Numeriese Metodes / *Numerical Methods* 324

Rekenaaropdrag 05 / *Computer Assignment* 05 - 2018

Sperdatum / *Deadline* - 04/05/2018

Let asseblief op:

- Opdragte word met die aanvang van die tutoriaalsessie op die sperdatum ingehandig. Laat inhandiging sal gepenaliseer word. Handig gedrukte weergawes in asb, geen elektroniese weergawes word aanvaar nie. **Jou opdrag moet beide jou kode en jou resultate insluit.**
- Alle werk ingehandig moet jou eie werk wees. Jy kan idees uitruil met ander studente, maar alle berekeninge en grafieke moet jou eie wees. **Stappe sal geneem word teen plagiaat.**
- Rekenaaropdragte moet in MATLAB of Python voltooi word. As jy 'n alternatiewe program wil gebruik, maak eers seker dat dit reg is met die dosent.
- Wanneer opdragte ingehandig word, is die voorkeurmetode MATLAB se `publish` of `livescript` funksie, maar dis nie verpligtend nie. Maar dit is heeltemal aanvaarbaar om jou kode in 'n MS Word dokument te plak of dit in LaTeX op te tik.

Please Note:

Computer assignments are handed in at the start of the tutorial session on the due date. There is a penalty if you hand in late. Please hand in hard copy, no electronic versions are accepted. 2**You should include both your code and your results.**

All work handed in must be your own work. You may exchange ideas with other students, but all calculations and graphs must be your own. **Action will be taken against plagiarism.**

Computer assignments must be completed using MATLAB or Python. If you would like to use an alternative, please clear this in advance with the course lecturer.

The preferred method for submitting assignments is MATLAB's `publish` or `livescript` function, but this is not compulsory. However, it is perfectly acceptable to paste your code to an MS Word document or to type it up in LaTeX.

Probleme:

1. Beskou die integraal

$$I = \int_0^1 e^x \, dx,$$

met die presiese oplossing $I = e - 1$. Gebruik die saamgestel trapezium, middelpuntreël, en Simpson reëls om die intergraal met $11, 21, 7, \ldots, 101$ punte te benader. Vir elk van die metodes plot (op die selfde grafiek met 'n log-log skaal) die absolute fout teenoor die aantal punte. Vergelyk jou resultate met die figuur geskep in Probleem 4 van Opdrag 04.

2. Met Mathematica, Wolfram Alpha, of `integral` in MATLAB bereken tot tenminste vyftiensyfer akkuraatheid

$$I = \int_0^1 e^{\sin \pi x} \, dx.$$

Herhaal Probleem 1 vir hierdie integrand, maar met $3, 5, 7, 9, 11$ punte hier. Vergelyk met die figuur van Probleem 1 en lewer kommentaar op/verduidelik die verskille.

Problems:

1. Consider the integral

$$I = \int_0^1 e^x \, dx,$$

with exact solution $I = e - 1$. Use the composite trapezium, midpoint, and Simpson rules to approximate the integral with $11, 21, 31, \ldots, 101$ points. For each of the methods plot (on the same figure using a log-log scale) the absolute errors against the number of points. Compare your result with the figure obtained in Problem 4 of Assignment 04.

2. Use Mathematica, Wolfram Alpha, or `integral` in MATLAB to compute to at least fifteen digits of accuracy

$$I = \int_0^1 e^{\sin \pi x} \, dx.$$

Repeat Problem 1 for this integrand, but here using $3, 5, 7, 9, 11$ points. Compare with the figure from Problem 1 and comment on/explain the differences.

3. (a) Laai die kode `romberg.m` of `romberg.py` af van die kursus webblad (of skryf jou eie).
(b) Skep die 5×5 Romberg tabel vir die integrale in Probleme 1 en 2.
(c) Maak twee nuwe matrikse wat die foute wys in elk van die twee tabelle. Watter inskrywing is mees akkuraat in elke geval? Verduidelik hoekom.

(a) Download the code `romberg.m` or `romberg.py` from the course website (or write your own).
(b) Create the 5×5 Romberg tables for the integrals in Problems 1 and 2.
(c) Make two new matrices showing the errors in each of these two tables. Which entry is the most accurate in each case? Explain why.

4. Laai `gauss.m` en `fejer.m` (of `.py`) af van die kursus webblad. Gebruik hierdie om die Gauss–Legendre en Fejèr benaderings te bereken vir die integraal in Probleem 1. Lewer kommentaar op die akkuraatheid van die benaderings in vergelyking met mekaar en met die resultate van die Romberg tabel in Probleem 3.

Download `gauss.m` and `fejer.m` (or `.py`) from the course webpage. Use these to compute the Gauss–Legendre and Fejèr approximations to the integral in Problem 1. Comment on the accuracy of these approximations compared to each other and to the results of the Romberg table in Problem 3.

5. Gebruik Gauss–Legendre en Gauss–Chebyshev (sien Lesing 27) om die 5-punt kwadratuur benaderings te bereken vir die integraal

Use Gauss–Legendre and Gauss–Chebyshev (see Lecture 27) to compute 5-point quadrature approximations to the integral

$$\int_{-1}^{1} \frac{e^x}{\sqrt{1-x^2}}.$$

Met Mathematica, Wolfram Alpha, of `integral` in MATLAB bereken tot tenminste tien syfers die integraal en kontroleer die akkuraatheid van jou twee benaderings. Lewer kommentaar op die resultate.

Use Mathematica, Wolfram Alpha, or `integral` in MATLAB to compute the integral to at least 10 digits and check the accuracy of your two approximations. Comment on the results.

6. (a) Gebruik die Euler metode met $n = 10, 100, 1000$ om die vryval-probleem met kwadratiese lugweerstand in Lesing 30 vir $0 \le t \le 10$ op te los, waar $g = 9.81$ en $c_2 = 1/1000$. Bereken die fout by $t = 10$ en maak 'n loglog skets van die fout vir elke n teenoor h om te bevestig dat die globale fout $\mathcal{O}(h)$ is.

(a) Use Euler's method with $n = 10, 100, 1000$ to solve the free-fall problem with quadratic air resistance from Lecture 30 for $0 \le t \le 10$ with $g = 9.81$ and $c_2 = 1/1000$. Compute the error at $t = 10$ and make a loglog plot of the error for each n versus h to confirm the global error is $\mathcal{O}(h)$.

(b) Herhaal met die eksplisiete trapezium en eksplisiete middelpuntreël. Plot al drie loglog plotte op die selfde figuur. Lewer kommentaar op die resultaat.

(b) Repeat using the explicit trapezium and explicit midpoint rules. Plot all three loglog plots on the same figure. Comment on the result.

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7. **Opsioneel:** Die veranderlike x kan uit die vergelyking

Optional: The equation:

$$\frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2}\, dt = 0.45$$

opgelos word deur Newton se metode met

can be solved for x by using Newtons method with

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-t^2/2}\, dt - 0.45 \qquad \text{en / and} \qquad f'(x) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}.$$

Om f te evalueer by die benadering p_k, het ons 'n kwadratuurformule nodig om die volgende te benader x_k

To evaluate f at the approximation x_k, we need a quadrature formula to approximate

$$f(x_k) = \frac{1}{\sqrt{2\pi}} \int_0^{x_k} e^{-t^2/2}\, dt - 0.45.$$

Vind 'n oplossing vir $f(x) = 0$ wat akkuraat tot en met 10^{-5} is deur Newton se metode te gebruik met $x_0 = 0.5$ en a kwadratuur oppergesag van jou keuse.

Find a solution to $f(x) = 0$ accurate to within 10^{-5} using Newtons method with $x_0 = 0.5$ and a quadrature rule of your choice.