CSE475 HW1, Tuesday, 02/04/2025, Due: Friday, 02/14/2025

1. Please note that you have to typeset your assignment using either LATEX or Microsoft Word, and produce a PDF file for submission. If you have difficulty typing mathematical equations or symbols, you can take photos of handwritten answers and then insert the photos to the file. Only legible handwritten answers will be accepted and graded. You need to submit an electronic version (in PDF form) on Canvas. You should name your file using the format CSE475-HW1-LastName-FirstName.

2. If you have any questions on the homework problems, you should post your question on the Canvas discussion board (under HW1 Q&A), instead of sending emails to the instructor or TA. We will answer your questions there. In this way, we can avoid repeated questions, and help the entire class stay on the same page whenever any clarification/correction is made.

1. [10pts] Consider a one-dimensional two-class classification problem with class-conditional PDFs and class prior probabilities as follows:

$$p(x|\omega_1) = \begin{cases} \frac{1}{2} & x \in [0,2] \\ 0 & \text{otherwise} \end{cases}, \quad p(x|\omega_2) = \begin{cases} \frac{3x^2}{8} & x \in [0,2] \\ 0 & \text{otherwise} \end{cases},$$

$P(\omega_1) = \frac{3}{7}$ (and thus $P(\omega_2) = \frac{4}{7}$). ω_1, ω_2 are two class labels.

(a) The minimum-error-rate classification (by Bayes decision rule) will decide class label ω_1 for feature x such that $x < a$, and this is the Bayesian decision boundary. What is the value of a?

(b) What is the average error using the Bayes decision rule? Note that the average error should be computed by $P(error) = \mathrm{E}[P(error \mid x)] = \int P(error \mid x)\, p(x) \mathrm{d}x$.

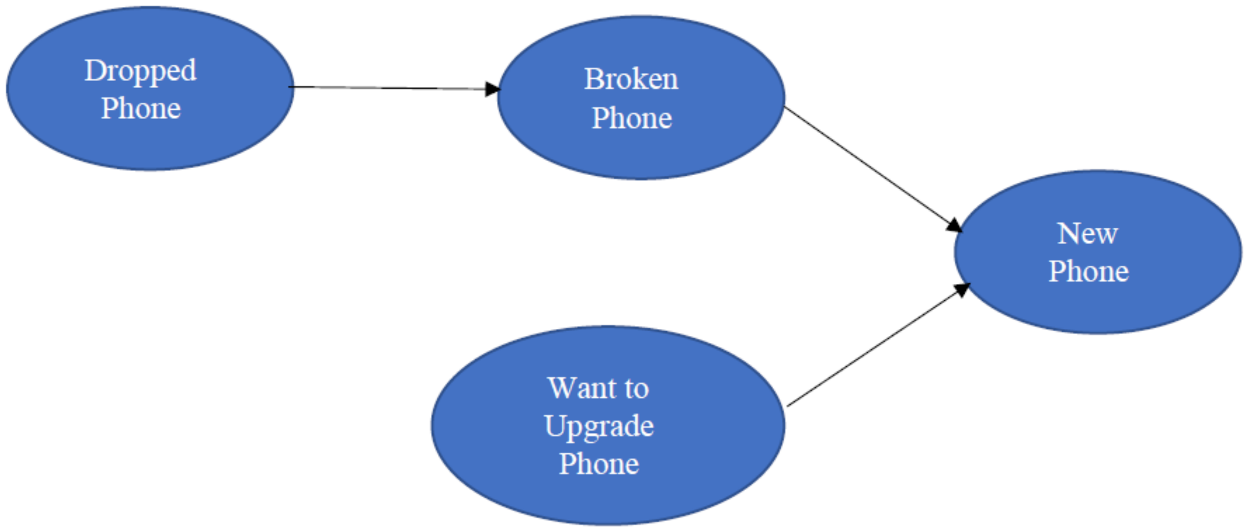
2. [10pts] Consider a 1-dimensional two-class classification problem, with prior probabilities $P(\omega_1) = \frac{1}{3}$ and $P(\omega_2) = \frac{2}{3}$. The class-conditional PDFs for the two classes are the normal densities $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, respectively. Note that these two PDFs have the same variance σ^2. $N(\mu, \sigma^2)$ denotes the normal density defined by

$$p(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}.$$

We further assume that the losses $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{21} = \frac{1}{4}$ and $\lambda_{12} = \frac{3}{4}$. Find the optimal decision rule for classifying any feature point x. [Hint: please refer to slides 50 and 51 of

CSE475-L03.pptx for definition of losses. You need to present your rule in the form of "Deciding ω_1 if x…; otherwise, deciding ω_2", and please specify the condition on x in the if statement].

3. [20pts] Consider the following simple Bayesian Network (BN), where all the nodes are assumed to be binary random variables.



Suppose we use the following abbreviations: $\mathrm{D} = \mathrm{Dropped\ Phone}, \mathrm{B} = \mathrm{Broken\ Phone}, \mathrm{W} = \mathrm{Want\ to\ Upgrade\ Phone}, \mathrm{N} = \mathrm{New\ Phone}$. This BN is fully specified if we are given the following (conditional) probabilities:

$P(\mathrm{W} = \mathrm{Yes}) = 0.4$
$P(\mathrm{D} = \mathrm{Yes}) = 0.2$
$P(\mathrm{B} = \mathrm{Yes}|\mathrm{D} = \mathrm{Yes}) = 0.8$
$P(\mathrm{B} = \mathrm{Yes}|\mathrm{D} = \mathrm{No}) = 0.1$
$P(\mathrm{N} = \mathrm{Yes}|\mathrm{B} = \mathrm{No}, \mathrm{W} = \mathrm{No}) = 0.1$
$P(\mathrm{N} = \mathrm{Yes}|\mathrm{B} = \mathrm{No}, \mathrm{W} = \mathrm{Yes}) = 0.3$
$P(\mathrm{N} = \mathrm{Yes}|\mathrm{B} = \mathrm{Yes}, \mathrm{W} = \mathrm{No}) = 0.8$
$P(\mathrm{N} = \mathrm{Yes}|\mathrm{B} = \mathrm{Yes}, \mathrm{W} = \mathrm{Yes}) = 0.9$

You can use the fact that D, W are independent.

(a) Suppose that you dropped your phone ($\mathrm{D} = \mathrm{Yes}$), but do not want to upgrade the phone ($\mathrm{W} = \mathrm{No}$), compute the probability that you will buy a new phone, i.e., $P(\mathrm{N} = \mathrm{Yes}|\mathrm{D} = \mathrm{Yes}, \mathrm{W} = \mathrm{No})$.

(b) Suppose that you bought a new phone ($\mathrm{N} = \mathrm{Yes}$), compute the probability that the old phone had dropped, i.e., $P(\mathrm{D} = \mathrm{Yes} | \mathrm{N} = \mathrm{Yes})$.