2025-2026 Semester I. CML1001. Tutorial 1. Due: 12.Aug.2025 (beginning of lecture)

1. What is the probability of finding a particle in a 1-D box of length L: (a) in the right half of the box (b) in the central third of the box and (c) between $x = 0$ and $x = L/n$ when it is in the n-th state? Does this probability depend on the quantum number labeling the state of the particle? Useful Integral: $\int \sin^2(by)\, dy = \frac{y}{2} - \frac{1}{4b}\sin(2by)$
2. Calculate the energy levels for n = 1, 2, and 3 for an electron in an infinite potential well of width 0.25 nm. (b) If an electron makes a transition from n = 2 to n =1 what will be the wavelength of the emitted radiation.
3. (i) To a crude first approximation, a π electron in linear polyene may be considered to behave as a particle in a one-dimensional box. The polyene in β- carotene contains 22 conjugated C atoms and the average internuclear distance is 140 pm. Each state upto n = 11 is occupied by two electrons. Calculate (a) the separation energy between the ground state and the first excited state in which one electron occupies the state with n = 12 and (b) The frequency of the radiation required to produce a transition between these two states.

   (ii) When β- carotene is oxidized, it breaks into half and forms two molecules of retinal (vitamin A) which is a precursor to the pigment in the retina responsible for vision. The conjugated system for retinal consists of 11 C atoms and one O atom. In the ground state of retinal, each level upto n = 6 is occupied by 2 electrons. Treating everything else to be similar repeat calculations for parts (a) and (b) of the previous problem keeping in mind that in this case the first excited state has one electron in the n = 7 state.
4. Consider a particle of mass m in a 1D box of length a. Its average energy is given by $\langle E \rangle = \frac{\langle p^2 \rangle}{2m}$. Using the uncertainty principle show that the energy must be at least as large as $\frac{\hbar^2}{8ma^2}$ because σ_x, the uncertainty in x, cannot be larger than a.
5. Set up the problem of a particle in a box with its walls located at $-L$ and $+L$. Show that the energies are equal to those of a box with walls located at 0 and $2L$. Show however that the wavefunctions are not the same. Comment on the results thus obtained by you.
6. Consider a particle in a 1D box defined by $V(x) = 0, a > x > 0$ and $V(x) = \infty, x \geq a, x \leq 0$. Explain whether the following functions are acceptable wavefunctions for this particle or not:

   (a) $A \cos(n\pi x/a)$ (b) $B\,(x+x^2)$ (c) $C\, x^3(x-a)$ (d) $D/\sin(n\pi x/a)$
7. Write down the Hamiltonian for the following systems: (a) a particle of mass m in a cubical box of side a; (b) a particle of mass m in a spherical box of radius a; (c) a particle of mass m moving on the x-axis subjected to a force directed towards the origin, of magnitude proportional to the distance from the origin; (d) an electron moving in the presence of a nuclear charge $+Ze$; (e) two electrons moving in the presence of a fixed nucleus of charge $+Ze$
8. Prove that if $\psi(x)$ is a solution to the Schrödinger equation, then any constant times $\psi(x)$ is also a solution.