

MPP-E1180 Lecture 9: Statistical Modeling with R

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Objectives for the class

- ▶ Assignment 3
- ▶ Review
- ▶ Intro to the general syntax for statistical modelling in R.
- ▶ Specific examples using:
 - ▶ Normal linear regression
 - ▶ Logistic regression
 - ▶ Panel data

Collaborative Research Project (1)

Purposes: Pose an interesting research question and try to answer it using data analysis and standard academic practices. Effectively communicate your results to a **variety of audiences** in a **variety of formats**.

Deadline:

- ▶ Presentation: In-class 2 December
- ▶ Website/Paper: 16 December 2016

Collaborative Research Project (2)

The project can be thought of as a 'dry run' for your thesis with multiple presentation outputs.

Presentation: 10 minutes **maximum**. **Engagingly** present your research question and key findings to a general academic audience (fellow students).

Paper: 5,000 words maximum. **Standard academic paper**, properly cited laying out your research question, literature review, data, methods, and findings.

Website: An engaging website designed to convey your research to a **general audience**.

Collaborative Research Project (3)

As always, you should **submit one GitHub repository** with all of the materials needed to **completely reproduce** your data gathering, analysis, and presentation documents.

Note: Because you've had two assignments already to work on parts of the project, I expect **high quality work**.

Collaborative Research Project (4)

Find one other group to be a **discussant** for your presentation.

The discussants will provide a quick (max 2 minute) critique of your presentation—ideas for things you can improve on your paper.

Office hours

I will have normal office hours every week for the rest of the term **except 9 December**.

Please take advantages of this opportunity to **improve your final project**.

Review

- ▶ What is **web scraping**? What are some of tools R has for web scraping?
- ▶ What are **regular expressions** (give at least two examples)? Why are they useful?
- ▶ What dplyr function can you use to create a **new variable** in a data frame by running a command on values from groups in that data frame?

Review

- ▶ What is the data-ink ratio? Why is it important for effective plotting.
- ▶ Why should you avoid using the **size** of circles to have meaning about continuous variables?
- ▶ Why not use red-green colour contrasts to indicate contrasting data?
- ▶ How many decimal places should you report in a table and why?

Statistical Modelling in R

Caveat: We are **definitely not** going to cover anywhere near R's full capabilities for statistical modeling.

We are also **not going to cover** all of the **modeling concerns/diagnostics** you need to consider when using a given model.

You will need to **rely on your other stats courses and texts**.

What are we going to do?

- ▶ Discuss the basic syntax and capabilities in R for estimating normal linear and logistic regressions.
- ▶ Basic model checking in R.
- ▶ Discuss basic ways of interpreting and presenting results.

The basic model

Most statistical models you will estimate are from a general class (**Generalised Linear Model**) that has **two parts**:

Stochastic Component (e.g. randomly determined) assumes the dependent variable Y_i is generated from as a random draw from the probability density function:

$$Y_i \sim f(\theta_i, \alpha)$$

- ▶ θ_i : parameter vector of the part of the function that **varies between observations**.
- ▶ α : matrix of **non-varying parameters**.

Sometimes referred to as the '**error structure**'.

The basic model

The **Systematic Component** indicating how θ_i varies across observations depending on values of the explanatory variables and (often) some constant:

$$\theta_i = g(X_i, \beta)$$

- ▶ X_i : a $1 \times k$ vector of **explanatory variables**.
- ▶ β : a $1 \times k$ vector of **parameters** (i.e. coefficients).
- ▶ $g(.,.)$: the **link function**, specifying how the explanatory variables and parameters are translated into θ_i .

Today

Today we will cover two variations of this general model:

- ▶ linear-normal regression (i.e. ordinary least squares)
- ▶ logit model

Linear-normal regression

For continuous dependent variables assume that Y_i is from the **normal distribution** ($N(., .)$).

Set the main parameter vector θ_i to the **scalar mean** of:

$$\theta_i = E(y_i) = \mu_i.$$

- ▶ **Scalar**: a real number (in R-language: a vector of length 1)

Assume the ancillary parameter matrix is the scalar homoskedastic variance: $\alpha = V(Y_i) = \sigma^2$.

- ▶ **Homoskedastic variance**: variance does not depend on the value of x . The standard deviation of the error terms is constant across values of x .

Set the systematic component to the linear form:

$$g(X_i, \beta) = X_i\beta = \beta_0 + X_{i1}\beta_1 + \dots$$

Linear-normal regression

So:

$$Y_i \sim N(\mu_i, \sigma^2), \quad \mu_i = X_i \beta$$

Logit regression

For binary data (e.g. 0, 1) we can assume that the stochastic component has a Bernoulli distribution.

The main parameter is $\pi_i = \Pr(Y_i = 1)$.

The systematic component is set to a logistic form: $\pi_i = \frac{1}{1+e^{-X_i\beta}}$.

So:

$$Y_i \sim \text{Bernoulli}(\pi_i), \quad \pi_i = \frac{1}{1 + e^{-X_i\beta}}$$

Example error structure families and link functions

Error Family	Canonical link
Normal	identity
binomial	logit
poisson	log

R syntax

The general syntax for estimating statistical models in R is:

```
response variable ~ explanatory variable(s)
```

Where '~' reads 'is modelled as a function of'.

In the Generalised Linear Model context, either explicitly or implicitly:

```
response variable ~ explanatory variable(s), family = error
```

Model functions

We use model functions to specify the model structure.

Basic model functions include:

- ▶ `lm`: fits a linear model where Y is assumed to be normally distributed and with homoskedastic variance.
- ▶ `glm`: allows the fitting of many Generalised Linear Models. Lets you specify the error family.
- ▶ `plm` (package and function): panel data Linear Models
- ▶ `pglm` (package and function): panel data Generalised Linear Models

Example of `lm`

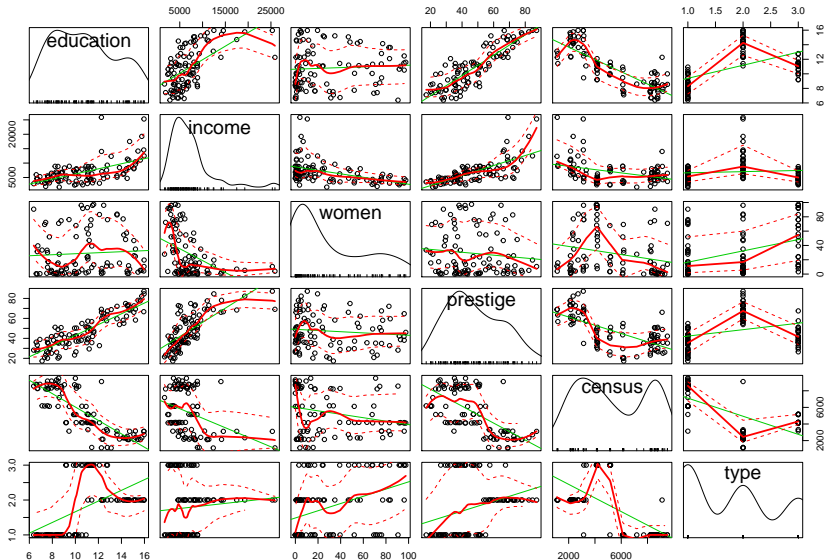
Example data *Prestige* (example based on <http://www.princeton.edu/~otorres/Regression101R.pdf>).

The observations are **occupations** and the dependent variable is a score of each occupation's **prestige**.

```
library(car)  
data(Prestige)
```

Examine correlation matrix

```
car::scatterplotMatrix(Prestige)
```



Example of `lm`

Estimate simple model (education is in years):

```
M1 <- lm(prestige ~ education, data = Prestige)
```

summary(M1)

##

Call:

lm(formula = prestige ~ education, data = Prestige)

##

Residuals:

##	Min	1Q	Median	3Q	Max
##	-26.0397	-6.5228	0.6611	6.7430	18.1636

##

Coefficients:

##		Estimate	Std. Error	t value	Pr(> t)
##	(Intercept)	-10.732	3.677	-2.919	0.00434 **
##	education	5.361	0.332	16.148	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1

##

Residual standard error: 9.103 on 100 degrees of freedom

Multiple R-squared: 0.7228, Adjusted R-squared: 0.72

F-statistic: 262.2 on 1 and 100 DF, p-value: 1.22e-10

Confidence intervals of parameter point estimates

Note: **Always prefer estimation intervals** over point estimates.

Deal with your **uncertainty**!

About **95%** of the time the population parameter will be within **about 2 standard errors** of the point estimate.

Using **Central Limit Theorem** (at least about 50 observations and the data is not extremely skewed):

$$CI_{95} = \text{point estimate} \pm 1.96 * SE$$

Confidence intervals of parameter point estimates

```
confint(M1)
```

```
##              2.5 %    97.5 %  
## (Intercept) -18.027220 -3.436744  
## education    4.702223  6.019533
```

Example of `lm`

Estimate model with categorical (factor) variable:

```
M2 <- lm(prestige ~ education + type,  
         data = Prestige)
```

summary(M2)

##

Call:

lm(formula = prestige ~ education + type, data = Prestig

##

Residuals:

##	Min	1Q	Median	3Q	Max
##	-19.410	-5.508	1.360	5.694	17.171

##

Coefficients:

##		Estimate	Std. Error	t value	Pr(> t)
##	(Intercept)	-2.6982	5.7361	-0.470	0.6392
##	education	4.5728	0.6716	6.809	9.16e-10 ***
##	typeprof	6.1424	4.2590	1.442	0.1526
##	typewc	-5.4585	2.6907	-2.029	0.0453 *

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1

##

Residual standard error: 7.214 on 64 degrees of freedom

Create categorical variable from continuous variable

Use the cut function to create a categorical (factor) variable from a continuous variable.

```
Prestige$income_cat <- cut(Prestige$income,  
                           breaks = c(0, 4999, 9999, 14999, 30000),  
                           labels = c('< 5,000', '< 10,000', '< 15,000',  
                                       '>= 15,000'))  
summary(Prestige$income_cat)
```

```
##    < 5,000  < 10,000  < 15,000  >= 15,000  
##          38          51           9           4
```

Note: cut excludes the left value and includes the right value, e.g. (0, 4999].

Example of lm

```
M3 <- lm(prestige ~ education + income_cat,  
         data = Prestige)  
confint(M3)
```

##		2.5 %	97.5 %
## (Intercept)	-10.914031	3.182500	
## education	3.350201	4.778444	
## income_cat< 10,000	6.085375	13.030546	
## income_cat< 15,000	9.584532	23.391854	
## income_cat>= 15,000	12.040936	29.902733	

Example of lm

Estimate models with polynomial transformations:

```
# Cubic polynomial transformation  
M4 <- lm(prestige ~ education + poly(income, 2),  
         data = Prestige)  
confint(M4)
```

##	2.5 %	97.5 %
## (Intercept)	-1.470988	13.33552
## education	3.132827	4.48515
## poly(income, 2)1	45.174019	81.39445
## poly(income, 2)2	-43.150740	-12.87994

Example of `lm`

Estimate models with (natural) logarithmic transformations:

```
# Cubic polynomial transformation  
M5 <- lm(prestige ~ education + log(income),  
         data = Prestige)
```


summary(M5)

##

Call:

lm(formula = prestige ~ education + log(income), data =

##

Residuals:

##	Min	1Q	Median	3Q	Max
----	-----	----	--------	----	-----

##	-17.0346	-4.5657	-0.1857	4.0577	18.1270
----	----------	---------	---------	--------	---------

##

Coefficients:

##		Estimate	Std. Error	t value	Pr(> t)
----	--	----------	------------	---------	----------

##	(Intercept)	-95.1940	10.9979	-8.656	9.27e-14 ***
----	-------------	----------	---------	--------	--------------

##	education	4.0020	0.3115	12.846	< 2e-16 ***
----	-----------	--------	--------	--------	-------------

##	log(income)	11.4375	1.4371	7.959	2.94e-12 ***
----	-------------	---------	--------	-------	--------------

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1

##

Residual standard error: 7.145 on 99 degrees of freedom

Multiple R-squared: 0.824 Adjusted R-squared: 0.827

Example of `lm`

Estimate model with interactions:

```
M6 <- lm(prestige ~ education * type,  
         data = Prestige)
```

summary(M6)

##

Call:

lm(formula = prestige ~ education * type, data = Prestig

##

Residuals:

##	Min	1Q	Median	3Q	Max
##	-19.7095	-5.3938	0.8125	5.3968	16.1411

##

Coefficients:

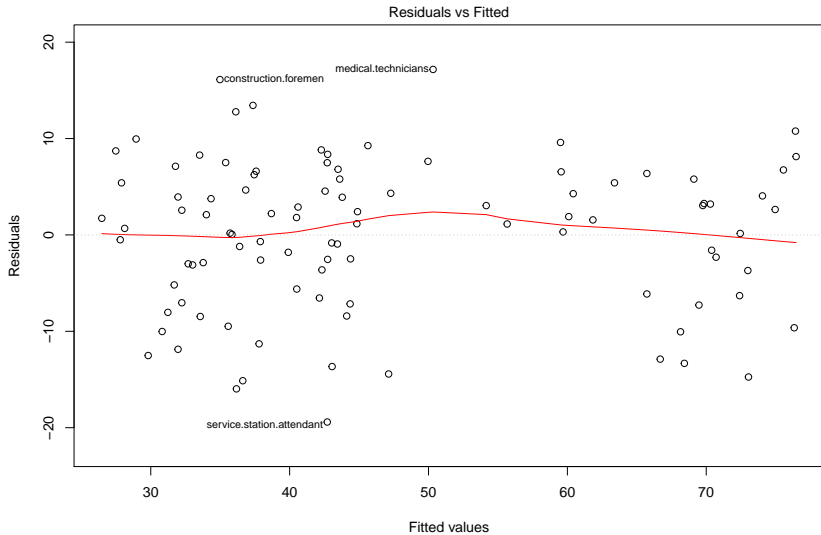
##	Estimate	Std. Error	t value	Pr(> t)
## (Intercept)	-4.2936	8.6470	-0.497	0.621
## education	4.7637	1.0247	4.649	1.11e-05
## typeprof	18.8637	16.8881	1.117	0.267
## typewc	-24.3833	21.7777	-1.120	0.266
## education:typeprof	-0.9808	1.4495	-0.677	0.500
## education:typewc	1.6709	2.0777	0.804	0.423

Signif. codes: 0.001 '***' 0.01 '**' 0.05 '*' 0.1 '.' 1 ' ' 0

Diagnose heteroscedasticity

Use `plot` on a model object to run visual diagnostics.

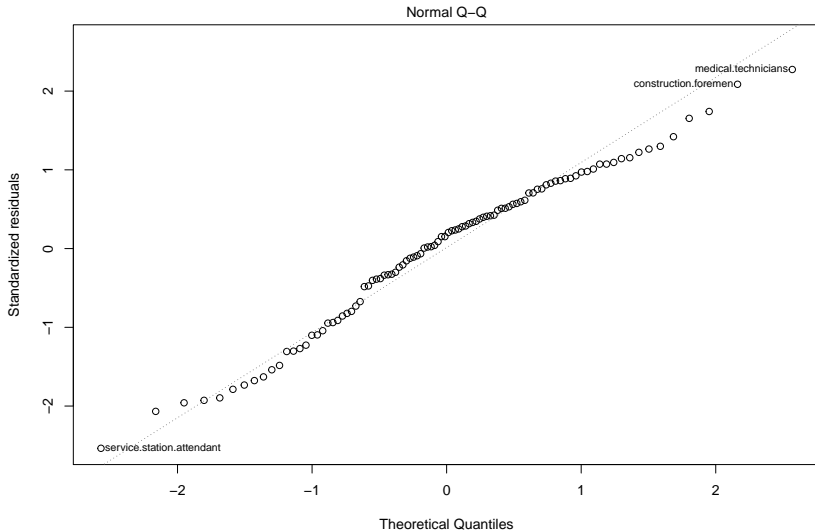
```
plot(M2, which = 1)
```



Diagnose non-normality of errors

plot to see if a model's errors are normally distributed.

```
plot(M2, which = 2)
```



Example of logistic regression with glm

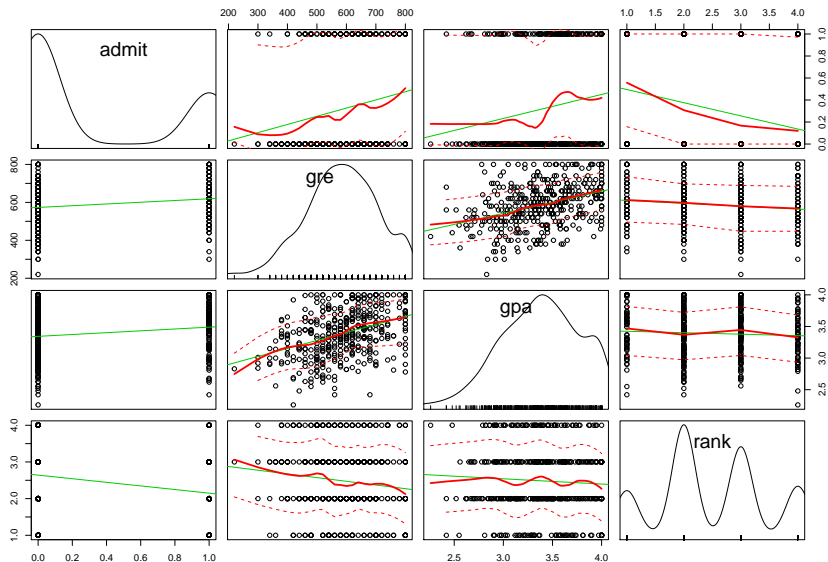
Example from UCLA IDRE.

Simulated data of admission to grad school.

```
# Load data  
URL <- 'http://www.ats.ucla.edu/stat/data/binary.csv'  
Admission <- read.csv(URL)
```

Example of logistic regression with glm

```
car::scatterplotMatrix(Admission)
```



Contingency table for school rank and admission

```
admit_table <- xtabs(~admit + rank, data = Admission)
admit_table
```

```
##      rank
## admit  1  2  3  4
##      0 28 97 93 55
##      1 33 54 28 12
```


Row and column proportions

```
# Row proportions
```

```
prop.table(admit_table, margin = 1)
```

```
##      rank
```

```
## admit          1          2          3          4
```

```
##      0 0.10256410 0.35531136 0.34065934 0.20146520
```

```
##      1 0.25984252 0.42519685 0.22047244 0.09448819
```

```
# Column proportions
```

```
prop.table(admit_table, margin = 2)
```

```
##      rank
```

```
## admit          1          2          3          4
```

```
##      0 0.4590164 0.6423841 0.7685950 0.8208955
```

```
##      1 0.5409836 0.3576159 0.2314050 0.1791045
```

Summary of contingency table for school rank and admission

```
summary(admit_table)
```

```
## Call: xtabs(formula = ~admit + rank, data = Admission)
## Number of cases in table: 400
## Number of factors: 2
## Test for independence of all factors:
##  Chisq = 25.242, df = 3, p-value = 1.374e-05
```

Example of logistic regression with glm

```
Logit1 <- glm(admit ~ gre + gpa + as.factor(rank),  
              data = Admission, family = 'binomial')
```

Note: Link function is assumed to be logit if family = 'binomial'.

Example of logistic regression with glm

```
confint(Logit1)
```

##		2.5 %	97.5 %
## (Intercept)		-6.2716202334	-1.792547080
## gre		0.0001375921	0.004435874
## gpa		0.1602959439	1.464142727
## as.factor(rank)2		-1.3008888002	-0.056745722
## as.factor(rank)3		-2.0276713127	-0.670372346
## as.factor(rank)4		-2.4000265384	-0.753542605

Interpreting logistic regression results

β 's in logistic regression are interpretable as **log odds**. These are weird.

If we exponentiate log odds we get **odds ratios**.

```
exp(cbind(OddsRatio = coef(Logit1), confint(Logit1)))
```

##	OddsRatio	2.5 %	97.5 %
## (Intercept)	0.0185001	0.001889165	0.1665354
## gre	1.0022670	1.000137602	1.0044457
## gpa	2.2345448	1.173858216	4.3238349
## as.factor(rank)2	0.5089310	0.272289674	0.9448343
## as.factor(rank)3	0.2617923	0.131641717	0.5115181
## as.factor(rank)4	0.2119375	0.090715546	0.4706961

These are **also weird**.

Interpreting logistic regression results

What we really want are **predicted probabilities**

First create a data frame of fitted values:

```
fitted <- with(Admission,  
               data.frame(gre = mean(gre),  
                           gpa = mean(gpa),  
                           rank = factor(1:4)))  
fitted
```

##		gre	gpa	rank
##	1	587.7	3.3899	1
##	2	587.7	3.3899	2
##	3	587.7	3.3899	3
##	4	587.7	3.3899	4

Interpreting logistic regression results

Second predict probability point estimates for each fitted value.

```
fitted$predicted <- predict(Logit1, newdata = fitted,  
                             type = 'response')
```

```
fitted
```

##	gre	gpa	rank	predicted
## 1	587.7	3.3899	1	0.5166016
## 2	587.7	3.3899	2	0.3522846
## 3	587.7	3.3899	3	0.2186120
## 4	587.7	3.3899	4	0.1846684

Showing results from regression models with simulations

King, Tomz, and Wittenberg (2001) argue that **post-estimation simulations** can be used to effectively communicate **results from regression models**.

Steps

1. Estimate our parameters' point estimates for $\hat{\beta}_{1...k}$.
2. Draw n values of the point estimates from multivariate normal distributions with means $\bar{\beta}_{1...k}$ and variances specified by the parameters' estimated co-variance.
3. Use the simulated values to calculate quantities of interest (e.g. predicted probabilities).
4. Plot the simulated distribution using **visual weighting**.

Notes

Post-estimation simulations allow us to effectively communicate our estimates and the **uncertainty around them**.

This method is broadly similar to a fully Bayesian approach with Markov-Chain Monte Carlo or bootstrapping. Just differ on **how the parameters are drawn**.

Implementation

1. Find the coefficient estimates from an estimated model with `coef`.
2. Find the co-variance matrix with `vcov`.
3. Draw point estimates from the multivariate normal distribution with `mvrnorm`.
4. Calculate the quantity of interest with the draws + fitted values using and plot the results.

Simulations: estimate model

First estimate your model as normal and create fitted values:

```
library(car) # Contains data  
library(dplyr) # Piping function  
M_prest <- lm

```
prestige ~ education + type,
```

  
      data = Prestige)  
  
# Find a range of education values  
range(Prestige$education)
```

```
## [1] 6.38 15.97
```

```
edu_range <- 6:16
```

Simulations: extract estimates

Extract point estimates (coefficients) and co-variance matrix:

```
mp_coef <- matrix(coef(M_prest))  
mp_vcov <- vcov(M_prest)
```

Now draw 1,000 simulations of your point estimates:

```
library(MASS) # contains the mvrnorm function  
drawn <- mvrnorm(n = 1000, mu = mp_coef, Sigma = mp_vcov) %  
          data.frame  
head(drawn)[1:3, ]
```

```
##      X.Intercept.  education typeprof      typewc  
## 1      -6.2547586    4.979766 3.183540 -4.9515917  
## 2      -0.8856289    4.222594 8.711044  0.5419205  
## 3      -3.9574295    4.735935 5.515092 -5.7230707
```

Simulations: merge in fitted values

Now we can add in our fitted values to the simulation data frame:

```
drawn_sim <- merge(drawn, edu_range)

# Rename the fitted value variable
drawn_sim <- dplyr::rename(drawn_sim, fitted_edu = y)

nrow(drawn)
```

```
## [1] 1000
```

```
nrow(drawn_sim)
```

```
## [1] 11000
```

Simulations: calculate quantity of interest

Using the normal linear regression formula ($\hat{y}_i = \hat{\alpha} + X_{i1}\hat{\beta}_1 + \dots$) we can find the quantity of interest for white collar workers:

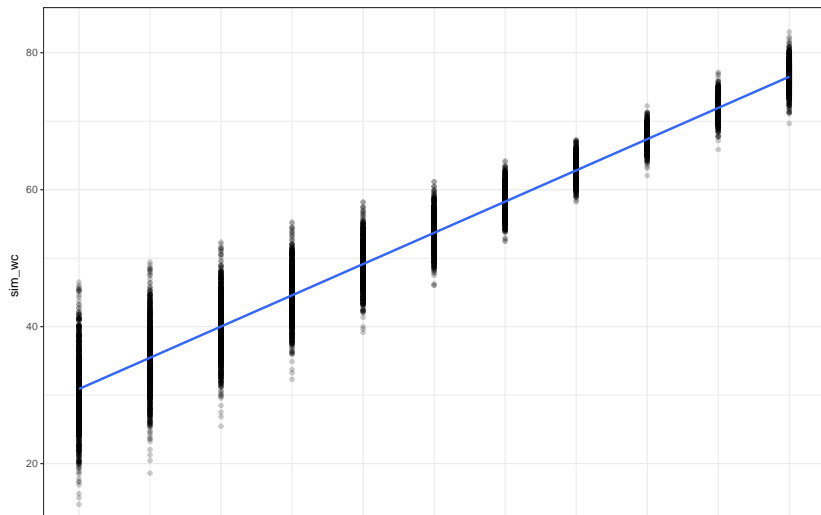
```
names(drawn_sim)
```

```
## [1] "X.Intercept." "education"      "typeprof"      "typewo  
## [5] "fitted_edu"
```

```
drawn_sim$sim_wc <- drawn_sim[, 1] + drawn_sim[, 2] * drawn  
                        drawn_sim[, 3]
```

Simulations: plot points

```
ggplot(drawn_sim, aes(fitted_edu, sim_wc)) +  
  geom_point(alpha = 0.2) + stat_smooth(se = FALSE)  
  theme_bw()
```

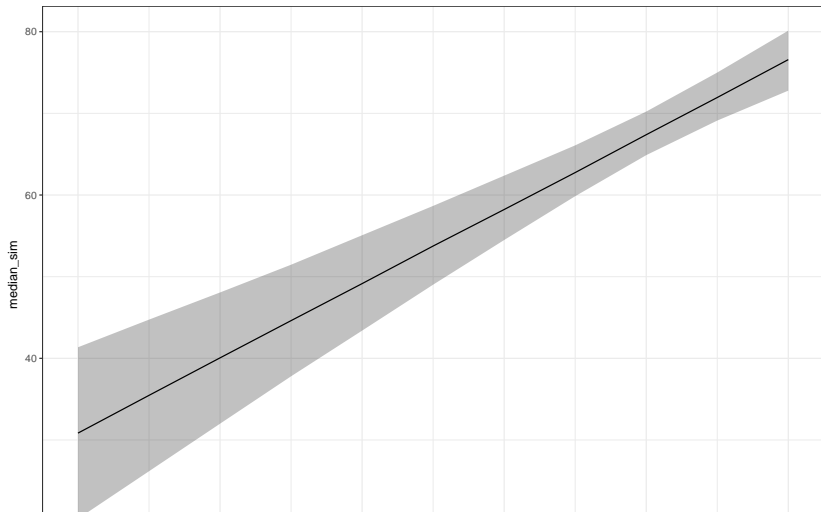


Simulations: find 95% central interval ribbons

```
# Find 95% central interval and median at each fitted value  
central <- drawn_sim %>% group_by(fitted_edu) %>%  
  summarise(median_sim = median(sim_wc),  
            lower_95 = quantile(sim_wc, probs = 0.025),  
            upper_95 = quantile(sim_wc, probs = 0.975)  
  )
```

Simulations: plot 95% central interval

```
ggplot(central, aes(fitted_edu, median_sim)) +  
  geom_ribbon(aes(ymin = lower_95, ymax = upper_95), alpha = 0.5) +  
  geom_line() + theme_bw()
```



Predictions from logistic regression

Use the same steps for simulating predicted outcomes from logistic regression models. The only difference is that the equation for the quantity of interest is:

$$P(y_i = 1) = \frac{\exp(\hat{\alpha} + X_{i1}\hat{\beta}_1 + \dots)}{1 + \exp(\hat{\alpha} + X_{i1}\hat{\beta}_1 + \dots)}$$

Easier Implementation

The Zelig package streamlines the simulation process.

Note: There appears to be a bug in Zelig version 5.0-13 when dealing with factor-level explanatory variables. So, use version 5.0-11:

```
devtools::install_version('Zelig', version = '5.0-11')
```

Zelig (1)

First estimate your regression model using zelig.

```
library(Zelig)

# Have to explicitly declare rank as factor
Admission$rank <- as.factor(Admission$rank)

Z1 <- zelig(admit ~ gre + gpa + rank, cite = FALSE,
            data = Admission, model = 'logit')
```

Zelig (2)

Then set the fitted values with `setx`.

```
setZ1 <- setx(Z1, gre = 220:800)
```

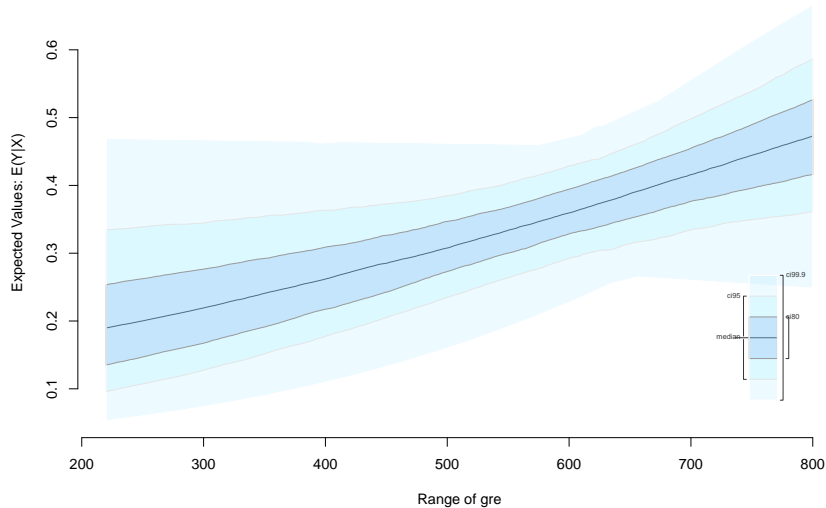
And run the simulations (1,000 by default) with `sim`.

```
simZ1 <- sim(Z1, x = setZ1)
```

Zelig (3)

Plot:

```
ci.plot(simZ1)
```



Seminar: modeling

Begin working on the statistical models for **your project**.

and/or

Out of Lecture Challenge: Estimate a normal regression model and **plot predicted values** across a range of fitted values. Bonus: do so with a measure of uncertainty.

References

King, Gary, Micheal Tomz, and Jason Wittenberg. 2001. "Making the Most of Statistical Analyses: Improving Interpretation and Presentation." *American Journal of Political Science* 22 (4): 341–255.