

Course: Computational Logic and Discrete Structure

Course Code: USIT104

Maximum marks: 75

Duration: 2 ½Hrs

**Instructions:**

All questions are compulsory and carry equal marks

Figures to the right indicate full marks

**Q.1 Attempt any three of the following**

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- a) Out of 450 students in a school, 198 students study science, 200 student study commerce, 80 students study neither. Find out of how many study both?
- b) Prove that  $(x^n - y^n)$  is divisible by  $x - y$ .
- c) Prove that  $1 + 3 + 5 + \dots + (2n - 1) = n^2$
- d) Using Venn diagram prove or disprove  $A \cap B \cap C = A - [(A - B) \cup (A - C)]$
- e) Prove by induction that the sum of cubes of three consecutive integers is divisible by 9.
- f) Let  $R = \{(a, b) : a \text{ is factor of } b, a, b \in \mathbb{Z}\}$  prove that R is an equivalence relation.

**Q.2 Attempt any three of the following**

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- a) If  $f: R \rightarrow R_1, f(x) = 2x + 3, g: R \rightarrow R_1, g(x) = 3x + 4, h: R \rightarrow R, h(x) = 4x$  find  $gof, fog, hof, gofoh$ .
- b) Determine whether the function  $f: I \rightarrow R^+$  defined by  $f(x) = 2x + 7$  is bijective or not?
- c) Explain complexity of algorithm.
- d) An urn contains 10 white balls and 3 black balls. Another urn contain 3 white and 5 black balls. Two balls are drawn at random from first urn and place in the second urn. Now one ball is taken from latter, what is the probability that it is white ball?
- e) If A and B are two events such that  $P(A) = 0.3, P(B) = 0.4, P(A \cap B) = 0.2$  Find i)  $P(A \cup B)$  ii)  $P(\bar{A}/B)$  iii)  $P(A/\bar{B})$
- f) In Binomial distribution the mean is 5 and the standard deviation is 3. Find fallacy, if any in this satenent

**Q.3 Attempt any three of the following**

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- a) Find out how many 5 digit number greater than 30000 can be formed from the digits 1, 2, 3, 4, 5.
- b) Out of 5 males and 6 females a committee of 5 is to be formed. Find the number of ways in which it can be formed so that among the person chosen in the committee there are at least 2 males and 1 female.
- c) Show that 7 colours are used to paint 50 bicycles, then at least 8 bicycles of same colour.

d) State and prove Inclusion-Exclusion principal.

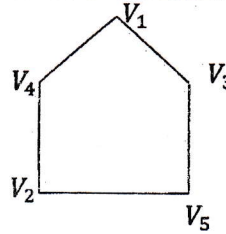
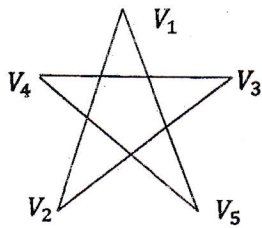
e) Solve the recurrence relation  $a_n = a_{n-1} + 2a_{n-2}$  with initial condition that  $a_0 = 2, a_1 = 7$

f) What is ordered and unordered partition? Find the possible partition of number 4.

**Q.4 Attempt any three of the following**

**15**

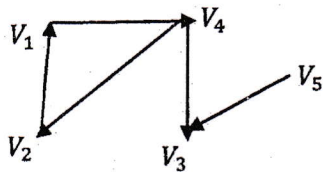
a) Define isomorphism of two graphs and check whether the following graphs are isomorphic or not



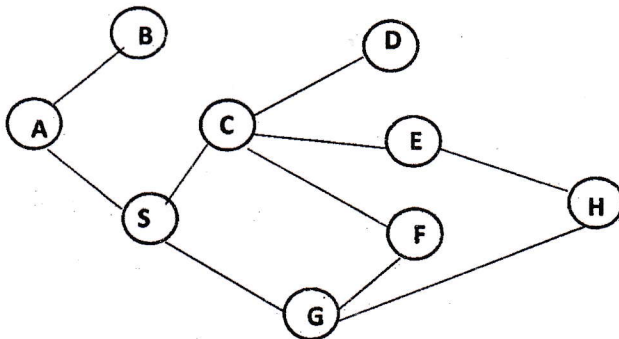
b) Find the directed graph from the following adjacency matrix

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

c) Use Warshall's algorithm to obtain path matrix from the following graph



d) Find shortest path using Depth First Search algorithm



e) Explain complete bipartite graph with example.

f) Define the following terms 1) walk 2) trail 3) path 4) circuit 5) connected graph

**Q.5 Attempt any three of the following**

- a) What is Binary tree? State properties of external binary tree.
- b) Explain Huffman's algorithm.
- c) Prove that the set of natural number under divisibility is POSET?
- d) Let  $A = \{1, 2, 3, 4\}$  relation  $R = \{(1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4)\}$  Then show That  $R$  is partial order.
- e) Draw Hasse diagram of  $D_{24}$ , where  $D$  denotes relation of division.
- f) Let  $A = \{2, 3, 5, 6, 10, 15, 30, 45\}$   $R \{(x, y): x \text{ divides } y\}$  Find 1) Maximal element 2) minimal element  
3) Is POSET  $(A, /)$  Lattice or not?