

Time: 3 hrs.

M. M.: 100

N.B.:

1. All questions are **compulsory**.
2. **Figures** to the **right** indicate **full** marks.
3. Draw **neat** diagrams wherever **necessary**.
4. Symbols have usual meaning unless otherwise stated.
5. Use of **non-programmable** calculator is allowed.

Q1. Attempt any **two**

- (i) State and prove Kepler's laws of planetary motion. **10**
- (ii) Discuss quantitatively the motion of a particle in an inverse square field. **10**

Show that the eccentricity of the particle is given by  $\epsilon = \sqrt{1 + \frac{2EL^2}{mK^2}}$ .

Give conditions on E and  $\epsilon$  for different shapes of orbits.

- (iii) What is a Foucault pendulum? Obtain equation of motion for it. Hence show that the pendulum precesses slowly clockwise in the northern hemisphere. **10**
- (iv) Consider a starred (rotating) reference frame rotating with angular velocity  $\vec{\omega}$  relative to the unstarred (fixed) frame with their origins O and O\* coinciding. Prove that for an arbitrary vector  $\vec{A}$ , **10**

$$\frac{d^2\vec{A}}{dt^2} = \frac{d^{*2}\vec{A}}{dt^2} + \vec{\omega} \times (\vec{\omega} \times \vec{A}) + 2\vec{\omega} \times \frac{d^*\vec{A}}{dt} + \frac{d^*\vec{\omega}}{dt} \times \vec{A}$$

Q2. Attempt any **two**

- (i) Starting with D'Alembert's Principle, obtain Lagrange's equations in terms of generalized coordinates. **10**
- (ii) What is meant by generalized co-ordinates? Derive an expression for generalized velocity and generalized kinetic energy. **10**
- (iii) A body of mass  $m_1$  can move on a smooth flat horizontal table top. It is connected to a string of length  $\ell$  which passes through a hole in the centre of the table. The other end of the string is connected to a mass  $m_2$  which is suspended vertically. Identify appropriate generalized coordinate for the system and obtain the equations of motion using D'Alembert's principle. **10**
- (iv) A double pendulum consists of two weightless rods connected to each other and a point of support. The masses  $m_1$  and  $m_2$  are not equal but the length of the rods are equal. Pendulums are free to swing only in one vertical plane. Write down the Lagrangian for the system. **10**

Q3. Attempt any **two**

- (i) For a moving fluid show that **10**
  - a)  $\frac{dp}{dt} = \frac{\partial p}{\partial t} + \vec{v} \cdot \vec{\nabla} p$
  - b)  $\frac{d}{dt} \delta V = \vec{\nabla} \cdot \vec{v} \delta V$

(Symbols have their usual meanings)
- (ii) Derive Bernoulli's theorem. Hence for a steady flow of the fluid show that, **10**

$$\frac{v^2}{2} + \frac{p}{\rho} - G + u = \text{constant} \quad (\text{Symbols have their usual meanings})$$

- (iii) With reference to rotations of rigid body explain setting of the Euler's angles. Draw suitable diagrams. Find expression for the Lagrangian of a heavy symmetric top. **10**
- (iv) Derive an expression for the moment of inertia tensor for a rigid body made up of N number of particles. **10**
- Q4** Attempt any **two**
- (i) Discuss numerical solutions of Duffing's equation for  $\gamma = 0.1$  and  $f = 0.5$  and 2)  $\gamma = 0.1$  and  $f = 3$ . Compare the nature of odd and even harmonics. **10**
- (ii) Consider an anharmonic oscillator with potential energy  $V(x) = K \left( \frac{x^2}{2} + \frac{\alpha x^4}{4} \right)$  where K is the spring constant and  $\alpha$  is anharmonic coefficient. Discuss the potential energy curve for positive and negative values of K and  $\alpha$ . Comment on confinement of motion. **10**
- (iii) Discuss fixed points of a logistic map, stability of fixed points and periodic attractors. Discuss logistic map for  $3 < \lambda < 4$  and explain the onset of chaos qualitatively. **10**
- (iv) Discuss numerical solutions of Duffing's equation for 1)  $\gamma = 0.1$  and  $f = 20$  and 2)  $\gamma = 0.1$  and  $f = 25$  **10**
- Q5.** Attempt any **four**
- (i) If a body of mass 100 kg is moving with a velocity of 10m/s; estimate the maximum Coriolis force experienced by the body. **05**
- (ii) The eccentricity of a planet's orbit about sun is 0.4. Find the ratio of the lengths of the semi major to the semi minor axes of the orbit of the planet. **05**
- (iii) Write down the Lagrangian for a simple pendulum and hence find its equation of motion. **05**
- (iv) Define constraints. With good examples, explain holonomic and non-holonomic constraints. **05**
- (v) Consider a fluid flow given by  $\vec{v} = cy\hat{i}$ . Show that the fluid is incompressible and non-irrotational. **05**
- (vi) What is a rigid body? Discuss the different types of rigid bodies with reference to the symmetry present in the body. **05**
- (vii) Two very close initial values of x on logistic map are 0.40000 and 0.40002 respectively. With  $\lambda=4$ , after 20 iterations the values are 0.14561 and 0.00170 respectively. Calculate Lyapunov exponent. **05**
- (viii) Discuss the nature of phase space diagram for 1)undamped oscillator and 2)damped oscillator. **05**

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