Exam 1:

Q1

1] True
2] False
3] True
4] False

Q2

1] Total number of parameters:
= 4 (features) x 2 (values per feature per class) x 2 (classes) + 1 (class prior)
= 4 x 2 x 2 + 1
= 17

The minimum number of parameters required is 17.

2] The parameters to estimate are:

- Class Prior Probability:
  $P(Y=1)$
- Feature likelihoods:

For X_1: $P(X_1=1 \mid Y=1)$, $P(X_1=2 \mid Y=1)$, $P(X_1=1 \mid Y=-1)$, $P(X_1=2 \mid Y=-1)$

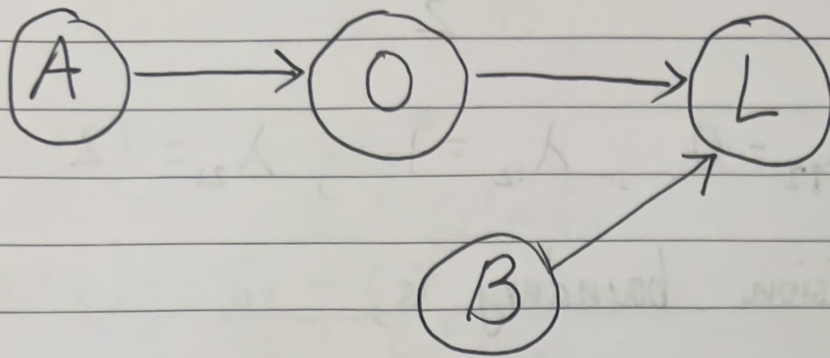
For X_2:
$P(X_2=1 \mid Y=1)$, $P(X_2=2 \mid Y=1)$,
$P(X_2=1 \mid Y=-1)$, $P(X_2=2 \mid Y=-1)$

For X_3:
$P(X_3=1 \mid Y=1)$, $P(X_3=2 \mid Y=1)$,
$P(X_3=1 \mid Y=-1)$, $P(X_3=2 \mid Y=-1)$

For X_4:
$P(X_4=1 \mid Y=1)$, $P(X_4=2 \mid Y=1)$,
$P(X_4=1 \mid Y=-1)$, $P(X_4=2 \mid Y=-1)$

Q3

1]



2] Joint Probability Distribution Factorized:

$$P(A, O, B, L) = P(A)\,P(O|A)\,P(B)\,P(L|O, B)$$

3] Prior Probabilities:

$P(A=1)$, $P(B=1)$

Conditional Probabilities:

$P(O=1|A=1)$, $P(O=1/A=0)$, $P(L=1|O=1, B=1)$

$P(L=1/O=1, B=0)$, $P(L=1|O=0, B=1)$, $P(L=1|O=0, B=0)$

The minimum number of parameter is 8.

4] $P(L=0/O=1) = P(L=0|O=1, B=1)P(B=1) +$

$P(L=0|O=1, B=0)\,P(B=0)$

~~Using Total~~

Q4

1] $\mu_1 = -1$, $\mu_2 = 1$, $\sigma^2 = 0.5$, $P(\omega_1) = P(\omega_2) = \frac{1}{2}$

The decision boundary is,

$$a = \frac{\mu_1 + \mu_2}{2} = 0$$

2] $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = 1$, $\lambda_{21} = 2$

The decision boundary is,

$$a = \frac{\mu_1 + \mu_2}{2} + \frac{\sigma^2}{\mu_2 - \mu_1} \ln\left(\frac{\lambda_{21}}{\lambda_{12}}\right)$$

$$= \frac{1}{4} \ln(2) \approx \boxed{0.173}$$

3] $\lambda_{12} = 1$, $\lambda_{21} = 2$, $P(\omega_1) = \frac{1}{3}$, $P(\omega_2) = \frac{2}{3}$

$$\lambda_{12} P(\omega_2 | x) = \lambda_{21} P(\omega_1 | x)$$

$$\frac{\lambda_{12}\, p(x|\omega_2) P(\omega_2)}{p(x)} = \frac{\lambda_{21}\, p(x|\omega_1) P(\omega_1)}{p(x)}$$

$$\frac{2}{3} p(x|\omega_2) = \frac{2}{3} p(x|\omega_1)$$

$$\Rightarrow p(x|\omega_2) = p(x|\omega_1)$$

This same as part a], $\therefore$ Decision Boundary a is 0.

Q5 1]

$$P(x) = P(x, \omega_1) + P(x, \omega_2)$$
$$= P(x|\omega_1)P(\omega_1) + P(x|\omega_2)P(\omega_2)$$
$$= 4x^3 \cdot \frac{9}{10} + (6x - 6x^2)\frac{1}{10}$$
$$= \frac{36x^3}{10} + \frac{6x}{10} - \frac{6x^2}{10}$$

$$P(\omega_1|x) = \frac{36x^3}{36x^3 - 6x^2 + 6x}$$
$$P(\omega_2|x) = \frac{6x - 6x^2}{36x^3 - 6x^2 + 6x}$$

Now we decide ω_1 if:
$$\cancel{36x}\ P_{\cancel{p}}(\omega_1|x) > P(\omega_2|x)$$

$$\Rightarrow \quad x > \frac{1}{3}, \text{ else we decide } \omega_2$$

$$\begin{cases} 0 \le x \le 1/3, & \text{decide } \omega_2 \\ 1/3 < x \le 1, & \text{decide } \omega_1 \end{cases}$$

~~2] Average Error~~ Value of $a = \frac{1}{3}$

2] Average Error, $P(\text{error}) = \int_0^1 p(\text{error}|x)\,p(x)\,dx$

$$= \int_0^{0.33} P(\omega_1|x)\,p(x)\,dx + \int_{0.33}^{1} p(\omega_2|x)\,p(x)\,dx$$

$$\doteq \frac{18}{5}\int_0^{0.33} x^3\,dx + \frac{1}{10}\int_{0.33}^{1} 6x - 6x^2\,dx$$

$$= 0.0107 + 0.0745$$

$$\boxed{P(\text{error}) = 0.0852}$$

Q6 IF $P(A|B) = P(A|C)$, then $P(B|A) = P(C|A)$

This is not always true, here's a counter example:

$$P(A|B) = P(A|C) = 0.5$$

$$P(B) = 0.8, \quad P(C) = 0.2, \quad \cancel{P(A) = 0.4}$$

$$P(B|A) = \frac{0.5 \times 0.8}{P(A)}$$

$$P(C|A) = \frac{0.5 \times 0.2}{P(A)}$$

Since $P(B|A) \neq P(C|A)$, we have a contradiction.

Q7 $P(l=1|o=1) = \sum_{c} \sum_{f} P(l=1|f)\, P(f|o=1, c)\, P(c)$

$$= \sum_{c=0}^{1} \sum_{f=0}^{1} P(l=1|f)\, P(f|o=1, c)\, P(c)$$

$$= \boxed{\begin{aligned} &P(l=1|f=0)\,(1 - P(f=1|o=1, c=0))\,P(c=0) \,+ \\ &P(l=1|f=1)\,P(f=1|o=1, c=0)\,P(c=0) \,+ \\ &P(l=1|f=0)\,(1 - P(f=1|o=1, c=1))\,P(c=1) \,+ \\ &P(l=1|f=1)\,P(f=1|o=1, c=1)\,P(c=1) \end{aligned}}$$

Q8 1] $P(F=F \mid S) = \frac{1}{3}$, $P(H=T \mid S) = \frac{7}{10}$

$P(F=F \mid H) = \frac{13}{14}$, $P(H=T \mid H) = \frac{9}{70}$

$$P(S \mid H=T, F=F) = \frac{P(S)\,P(F=F \mid S)\,P(H=T \mid S)}{P(S)P(F=F \mid S)P(H=T \mid S) + P(H)P(F=F \mid H)P(H=}$$

$$= \frac{0.3 \times 0.33 \times 0.7}{0.3 \times 0.33 \times 0.7 + 0.7 \times 0.93 \times 0.13}$$

$$= \boxed{0.4558}$$

2] $P(F=T \mid S) = \frac{2}{3}$, $P(H=F \mid S) = \frac{3}{10}$

$P(F=T \mid H) = \frac{1}{14}$, $P(H=F \mid H) = \frac{61}{70}$

$$P(S \mid F=T, H=F) = \frac{P(S)P(F=T \mid S)\,P(H=F \mid S)}{P(S)P(F=T \mid S)\,P(H=F \mid S) + P(H)P(F=T \mid H)P(H=F \mid H)}$$

$$= \frac{0.3 \times 0.66 \times 0.3}{0.3 \times 0.66 \times 0.3 + 0.7 \times 0.07 \times 0.87}$$

$$= \boxed{0.5793}$$