# Application #3
# Distribution by Monte Carlo

## 1 Data

- No data is needed for this exercise.

## 2 Simulation

We want to simulate the S&P 500 returns, assuming the following:

- The log-returns are normally distributed, with mean of 0.09 and standard deviation of 0.16.
- Returns are independent across time. That is, each period we just get a new log-return from the distribution described above.

1. Simulate a time-series of the log-returns with length $T = 10$. Check that your simulated vector of length $T \times 1$ seems reasonable.

2. Expand this to now simulate $N = 100$ paths—each of length $T = 10$. (With randn, there is no need to use a loop here.)

## 3 Plots

1. Make a histogram of all the simulated returns. (That is, include all paths and all time-periods in the histogram.)

2. Try using the function, histfit to see a histogram along with an estimated distribution. Specifically, enter histfit(data,nbins,'normal'). Note that nbins is simply the number of bins you want in the histogram. Try using 100.

3. Make a plot showing the first 10 time-series paths. (Hint: no need for a loop to plot multiple time-series all on the same plot.)

## 4 Probability Distribution

1. Calculate the total cumulative *log* return of each path. (No need to calculate the history of cumulative returns—just the final cumulative return of each path.)

2. Using your simulated sample, what is the mean and standard deviation of the total cumulative log return?

3. How does this distribution change if we use $N = 1,000$ paths. What if we use $T = 20$?

## 5 Underperformance

Suppose a client is concerned about the probability that the stock index under-performs a bond index which has mean log-returns of 0.06 and standard deviation of 0.04.

1. Simulate $N$ paths of length $T$ for the bonds. Assume that the bond returns are independent of the stock returns—so you can do this simulation completely separate from the stock simulation.

2. Based on your simulations, what is the probability that the stock index under-performs the bond index? Report this probability for $T = 10, 20, 30$.

## 6 Extra

1. Re-do the section on under-performance, but this time assume that the bond and stock index are not independent. Rather, their log returns have covariance of

$$\begin{bmatrix} 0.0256 & -0.0019 \\ -0.0019 & 0.0015 \end{bmatrix}$$

   To do this, try using mvnrnd(meanRowVec, sigmaMat, N). This will give you only the first period of each simulated path—use a loop to fill out the rest of the path.

2. What is the estimated chance of under-performance? How does this compare to above?