Numeriese Metodes / *Numerical Methods* 324

Rekenaaropdrag 01 / *Computer Assignment* 02 - 2018

Sperdatum / *Deadline* - 02/03/2018

Let asseblief op:

- Opdragte word met die aanvang van die tutoriaalsessie op die sperdatum ingehandig. Laat inhandiging sal gepenaliseer word. Handig gedrukte weergawes in asb, geen elektroniese weergawes word aanvaar nie. **Jou opdrag moet beide jou kode en jou resultate insluit.**
- Alle werk ingehandig moet jou eie werk wees. Jy kan idees uitruil met ander studente, maar alle berekeninge en grafieke moet jou eie wees. **Stappe sal geneem word teen plagiaat.**
- Rekenaaropdragte moet in MATLAB of Python voltooi word. As jy 'n alternatiewe program wil gebruik, maak eers seker dat dit reg is met die dosent.
- Wanneer opdragte ingehandig word, is die voorkeurmetode MATLAB se `publish` of `livescript` funksie, maar dis nie verpligtend nie. Maar dit is heeltemal aanvaarbaar om jou kode in 'n MS Word dokument te plak of dit in LaTeX op te tik.

Please Note:

Computer assignments are handed in at the start of the tutorial session on the due date. There is a penalty if you hand in late. Please hand in hard copy, no electronic versions are accepted. **You should include both your code and your results.**

All work handed in must be your own work. You may exchange ideas with other students, but all calculations and graphs must be your own. **Action will be taken against plagiarism.**

Computer assignments must be completed using MATLAB or Python. If you would like to use an alternative, please clear this in advance with the course lecturer.

The preferred method for submitting assignments is MATLAB's `publish` or `livescript` function, but this is not compulsory. However, it is perfectly acceptable to paste your code to an MS Word document or to type it up in LaTeX.

Probleme:

1 Heroorweeg die vaste punt iterasies van tutoriaal 1, naamlik:

$$\text{(a)}\quad x \to \frac{1}{2}x + \frac{1}{x}, \qquad \text{(b)}\quad x \to \frac{2}{3}x + \frac{2}{3x}, \qquad \text{(c)}\quad x \to \frac{3}{4}x + \frac{1}{2x}.$$

Kies 'n begin raaiskoot (skatting) van $x = 1$ en produseer 'n tabel van die eerste 10 iterasies vir elke een van die funksies hierbo. Verbind die koers van konvergensie / divergensie na jou resultate van Probleem 4 van tutoriaal 1.

2a Die skopper van die Maties rugby span will graag die toeskouers beindruk/frustreer deur al die balle wat hy skop op die pale se dwarslat te laat hop. Die bekende vergelyking vir projektiele beweging,

$$y = x\tan(\theta) - \frac{1}{2}\frac{x^2 g}{v_0^2}\frac{1}{\cos^2(\theta)},$$

regeer die beweging van die bal na dit geskop is. As die pale 20m ver is, die dwarslat 3m van die grond is en die aanvanklike snelheid $v_0 = 17$m/s is, teen watter hoek θ moet die speler die bal skop? (gebruik enige metode wat jy wil, insluitend `fzero`, los die relevante vergelyking op en neem $g = 9.81$.)

Problems:

Revisit the fixed point iterations from tutorial 1, namely:

Choose a starting guess of $x = 1$ and produce a table of the first 10 iterations for each of the functions above. Relate the rate of convergence / divergence to your results from Problem 4 of Tutorial 1.

The kicker for the Maties rugby team wishes to impress/frustrate the crowd by having all his kicks bounce off the crossbar of the posts. The familiar equation of projectile motion,

governs the motion of the ball after it is kicked. If the posts are 20m away, the crossbar is 3m from the ground, and the initial velocity is $v_0 = 17$m/s, at what angle θ must the player kick the ball? (Use any method you like, including `fzero`, the solve the relevant equation, and take $g = 9.81$.)

2b [Optional] Plot the solution obtained above. Does it seem physically accurate? Implement instead the projectile motion with damping (TW244 Lecture 20) taking $r = 0.1$. Show that in this case there are two solutions for θ; one near $\pi/5$ and another near $\pi/3$. Plot all three solutions on the same graph.

3a Die funksie $f(x) = xe^{x-1} - 1$ het die eksakte wortel $r = 1$. Skryf die Newton formule neer vir die berekening van hierdie wortel en implementeer dit in jou sagteware stelsel. Kies 'n aanvanklike skatting naby $r = 1$ en lys die benaderings tot alle syfers. Onderstreep korrekte syfers en lei vervolgens af dat die konvergensie kwadraties is.

The function $f(x) = xe^{x-1} - 1$ has the exact root $r = 1$. Write down the Newton formula for the computation of this root and implement it in your software system. Pick an initial guess close to $r = 1$ and list the approximations to all digits. Underline correct digits and hence deduce that the convergence is quadratic.

3b Herhaal Probleem 1 vir die funksie $g(x) = -xe^{1-x} + 1$. Lei af dat in hierdie geval is die konvergensie slegs lineer.

Repeat Problem 1 for the function $g(x) = -xe^{1-x} + 1$. Deduce that in this case the convergence is only linear.

3c Deur afgeleides (met die hand) te neem, bereken die veelvuldigheid a van die wortel $r = 1$, vir beide funksies $f(x)$ and $g(x)$. Lei af dat die berekening van hierdie wortel van $f(x)$ goed-geaard is, maar sleg-geaard vir $g(x)$.

By taking derivatives (by hand), compute the multiplicity a of the root $r = 1$ for both functions $f(x)$ and $g(x)$. Deduce that the computation of this root is a well-conditioned problem for $f(x)$ but ill-conditioned for $g(x)$.

3d In Stelling 1.13 in Sauer word 'n gewysigde Newton metode aangebied. Gebruik hierdie metode om die wortel $r = 1$ van $g(x)$ te bereken. Lys die benaderings tot volle akkuraatheid en onderstreep korrekte syfers. Word kwadratiese konvergensie herwin soos beweer in die stelling? Kan die wortel tot volle akkuraatheid bereken word met hierdie metode? Maw, is die sleg-geaardheid oorkom? (Wenk: Die gewysigde metode is om $x_{n+1} = x_n - mf(x_n)/f'(x_n)$ te neem, waar m die veelvoud van die wortel is.)

In Theorem 1.13 in Sauer a modified Newton method is presented. Use this method to compute the root $r = 1$ of $g(x)$. List the approximations to full accuracy and underline correct digits. Is quadratic convergence restored as claimed in the Theorem? Can the root be computed to full precision with this method? I.e., has the ill-conditioning been thwarted? (Hint: The modified method is to take $x_{n+1} = x_n - mf(x_n)/f'(x_n)$, where m is the multiplicity of the root.)

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4 Implimenteer sewe iterasies van die Secant metode vir die probleem van $\sqrt{1/9}$ vind (met begin raaiskote (skattings) $x_0 = 1$, $x_1 = 0.5$) en vandaar produseer weer die tabel op bladsy 8 van les 7. Pas Newton se metode toe op die selfde probleem en toon die resultate langs mekaar aan.

Implement seven iterations of the Secant method for the problem of finding $\sqrt{1/9}$ (with starting guesses $x_0 = 1$, $x_1 = 0.5$) and hence reproduce the table from slide 8 of Lecture 7. Apply Newton's method to the same problem and display the results side by side.

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5 Gebruik Newton se metode om die nie-lineêre stelsel van vergelykings optelos:

Use Newton's method to solve the nonlinear system of equations:

$$xe^y = 7 - y, \quad \text{en/and} \quad \sin(x) = \cos(y).$$

Kies 'n begin punt vir $x_0 = [0, 0]^T$ en voer 8 iterasies uit. Wys jou finale antwoord, x^*, sowel as die terugwaartse fout, $f(x^*)$.

Choose a starting for $x_0 = [0, 0]^T$ and perform 8 iterations. Show your final solution, x^*, as well as the backward error, $f(x^*)$.

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6 [Opsioneel] Verminder die stelsel van probleem 5 na 'n ekwivalente probleem met 'n enkele onbekende en gebruik `fzero` of iets ekwivalent om dit op te los.

[Optional] Reduce system from Problem 5 to an equivalent problem in a single unknown and use `fzero` or some equivalent to solve it.

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7 [Opsioneel] Implementeer inverse iterasie om $x = \sqrt{1/9}$ te vind met die begin raaiskote (skattings) as $x_0 = 1$, $x_1 = 0.5$, $x_2 = 11/27$. Vergelyk dit met jou resultate van probleem 3. (Wenk: Gebruik `polyfit`.)

[Optional] Implement inverse iteration to find $x = \sqrt{1/9}$ with initial guesses $x_0 = 1$, $x_1 = 0.5$, $x_2 = 11/27$. Compare with your results from Problem 3. (Hint: Use `polyfit`.)