Hierdie opdrag moet **15:50 08 Augustus** in harde kopie ingehandig word. Geen elektroniese weergawes word aanvaar nie. Laat inhandiging sal gepenaliseer word. Samewerking word beperk tot die uitruil van enkele idees. **Die uitruil van kode, grafieke of besonderhede van wiskundige berekeninge word nie toegelaat nie.** Wat jy inhandig moet jou eie werk wees. **Plagiaat in opdragte sal ernstige gevolge hê.**

This assignment must be submitted on **15:50 08 August** in hard copy. No electronic versions will be accepted. Late submissions will be penalized. Cooperation is limited to the exchange of a few ideas. **The exchange of code, graphs or details of mathematical calculations are not allowed.** What you submit must be your own work. **Plagiarism in assignments will have severe consequences.**

Probleem 1: Beskou die volgende outonome 1ste-orde DV:

Problem 1: Consider the following autonomous 1st-order DE:

$$\frac{dP}{dt} = P^3 - 4P^2 + 4P.$$

(a) Vind al die kritieke oplossings van hierdie DV.
(b) Sonder om die DV op te los, bepaal gebiede waar oplossings stygend/dalend is en ook waar dit konkaaf-op/konkaaf-af is.
(c) Skets met die hand 'n klompie tipiese oplossingskrommes in die tP-vlak.
(d) Gebruik dan jou skets om elke kritieke oplossing as stabiel, onstabiel of semi-stabiel te klassifiseer.
(e) Bevestig jou handskets van deel (c) met behulp van `dfield8`. (Wenk: 'n goeie plotstreek is $[0, 4]$ in t en $[-2, 4]$ in P.)

(a) Find all the critical solutions of this DE.
(b) Without solving the DE, determine regions where solutions increase/decrease and also where they are concave-up/concave-down.
(c) Sketch by hand a few typical solution curves in the tP-plane.
(d) Use your sketch to classify every critical point as stable, unstable, or semi-stable.
(e) Verify your hand sketch from part (c) with the aid of `dfield8`. (Hint: a good plotting region is $[0, 4]$ in t and $[-2, 4]$ in P.)

Probleem 2: Beskou die aanvangswaardeprobleem

Problem 2: Consider the differential equation

$$\frac{dy}{dx} = e^{-x}y.$$

(a) Gebruik skeiding van veranderlikes of die integrasiefaktor metode om te wys dat die oplossing vir hierdie metode gegee word deur $y(x) = ce^{-e^{-x}}$.

(a) Use separation of variables or the integrating factor method to show that the solution to this DE is given by $y(x) = ce^{-e^{-x}}$.

Beskou nou die aanvangswaardeprobleem

Consider now the initial value problem

$$\frac{dy}{dx} = e^{-x}y, \qquad y(0) = 1.$$

(b) Wys dat die oplossing gegee word deur

(b) Show that the solution is given by

$$y(x) = e^{1-e^{-x}},$$

en gebruik MATLAB of Python om hierdie funksie te skets vir $0 \leq x \leq 4$.

and use MATLAB or Python to plot this function of $0 \leq x \leq 4$.

Veronderstel nou dat die eksakte oplossing onbekend is en dat ons die probleem numeries wil oplos.

Now suppose that the exact solution is not known and that we wish to solve the problem numerically.

(c) Gebruik Euler se metode met staplengte $h = 1$ om waardes vir $y(1)$, $y(2)$, $y(3)$, $y(4)$ te bereken. Vergelyk jou resultate met die eksakte funksiewaardes (soos bereken met die gegewe oplossing) in beide 'n tabel en 'n grafiek.

(d) Gebruik die aangepaste Euler metode (soos in die klas bespreek) om dieselfde waardes as hierbo te bereken. Weereens, vergelyk die eksakte waardes. Is hierdie benaderings beter of slegter as die van deel (a)?

(e) Gebruik die `dfield8` sagteware om die rigtingsveld vir die bostaande DV te skets. Voeg die oplossingskurwe wat ooreenstem met aanvangswaarde $y(0) = 1$ by.

(c) Use Euler's method with step size $h = 1$ to calculate approximate values for $y(1)$, $y(2)$, $y(3)$, $y(4)$. Compare your results to the exact function values (as calculated with the given solution) in both a table and a graph.

(d) Use the modified Euler method (as discussed in class) to approximate the same values as above. Again, compare to the exact values. Are these approximations better or worse than those of part (a)?

(e) Use the `dfield8` software to plot the direction field for the DE above. Include the solution curve corresponding the initial condition $y(0) = 1$.
