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3CS09

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Parameter Estimation

1. Let $(x_1, x_2 - -)$ be random sample of size n taken from normal population with parameter mean $= \theta_1$ & variance $= \theta_2$. Find the maximum likelihood estimates of these 2 parameters.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$x_1, x_2, - - - x_n$ } sample of size n

$$L = (x_1, x_2, - - x_n) = f(x_1).f(x_2) - - - f(x_n)$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) - - -$$

Taking ln on both sides.

$$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n}\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

Taking derivative w.r.t μ.

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} - \left(\frac{2(x_i - \mu)}{2\sigma^2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n}(x_i - \mu) = 0$$

$$\Rightarrow n\bar{x} - n\mu = 0$$

$$\Rightarrow \bar{x} = \mu$$

Hence, $\theta_1 = \bar{x}$ i.e. sample mean

$$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) + \sum_{i=1}^{n}\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)$$

Taking derivative ~~on~~ ~~to~~ w.r.t σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1}^{n} - \frac{(x_i-\mu)^2}{2(\sigma^2)^2} = 0$$

$$-n + \sum_{i=1}^{n} - \frac{(x_i-\mu)^2}{\sigma^2} =$$

$$n = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

Hence $\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$

2. Let $x_1, x_2, - - x_n$ be random sample from $B(m, \theta)$ distribution where $\theta \in \Theta = (0,1)$ is unknown & 'm' is +ve integer. compute value of θ using MLE Binomial distribution,

$${}^{n}C_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

$$L = \prod_{i=1}^{n} {}^{n}C_{x_i} \theta^{x_i} (1-\theta)^{n-x_i}$$

Taking log on both sides

$$\log L = \sum_{i=1}^{n} \left(\left\{ \log ({}^{n}C_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i} \right) \right.$$

diff. w.r.t. θ

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \Sigma x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \Sigma x_i = 0$$

$$\frac{1}{\theta(\theta)} \Sigma x_i = \frac{n^2}{1-\theta}$$

$$\frac{\Sigma x_i}{\theta} = n^2$$

$$\Rightarrow \boxed{\theta = \frac{\Sigma x_i}{n^2}}$$