Numeriese Metodes / *Numerical Methods* 324

Rekenaaropdrag 04 / *Computer Assignment* 04 - 2018

Sperdatum / *Deadline* - 13/04/2018

Let asseblief op:

- Opdragte word met die aanvang van die tutoriaalsessie op die sperdatum ingehandig. Laat inhandiging sal gepenaliseer word. Handig gedrukte weergawes in asb, geen elektroniese weergawes word aanvaar nie. **Jou opdrag moet beide jou kode en jou resultate insluit.**
- Alle werk ingehandig moet jou eie werk wees. Jy kan idees uitruil met ander studente, maar alle berekeninge en grafieke moet jou eie wees. **Stappe sal geneem word teen plagiaat.**
- Rekenaaropdragte moet in MATLAB of Python voltooi word. As jy 'n alternatiewe program wil gebruik, maak eers seker dat dit reg is met die dosent.
- Wanneer opdragte ingehandig word, is die voorkeurmetode MATLAB se `publish` of `livescript` funksie, maar dis nie verpligtend nie. Maar dit is heeltemal aanvaarbaar om jou kode in 'n MS Word dokument te plak of dit in LaTeX op te tik.

Please Note:

Computer assignments are handed in at the start of the tutorial session on the due date. There is a penalty if you hand in late. Please hand in hard copy, no electronic versions are accepted. 2**You should include both your code and your results.**

All work handed in must be your own work. You may exchange ideas with other students, but all calculations and graphs must be your own. **Action will be taken against plagiarism.**

Computer assignments must be completed using MATLAB or Python. If you would like to use an alternative, please clear this in advance with the course lecturer.

The preferred method for submitting assignments is MATLAB's `publish` or `livescript` function, but this is not compulsory. However, it is perfectly acceptable to paste your code to an MS Word document or to type it up in LaTeX.

Probleme:

1. Beskou die numeriese differensiasie formule herlei mbv ekstrapolasie in Lesing 18, naamlik

Problems:

1. Consider the numerical differentiation formula derived by extrapolation in Lecture 18, namely

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h} + O(h^4).$$

Vir die toetsprobleem

For the test problem

$$f(x) = \sqrt{1 - 2\sin x}, \qquad f'(0) = -1.$$

(a) Implementeer die formule en bereken absolute foute vir 100 staplengtes tussen 10^{-7} en 10^{-1}. (Wenk: Sien `logspace` in MATLAB.) Op 'n loglog plot, vertoon die fout as funksie van die staplengte en lei af wat die orde van konvergensie is op grond van die figuur. Is dit in ooreenstemming met die $O(h^4)$ voorspel deur die teorie?

(a) Implement the formula and compute absolute errors for a 100 step sizes between 10^{-7} and 10^{-1}. (Hint: See `logspace` in MATLAB.) On a loglog plot, display the error as function of the step size and deduce the order of convergence from the plot. Does your conclusion match the $O(h^4)$ convergence predicted by the theory?

(b) Herhaal die afrondingsfout analiese in Lesing XX vir hierdie probleem om te wys dat

(b) Repeat the roundoff error analysis in Lecture XX for this problem to show that

$$|f_{\text{exact}} - f_{\text{machine}}| \le \frac{18\epsilon}{12h} + Mh^4.$$

Plot die fout grens hierbo op die selfde figuur as in deel (a), neem $M = 11$.

Plot the error bound above on the same figure as in part (a) taking $M = 11$.

2. Verifieer dat Simpson se reël polynomiese mate van akkuraatheid het deur te wys dat dit presies is vir die integraall

2. Verify that Simpson's rule has polynomial degree of precision by showing it is exact for the integral

$$\int_a^b x^3\,dx.$$

3. (a) Gebruik die metode van ekstrapolasie om die tweede afgeleide benadering te bereken

(a) Use the method of extrapolation to derive the second derivative approximation

$$f''(x) = \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2} + O(h^4).$$

(b) Herhaal Probleem 1(a) en 1(b) om die tweede afgeleide van die selfde funkise $f(x)$ te bereken (Wenk: $f''(0) = 1$.)

(b) Repeat Problem 1(a) and 1(b) to approximate the second derivative of the same function $f(x)$ (Hint: $f''(0) = 1$.)

4. Beskou die integraal

Consider the integral

$$I(h) = \int_0^h e^x \, dx,$$

met die presiese oplossing $I(h) = e^h - 1$. Gebruik die trapezium, middelpuntreël, en Simpson reëls om die intergraal met $h = 1, 0.1,$ and 0.01 te benader. Bereken die fout in elke geval en stel die resultate voor in 'n tabel. Vir elk van die metodes plot (op die slefe figuur deur gebruik te maak van 'n log-log skaal) die absolute foute teenoor h. Stem die koers van konvergensie ooreen met die teoretiese benaderings bereken in die lesings?

with exact solution $I(h) = e^h - 1$. Use the trapezium, midpoint, and Simpson rules to approximate the integral with $h = 1, 0.1,$ and 0.01. Compute the errors in each case and display the results in a table. For each of the methods plot (on the same figure using a log-log scale) the absolute errors against h. Do the rates of convergence agree with the theoretical approximations derived in lectures?

5. Beskou die sentrale verskil benadering

Consider the central difference approximation

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2},$$

wat as 'n vektor-vektor vermenigvuldiging geskryf kan word

which can be written as a vector-vector multiplication

$$f''(x) \approx \frac{1}{h^2} \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} f(x-h) \\ f(x) \\ f(x+h) \end{bmatrix}.$$

As $\underline{x}$ 'n vektor is van n 'n eweredige gespasieerde waardes in $[0, 1]$ (bv. geskep deur `linspace(0,1,n)'`) sodat $x_1 = 1$ en $x_{j+1} = x_j + h$ waar $h = 1/n$, en $\underline{f}$ is 'n vektor $f(\underline{x})$, dan kan ons vir $j = 2, \ldots, n-1$ skryf

If $\underline{x}$ is a vector of n equally-spaced values in $[0, 1]$ (e.g., created by `linspace(0,1,n)'`) so that $x_1 = 1$ and $x_{j+1} = x_j + h$ where $h = 1/(n-1)$, and $\underline{f}$ is the vector $f(\underline{x})$, then for $j = 2, \ldots, n-1$ we could write

$$f''(x_j) \approx \frac{1}{h^2} \begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} f_{j-1} \\ f_j \\ f_{j+1} \end{bmatrix},$$

of in matriks vorm

or in matrix form

$$f''\left(\begin{bmatrix} x_2 \\ x_4 \\ \vdots \\ x_{n-1} \end{bmatrix} \right) \approx \frac{1}{h^2} \begin{bmatrix} 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{n-1} \\ f_n \end{bmatrix}.$$

(a) Skep die 6×8 matriks hierbo met $n = 8$ om die afgeleide van $f(x) = e^x$ te benader.

(a) Form the 6×8 matrix above with $n = 8$ to approximate the derivative of $f(x) = e^x$.

(b) [Optioneel] Gebruik die matriks van (a) om die grenswaarde probleem GDV op te los

(b) [Optional] Use the matrix from (a) to solve the boundary value problem ODE

$$u_{xx} + u = e^x, \qquad u(0) = u(1) = 0.$$