

Course: Mathematics Paper-III

Course Code: USMT303

Maximum marks: 75

Duration: 2 ½Hrs

Instructions:

All questions are compulsory and carry equal marks

Use of scientific calculator is allowed

Figures to the right indicate full marks

Q.1 Attempt any four of the following

20

- State general linear differential equation.
- Show that linear combination of solution of linear homogeneous differential equation is also solution
- What are linear independence and dependence?
- Find the Wronskian of 1) $y_1 = \cos x$, $y_2 = \sin x$ 2) $y_1 = e^x$, $y_2 = e^{-x}$
- Evaluate $(D^2 + a^2)^{-1} \sin ax$

Q.2 Attempt any four of the following

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- Replace the differential equation 1) $y'' - x^2 y' - xy = 0$ 2) $y''' - y'' - x^2 (y')^2$ by an equivalent system of first order differential equation
- If $w(t)$ is the wronskian of two solutions $x = x_1(t)$, $y = y_1(t)$, and $x = x_2(t)$, $y = y_2(t)$
 $x_1(t) = x_2(t) = x(t)$, $y_1(t) = y_2(t) = y(t)$ Of homogeneous system,
 $\frac{dx}{dt} = a_1(t)x + b_1(t)y$ $\frac{dy}{dt} = a_2(t)x + b_2(t)y$
- Explain the linear system of first order differential equation.
- State Existence and Uniqueness theorem of linear system of equation
- Find general solution of $\frac{dx}{dt} = -3x + 4y$, $\frac{dy}{dt} = -2x + 3y$

Q.3 Attempt any four of the following

20

- Explain Taylor series method
- Explain Runge Kutta method of fourth order
- Solve by Euler's method $y' = -y$ with initial condition that $y(0) = 1$
- Determine the value of x when $x=0.1$ given that $y(0) = 1$, $y' = x^2 + y$
- Use Runge Kutta method of second order method to find an approximate value of f at $x=0.1$, 0.2 if
 $\frac{dy}{dx} = y - x$ given that $y = 2$ at $x = 0$

Q.4 Attempt any three of the following

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- Find the general solution of $y'' = 4$
- Explain solution of Legendre's linear differential equation
- Explain homogeneous linear system with constant coefficient
- Given $y' = 2xy$, $y(1) = 2$ estimate $y(1.75)$ using modified Euler's method