

# Shri S. H. Kelkar College of Arts, Commerce and Science, Devgad

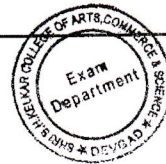
## S.Y.BSC SEMISTER-IV Examination-March 2023

Course: Mathematics Paper-II

Course Code: USMT421402

Maximum marks: 75

Duration: 2 ½ Hrs



### Instructions:

All questions are compulsory and carry equal marks

Figures to the right indicate full marks

#### Q.1 Attempt any four of the following

20

- Define Linear Transformation and explain identity transformation.
- Let  $T: U \rightarrow V$  be linear transformation, Prove that  $\text{Ker}T$  is subspace of vector space  $U$
- Check whether the linear transformation defined by  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$   $T(x, y, z) = (x, y + z)$  is linear or not?
- Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be defined as  $T(x, y) = (x + y, x - y)$  verify Rank-Nullity theorem.
- Check whether the linear transformation  $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ,  $L(x, y, z) = (x + y, x - z, y + 2z)$  is invertible or not.

#### Q.2 Attempt any four of the following

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- Let  $V$  be set of continuous real valued function define on  $[-\pi, \pi]$  then addition in  $V$  is defined as  $(f + g)(t) = f(t) + g(t) \forall t \in [-\pi, \pi]$  and  $(\alpha.f)(t) = \alpha f(t) \forall t \in [-\pi, \pi]$  Define  $\langle f, g \rangle = \int_{-\pi}^{\pi} f(t).g(t)dt$  Show that  $V$  forms inner product space.
- Consider vectors  $u$  and  $v$  in Euclidean inner product space, compute  $\langle u, v \rangle$ ,  $\|u\|$ ,  $d(u, v)$  for  $u=(1, 1, 3)$ ,  $v=(2, -2, 7)$ .
- Define the following  
1) Orthogonal set 2) Orthonormal Set 3) Unit vector
- Find the projection vector of  $u$  along  $v$  where  $u=(6, 1, -1, 4)$ ,  $v=(1, -1, 0, 0)$ .
- Let  $W$  be subspace of  $V$  then  $W^T$  is also subspace of  $V$

#### Q.3 Attempt any four of the following

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- State Cayley Hamilton theorem and verify it for matrix  $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & 0 & 4 \end{bmatrix}$
- Define Characteristic polynomial, characteristic equation and hence determine it for  $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$
- Show that two matrices  $A, PAP^{-1}$  have same characteristic roots
- For the given matrix find Algebraic and geometric multiplicity where  $B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$
- If  $v_1, v_2, \dots, v_p$  be eigen vectors of matrix  $A$  corresponding to distinct eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_p$  Respectively, then  $v_1, v_2, \dots, v_p$  are linearly independent.



**Q.4 Attempt any three of the following**

**15**

- a) Let  $T: R^3 \rightarrow R^3$  be a linear transformation defined by  $T(x, y, z) = (x + y, y - z)$  Find matrix of  $T$  with natural basis.
- b) Explain concept of linear isomorphism
- c) Using Gram Schmidt process construct orthonormal basis for  $\{(1, 0, 1), (1, 0, -1), (0, 3, 4)\}$
- d) Using Caley Hamilton theorem find inverse of  $A$  for  $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & 2 \\ 2 & 0 & 3 \end{bmatrix}$