Assignment 2

Title of Assignment : Differential Calculus

Overview of the assignment

- Differential calculus provides the necessary tools to model and solve engineering problem. This assignment is focusing on some of those tools and techniques of differential calculus and some direct applications like error estimation and Optimization.

Learning Goals of the assignment

- To develop an understanding of function of several variables and thus the concept of partial differentiation.
- Modelling the real life problems mathematically.
- Computing the partial derivatives.
- Applying the tools of differential calculus in solving real life problems of error estimation and maxima-minima.
- Solving the problems of vector differentiation.

Exercises

1. The wave equation $\frac{\partial^2 U}{\partial t^2}=\frac{\partial^2 U}{\partial x^2}$ governs the motion of a sound or a light wave. Prove that the function $U\left(t,x\right)=\sin\left(x-t\right)+\sin\left(x+t\right)$ satisfies the wave equation.
2. The time T of a complete oscillation of a simple pendulum of length L is governed by the equation $T=2\pi\sqrt{\frac{L}{g}}$ where g is a constant. Now find the error in the calculated value of T corresponding to an error of 2% in the value of L. Also by what percentage should the length be changed in order to correct a loss of 2 minutes per day?
3. Calculate the percentage error in calculating the volume of a right circular cone whose altitude is same as the base radius and is measured as 5 inches with a possible error of 0.02 inches.
4. Suppose that we want to design a rectangular building having a volume of 147,840 cubic feet. Assuming that the daily loss of heat is given by $w=11xy+14yz+15xz$, where x, y and z are respectively, the length, width and height of the building, find the dimensions of the building for which the daily heat loss is minimal.

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5. Compute the directional derivative of $f(x, y, z) = x^2 y^2 z^2$ at the point (1, 1, -1) in the direction parallel to the vector $\hat{i} + 2\hat{j} + \hat{k}$.
6. A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 3$ where t is the time. Find the components of velocity and acceleration at t=1 in the direction of the vector $\hat{i} + \hat{j} + \hat{k}$.
7. Check whether the following vector functions are solenoidal or irrotational -
    a. $\vec{A} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$
    b. $\vec{B} = (6xy + z^2)\hat{i} + (3x^2 y - z)\hat{j} + (3xz^3 - y)\hat{k}$

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Case Study - Migrating Birds

Certain birds tend to avoid flights over large bodies of water during daylight hours. It is believed that more energy is required to fly over water than land because air generally rises over land and falls over water during the day. A bird with these tendencies is released from point A on an island that is P kms from the nearest point B along a straight shoreline. The nesting point of the bird is at C, which is 2P kms from B. Thus the bird starts at A and concludes its journey at C. See the Figure below. (Here P is your two digit Roll No.)

Suppose it takes 10 units of energy per Km to fly over land (along the shoreline) and 14 units of energy per Km to fly over water. Assume that birds instinctively choose a path that minimizes energy spent in flying.

1. Write down the mathematical equation which upon minimization will give you the path that the bird would take.
2. Find the path the bird would take to minimize the energy spent.