}$$

$PI = \dfrac{1}{D^2 - 2DD' + D'^2} \sin x$

$PI = \dfrac{1}{D^2 - 2DD' + D'^2} \sin(x + 0y)$

Put $D = 1$, $D' = 0$, $x + 0y = u$

$PI = \dfrac{1}{1} \iint \sin u \, du$

$PI = \int -\cos u \, du$

$PI = -\sin u$
$PI = -\sin(x + 0y)$
$PI = -\sin x$

Complete Solution :- $Z = CF + PI$
$Z = f_1(y+x) + x f_2(y+x) - \sin x$

Ans

PDE : DPP-10

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-I (Type-2)

Q .1 *Solve the linear partial differential equation* $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 4\sin(2x+y)$.

Ans $f_1(y) + f_2(y+2x) + x f_3(y+2x) - x^2\cos(2x+y)$

Q .2 Solve : $2r - s - 3t = 5\frac{e^x}{e^y}$

Ans $z = f_1(y-x) + f_2\left(y+\frac{3}{2}x\right) + xe^{x-y}$

Q .3 Solve : $\frac{\partial^3 z}{\partial x^3} - 7\frac{\partial^3 z}{\partial x \partial y^2} - 6\frac{\partial^3 z}{\partial y^3} = \sin(x+2y) + e^{3x+y}$. *(UPTU-2014)*

Ans $z = f_1(y-x) + f_2(y-2x) + f_3(y+3x) - \frac{1}{75}\cos(x+2y) + \frac{x}{20}e^{3x+y}$

Maths - IV

DPP - 10

Topic - Particular Integral
Case -1 (Type-2)

Q-1: Solve the linear partial differential equation

$$\frac{\delta^3 z}{\delta x^3} - 4\frac{\delta^3 z}{\delta x^2 \delta y} + 4\frac{\delta^3 z}{\delta x \delta y^2} = 4\sin(2x+y)$$ {GBTU-2011}

$$(D^3 - 4D^2D' + 4DD'^2)z = 4\sin(2x+y)$$

$$D(D^2 - 4DD' + 4D'^2)z = 4\sin(2x+y)$$

Auxiliary Equation

$D = 0$

C.F. $= f_1(y)$

$D^2 - 4DD' + 4D'^2 = 0$

Replace $D = m$ and $D' = 1$

$$m^2 - 4m + 4 = 0$$
$$(m-2)^2 = 0$$
$$m = 2, 2$$
$$C.F = f_2(y+2x) + x f_3(y+2x)$$

$$\boxed{C.F. = f_1(y) + f_2(y+2x) + x f_3(y+2x)}$$

$$PI = \frac{1}{D^3 - 4D^2D' + 4DD'^2} 4\sin(2x+y)$$

Put $D = 2$, $D' = 1$

$$PI = \frac{1}{2^3 - 4\times4\times1 + 4\times2\times1} 4\sin(2x+y)$$

$$PI = \frac{1}{8-16+8} 4\sin(2x+y)$$

{Case of failure}

Differentiate w.r.t 'D' and multiply with x.

$$PI = \frac{x}{3D^2 - 8DD' + 4D'^2} 4\sin(2x+y)$$

Put $D = 2$, $D' = 1$

$$PI = \frac{x}{3\times4 - 8\times2 + 4} 4\sin(2x+y)$$

$$PI = \frac{x}{12-16+4} 4\sin(2x+y)$$

{Again Case of failure}

Differentiate w.r.t 'D' and multiply with x.

$$PI = \frac{x^2}{6D - 8D'} 4\sin(2x+y)$$

Put $D = 2$, $D' = 1$, $2x+y = u$

$$PI = 4x^2 \times \frac{1}{12-8} \int \sin u \, du$$

$$PI = 4x^2 \times \frac{1}{4} \times -\cos u$$

$$PI = -x^2 \cos u$$

$$PI = -x^2 \cos(2x+y)$$

Complete solution $\Rightarrow$ $Z = CF + PI$.

$$\boxed{Z = f_1(y) + f_2(y+2x) + x f_3(y+2x) - x^2 \cos(2x+y)}$$

Q-2 : Solve : $2r - s - 3t = 5\frac{e^x}{e^y}$

$$2r - s - 3t = 5e^{x-y}$$

$$2\frac{\delta^2 z}{\delta x^2} - \frac{\delta^2 z}{\delta x \delta y} - 3\frac{\delta^2 z}{\delta y^2} = 5e^{x-y}$$

$$(2D^2 - DD' - 3D'^2)Z = 5e^{x-y}$$

Auxiliary Equation $\Rightarrow$ $2D^2 - DD' - 3D'^2 = 0$

Put $D = m$, $D' = 1$

$$2m^2 - m - 3 = 0$$

$$2m^2 - 3m + 2m - 3 = 0$$

$$m(2m-3) + 1(2m-3) = 0$$

$$(m+1)(2m-3) = 0$$

$$m = -1, 3/2$$

$$\boxed{C.F. = f_1(y-x) + f_2\left(y+\frac{3}{2}x\right)}$$

$$PI = \frac{1}{2D^2 - DD' - 3D'^2} 5e^{x-y}$$

Put $D = 1$ and $D' = -1$

$$PI = \frac{1}{2+1-3} 5e^{x-y}$$

{ Case of failure }

Diffrentiate w.r.t D and multiply with x

$$PI = x \times \frac{1}{4D - D'} 5e^{x-y}$$

Put $D = 1$, $D' = -1$, $u = x - y$

$$PI = 5x \times \frac{1}{4+1} \int e^u du$$

$$PI = 5x \times \frac{1}{5} \times e^u$$

$$PI = xe^u$$

$$PI = xe^{x-y}$$

Complete solution :- $z = CF + PI$

$$\boxed{z = f_1(y-x) + f_2\left(y + \frac{3}{2}x\right) + xe^{x-y}}$$

Q-3:- Solve : $\frac{\delta^3 z}{\delta x^3} - 7\frac{\delta^3 z}{\delta x \delta y^2} - 6\frac{\delta^3 z}{\delta y^3} = \sin(x+2y) + e^{3x+y}$

Sol:- $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x+2y) + e^{3x+y}$

Auxiliary Equation :- $D^3 - 7DD'^2 - 6D'^3 = 0$

Put $D = m$, $D' = 1$

$$m^3 - 7m - 6 = 0$$
$$(m+1)(m+2)(m-3) = 0$$
$$m = -1, -2, 3$$

$$\boxed{C.F. = f_1(y-x) + f_2(y-2x) + f_3(y+3x)}$$

$$PI = \frac{1}{D^3 - 7DD'^2 - 6D'^3}\left[\sin(x+2y) + e^{3x+y}\right]$$

$$PI = \frac{1}{D^3 - 7DD'^2 - 6D'^3}\sin(x+2y) + \frac{1}{D^3 - 7DD'^2 - 6D'^3}e^{3x+y}$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{D^3 - 7DD'^2 - 6D'^3}\sin(x+2y)$$

Put $D = 1$, $D' = 2$, $\sin(x+2y) = u$

$$P_1 = \frac{1}{(1)^3 - 7(1)(2)^2 - 6(2)^3} \iiint \sin u \, du\, du\, du$$

$$P_1 = \frac{-1}{75} \cos u$$

$$\boxed{P_1 = \frac{-1}{75} \cos(x+2y)}$$

$$P_2 = \frac{1}{D^3 - 7DD'^2 - 6D'^3} e^{3x+y}$$

Put $D = 3$, $D' = 1$

$$P_2 = \frac{1}{27 - 21 - 6} e^{3x+y}$$

{ Case of failure }

Differentiate w.r.t 'D' and multiply with x

$$P_2 = x \times \frac{1}{3D^2 - 7D'^2} e^{3x+y}$$

Put $D = 3$, $D' = 1$, $3x + y = u$

$$P_2 = x \times \frac{1}{3 \times 9 - 7 \times 1} e^{3x+y}$$

$$P_2 = x \times \frac{1}{20} \iint e^u \, du\, du$$

$$P_2 = \frac{x}{20} e^u$$

$$\boxed{P_2 = \frac{x}{20} e^{3x+y}}$$

$$PI = P_1 + P_2$$

$$PI = \frac{-1}{75} \cos(x+2y) + \frac{x}{20} e^{3x+y}$$

Complete solution is

$$Z = CF + PI$$

$$\boxed{Z = f_1(y-x) + f_2(y-2x) + f_3(y+3x) - \frac{1}{75}\cos(x+2y) + \frac{x}{20} e^{3x+y}}$$

Ans.

PDE : DPP-11

Topic : Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral(PI) : Case-II and Case-III

Q.1 Solve $[D^2 + (a+b)DD' + ab\,D'^2]z = xy$ (GBTU-2011 & 2013)

$$z = f_1(y-ax) + f_2(y-bx) + \frac{1}{6}x^3y - \frac{(a+b)}{24}x^4$$

Q.2 Solve : $(D^3 + 2D^2D' - DD'^2 - 2D'^3)z = (y+2)e^x.$

$$z = f_1(y+x) + f_2(y-x) + f_3(y-2x) + ye^x$$

Q. Solve : $(D^2 + DD' - 6\ D'^2)z = y\sin x.$ (MTU-2011 & 2012)

$$z = f_1(y+2x) + f_2(y-3x) - y\sin x - \cos x$$

DPP-11 Topic - Particular Integral (PI)
Case-II and case-III

Q-1: Solve $[D^2+(a+b)DD'+abD'^2]z = xy$

$[D^2+(a+b)DD'+abD'^2]z = xy$

Put $D=m$, $D'=1$

$[m^2+(a+b)m+ab]z = xy$

Auxiliary Equation $\Rightarrow m^2+(a+b)m+ab=0$

$m(m+b)+a(m+b)=0$

$(m+a)(m+b)=0$

$m=-a,-b$

$$\boxed{C.F. = f_1(y-ax)+f_2(y-bx)}$$

$$PI = \frac{1}{D^2+(a+b)DD'+abD'^2}xy$$

$$PI = \frac{1}{D^2\left[1+(a+b)\frac{D'}{D}+ab\frac{D'^2}{D^2}\right]} \times xy$$

$$PI = \frac{1}{D^2}\left[1+\left((a+b)\frac{D'}{D}+ab\frac{D'^2}{D^2}\right)\right]^{-1} \times xy$$

Using expansion

$$PI = \frac{1}{D^2}\left[1-\left((a+b)\frac{D'}{D}+ab\frac{D'^2}{D^2}\right)+\left((a+b)\frac{D'}{D}+ab\frac{D'^2}{D^2}\right)^2 \cdots\right]xy$$

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$$PI = \frac{1}{D^2}\left[1 - (a+b)\frac{D'}{D}\right] xy$$

$$PI = \frac{1}{D^2}\left[xy - (a+b)\frac{D'}{D}(xy)\right]$$

$$PI = \frac{1}{D^2}\left[xy - (a+b)\frac{x}{D}\right]$$

$$PI = \frac{1}{D^2}\left[xy - (a+b)\frac{x^2}{2}\right]$$

$$PI = \frac{1}{D}\left[\int\left[xy - (a+b)\frac{x^2}{2}\right] dx\right]$$

$$PI = \frac{1}{D}\left[\int xy\,dx - \int (a+b)\frac{x^2}{2}\,dx\right]$$

$$PI = \frac{1}{D}\left[\frac{x^2 y}{2} - \frac{(a+b)}{2} \times \frac{x^3}{3}\right]$$

$$PI = \int \frac{x^2 y}{2}\,dx - \int \frac{(a+b)}{6} x^3\,dx$$

$$\boxed{PI = \frac{x^3}{6} y - \frac{(a+b) x^4}{24}}$$

Complete Solution $\Rightarrow$ $Z = CF + PI$

$$\boxed{Z = f_1(y - ax) + f_2(y - bx) + \frac{x^3}{6} y - \frac{(a+b) x^4}{24}}$$

Q-2÷ Solve : $(D^3+2D^2D'-DD'^2-2D'^3)z = (y+2)e^x$

$$(D^3+2D^2D'-DD'^2-2D'^3)z = (y+2)e^x$$

Put $D = m$, $D' = 1$

$$(m^3+2m^2-m-2)z = (y+2)e^x$$

Auxiliary Equation $\Rightarrow$ $m^3+2m^2-m-2 = 0$

$$(m-1)(m+1)(m+2) = 0$$
$$m = 1, -1, -2$$

$$\boxed{C.F. = f_1(y+x) + f_2(y-x) + f_3(y-2x)}$$

$$PI = \frac{1}{D^3+2D^2D'-DD'^2-2D'^3}(y+2)e^x$$

$$PI = \frac{1}{(D-D')(D+D')(D+2D')}(y+2)e^x$$

$$PI = \frac{1}{(D-D')(D+2D')}\left[\frac{1}{D+D'}(y+2)e^x\right]$$

Put $y = c+x$

$$PI = \frac{1}{(D-D')(D+2D')}\int \underset{1}{(c+x+2)}\underset{2}{e^x}\,dx$$

$$PI = \frac{1}{(D-D')(D+2D')}\left[(c+x+2)e^x - 1\times e^x\right]$$

$$PI = \frac{1}{(D-D')(D+2D')}\left[(c+x+2-1)e^x\right]$$

$$PI = \frac{1}{(D-D')(D+2D')}\left[(c+x+1)e^x\right]$$

$$PI = \frac{1}{(D-D')(D+2D')}(y+1)e^x$$

$$PI = \frac{1}{D+2D'}\left[\frac{1}{D-D'}(y+1)e^x\right]$$

Put $y = c-x$

$$PI = \frac{1}{D+2D'}\int \underset{I}{(c-x+1)}\underset{II}{e^x}\,dx$$

$$PI = \frac{1}{D+2D'}\left[(c-x+1)e^x - (-1)e^x\right]$$

$$PI = \frac{1}{D+2D'}\left[(c-x+1+1)e^x\right]$$

$$PI = \frac{1}{D+2D'}(c-x+2)e^x$$

$$PI = \frac{1}{D+2D'}(y+2)e^x$$

Put $y = c+2x$

$$PI = \int \underset{I}{(c+2x+2)}\underset{II}{e^x}\,dx$$

$$PI = (c+2x+2)e^x - 2e^x$$

$$PI = (c+2x+2-2)e^x$$

$$PI = (c+2x)e^x$$

$$\boxed{PI = ye^x}$$

Complete Solution $\Rightarrow$ $z = CF + PI$

$$\boxed{z = f_1(y+x) + f_2(y-x) + f_3(y-2x) + ye^x}$$

Q-3: Solve $(D^2+DD'-6D'^2)z = y\sin x$

$$(D^2+DD'-6D'^2)z = y\sin x$$

Put $D = m$ $D' = 1$

$$(m^2+m-6)z = y\sin x$$

Auxiliary Equation $\Rightarrow$ $m^2+m-6=0$

$$m^2+3m-2m-6=0$$

$$m(m+3)-2(m+3)=0$$

$$(m-2)(m+3)=0$$

$$m = 2, -3$$

$$\boxed{C.F. = f_1(y+2x) + f_2(y-3x)}$$

$$PI = \frac{1}{(D+3D')(D-2D')} y \sin x$$

$$PI = \frac{1}{D+3D'}\left[\frac{1}{D-2D'} y \sin x\right]$$

$$PI = \frac{1}{D+3D'} \int \underset{\text{I}}{(c-2x)} \underset{\text{II}}{\sin x}\, dx$$

$$PI = \frac{1}{D+3D'}\left[(c-2x)(-\cos x) - \int (-2)(-\cos x)\right]$$

$$PI = \frac{1}{D+3D'}\left[-(c-2x)\cos x - \int 2\cos x\right]$$

$$PI = \frac{1}{D+3D'}\left[-(c-2x)\cos x - 2\sin x\right]$$

$$PI = \frac{1}{D+3D'}\left[-y\cos x - 2\sin x\right]$$

$$PI = \int\left[-(c+3x)\cos x - 2\sin x\right] dx$$

$$PI = -\int \underset{\text{I}}{(c+3x)} \underset{\text{II}}{\cos x}\, dx - 2\int \sin x\, dx$$

$$PI = -\left[(c+3x)\sin x - \int 3\sin x\right] - 2(-\cos x)$$

$$PI = -\left[(c+3x)\sin x - 3(-\cos x)\right] + 2\cos x$$

$PI = -\left[(c+3x)\sin x + 3\cos x\right] + 2\cos x$

$PI = -(c+3x)\sin x - 3\cos x + 2\cos x$

$PI = -(c+3x)\sin x - \cos x$

$\boxed{PI = -y\sin x - \cos x}$

Complete solution $\Rightarrow$ $z = CF + PI$

$\boxed{z = f_1(y+2x) + f_2(y-3x) - y\sin x - \cos x}$

Ans

PDE : DPP-12

Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Complementary Function (CF)

Q .1 *Solve* $2s + t - 3q = 0$

Q .2 Solve $DD'(2D + D' + 11)^2 z = 0$

DPP - 12 Maths - IV Unit - 1

Topic - Complementary Function (CF)

Q-1: Solve $2s + t - 3q = 0$

Sol: $2\frac{\delta^2 z}{\delta x \delta y} + \frac{\delta^2 z}{\delta y^2} - 3\frac{\delta z}{\delta y} = 0$

$(2DD' + D'^2 - 3D')z = 0$

$D'\left(D + \frac{1}{2}D' - \frac{3}{2}\right)z = 0$

CF corresponding to $D' = f_1(x)$

CF corresponding to $\left(D + \frac{1}{2}D' - \frac{3}{2}\right) = e^{3x/2} f_2\left(y - \frac{x}{2}\right)$

$PI = 0$

Complete solution $\Rightarrow Z = CF + PI$

$$\boxed{Z = f_1(x) + e^{3x/2} f_2\left(y - \frac{x}{2}\right)}$$

Q-2: Solve $DD'(2D + D' + 11)^2 z = 0$

$DD'\left(D + \frac{D'}{2} + \frac{11}{2}\right)^2 z = 0$

CF corresponding to $D = f_1(y)$

CF corresponding to $D' = f_2(x)$

CF corresponding to $\left(D + \frac{D'}{2} + \frac{11}{2}\right)^2 = e^{-11x/2} f_3\left(y - \frac{x}{2}\right) + x e^{-11x/2} f_4\left(y - \frac{x}{2}\right)$

$PI = 0$

Complete solution $\Rightarrow$ $Z = CF + PI$

$$\boxed{Z = f_1(y) + f_2(x) + e^{-11x/2} f_3\left(y - \frac{x}{2}\right) + x e^{-11x/2} f_4\left(y - \frac{x}{2}\right)}$$

PDE : DPP-13

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-1 and Case-2

Q .1 *Solve:* $(D^3 - 3DD' + D' + 4)z = e^{2x+y}$.

Q .2 *Solve* $(D^2 - D'^2 - 3D + 3D')z = e^{x-2y}$

Q .3 *Find the particular integral of* $2s + t - 3q = 5\cos(3x - 2y)$.

Q .4 *Solve the following partial differential equations:* $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$

DPP-13 Maths - IV Unit - 1

Topic- Non Homogeneous Partial Differential Equation with constant coefficient

Q-1:- Solve $(D^3-3DD'+D'+4)z = e^{2x+y}$

$$(D^3-3DD'+D'+4)z = e^{2x+y}$$

$$(D^2-3DD'+D'+4)z=0 \text{ --- } (1)$$

Here equation (1) cannot be resolved into linear factors in D and D'.

Consider a trial solution-

$$z = Ae^{hx+ky}$$

From eq (1)

$$(D^3-3DD'+D'+4)Ae^{hx+ky}=0$$

$$A\left[D^3e^{hx+ky}-3DD'e^{hx+ky}+D'e^{hx+ky}+4e^{hx+ky}\right]=0$$

$$A\left[h^3e^{hx+ky}-3hke^{hx+ky}+ke^{hx+ky}+4e^{hx+ky}\right]=0$$

$$Ae^{hx+ky}\left[h^3-3hk+k+4\right]=0$$

$$Ae^{hx+ky} \neq 0$$

$$\therefore h^3-3hk+k+4=0$$

$\boxed{C.F. = \sum A e^{hx+ky}}$, where $h^3 - 3hk + k + 4 = 0$

$$PI = \frac{1}{D^3 - 3DD' + D' + 4} e^{2x+y}$$

Replace $D = 2$, $D' = 1$

$$PI = \frac{1}{8 - 6 + 1 + 4} e^{2x+y}$$

$$\boxed{PI = \frac{1}{7} e^{2x+y}}$$

Complete solution is :-

$$Z = CF + PI$$

$$\boxed{Z = \sum A e^{hx+ky} + \frac{1}{7} e^{2x+y}}$$, where $h^3 - 3hK + K + 4 = 0$

Q-2 :- Solve $(D^2 - D'^2 - 3D + 3D')z = e^{x-2y}$

$$(D^2 - D'^2 - 3D + 3D')z = e^{x-2y}$$

$$[(D-D')(D+D') - 3(D-D')] = e^{x-2y}$$

$$(D-D')(D+D'-3) = e^{x-2y}$$

$$\boxed{C.F. = f_1(y+x) + e^{3x} f_2(y-x)}$$

$$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x-2y}$$

Put $D = 1$, $D' = -2$

$$PI = \frac{1}{1-4-3-6} e^{x-2y}$$

$$\boxed{PI = -\frac{1}{12} e^{x-2y}}$$

Complete solution- $Z = CF + PI$

$$\boxed{Z = f_1(y+x) + e^{3x} f_2(y-x) - \frac{1}{12} e^{x-2y}}$$

Q-3-: Find the Particular Integral of $2s + t - 3q = 5\cos(3x - 2y)$

$$2s + t - 3q = 5\cos(3x-2y)$$

$$(2DD' + D'^2 - 3D')z = 5\cos(3x-2y)$$

$$PI = \frac{1}{2DD' + D'^2 - 3D'} 5\cos(3x-2y)$$

Replace $D^2 = -9$, $D'^2 = -4$, $DD' = 6$

$$PI = \frac{1}{2(6) - 4 - 3D'} 5\cos(3x-2y)$$

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$$PI = \frac{1}{8-3D'} 5\cos(3x-2y)$$

Now multiply by conjugate,

$$PI = \frac{1}{8-3D'} \times \frac{8+3D'}{8+3D'} 5\cos(3x-2y)$$

$$PI = \frac{8+3D'}{64-9D'^2} 5\cos(3x-2y)$$

Again put $D'^2 = -4$

$$PI = \frac{8+3D'}{64-(-4)9} 5\cos(3x-2y)$$

$$PI = \frac{5}{100}(8+3D')\cos(3x-2y)$$

$$PI = \frac{1}{20}\left[8\cos(3x-2y) + 3D'\cos(3x-2y)\right]$$

$$PI = \frac{1}{20}\left[8\cos(3x-2y) + 3(-2)(-\sin(3x-2y)\right]$$

$$\boxed{PI = \frac{1}{10}\left[4\cos(3x-2y) + 3\sin(3x-2y)\right]}$$

Ans

Q-4:- Solve the following partial differential equation

$$(D^2 - DD' + D' - 1)z = \cos(x+2y)$$

Sol:- $(D^2 - DD' + D' - 1)z = \cos(x+2y)$

$[D^2 - 1 - D'(D-1)]z = \cos(x+2y)$

$[(D+1)(D-1) - D'(D-1)]z = \cos(x+2y)$

$(D-1)[D - D' + 1]z = \cos(x+2y)$

$$\boxed{C.F. = e^{x} f_1(y) + e^{-x} f_2(y+x)}$$

$$PI = \frac{1}{D^2 - DD' + D' - 1} \cos(x+2y)$$

Put $D^2 = -1$, $D'^2 = -4$, $DD' = -2$

$$PI = \frac{1}{-1-(-2)+D'-1} \cos(x+2y)$$

$$PI = \frac{1}{\cancel{-1}+\cancel{2}+D'-\cancel{1}} \cos(x+2y)$$

$$PI = \frac{1}{D'} \cos(x+2y)$$

$$\boxed{PI = \frac{\sin(x+2y)}{2}}$$

Complete solution $\Rightarrow$ $z = CF + PI$

$$\boxed{z = e^{x} f_1(y) + e^{-x} f_2(y+x) + \frac{1}{2}\sin(x+2y)}$$

Ans

PDE : DPP-14

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

Q .1 *Solve the linear partial differential equation :* $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$. (GBTU-2011)

Q .2 *Solve* $(D + D' - 1)^2 z = xy$ (GBTU-2011)

Q .3 *Solve :* $(D - 3D' - 2)^3 z = 6e^{2x} \sin(3x + y)$.

Q .4 *Solve :* $r - 4s + 4t + p - 2q = e^{x+y}$.

DPP - 14 Maths - IV Unit - 1

Topic - Particular Integral (PI) : Case 3 and Case 4

Q-1 - Solve the linear partial differential equation
$$(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$$

Sol- $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$

$$[(D+D')(D-D') - 3(D-D')]z = xy + e^{x+2y}$$

$$(D-D')[D+D'-3]z = xy + e^{x+2y}$$

$$\boxed{C.F. = f_1(y+x) + e^{3x} f_2(y-x)}$$

$$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} xy + e^{x+2y}$$

$$PI = \frac{1}{D^2 - D'^2 - 3D + 3D'} xy + \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x+2y}$$

$$PI = P_1 + P_2$$

$$P_1 = \frac{1}{(D-D')(D+D'-3)} xy$$

$$P_1 = \frac{1}{D\left(1 - \frac{D'}{D}\right) - 3\left(1 - \left(\frac{D}{3} + \frac{D'}{3}\right)\right)} xy$$

$$P_1 = \frac{-1}{3D}\left(1-\frac{D'}{D}\right)^{-1}\left[1-\left(\frac{D}{3}+\frac{D'}{3}\right)\right]^{-1}$$

$$P_1 = \frac{-1}{3D}\left(1+\frac{D'}{D}+\frac{D'^2}{D^2}+\dots\right)\left(1+\frac{D}{3}+\frac{D'}{3}+\left(\frac{D}{3}+\frac{D'}{3}\right)^2+\dots\right)(xy)$$

$$P_1 = \frac{-1}{3D}\left(1+\frac{D'}{D}+\cancel{\frac{D'^2}{D^2}}+\dots\right)\left(1+\frac{D}{3}+\frac{D'}{3}+\cancel{\frac{D^2}{9}}+\cancel{\frac{D'^2}{9}}+\frac{2DD'}{9}+\dots\right)(xy)$$

$$P_1 = \frac{-1}{3D}\left(1+\frac{D}{3}+\frac{D'}{3}+\frac{2DD'}{9}+\frac{\cancel{D}D'}{3\cancel{D}}+\cancel{\frac{D'^2}{3D}}+\cancel{\frac{2DD'^2}{9D}}+\frac{D'}{D}\right)(xy)$$

$$P_1 = \frac{-1}{3D}\left(1+\frac{D}{3}+\frac{D'}{3}+\frac{2DD'}{9}+\frac{D'}{D}+\frac{D'}{3}\right)(xy)$$

$$PI = \frac{-1}{3D}\left[xy+\frac{D}{3}(xy)+\frac{D'}{3}(xy)+\frac{2DD'}{9}(xy)+\frac{D'}{D}(xy)\right.$$
$$\left.+\frac{D'}{3}(xy)\right]$$

$$P_1 = \frac{-1}{3D}\left[xy+\frac{y}{3}+\frac{x}{3}+\frac{2}{9}+\frac{x^2}{2}+\frac{x}{3}\right]$$

$$P_1 = \frac{-1}{3D}\left[xy+\frac{y}{3}+\frac{2x}{3}+\frac{x^2}{2}+\frac{2}{9}\right]$$

$$\boxed{P_1 = \frac{-1}{3}\left[\frac{x^2y}{2}+\frac{xy}{3}+\frac{x^2}{3}+\frac{x^3}{6}+\frac{2}{9}x\right]}$$

$P_2 = \frac{1}{D^2 - D'^2 - 3D + 3D'} e^{x+2y}$

Put $D = 1$, $D' = 2$

$P_2 = \frac{1}{1 - 4 - 3 + 6} e^{x+2y}$

{ Case of failure }

Now multiply with x and differentiate w.r.t D

$P_2 = \frac{x}{2D - 3} e^{x+2y}$

$\boxed{P_2 = -x e^{x+2y}}$

$PI = \frac{-1}{3}\left[\frac{x^2}{2}y + \frac{xy}{3} + \frac{x^2}{3} + \frac{x^3}{6} + \frac{2x}{9}\right] - x e^{x+2y}$

Complete solution $\Rightarrow$ $Z = CF + PI$

$$\boxed{Z = f_1(y+x) + e^{3x} f_2(y-x) - \frac{1}{3}\left[\frac{x^2}{2}y + \frac{xy}{3} + \frac{x^2}{3} + \frac{x^3}{6} + \frac{2x}{9}\right] - x e^{x+2y}}$$

Ans

Q-2 :- Solve: $(D+D'-1)^2 z = xy$

$$(D+D'-1)^2 z = xy$$

$$CF = e^x f_1(y-x) + x e^x f_2(y-x)$$

$$PI = \frac{1}{(D+D'-1)^2} xy$$

$$PI = \frac{1}{[1-(D+D')]^2} xy$$

$$PI = [1-(D+D')]^{-2} xy$$

Using expansion :-

$$(1-x)^{-2} = 1+2x+3x^2+4x^3+ \cdots\cdots$$

$$PI = \left[1+2(D+D')+3(D+D')^2+ \cdots\cdots\right] xy$$

$$PI = \left[1+2D+2D'+\cancel{3D^2}+\cancel{3D'^2}+6DD'+ \cdots\right] xy$$

$$PI = \left[xy+2D(xy)+2D'(xy)+6DD'(xy)\right]$$

$$\boxed{PI = xy+2y+2x+6}$$

Complete solution $\Rightarrow$ $Z = CF + PI$

$$\boxed{Z = e^x f_1(y-x) + x e^x f_2(y-x) + xy + 2y + 2x + 6}$$

Que-3 :- Solve : $(D-3D'-2)^3 z = 6e^{2x}\sin(3x+y)$

$$(D-3D'-2)^3 z = 6e^{2x}\sin(3x+y)$$

$$\boxed{C.F. = e^{2x}f_1(y+3x) + xe^{2x}f_2(y+3x) + x^2e^{2x}f_3(y+3x)}$$

$$P.I. = \frac{1}{(D-3D'-2)^3}\, 6e^{2x}\sin(3x+y)$$

Replace D by D+2

$$P.I. = 6e^{2x}\frac{1}{(D+2-3D'-2)^3}\sin(3x+y)$$

$$P.I. = 6e^{2x}\frac{1}{(D-3D')^3}\sin(3x+y)$$

Put $D \to 3$, $D' \to 1$

{Case of failure}

$$PI = 6e^{2x} \times x \times \frac{1}{3(D-3D')^2}\sin(3x+y)$$

Again Put $D \to 3$, $D' \to 1$

Again case of failure

$$PI = 6e^{2x} \times x^2 \times \frac{1}{6(D-3D')}\sin(3x+y)$$

Put $D \to 3$, $D' \to 1$

{ Again case of failure }

$$PI = \cancel{6} e^{2x} \times x^3 \times \frac{1}{\cancel{6}} \sin(3x+y)$$

$$\boxed{PI = x^3 e^{2x} \sin(3x+y)}$$

Complete solution $\Rightarrow$ $z = CF + PI$

$$\boxed{\begin{aligned} z = e^{2x} f_1(y+3x) + x e^{2x} f_2(y+3x) + x^2 e^{2x} f_3(y+2x) \\ + x^3 e^{2x} \sin(3x+y) \end{aligned}}$$

Q-4 :- Solve $r - 4s + 4t + p - 2q = e^{x+y}$

$$r - 4s + 4t + p - 2q = e^{x+y}$$

$$(D^2 - 4DD' + 4D'^2 + D - 2D') z = e^{x+y}$$

$$[(D-2D')^2 + (D-2D')] z = e^{x+y}$$

$$(D-2D')(D-2D'+1) z = e^{x+y}$$

$$\boxed{C.F. = f_1(y+2x) + e^{-x} f_2(y+2x)}$$

$$PI = \frac{1}{D^2 - 4DD' + 4D'^2 + D - 2D'} e^{x+y}$$

Put $D \to 1$, $D' \to 1$

{Case of failure}

$$PI = x \times \frac{1}{2D-4D'+1} e^{x+y}$$

$D \to 1,\ D' \to 1$

$$PI = x \times \frac{1}{2-4+1} e^{x+y}$$

$$\boxed{PI = -xe^{x+y}}$$

Complete solution $\Rightarrow$ $z = CF + PI$

$$\boxed{z = f_1(y+2x) + e^{-x}(y+2x) - xe^{x+y}}$$

PDE : DPP-15

➢ Topic : Non-Homogeneous Linear Partial Differential Equation with constant coefficient

Particular Integral (PI) : Case-3 and Case-4

Q .1 *Solve the linear partial differential equation :* $(x^2D^2 + 2xyDD' + y^2D'^2)z = x^my^n$. AKTU-2020-21

DPP-15 Maths - IV Unit -1

Topic →

Q-1-: Solve $(x^2D^2+2xyDD'+y^2D'^2)z = x^m y^n$

Let $x = e^u$, $y = e^v$

$u = \log x$, $v = \log y$

$x^2 \frac{\delta^2 z}{\delta x^2} = D(D-1)z$

$y^2 \frac{\delta^2 z}{\delta y^2} = D'(D'-1)z$

$xy \frac{\delta^2 z}{\delta x \delta y} = DD'z$

Given equation reduced into

$[D(D-1)+2DD'+D'(D'-1)]z = e^{mu+nv}$

$(D^2-D+2DD'+D'^2-D')z = e^{mu+nv}$

$[(D+D')^2-(D+D')]z = e^{mu+nv}$

$(D+D')(D+D'-1)z = e^{mu+nv}$

C.F. $= f_1(v-u)+e^u f_2(v-u)$

$$C.F. = f_1(\log y - \log x) + x f_2(\log y - \log x)$$

$$= f_1\left(\log \frac{y}{x}\right) + x f_2\left(\log \frac{y}{x}\right)$$

$$\boxed{C.F. = g_1\left(\frac{y}{x}\right) + x g_2\left(\frac{y}{x}\right)}$$

$$P.I. = \frac{1}{(D+D')(D+D'-1)} e^{mu+nv}$$

$$P.I. = \frac{1}{D+D'}\left[\frac{1}{D+D'-1} e^{mu+nv}\right]$$

$$P.I = \frac{1}{D+D'}\left[\frac{1}{m+n-1} e^{mu+nv}\right]$$

$$P.I. = \frac{1}{(m+n)(m+n-1)} e^{mu+nv}$$

$$\boxed{P.I. = \frac{x^m y^n}{(m+n)(m+n-1)}}$$

Complete solution $\Rightarrow$ $Z = C.F + PI$

$$\boxed{Z = g_1\left(\frac{y}{x}\right) + x g_2\left(\frac{y}{x}\right) + \frac{x^m y^n}{(m+n)(m+n-1)}}$$