(3 Hours) **[Total Marks: 100]**

N.B. : (1) **All** questions are **compulsory**.
(2) **Figures** to the **right** indicate **full** marks.
(3) Draw **neat** diagrams wherever **necessary**.
(5) Symbols have usual meaning unless otherwise stated.
(5) Use of **non-programmable** calculator is allowed.

Q1. Attempt any **two:---**

(i) Explain the fundamental principle of counting with a suitable example. **10**

(ii) What is Bernoulli's trial? Explain Binomial Probability function and corresponding cumulative distribution function. **10**

(iii) Consider an experiment of tossing two dices and write uniform sample space. What is a random variable? Consider x = sum of the numbers on the dice and explain the probability function $f(x_i)$ for the random variable. **10**

(iv) Explain Poisson's distribution. Derive expression for it considering number of particles emitted by a radioactive substance. **10**
Consider an experiment in which number particles emitted each minute by a radioactive source is recorded for a period of 15 hrs. A total of 2700 counts are registered. During how many 1-minute intervals should we expect to observe no particles?

Q2 Attempt any **two:---**

(i) Define sin z and cos z in terms of exponential functions of z. Using these definitions **10**
(a) Find the value of $sin\ (\pi/2 + i\ ln\ 2)$
(b) Prove that $sin^2 z + cos^2 z = 1$
(c) Prove that $d/dz\ (sin\ z) = cos\ z$

(ii) Find impedance of the circuit in which R and L and C are in series. Also find ω in terms of R, L and C at resonance. **10**

(iii) The vertical motion of a particle of mass m on a spring with spring constant k is described by the following differential equation: **10**

$$my'' = -ky + mg \quad where\ (y(0) = y_0\ and\ y'(0) = 0)$$

Solve this equation for the position of the particle as a function of time.

(iv) Solve the equation $\frac{\partial^2 z(x,y)}{\partial x\,\partial y} = x^2 y$ **10**
subject to the conditions

$$z(x,0) = x^2$$
$$and\ z(1,y) = cos\ y$$

Q3 Attempt any **two:**---

(i) What is Boltzmann distribution? Derive its expression. **10**

(ii) What is a Canonical Ensemble? Express canonical partition function Q. Hence obtain its relation with q for an ideal gas? How does this relationship differ for distinguishable and indistinguishable particles? **10**

(iii) What is entropy? Derive the Boltzmann formula $dS = k\, d(lnW)$. **10**

(iv) Obtain the relation between β and temperature T. What are the units of kT where k is Boltzmann constant? **10**

Q4 Attempt any **two:**---

(i) Consider a large box of area A divided into k cells of area a_1, a_2 , …a_k. N identical balls are thrown in a completely random manner. Obtain the most probable distribution of N balls in the k cells. **10**

(ii) Derive Rayleigh Jeans formula to explain black body radiation. **10**

(iii) Using Maxwell's distribution of velocity, derive an expression for the average velocity and most probable velocity. **10**

(iv) Derive Fermi-Dirac distribution law. **10**

Q5. Attempt any **four:**--- **20**

(i) Explain the terms mean value, standard deviation and variance of an experimental data. **05**

(ii) Consider tossing of a coin 5 times. Find the probability of getting a particular event, say, thhth where 't' and 'h' indicate tail and head on the top face of the coin. Also give the probability of getting 3 heads and 2 tails. **05**

(iii) If $z = \cos^{-1}2$, find all values of z. **05**

(iv) Solve y" – 2y' + y = 2 cos x by finding the complementary and particular solution. **05**

(v) Determine the total energy of a canonical ensemble consisting of N particles that have only two energy levels separated by $h\nu$. **05**

(vi) Write a short note on translational partition function. **05**

(vii) Calculate the number of modes of vibration per unit volume in a black body cavity for the wavelengths between 6000 AU and 6010 AU. **05**

(viii) When the temperature of black body is 60°C, it emits maximum energy at wavelength 8.71×10^{-6} m. If its temperature increased to 100°C, at what wavelength will the maximum energy be emitted? **05**
