



Course: Mathematics Paper-I

Course Code: USMT101

Maximum marks: 75

Duration: 2 ½ Hrs

Instructions:

All questions are compulsory and carry equal marks

Figures to the right indicate full marks

Q.1 Attempt any four of the following

20

- Define the terms 1) Least Upper Bound 2) Greatest Lower Bound 3) Neighbourhood of a point.
- By applying Hausdroff property Find disjoint neighbourhoods of 1.5 and 1.6
- Find the least upper bound of set $S = \{x \in \mathbb{R} \mid |x + 12| \leq 12\}$
- Show that if $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ then $\inf S = 0$
- State and prove Cauchy Schwarz inequality.

Q.2 Attempt any four of the following

20

- Examine whether the sequences 1) $x_n = \frac{4}{n+1}$ 2) $x_n = 3n^2 + 2$ is bounded or not?
- Using definition of limit show that $\lim_{n \rightarrow \infty} \left(\frac{2n+3}{n+1}\right) = 2$
- State and prove Sandwich theorem
- Define the following terms 1) Monotonic Sequences 2) Cauchy Sequences
- Find the convergence of $a_n = \left(1 + \frac{3}{n}\right)^{4n}$

Q.3 Attempt any four of the following

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- Explain ordinary differential equation with suitable example
- Solve $(2x^3 + 3y)dx + (3x + y - 1)dy = 0$
- Solve $y^2 dx + (x^2 - xy)dy = 0$
- Find the solution of $\frac{dy}{dx} + y = x$
- Determine the orthogonal trajectories of hyperbolas $xy = c$

Q.4 Attempt any three of the following

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- State 1) Arithmetic-Geometric mean inequality 2) Archimedean property 3) Hausdroff property
- Prove that a converging sequence has unique limit
- By using Sandwich theorem show that the sequence $x_n = \left(\frac{\sin n}{n}\right)$ is convergent
- Find the particular solution of equation $\frac{dy}{dx} \left(\frac{d^2 y}{dx^2}\right) - x = 0$ given $y(1) = 2, y'(1) = 1$