Hierdie opdrag moet **15:50 10 Oktober** in harde kopie ingehandig word. Geen elektroniese weergawes word aanvaar nie. Laat inhandiging sal gepenaliseer word. Samewerking word beperk tot die uitruil van enkele idees. **Die uitruil van kode, grafieke of besonderhede van wiskundige berekeninge word nie toegelaat nie.** Wat jy inhandig moet jou eie werk wees. **Plagiaat in opdragte sal ernstige gevolge hê.**

This assignment must be submitted on **15:50 10 October** in hard copy. No electronic versions will be accepted. Late submissions will be penalized. Cooperation is limited to the exchange of a few ideas. **The exchange of code, graphs or details of mathematical calculations are not allowed.** What you submit must be your own work. **Plagiarism in assignments will have severe consequences.**

Probleem 1: Verskillende soorte demping.

(a) Skryf die volgende veer-massa stelsel as 'n stelsel van eerste-orde DVs en los op vir $t \in [0, 10]$ m.b.v. `ode45`. Stip twee figure; (t, x) en (x, x').

Problem 1: Different kinds of damping.

(a) Write the following spring-mass system as a system of first-order DEs and solve with `ode45` for $t \in [0, 10]$. Plot two figures; (t, x) and (x, x').

$$x'' = -4x - x', \quad x(0) = 1, \quad x'(0) = 0.$$

(b) Herhaal (a) vir die onderstaande Duffing-tipe nie-lineêre veer-massa stelsel.

(b) Repeat (a) for the Duffing-type nonlinear spring-mass system below.

$$x'' = -4x - x^3, \quad x(0) = 1, \quad x'(0) = 0.$$

(c) Herhaal (a) vir die onderstaande veer-massa stelsel met gegewe nie-lineêre demping. Waarom is die dempingsterm nie net eenvoudig $-(x')^2$ nie?

(c) Repeat (a) for the spring-mass system with non-linear damping given below. Why is the damping term not simply $-(x')^2$?

$$x'' = -4x - |x'|x', \quad x(0) = 1, \quad x'(0) = 0.$$

Wenk: Dit is genoeg om al drie grafieke van (a)-(c) op twee figure in totaal te stip (i.e., (t, x) en (x, x') vir elke probleem). Benoem die grafiek ('labels') en gee 'n geskikte titel en name vir die asse.

Hint: It is sufficient to plot all three graphs from (a)-(c) on two figures total (i.e., (t, x) and (x, x') for each problem). Include suitable axis labels, title, and a legend.

Probleem 2: Stuksgewyse aangedrew.

Beskou die volgende ongedempte veer-massa stelsel met 'n stuksgewyse aangedrewe funksie:

Problem 2: Piecewise forcing.

Consider the following undamped spring-mass system with a piecewise forcing function:

$$x'' + 16x = \begin{cases} 0, & t < 2\pi \\ \sin(4t), & 2\pi \le t \le 4\pi \\ 0, & t > 4\pi \end{cases}, \quad x(0) = 1, \quad x'(0) = 0.$$

(a) Gebruik `ode45` om dit op te los. Skets (t, x) en (x, x') cir $0 \le t \le 25$ en lewer kommentaar oor die gedrag van die oplossing.

(a) Solve using `ode45`. Plot (t, x) and (x, x') for $0 \le t \le 25$ and comment on the behaviour of the solution.

(b) [**Bonus**] Skryf die Green's funksie vir hierdie DV neer en gebruik dit om die partikuliere oplossing van die nie-homogene DV hierbo te verkry. Toon dus aan dat die analitiese oplossing vir die AWP gegee word deur:

(b) [**Bonus**] Write down the Green's function for this DE and use it to obtain a particular solution for the inhomogeneous DE above. Hence show that the analytic solution to the IVP is given by:

$$x(t) = \cos(4t) + \begin{cases} 0, & 0 \le t < 2\pi, \\ \frac{1}{32}(\sin(4t) + (8\pi - 4t)\cos(4t)), & 2\pi \le t < 4\pi, \\ -\frac{\pi}{4}\cos(4t), & t > 4\pi. \end{cases}$$

(c) Bevestig die analitiese oplossing hiebo deur om dit te stip en vergelyk dit met die `ode45` oplossing.

(c) Confirm the analytic solution above by plotting it and comparing it to the `ode45` solution.

Probleem 3: Resonansie en "beats".

(a) Beskou die ongedempte maar aangedrewe veer-massa stelsel:

$$x'' = -4x + \cos((2+1/10)t), \quad x(0) = 0, \quad x'(0) = 0.$$

Wat verwag jy van die gedrag van hierdie stelsel? (Wenk: Onthou lesing 26.) Gebruik `ode45` om die AWP op te los vir $0 \le t \le 250$, en stip (t, x) op nuwe figure. Met verwysing na Lesing 26, voeg die lae frekwensie "omhulsel" as 'n stippellyn by jou figuur.

(b) Herhaal deel **(a)** vir die AWP

$$x'' = -4x + \cos(2t), \quad x(0) = 0, \quad x'(0) = 0.$$

(c) In klas het ons slegs die geval van 'suiwer resonansie', i.e., wanneer daar geen demping teenwoordig was nie. Sekere resonansie kan nog steeds voorkom, as die DV $x'' + 2\gamma x' + \omega^2 x = f(t)$ gedrewe is by $f(t) = F_0 \cos(\omega_f x)$ waar $\omega_f \approx \omega$. Demonstreer dit deur die volgende DV op te los soos in deel (a) (maar moet nie probeer om die omhulsel te stip nie).

$$x'' = -4x - \frac{1}{10}x' + \cos(2t), \quad x(0) = 0, \quad x'(0) = 0.$$

Vergelyk die skets van deel (c) met die van deel (b) en lewer kommentaar oor die verskille.

(d) [Opsioneel maar koel] MATLAB het 'n `sound` funksie, wat gebruik kan word om 'n vektor as klank te speel. Hoe dink jy sal die resultate van (a)–(c) klink? Probeer dit deur `sound(x, 1000)` uit te voer, waar `x` die eerste kolom van `ode45` se oplossing is.

Problem 3: Resonance and beats.

(a) Consider the undamped but forced spring-mass system:

How do you expect this system to behave? (Hint: Recall lecture 26.) Use `ode45` to solve the IVP for $0 \le t \le 250$ and plot (t, x). With reference to Lecture 26, add the low frequency "envelope" as a dashed line to your figure.

(b) Repeat part **(a)** for the IVP

(c) In class we considered only the case of 'pure resonance', i.e., when there was no damping present. Some resonance can still occur if the DE $x'' + 2\gamma x' + \omega^2 x = f(t)$ is forced at $f(t) = F_0 \cos(\omega_f x)$ where $\omega_f \approx \omega$. Demonstrate this by solving the following DE as in part (a) (but do not attempt to plot an envelope):

Compare your plot for part (c) to that of part (b) and comment on the differences.

(d) [Optional but cool] MATLAB has a `sound` function, which can be used to play a vector as a sound. What do you think the results of (a)–(c) would sound like? Try it out by running `sound(x, 1000)`, where `x` is the first column in your solution from `ode45`.

Probleem 4: Parametriese osilasie.

Nog 'n manier om resonansie te produseer in 'n veer-massa stelsel is deur sogenaamde "parametric forcing". Dus, inplaas daarvan om 'n eksterne krag te verskaf soos beskryf in Lesing 25, die dempingskrag of veer 'konstante' word verander met tyd om stelsel op te wek. Dit is soortgelyk aan hoe 'n kind (of volwassenne, ek oordeel nie) hulself op 'n swaai voortdryf deur om die middelpunt van hul massa periodies te verlaag en te verhoog.

Los die "pumped" veer-massa stelsel op

$$x'' = -4x(1 + \sin(2t)), \quad x(0) = 0.1, \quad x'(0) = 0,$$

deur gebruik te maak van `ode45` of iets soortgelyk en stip (t, x) vir $0 \le t \le 25$. Jy moet bevind dat die amplitude van hierdie osilasies eksponensieel groei. Bevestig dit deur 'n omhulsel $(t, 0.1e^{2t/11})$ as 'n stippellyn op jou figuur te stip.

Comment: Convince yourself that for this kind of forcing the system cannot start at rest, hence we take $x(0) = 0.1$.

Problem 4: Parametric oscillation.

Another way to produce resonance in spring-mass systems is through so-called "parametric forcing". Here, rather than supply and external force as described in Lecture 25, the damping-force or spring 'constant' is changed with time to excite the system. This is similar to the way a child (or adult, I don't judge) on a swing propels themselves by periodically raising or lowering their centre of mass.

Solve the 'pumped' spring-mass system

using `ode45` or some equivalent and plot (t, x) for $0 \le t \le 25$. You should find that the amplitude of these oscillations grow exponentially. Verify this by including the envelope $(t, 0.1e^{2t/11})$ as a dashed line on your plot.

Comment: Convince yourself that for this kind of forcing the system cannot start at rest, hence we take $x(0) = 0.1$.