



Course: Mathematics Paper-I

Course Code: USMT301

Maximum marks: 75

Duration: 2 ½ Hrs

**Instructions:**

All questions are compulsory and carry equal marks

Figures to the right indicate full marks

**Q.1) Attempt any four of the following**

**20**

- Explain infinite series and its convergence.
- Define alternating series and state Leibnitz Theorem.
- Prove that if  $\sum_{n=1}^{\infty} a_n$  is convergent, then  $\lim_{n \rightarrow \infty} a_n = 0$
- Discuss the convergence of  $\sum_{n=1}^{\infty} \frac{7}{2n^2+9n+8}$
- Discuss the absolute convergence of series  $1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \dots$

**Q.2) Attempt any four of the following**

**20**

- Explain Upper, Lower Riemann sum of bounded function and Partition
- Let  $f: [0, 1] \rightarrow R$  defined by  $f(x) = 1 - x$ ,  $P = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$  Find  $L(P, f)$  and  $U(P, f)$
- Define Upper and Lower integral of  $f$  and state its properties
- Let  $f: [a, b] \rightarrow R$  be bounded function. Let  $P$  be partition then  $L(P, f) \leq U(P, f)$
- Prove that the constant function is Riemann Integrable.

**Q.3) Attempt any four of the following**

**20**

- States properties of  $\beta$  function
- Examine the convergence of  $\int_2^{\infty} \frac{1}{x^3} dx$
- Prove that if  $a > 0$ ,  $\int_a^{\infty} x^p dx$  is convergent for all  $p < -1$  and divergent for  $p \geq -1$
- Prove that  $\Gamma(n+1) = n!$
- Solve  $\int_{-1}^1 \frac{1}{x^2} dx$

**Q.4) Attempt any three of the following**

**15**

- State P-series test and limit comparison test
- Define absolute convergence and conditional convergence of the series
- Explain norm of partition with suitable example
- Evaluate  $\int_0^1 \frac{dx}{\sqrt{1-x}}$