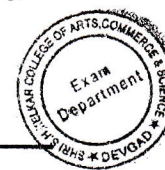


# Shri S. H. Kelkar College of Arts, Commerce and Science, Devgad

## S.Y.BSC SEMISTER-IV Examination-March 2023



Course: Mathematics Paper-I

Course Code: USMT401

Maximum marks: 75

Duration: 2 ½ Hrs

### Instructions:

All questions are compulsory and carry equal marks

Figures to the right indicate full marks

#### Q.1 Attempt any four of the following

20

- Explain any three properties of norm
- Define open set in  $R^n$  and Sequence in  $R^n$
- Check the convergence of sequence  $x_m = \left( \frac{4m-3}{8m-9}, \frac{3m^2}{4m^2+3m+1}, \frac{7m}{6m+5} \right)$
- Find directional derivatives of  $f(x, y) = x^2 + y^2 + xy + y$  at  $(1, -2)$  in the direction of  $u = (3, -5)$
- Find  $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$  where  $f(x, y, z) = 3x^2 + 4xy + 8y + 6x$

#### Q.2 Attempt any four of the following

20

- Prove that  $f(x, y) = \sqrt{x^2 + y^2}$  is not differentiable at  $(0, 0)$
- Define gradient of scalar field and determine  $\nabla f$  for  $f: R^3 \rightarrow R$  given by  $f(x, y, z) = 4x^3 - 9x^2yz + 3xy^2$  at  $(1, 2, -3)$
- Find the direction in which maximum rate of change occurs for the function  $f(x, y) = 9x^3 + 5y^2$  at  $(2, 1)$ .
- Define Level curves and tangent plane
- Find the equation of tangent plane and normal line to the surfaces  $z = 9x^2 + y^2 + 6x - 3y + 5$  at  $(1, 2, 3)$

#### Q.3 Attempt any four of the following

20

- Let  $f(x, y) = \sin xy + \log(x + y)$ , find the Taylors polynomial of degree 2 about point  $(1, 0)$  for  $(x, y)$
- Find the linearization  $L(x, y)$  of the function  $f(x, y) = x\sqrt{y}$  at  $(1, 4)$
- Let  $f: R^4 \rightarrow R^2$  be defined by  $f(x, y, z, w) = (x^2 + 2y, x + z^2 + 3w^2)$  find  $Jf(3, 1, 0, -1)$
- Use chain rule to find  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$  when  $s = 1, t = -1$  where  $z = e^{xy} \tan y$  and  $x = s + 4t^2, y = st$
- State Mean Value inequality of vector fields

#### Q.4 Attempt any four of the following

15

- Evaluate limit by converting to polar coordinates,  $\lim_{(x,y) \rightarrow (0,0)} e^{\frac{-1}{x^2+y^2}}$
- Find the real value  $\theta \in (0, 1)$  such that  $f(a + u) - f(a) = D_u f(a + \theta u)$  where  $f(x, y) = x^2 + y^2$  at  $a = (1, 0)$  and  $u = (-1, 1)$
- Let  $f: R^2 \rightarrow R$  given by  $f(x, y) = 3 \sin x + y \cos x$  find  $f_x, f_y$  and  $f_{xy}$
- Find all local maxima, local minima and saddle points of the function  $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4$