2025-2026 Semester I. CML1001. Tutorial 3. Due: 8.Sep.2025 (beginning of lecture)

1. Show that Hermitian operators guarantee real eigenvalues and orthogonal eigenfunctions. Worry only about the non-degenerate case.
2. The possible values obtained from a measurement of a discrete variable, x, are 1, 2, 3, and 4.
   (a) If the respective probabilities are 1/4, 1/4, 1/4, and 1/4, calculate the expectation values of x and x^2. (b) If the respective probabilities are 1/12, 5/12, 5/12, and 1/12, calculate the expectation values of x and x^2.
3. If $[\hat{A}, \hat{B}] = 0$ and $[\hat{A}, \hat{C}] = 0$, then which of the following necessarily holds
   a) $[\hat{B}, \hat{C}] = 0$ b) $[\hat{A}, \widehat{BC}] = 0$ c) $[\hat{B}, \widehat{AC}] = 0$ d) $[\hat{C}, \widehat{AB}] = 0$
4. For a Hermitian operator $\hat{A}$, which does not commute with the Hamiltonian $\hat{H}$, let ψ_1 be an eigenfunction of $\hat{A}$ and ψ_2 be an eigenfunction of $\hat{H}$. The correct statement is
   (a) both $\langle \psi_1 | [A, H] | \psi_1 \rangle$ and $\langle \psi_2 | [A, H] | \psi_2 \rangle$ are non-zero
   (b) only $\langle \psi_1 | [A, H] | \psi_1 \rangle$ is zero, but $\langle \psi_2 | [A, H] | \psi_2 \rangle$ is non-zero
   (c) only $\langle \psi_2 | [A, H] | \psi_2 \rangle$ is zero, but $\langle \psi_1 | [A, H] | \psi_1 \rangle$ is non-zero
   (d) both $\langle \psi_1 | [A, H] | \psi_1 \rangle$ and $\langle \psi_2 | [A, H] | \psi_2 \rangle$ are zero
5. Determine $\psi^* \psi$ for the following wave functions: (a) $\cos\theta + i\sin\theta$ and (b) $\exp -x^2$.
6. Consider a particle of mass *m* confined in a 3D (cubic box) of length *d*. (i) What values of n_x, n_y and n_z correspond to the **zero point energy**? (ii) Write down the expression for the zero-point energy of the system.
7. As a variant on the free-electron model applied to benzene, assume that the six π electrons are delocalized within a square plate of side '*a*'. Calculate the value of '*a*' that would account for the 268 nm ultraviolet absorption in benzene.
8. The function $\psi(x) = Ax[1 - \frac{x}{a}]$ is an acceptable wavefunction for the particle in the one dimensional infinite depth box of length a. Calculate the normalization constant A and the expectation values $\langle x \rangle$ and $\langle x^2 \rangle$.