Hierdie opdrag moet **15:50 22 Augustus** in harde kopie ingehandig word. Geen elektroniese weergawes word aanvaar nie. Laat inhandiging sal gepenaliseer word. Samewerking word beperk tot die uitruil van enkele idees. **Die uitruil van kode, grafieke of besonderhede van wiskundige berekeninge word nie toegelaat nie.** Wat jy inhandig moet jou eie werk wees. **Plagiaat in opdragte sal ernstige gevolge hê.**

This assignment must be submitted on **15:50 22 August** in hard copy. No electronic versions will be accepted. Late submissions will be penalized. Cooperation is limited to the exchange of a few ideas. **The exchange of code, graphs or details of mathematical calculations are not allowed.** What you submit must be your own work. **Plagiarism in assignments will have severe consequences.**

Probleem 1: Die tabel in Oefening 3.2.4 van Z&W (gereproduseer op die volgende bladsy) gee sensusdata van die Verenigde State bevolking vir 1790–1950. Jou taak is om 'n logistiese model vir die VS bevolking te konstrueer deur gebruik te maak van regressie (of "kleinste-kwadrate") gebaseer op die data in die tabel. Een manier om dit te doen is om $\frac{1}{P(t)}\frac{dP}{dt}$ te bereken in die logistiese model deur die voorwaartste verskil kwosiënt

$$Q(t) = \frac{1}{P(t)}\frac{P(t+10)-P(t)}{10}.$$

(a) Laat t die aantal verbygaande jare vanaf 1790 voorstel. Maak 'n tabel wat die t waardes, $P(t)$ en $Q(t)$ voorstel deur $t = 0, 10, 20, \ldots, 150$ te gebruik. (Let daarop dat ons nie tot $t = 160$ kan gaan nie, aangesien ons nie die bevolking van 1960 in ons datastel het nie). Wenk: $Q(0) = \frac{1}{P(0)}\frac{P(10)-P(0)}{10} = 0.035$.

Plot die data $(t + 1790, P(t))$ as sirkels op Figuur 1. Plot dan die data $(P(t), Q(t))$ op Figuur 2, ook as sirkels. Benoem die grafiek ('labels') en gee geskikte name vir die asse.

(b) Gebruik Matlab se `polyfit` funksie (of enige ander funksie) en bepaal die koëffisiënte a en b sodat $Q \approx a - bP$ in terme van kleinste-kwadrate vir die data in die bostaande tabel. (Wenk: Sien `help polyfit`). Superimponeer die kontinue lyn $Q = a - bP$ op jou plot in Figuur 2 vir jou berekende a en b waardes en P in die reeks $(0, 150)$.

(c) Los die logistiese model vir hierdie waardes van a en b op, met $P(0) = 3.929$. (Wenk: Sien Lesing 11.) Superimponeer 'n plot van die oplossing op Figuur 1 as 'n kontinue lyn tot en met die jaar 2018.

(d) Wat voorspel jou model vir die VS bevolking in 2018? Hoe vergelyk hierdie met die werklike bevolking? (Wenk: Gebruik Google!) Wat is die drakapasiteit van die VS volgens jou model? Dink jy die logistiese model is 'n goeie model vir hierdie data?

Problem 1: The table in Exercise 3.2.4 of Z&W (reproduced on the next page) gives census data for the population of the United States from 1790–1950. Your task is to construct a logistic model for the population of the US using regression (or "least squares") based on the data in the table. One way of doing this is to approximate $\frac{1}{P(t)}\frac{dP}{dt}$ in the logistic model by the forward difference quotient

(a) Letting t be the number of years that have passed since 1790, make a table of the values t, $P(t)$, and $Q(t)$ using $t = 0, 10, 20, \ldots, 150$. (Note we cannot go to $t = 160$ as this requires the population in 1960, which is not in our data set). Hint: $Q(0) = \frac{1}{P(0)}\frac{P(10)-P(0)}{10} = 0.035$.

On Figure 1, plot the data $(t + 1790, P(t))$ as circles. On Figure 2, plot the data $(P(t), Q(t))$, also as circles. Include suitable legends and axis labels.

(b) Using Matlab's `polyfit` function (or otherwise) determine the coefficients a and b so that $Q \approx a - bP$ in the least-squares sense for the data in your table above. (Hint: See `help polyfit`). For your computed values of a and b, superimpose the continuous line $Q = a - bP$ on your plot in Figure 2 for P in the range $(0, 150)$.

(c) Solve the logistic model for these values of a and b, with $P(0) = 3.929$. (Hint: See Lecture 11.) Superimpose a plot of the solution on to Figure 1 as a continuous line up until the year 2018.

(d) What does your model predict for the US population in 2018? How does this compare to the actual population? (Hint: Use Google!) According to your model, what is the carrying capacity of the US? Do you think the logistic model is a good model for this data?

Probleem 2: Beskou die Gompertz aanpassing (sien bl. 98 van Z&W) van die logistiese vergelyking:

$$\frac{dP}{dt} = P(a - b\ln P)$$

(a) Maak gebruik van skeiding van veranderlikes om hierdie DV op te los, en wys dat die oplossing geskryf kan word as

$$P(t) = e^{a/b + ce^{-bt}}.$$

Wenk: Jy mag die vervanging van $P = e^u$ nuttig vind.) Gebruik die aanvangsvoorwaarde $P(0) = 3.929$ om te wys dat $c = \log(3.929) - a/b$.

(b) Herhaal Probleem 1(b) maar bepaal nou a en b sodat $Q \approx a - b\ln P$. Voeg hierdie kurwe by Figuur 2.

(c) Herhaal Probleem 1(c) deur jou oplossing tot die Gompertz model te gebruik. Voeg die oplossingskurwe by Figuur 1.

(d) Wat is die beperking van kapasiteit (d.w.s., soos $t \to \infty$) vir hierdie model? Lewer kommentaar op die verskil tussen die twee modelle en hulle toepaslikheid tot hierdie sensusdata.

(e) Gebruik je model om te schatten wanneer de VS bevolking zal bereiken 600 miljoen.

Problem 2: Consider the Gompertz modification (see p. 98 of Z&W) of the logistic equation:

(a) Use separation of variables to solve this DE and show the solution may be written as

(Hint: You may find the substitution $P = e^u$ useful.) Use the initial condition $P(0) = 3.929$ to show that $c = \log(3.929) - a/b$.

(b) Repeat Problem 1(b) but now find a and b so that $Q \approx a - b\ln P$. Add this curve to Figure 2.

(c) Repeat Problem 1(c) but now using your computed solution to the Gompertz model. Add the solution curve to Figure 1.

(d) What is the limiting capacity (i.e., as $t \to \infty$) under this model? Comment on the difference between the two models and their applicability to this census data.

(e) Use your model to estimate when the US population will reach 600 million.

Problem 3: [Opsioneel] Besoek die onderstaande URL en voeg meer onlangse data by Figuur 1.

Problem 3: [Optional] Visit the URL below and add more recent data to Figure 1.

http://www.worldometers.info/world-population/us-population/

TABLE 3.2.1

Year	Population (in millions)
1790	3.929
1800	5.308
1810	7.240
1820	9.638
1830	12.866
1840	17.069
1850	23.192
1860	31.433
1870	38.558
1880	50.156
1890	62.948
1900	75.996
1910	91.972
1920	105.711
1930	122.775
1940	131.669
1950	150.697