2025-2026 Semester I. CML1001. Tutorial 2. Due: 25.Aug.2025 (beginning of lecture)

If an operator $\hat{A}$ operates on function $f(x)$and returns the same function multiplied by a constant, i.e. $\boxed{\hat{A}f(x) = af(x)}$, the equation is called an eigenvalue equation, where a is the eigenvalue and the corresponding eigenfunction is $f(x)$.

1. Given $\hat{A} = d/dx$ and $\hat{E} = x^2$, show (a) $\hat{A}^2 f(x) \neq [\hat{A}f(x)]^2$ and (b) $\hat{A}\hat{E}f(x) \neq \hat{E}\hat{A}f(x)$ for arbitrary f(x).

2. Identify which of the following functions are eigenfunctions of the operators d/dx and d^2/dx^2: (a) e^{ikx} (b) cos kx (c) k (d) kx (e) e^{-ax2}. Give the corresponding eigenvalue where appropriate.

3. Find the eigenvalue in the following cases:

$\hat{A}$ (operator)	f(x)	Eigenvalue
d^2/dx^2	$\cos \omega x$	
d/dt	$\exp(i\omega t)$	
$\frac{d^2}{dx^2} + 2\frac{d}{dx} + 3$	$\exp(\alpha x)$	
$\partial/\partial y$	$x^2 \exp(6y)$	

4. Show that $(\cos ax)(\cos by)(\cos cz)$ is an eigenfunction of the operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ which is known as the Laplacian Operator.

5. Write out the operator for $\hat{A}^2$ for $\hat{A}$ =
   (a) $\frac{d^2}{dx^2}$ (b) $\frac{d}{dx} + x$ (c) $\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 1$

6. Find the eigenfunctions of the operator d/dx.

7. In algebra it can be easily shown that $(P + Q)(P - Q) = P^2 - Q^2$. What is the value of $(P + Q)(P - Q)$ if P and Q are operators? Under what conditions will this result be equal to $P^2 - Q^2$?

8. (a) Find $[d/dx, x]$, $[z^3, d/dz]$, $[d^2/dx^2, ax^2 + bx + c]$, and $[a, a^\#]$ where $a = (x + ip)/\sqrt{2}$ and $a^\# = (x - ip)/\sqrt{2}$. (b) Determine whether the operators SQR and SQRT commute.

9. Evaluate the commutator $[\hat{A}, \hat{B}]$, where $\hat{A}$ and $\hat{B}$ are given below:

	$\hat{A}$	$\hat{B}$
(a)	$\frac{d}{dx} - x$	$\frac{d}{dx} + x$
(b)	$\frac{d^2}{dx^2} - x$	$\frac{d}{dx} + x^2$

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10. Normalize the following wavefunctions to unity:
    (a) $\sin(n\pi x/L)$ for the range $0 \le x \le L$.
    (b) c, a constant in the range $-L \le x \le L$.

11. Prove that if $\psi(x)$ is a solution to the Schrödinger equation, then any constant times $\psi(x)$ is also a solution.

12. Which of the following candidates for wavefunctions are normalizable over the indicated intervals? Normalize those that can be normalized.

(a) $\exp\left(-x^2/2\right)$ $(-\infty, \infty)$ (b) e^x $(0, \infty)$ (c) $\exp(i\theta)$ $(0, 2\pi)$ (d) xe^x $(0, \infty)$ (e) $\exp\left[-\left(\frac{x^2+y^2}{2}\right)\right]$ (x,y: $0, \infty$)

13. Write down the Schrödinger equation for the following systems: (a) a particle of mass m in a cubical box of side a; (b) a particle of mass m in a spherical box of radius a; (c) a particle of mass m moving on the x-axis subjected to a force directed towards the origin, of magnitude proportional to the distance from the origin; (d) an electron moving in the presence of a nuclear charge +Ze; (e) two electrons moving in the presence of a fixed nucleus of charge +Ze

14. The wave function for a system can be written as $\psi(x) = \frac{1}{2}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{3+i\sqrt{2}}{4}\phi_3(x)$ with $\phi_1(x)$, $\phi_2(x)$ and $\phi_3(x)$ being normalized eigenfunctions of the kinetic energy operator with eigenvalues E_1, $3E_1$ and $7E_1$ respectively. (a) Verify that $\psi(x)$ is normalized. (b) What are the possible values of KE you will obtain in identically prepared systems. (c) What is the probability of measuring each of these eigenvalues? (d) What is the average value of kinetic energy that you would obtain from a large number of measurements.

15. Suppose that a vibrating atom is described by the wavefunction $\boldsymbol{\psi = Nxe^{-x^2/2a^2}}$. Where is the most probable location of the atom?

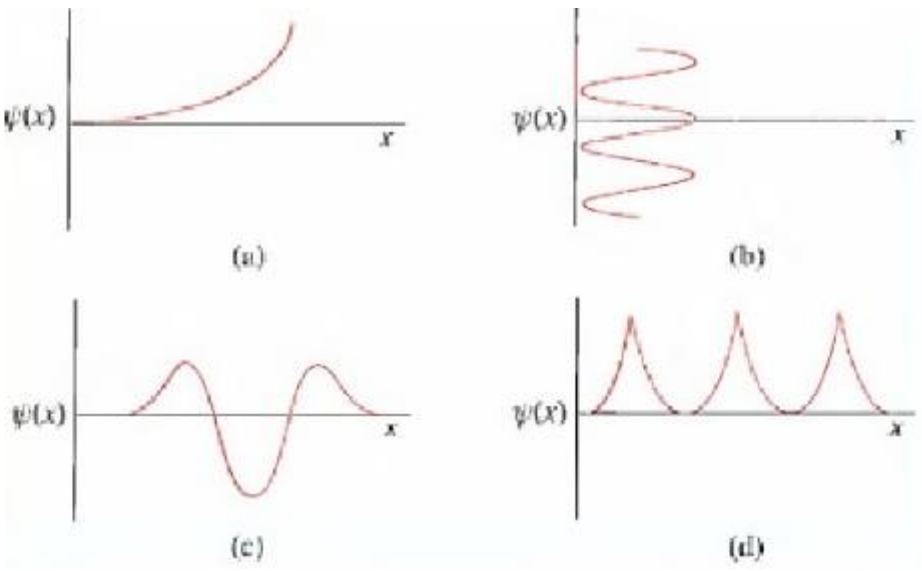
16. Consider a particle whose normalized wave function is

$$\psi(x) = 2\alpha\sqrt{\alpha}xe^{-\alpha x} \qquad x > 0$$

$$\psi(x) = 0 \qquad x < 0$$

(a) For what value of x does $P(x)$ peak?
(b) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$.
(c) What is the probability that the particle is found between $x = 0$ and $x = \frac{1}{\alpha}$.

21.



Which of the wavefunctions shown in the figure above are well behaved? Give reasons for your answer.