UNIVERSITY OF CHICAGO | FINANCIAL MATHEMATICS

Vasicek

Michael Beven - 455613

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The Vasicek model is given by:

$$dr(t) = \alpha(\mu - r(t))\mathrm{d}t + \sigma \mathrm{d}\tilde{W}(t)$$

Where $r(t)$ is the risk-free rate of interest. For $s > 0$, $r(t+s)$ given $r(t)$ is Normally distributed under the physical probability measure with mean $\mu + (r(t) - \mu)e^{-\alpha s}$ and variance $\frac{\sigma^2(1-e^{-2\alpha s})}{2\alpha}$.
Prices for zero-coupon bonds are given by:

$$P(t,T) = e^{A(t,T)-B(t,T)r(t)}$$

where

$$B(t,T) = \frac{1-e^{-\alpha(T-t)}}{\alpha}$$
$$A(t,T) = (B(t,T) - (T-t))\left(\mu - \frac{\sigma^2}{2\alpha^2}\right) - \frac{\sigma^2}{4\alpha}B(t,T)^2$$

The price of the European call which matures at time S with strike K and exercise date T (with $T < S$) is:

$$V_c(t) = P(t,S)\Phi(d_1) - KP(t,T)\Phi(d_2)$$

where

$$d_1 = \frac{1}{\sigma_p} log \frac{P(t,S)}{KP(t,T)} + \frac{\sigma_p}{2}, \qquad d_2 = d_1 - \sigma_p$$
$$\sigma_p = \frac{\sigma}{\alpha}(1 - e^{-\alpha(S-T)})\sqrt{\frac{1-e^{-2\alpha(T-t)}}{2\alpha}}$$

By put-call parity, we also have the price of the corresponding European put as:

$$V_p(t) = KP(t,T)\Phi(-d_2) - P(t,S)\Phi(-d_1)$$