

Numeriese Metodes / *Numerical Methods* 324

Rekenaaropdrag 05 / *Computer Assignment* 05 - 2018
Sperdatum / *Deadline* - 04/05/2018

Let asseblief op:

- Opdragte word met die aanvang van die tutori-aalsessie op die sperdatum ingehandig. Laat in-handiging sal geenaliseer word. Handig gedrukte weergawes in asb, geen elektroniese weergawes word aanvaar nie. **Jou opdrag moet beide jou kode en jou resultate insluit.**
- Alle werk ingehandig moet jou eie werk wees. Jy kan idees uitruil met ander studente, maar alle berekeninge en grafieke moet jou eie wees. **Stappe sal geneem word teen plagiaat.**
- Rekenaaropdragte moet in MATLAB of Python voltooi word. As jy 'n alternatiewe program wil ge-bruik, maak eers seker dat dit reg is met die dosent.
- Wanneer opdragte ingehandig word, is die voorkeurmetode MATLAB se `publish` of `livescript` funksie, maar dis nie verpligtend nie. Maar dit is heeltemal aanvaarbaar om jou kode in 'n MS Word dokument te plak of dit in LaTeX op te tik.

Probleme:

1. Beskou die integraal

met die presiese oplossing $I = e - 1$. Gebruik die saamgestel trapezium, middelpuntreël, en Simpson reëls om die integraal met 11, 21, 7, ..., 101 punte te benader. Vir elk van die metodes plot (op die selfde grafiek met 'n log-log skaal) die absolute fout teenoor die aantal punte. Vergelyk jou resultate met die figuur geskep in Probleem 4 van Opdrag 04.

2. Met Mathematica, Wolfram Alpha, of `integral` in MATLAB bereken tot tenminste vyftiensyfer akku- raatheid

Herhaal Probleem 1 vir hierdie integrand, maar met 3, 7, 11, 15, 19 punte hier. Vergelyk met die figuur van Probleem 1 en lewer kommentaar op/verduide- lik die verskille.

Please Note:

Computer assignments are handed in at the start of the tutorial session on the due date. There is a penalty if you hand in late. Please hand in hard copy, no electronic versions are accepted. **You should include both your code and your re- sults.**

All work handed in must be your own work. You may exchange ideas with other students, but all calculations and graphs must be your own. **Action will be taken against plagiarism.**

Computer assignments must be completed using MATLAB or Python. If you would like to use an alternative, please clear this in advance with the course lecturer.

The preferred method for submitting assignments is MATLAB's `publish` or `livescript` function, but this is not compulsory. However, it is perfectly acceptable to paste your code to an MS Word document or to type it up in LaTeX.

Problems:

Consider the integral

$$I = \int_0^1 e^x dx,$$

with exact solution $I = e - 1$. Use the composite trapezium, midpoint, and Simpson rules to approx- imate the integral with 11, 21, 31, ..., 101 points. For each of the methods plot (on the same figure using a log-log scale) the absolute errors against the number of points. Compare your result with the figure obtained in Problem 4 of Assignment 04.

Use Mathematica, Wolfram Alpha, or `integral` in MATLAB to compute to at least fifteen digits of accuracy

$$I = \int_0^1 e^{\sin 2\pi x} dx.$$

Repeat Problem 1 for this integrand, but here using 3, 5, 7, 9, 11 points. Compare with the figure from Problem 1 and comment on/explain the differences.

3. (a) Laai die kode `romberg.m` of `romberg.py` af van die kursus webblad (of skryf jou eie).
 (b) Skep die 5×5 Romberg tabel vir die integrale in Probleme 1 en 2.
 (c) Maak twee nuwe matrikse wat die foute wys in elk van die twee tabelle. Watter inskrywing is mees akkuraat in elke geval? Verduidelik hoekom.
4. Laai `gauss.m` en `fejer.m` (of `.py`) af van die kursus webblad. Gebruik hierdie om die **5-punte** Gauss–Legendre en Fejèr benaderings te bereken vir die integraal in Probleem 1. Lewer kommentaar op die akkuraatheid van die benaderings in vergelyking met mekaar en met die resultate van die Romberg tabel in Probleem 3.
5. Gebruik Gauss–Legendre en Gauss–Chebyshev (sien Lesing 27) om die 5-punt kwadratuur benaderings te bereken vir die integraal

$$\int_{-1}^1 \frac{e^x}{\sqrt{1-x^2}}.$$

Met Mathematica, Wolfram Alpha, of `integral` in MATLAB bereken tot tenminste tien syfers die integraal en kontroleer die akkuraatheid van jou twee benaderings. Lewer kommentaar op die resultate.

6. (a) Gebruik die Euler metode met $n = 10, 100, 1000$ om die vryval-probleem met kwadratiese lugweerstand in Lesing 30 vir $0 \leq t \leq 10$ op te los, waar $g = 9.81$ en $c_2 = 1/1000$. Bereken die fout by $t = 10$ en maak 'n loglog skets van die fout vir elke n teenoor h om te bevestig dat die globale fout $\mathcal{O}(h)$ is.
- (b) Herhaal met die eksplisiete trapezium en eksplisiete middelpuntreël. Plot al drie loglog plotte op die selfde figuur. Lewer kommentaar op die resultaat.

- (a) Download the code `romberg.m` or `romberg.py` from the course website (or write your own).
 (b) Create the 5×5 Romberg tables for the integrals in Problems 1 and 2.
 (c) Make two new matrices showing the errors in each of these two tables. Which entry is the most accurate in each case? Explain why.

Download `gauss.m` and `fejer.m` (or `.py`) from the course webpage. Use these to compute the **5-point** Gauss–Legendre and Fejèr approximations to the integral in Problem 1. Comment on the accuracy of these approximations compared to each other and to the results of the Romberg table in Problem 3.

Use Gauss–Legendre and Gauss–Chebyshev (see Lecture 27) to compute 5-point quadrature approximations to the integral

Use Mathematica, Wolfram Alpha, or `integral` in MATLAB to compute the integral to at least 10 digits and check the accuracy of your two approximations. Comment on the results.

- (a) Use Euler's method with $n = 10, 100, 1000$ to solve the free-fall problem with quadratic air resistance from Lecture 30 for $0 \leq t \leq 10$ with $g = 9.81$ and $c_2 = 1/1000$. Compute the error at $t = 10$ and make a loglog plot of the error for each n versus h to confirm the global error is $\mathcal{O}(h)$.
- (b) Repeat using the explicit trapezium and explicit midpoint rules. Plot all three loglog plots on the same figure. Comment on the result.

7. **Opsioneel:** Die veranderlike x kan uit die vergelyking

$$\frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt = 0.45$$

opgelos word deur Newton se metode met

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2} dt - 0.45$$

Om f te evalueer by die benadering p_k , het ons 'n kwadratuurformule nodig om die volgende te benader x_k

$$f(x_k) = \frac{1}{\sqrt{2\pi}} \int_0^{x_k} e^{-t^2/2} dt - 0.45.$$

Vind 'n oplossing vir $f(x) = 0$ wat akkuraat tot en met 10^{-5} is deur Newton se metode te gebruik met $x_0 = 0.5$ en a kwadratuur oppergesag van jou keuse.

Optional: The equation:

can be solved for x by using Newton's method with en / and $f'(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$.

To evaluate f at the approximation x_k , we need a quadrature formula to approximate

Find a solution to $f(x) = 0$ accurate to within 10^{-5} using Newton's method with $x_0 = 0.5$ and a quadrature rule of your choice.