(3 Hours) **[Total Marks: 100]**

N.B. : (1) **All** questions are **compulsory**.

(2) **Figures** to the **right** indicate **full** marks.

(3) Draw **neat** diagrams wherever **necessary**.

(5) Symbols have usual meaning unless otherwise stated.

(5) Use of **non-programmable** calculator is allowed.

Q1. Attempt any **two**:---

(i) What is meant by a sample space? Give its types with suitable examples. Consider an experiment of tossing two dices and write a uniform sample space showing all possible events. Find the probability of each sample point. Also find probability that sum of the numbers on the dice is 5. **10**

(ii) What is a random variable? Explain what you mean by probability function and cumulative distribution function. **10**

(iii) Explain the terms mean value, standard deviation and variance. Consider an experiment of tossing 3 coins. Write the sample space for a variable $x = number\ of\ heads$ and find mean, standard deviation and variance. **10**

(iv) What are Bernoulli trials? Explain Binomial distribution. If n=8 and $p = q = \frac{1}{2}$ find binomial probability function for $x = 2\ and\ x = 4$. **10**

Q2 Attempt any **two**:---

(i) Explain hyperbolic functions of complex numbers. Using the definition of hyperbolic functions find the values of **10**

(a) $\cosh^2 z - \sinh^2 z$

(b) $\frac{d(\cos h\, z)}{dz}$

(ii) An ac source is applied across a resistor, inductor and capacitor in series. Show that to calculate the total impedance of the circuit, it is much simpler if we take current $I = I_0\, e^{i\omega t}$ rather than $I = I_0 \sin \omega t$. **10**

(iii) Explain the successive integration method for solving a second order nonhomogeneous equation with constant coefficients. Hence solve the differential equation: **10**

$$y'' + 3y' + 2y = e^{-x}$$

(iv) Solve the two-dimensional Laplace equation using the method of separation of variables. **10**

Q3 Attempt any **two**:---

(i) Define "weight" of a configuration. Explain how one calculates the number of microstates associated with a given configuration with an example of an oscillator. **10**

	(ii)	Derive the expression of total energy E and internal energy U for canonical ensemble, $$E = NkT^2\left(\frac{d\,lnq}{dT}\right), \qquad U = -\left(\frac{\partial \ln Q}{\partial \beta}\right)_V$$	**10**
	(iii)	What is degeneracy? Explain with example how degeneracy influence probability of occupying a given energy level.	**10**
	(iv)	Derive the relation between entropy S and canonical partition function Q, $$S = \frac{U}{T} + k\,lnQ$$	**10**
Q4		Attempt any **two:---**	
	(i)	Derive Maxwell Boltzmann distribution law in terms of α and β, and evaluate $e^{-\alpha}$.	**10**
	(ii)	Consider a large box of area A divided into k cells of area a_1, a_2 , …a_k, N identical balls are thrown in a completely random manner. Show that the most probable no. of balls in any cell is equal to the average density of ball (N/ A) multiplied by the area of cell.	**10**
	(iii)	Apply the Bose-Einstein statistics to photons and obtain Planck's law for black body radiation.	**10**
	(iv)	Derive Bose Einstein's distribution law.	**10**
Q5.		Attempt any **four:---**	**20**
	(i)	Consider tossing of dice 5 times. Find the probability of getting exactly 3 aces.	**05**
	(ii)	Consider two successive events A and B.Show that probability of the compound event "A and B" is the product of the probability that A will happen times the probability that B will happen if A does.	**05**
	(iii)	Describe the motion of a particle given by $z = 1 + 3\,e^{2it}$.	**05**
	(iv)	Show that $u = cosx\,cosat$ is a solution of the partial differential equation $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$	**05**
	(v)	Describe what is meant by the phrase "the dominant configuration".	**05**
	(vi)	Write a note on equipartition theorem.	**05**
	(vii)	Calculate the temperature in which the average speed of hydrogen molecule will be same as the average speed of nitrogen molecule at 27°C. The molecular weight of nitrogen and hydrogen is 28 and 2 respectively.	**05**
	(viii)	Solar spectrum shows that maximum wavelength is emitted at wavelength 5500 AU. Assuming the sun as a black body, estimate its temperature. (Given Wien's constant = 2.9 x 10^{-3} meter-K)	**05**
