Shri S. H. Kelkar College of Arts, Commerce and Science, Devgad

S.Y.BSC SEMISTER-IV Examination-March 2023

Course: Mathematics Paper-II | **Course Code: USMT4~~21~~402**

Maximum marks: 75 | **Duration: 2 ½Hrs**

Instructions:

All questions are compulsory and carry equal marks

Figures to the right indicate full marks

Q.1 Attempt any four of the following **20**

a) Define Linear Transformation and explain identity transformation.

b) Let $T: U \to V$ be linear transformation ,Prove that KerT is subspace of vector space U

c) Check whether the linear transformation defined by $T: R^2 \to R^2$ $T(x, y, z) = (x, y + z)$ is linear or not?

d) Let $T: R^2 \to R^2$ $be\ defined\ as$ $T(x, y) = (x + y, x - y)$verify Rank-Nullity theorem.

e) Check whether the linear transformation $L: R^3 \to R^3$, $L(x, y, z) = (x + y, x - z, y + 2z)$ is invertible or not.

Q.2 Attempt any four of the following **20**

a) Let V be set of continuous real valued function define on $[-\pi, \pi]$ then addition in V is defined as $(f + g)(t) = f(t) + g(t)\ \forall\ t \in [-\pi, \pi]\ and\ (\alpha.f)(t) = \alpha f(t)\ \ \forall\ t \in [-\pi, \pi]\ \ Define$

$< f, g > = \int_{-\Pi}^{\Pi} f(t).g(t)dt$ Show that V forms inner product space.

b) Consider vectors u and v in Euclidean inner product space, compute <u, v>, ||u||, d(u, v) for u=(1, 1, 3), v=(2, -2, 7).

c) Define the following
1) Orthogonal set 2) Orthonormal Set 3) Unit vector

d) Find the projection vector of u along v where u=(6, 1, -1, 4), v=(1, -1, 0, 0).

e) Let W be subspace of V then W^T is also subspace of V

Q.3 Attempt any four of the following **20**

a) State Caley Hamilton theorem and verify it for matrix $A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 0 & 1 \\ 0 & 0 & 4 \end{bmatrix}$

b) Define Characteristic polynomial, characteristic equation and hence determine it for $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

c) Show that two matrices A, PAP^{-1} have same characteristic roots

d) For the given matrix find Algebraic and geometric multiplicity where $B = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$

e) If $v_1, v_2, \ldots.. v_p$ be eigen vectors of matrix A corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \ldots.\lambda_p$ Respectively, then $v_1, v_2, \ldots.. v_p$ are linearly independent.

Q.4 Attempt any three of the following **15**

a) Let y $T: R^3 \rightarrow R^3$ be a linear transformation defined by $T(x, y, z) = (x + y, y - z)$ Find matrix of T with natural basis.

b) Explain concept of linear isomorphism

c) Using Gram Schmidth process construct orthonormal basis for {(1, 0, 1), (1, 0, -1), (0, 3, 4)}

d) Using Caley Hamilton theorem find inverse of A for $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & -1 & 2 \\ 2 & 0 & 3 \end{bmatrix}$