Predictive Analysis using Stats

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Assignment - Parameter Estimation

Ans: given - Random Sample $(x_1, \ldots x_n)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\left(\frac{(x_i-\mu)^2}{2\sigma^2}\right)\right)}$$

Taking natural log of Likelihood fx^n

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2)\right)$$

To find MLE, diff. Log Likelihood w.r.t θ_1, θ_2.

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}\left(\frac{x_i-\mu}{\sigma^2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0$$

$$\frac{\theta_1}{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

For θ_2 $$\frac{\partial}{\partial\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n}\left(-\frac{(x_i-\theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n}\frac{(x_i-\theta_1)^2}{\theta_2^2} - \frac{n}{\theta_2} = 0$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$ Sample Variance.

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Ans 2

To find the MLE of θ for a bionomial distribution $B(m,\theta)$ where m is a known +ve integer.

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking ln

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln \binom{m}{x_i} + X_i \ln(\theta) + (m-X_i)\ln(1-\theta) \right)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{X_i}{\theta} - \frac{m-X_i}{1-\theta} \right) = 0$$

Solving for θ

$$\sum_{i=1}^{n} \frac{X_i}{\theta} = \sum_{i=1}^{n} \frac{m-X_i}{1-\theta}$$

$$\sum_{i=1}^{n} X_i(1-\theta) = \sum_{i=1}^{n} (m-X_i)\theta$$

$$\theta \sum_{i=1}^{n} X_i = m \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^{n} X_i$$

$\therefore$ MLE of θ is sample mean of observations.